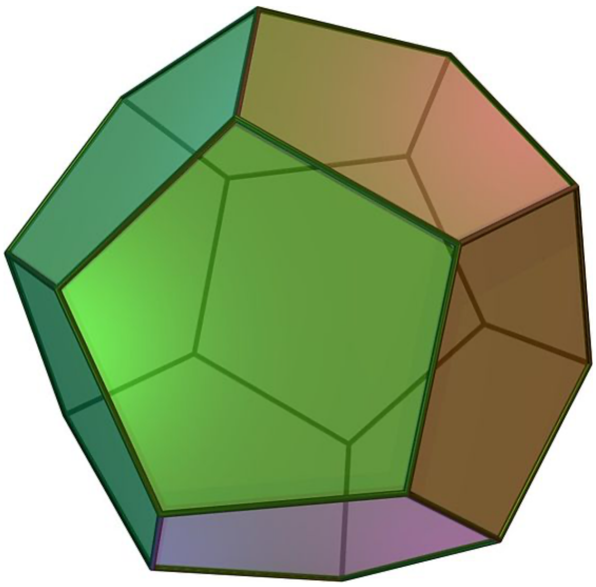
Regular dodecahedron

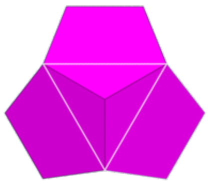
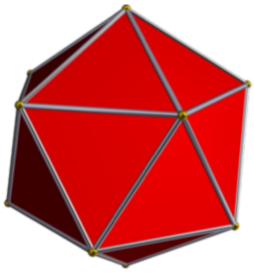
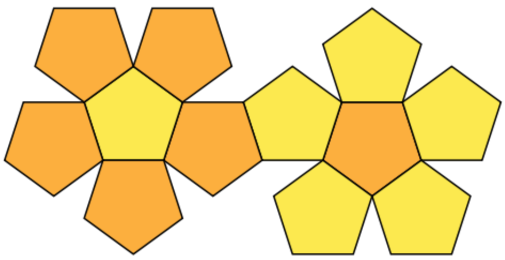
A **regular dodecahedron** or **pentagonal dodecahedron** is a dodecahedron that is regular, which is composed of twelve regular pentagonal faces, three meeting at each vertex. It is one of the five Platonic solids. It has 12 faces, 20 vertices, 30 edges, and 160 diagonals (60 face diagonals, 100 space diagonals).[1] It is represented by the Schläfli symbol {5,3}.

Regular dodecahedron	
 (Click here for rotating model)	
Type	Platonic solid
Elements	$F = 12$, $E = 30$ $V = 20$ ($\chi = 2$)
Faces by sides	12{5}
Conway notation	D
Schläfli symbols	{5,3}
Face configuration	V3.3.3.3.3
Wythoff symbol	3 | 2 5
	

Contents

Dimensions

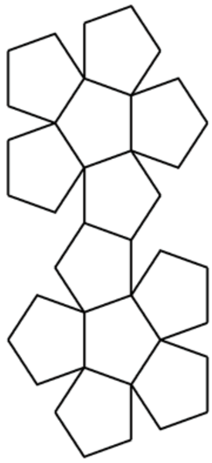
If the edge length of a regular dodecahedron is a, the radius of a circumscribed sphere (one that touches the regular dodecahedron at all vertices) is

Coxeter diagram	• - •
Symmetry	I_h, H_3, [5,3], (*532)
Rotation group	I, $[5,3]^+$, (532)
References	U_{23}, C_{26}, W_5
Properties	regular, convex
Dihedral angle	116.56505° = arccos($-\frac{1}{\sqrt{5}}$)
 5.5.5 (Vertex figure)	 Regular icosahedron (dual polyhedron)
 Net	

$$r_u = a\frac{\sqrt{3}}{4}\left(1+\sqrt{5}\right) \approx 1.401\,258\,538 \cdot a$$

OEIS: A179296

and the radius of an inscribed sphere (tangent to each of the regular dodecahedron's faces) is



Animation of a net of a regular (pentagonal) dodecahedron being folded

$$r_i = a\frac{1}{2}\sqrt{\frac{5}{2}+\frac{11}{10}\sqrt{5}} \approx 1.113\,516\,364 \cdot a$$

while the midradius, which touches the middle of each edge, is

$$r_m = a\frac{1}{4}\left(3+\sqrt{5}\right) \approx 1.309\,016\,994 \cdot a$$

These quantities may also be expressed as

$$r_u = a\,\frac{\sqrt{3}}{2}\phi$$

$$r_i = a\,\frac{\phi^2}{2\sqrt{3-\phi}}$$

$$r_m = a\,\frac{\phi^2}{2}$$

where ϕ is the golden ratio.

Note that, given a regular dodecahedron of edge length one, r_u is the radius of a circumscribing sphere about a cube of edge length ϕ, and r_i is the apothem of a regular pentagon of edge length ϕ.

Surface area and volume

The surface area A and the volume V of a regular dodecahedron of edge length a are:

$$A = 3\sqrt{25 + 10\sqrt{5}}a^2 \approx 20.645\,728\,807a^2$$

$$V = \frac{1}{4}(15 + 7\sqrt{5})a^3 \approx 7.663\,118\,9606a^3$$

Additionally, the surface area and volume of a regular dodecahedron are related to the golden ratio. A dodecahedron with an edge length of one unit has the properties:[2]

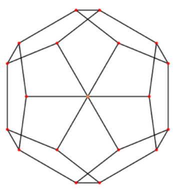
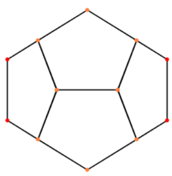
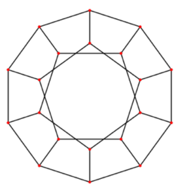
$$A = \frac{15\varphi}{\sqrt{3 - \varphi}}$$

$$V = \frac{5\varphi^3}{6 - 2\varphi}$$

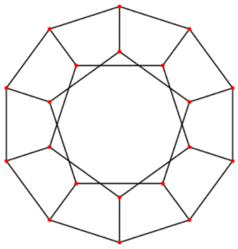
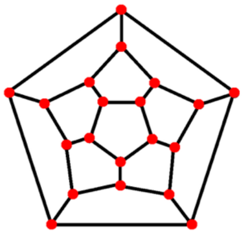
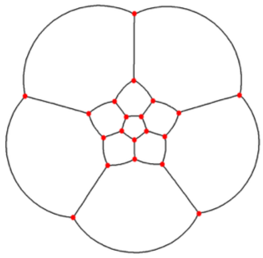
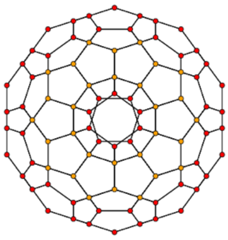
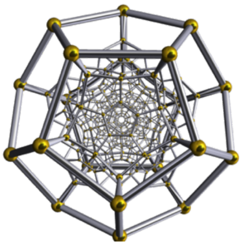
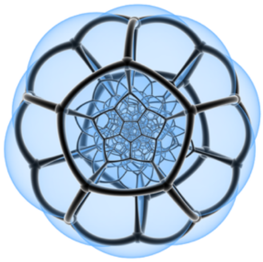
Two-dimensional symmetry projections

The *regular dodecahedron* has two special orthogonal projections, centered, on vertices and pentagonal faces, correspond to the A_2 and H_2 Coxeter planes.

Orthogonal projections

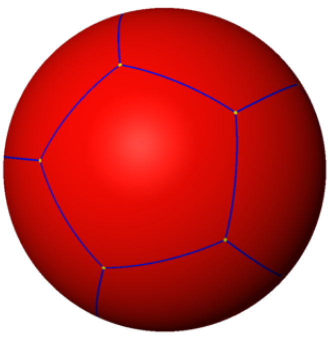
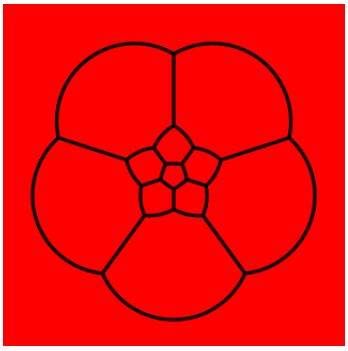
Centered by	Vertex	Edge	Face
Image			
Projective symmetry	[[3]] = [6]	[2]	[[5]] = [10]

In perspective projection, viewed on top of a pentagonal face, the regular dodecahedron can be seen as a linear-edged Schlegel diagram, or stereographic projection as a spherical polyhedron. These projections are also used in showing the four-dimensional 120-cell, a regular 4-dimensional polytope, constructed from 120 dodecahedra, projecting it down to 3-dimensions.

Projection	Orthogonal projection	Perspective projection	
		Schlegel diagram	Stereographic projection
Regular dodecahedron			
Dodecaplex (120-cell)			

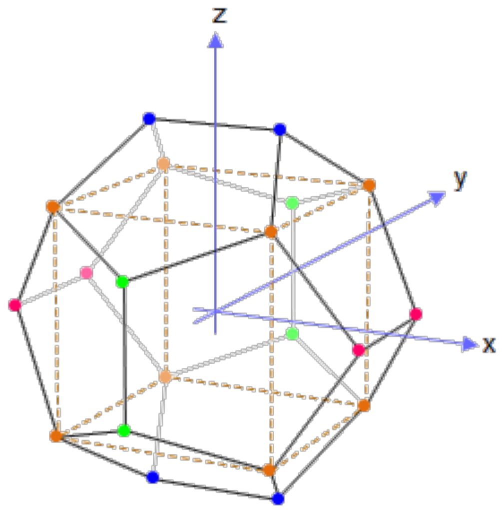
Spherical tiling

The regular dodecahedron can also be represented as a spherical tiling.

	
Orthographic projection	Stereographic projection

Cartesian coordinates

The following Cartesian coordinates define the 20 vertices of a regular dodecahedron centered at the origin and suitably scaled and oriented:[3]



$$(\pm 1, \pm 1, \pm 1)$$
$$(0, \pm\phi, \pm 1\phi)$$
$$(\pm 1\phi, 0, \pm\phi)$$
$$(\pm\phi, \pm 1\phi, 0)$$

where $\phi = 1 + \sqrt{5}2$ is the golden ratio (also written τ) ≈ 1.618. The edge length is $2\phi = \sqrt{5} - 1$. The circumradius is $\sqrt{3}$.

Vertex coordinates:	
	The orange vertices lie at (±1, ±1, ±1) and form a cube (dotted lines).
	The green vertices lie at (0, ±φ, ±1φ) and form a rectangle on the *yz*-plane.
	The blue vertices lie at (±1φ, 0, ±φ) and form a rectangle on the *xz*-plane.
	The pink vertices lie at (±φ, ±1φ, 0) and form a rectangle on the *xy*-plane.
The distance between adjacent vertices is 2φ, and the distance from the origin to any vertex is $\sqrt{3}$. $\varphi = 1 + \sqrt{5}2$ is the golden ratio.	

Facet-defining equations

Similar to the symmetry of the vertex coordinates, the equations of the twelve facets of the regular dodecahedron also display symmetry in their coefficients:

$$x \pm \phi y = \pm\phi^2$$
$$y \pm \phi z = \pm\phi^2$$
$$z \pm \phi x = \pm\phi^2$$

Properties

- The dihedral angle of a regular dodecahedron is 2 arctan(ϕ) or approximately 116.565° (where again $\phi = 1 + \sqrt{5}2$, the golden ratio). OEIS: A137218 Note that the tangent of the dihedral angle is exactly -2.

- If the original regular dodecahedron has edge length 1, its dual icosahedron has edge length ϕ.
- If the five Platonic solids are built with same volume, the regular dodecahedron has the shortest edges.
- It has 43,380 nets.
- The map-coloring number of a regular dodecahedron's faces is 4.
- The distance between the vertices on the same face not connected by an edge is ϕ times the edge length.
- If two edges share a common vertex, then the midpoints of those edges form a 36-72-72 triangle with the body center.

Geometric relations

The *regular dodecahedron* is the third in an infinite set of truncated trapezohedra which can be constructed by truncating the two axial vertices of a pentagonal trapezohedron.

The stellations of the regular dodecahedron make up three of the four Kepler–Poinsot polyhedra.

A rectified regular dodecahedron forms an icosidodecahedron.

The regular dodecahedron has icosahedral symmetry I_h, Coxeter group [5,3], order 120, with an abstract group structure of $A_5 \times Z_2$.

Relation to the regular icosahedron

When a regular dodecahedron is inscribed in a sphere, it occupies more of the sphere's volume (66.49%) than an icosahedron inscribed in the same sphere (60.55%).

A regular dodecahedron with edge length 1 has more than three and a half times the volume of an icosahedron with the same length edges (7.663... compared with 2.181...), which ratio is approximately

3.51246117975, or in exact terms: $35(3\phi + 1)$ or $(1.8\phi + 0.6)$.

A regular dodecahedron has 12 faces and 20 vertices, whereas a regular icosahedron has 20 faces and 12 vertices. Both have 30 edges.

Relation to the nested cube

A cube can be embedded within a regular dodecahedron, affixed to eight of its equidistant vertices, in five different positions.[4] In fact, five cubes may overlap and interlock inside the regular dodecahedron to result in the compound of five cubes.

The ratio of the edge of a regular dodecahedron to the edge of a cube embedded inside such a regular dodecahedron is $1 : \phi$, or $(\phi - 1) : 1$.

The ratio of a regular dodecahedron's volume to the volume of a cube embedded inside such a regular dodecahedron is $1 : 22 + \phi$, or $1 + \phi 2 : 1$, or $(5 + \sqrt{5}) : 4$.

For example, an embedded cube with a volume of 64 (and edge length of 4), will nest within a regular dodecahedron of volume $64 + 32\phi$ (and edge length of $4\phi - 4$).

Thus, the difference in volume between the encompassing regular dodecahedron and the enclosed cube is always one half the volume of the cube times ϕ.

From these ratios are derived simple formulas for the volume of a regular dodecahedron with edge length a in terms of the golden mean:

$$V = (a\phi)^3 \cdot 14(5 + \sqrt{5})$$
$$V = 14(14\phi + 8)a^3$$

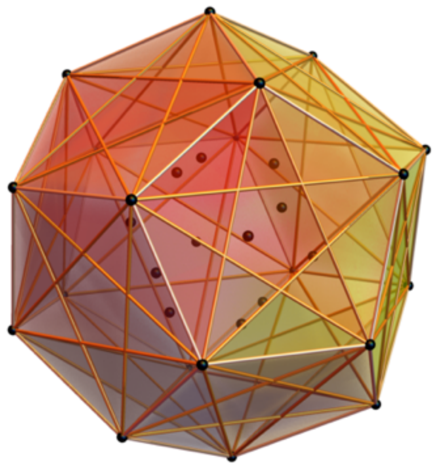
Relation to the golden rectangle

Golden rectangles of ratio $(\phi + 1) : 1$ and $\phi : 1$ also fit perfectly within a regular dodecahedron.[5] In proportion to this golden rectangle, an enclosed cube's edge is ϕ, when the long length of the rectangle is $\phi + 1$ (or ϕ^2) and the short length is 1 (the edge shared with the regular dodecahedron).

In addition, the center of each face of the regular dodecahedron form three intersecting golden rectangles.[6]

Relation to the 6-cube and rhombic triacontahedron

It can be projected to 3D from the 6-dimensional 6-demicube using the same basis vectors that form the hull of the rhombic triacontahedron from the 6-cube. Shown here including the inner 12 vertices, which are not connected by the outer hull edges of 6D norm length $\sqrt{2}$, form a regular icosahedron.



Projection of 6-demicube into regular dodecahedral envelope

The 3D projection basis vectors [u,v,w] used are:

$$u = (1, \varphi, 0, -1, \varphi, 0)$$
$$v = (\varphi, 0, 1, \varphi, 0, -1)$$
$$w = (0, 1, \varphi, 0, -1, \varphi)$$

History and uses

Regular dodecahedral objects have found some practical applications, and have also played a role in the visual arts and in philosophy.

Iamblichus states that Hippasus, a Pythagorean, perished in the sea, because he boasted that he first divulged "the sphere with the twelve pentagons."[7] In *Theaetetus*, a dialogue of Plato, Plato was able to prove that there are just five uniform regular solids; these later became known as the platonic solids. Timaeus (c. 360 B.C.), as a personage of Plato's dialogue, associates the other four platonic solids with the four classical elements, adding that there is a fifth solid pattern which, though commonly associated with the regular dodecahedron, is never directly mentioned as such; "this God used in the delineation of the universe."[8] Aristotle also postulated that the heavens were made of a fifth element, which he called aithêr (*aether* in Latin, *ether* in American English).

Regular dodecahedra have been used as dice and probably also as divinatory devices. During the hellenistic era, small, hollow bronze Roman dodecahedra were made and have been found in various Roman ruins in Europe. Their purpose is not certain.

In 20th-century art, dodecahedra appear in the work of M. C. Escher, such as his lithographs *Reptiles* (1943) and *Gravitation* (1952). In Salvador Dalí's painting *The Sacrament of the Last Supper* (1955), the room is a hollow regular dodecahedron. Gerard Caris based his entire artistic oeuvre on the regular dodecahedron and the pentagon, which is presented as a new art movement coined as Pentagonism.

In modern role-playing games, the regular dodecahedron is often used as a twelve-sided die, one of the more common polyhedral dice.

Some quasicrystals have dodecahedral shape (see figure). Some regular crystals such as garnet and diamond are also said to exhibit "dodecahedral" habit, but this statement actually refers to the rhombic dodecahedron shape.[9]

Immersive Media, a camera manufacturing company, has made the Dodeca 2360 camera, the world's first 360° full-motion camera which captures high-resolution video from every direction simultaneously at more than 100 million pixels per second or 30 frames per second. It is based on regular dodecahedron.

The Megaminx twisty puzzle, alongside its larger and smaller order analogues, is in the shape of a regular dodecahedron.

In the children's novel *The Phantom Tollbooth*, the regular dodecahedron appears as a character in the land of Mathematics. Each of his faces wears a different expression – *e.g.* happy, angry, sad – which he swivels to the front as required to match his mood.

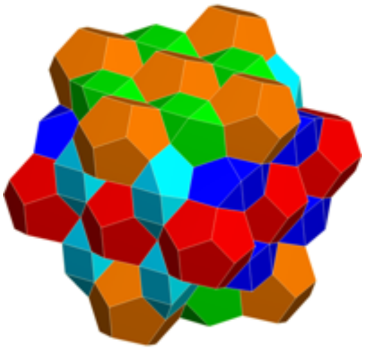
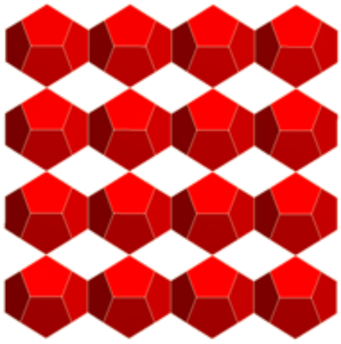
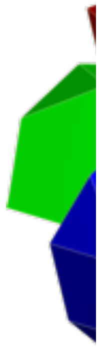
Shape of the universe

Various models have been proposed for the global geometry of the universe. In addition to the primitive geometries, these proposals include the Poincaré dodecahedral space, a positively curved space consisting of a regular dodecahedron whose opposite faces correspond (with a small twist). This was proposed by Jean-Pierre Luminet and colleagues in 2003,[10][11] and an optimal orientation on the sky for the model was estimated in 2008.[12]

In Bertrand Russell's 1954 short story "The Mathematician's Nightmare: The Vision of Professor Squarepunt," the number 5 said: "I am the number of fingers on a hand. I make pentagons and pentagrams. And but for me dodecahedra could not exist; and, as everyone knows, the universe is a dodecahedron. So, but for me, there could be no universe."

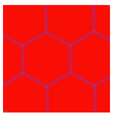
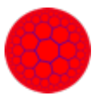
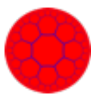
Space filling with cube and bilunabirotunda

Regular dodecahedra fill space with cubes and bilunabirotundae (Johnson solid 91), in the ratio of 1 to 1 to 3.[13][14] The dodecahedra alone make a lattice of edge-to-edge pyritohedra. The bilunabirotundae fill the rhombic gaps. Each cube meets six bilunabirotundae in three orientations.

 Block model		 Lattice of dodecahedra	 6 bilun cube

Related polyhedra and tilings

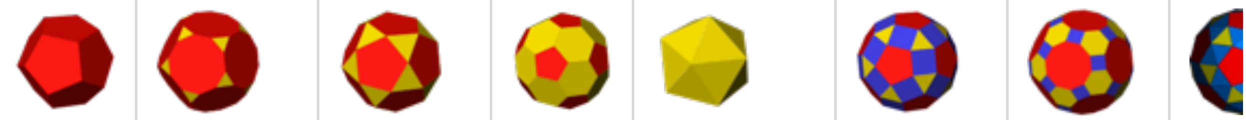
The regular dodecahedron is topologically related to a series of tilings by vertex figure n^3.

show*n32 symmetry mutation of regular tilings: {n,3}								
Spherical				**Euclidean**	**Compact hyperb.**		**Paraco.**	**No**
								
{2,3}	{3,3}	{4,3}	{5,3}	{6,3}	{7,3}	{8,3}	{∞,3}	{12i,3}

The regular dodecahedron can be transformed by a truncation sequence into its dual, the icosahedron:

showFamily of uniform icosahedral polyhedra

Symmetry: [5,3], (*532)							[5 (
{5,3}	t{5,3}	r{5,3}	t{3,5}	{3,5}	rr{5,3}	tr{5,3}	sr{

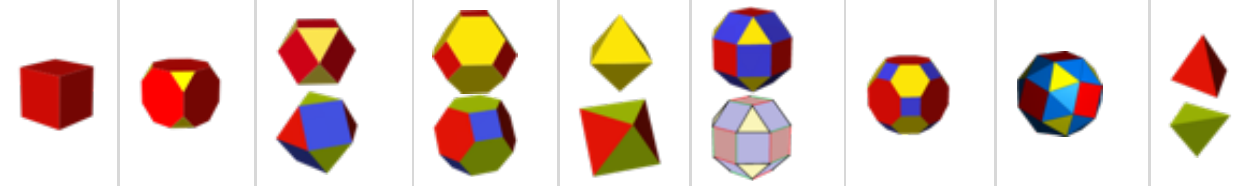


Duals to uniform polyhedra

V5.5.5	V3.10.10	V3.5.3.5	V5.6.6	V3.3.3.3.3	V3.4.5.4	V4.6.10	V3.

showUniform octahedral polyhedra

Symmetry: [4,3], (*432)							$[4,3]^+$ (432)	[1
{4,3}	t{4,3}	r{4,3} r$\{3^{1,1}\}$	t{3,4} t$\{3^{1,1}\}$	{3,4} $\{3^{1,1}\}$	rr{4,3} $s_2\{3,4\}$	tr{4,3}	sr{4,3}	h{4, {3,3
		=	=	=				= o



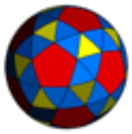
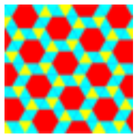
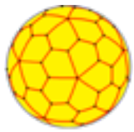
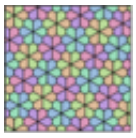
Duals to uniform polyhedra

$V4^3$	$V3.8^2$	$V(3.4)^2$	$V4.6^2$	$V3^4$	$V3.4^3$	V4.6.8	$V3^4.4$	$V3^3$

The regular dodecahedron is a member of a sequence of otherwise non-uniform polyhedra and tilings, composed of pentagons with face configurations (V3.3.3.3.n). (For n > 6, the sequence consists of tilings of the hyperbolic plane.) These face-transitive figures have (n32) rotational symmetry.

shown$n$32 symmetry mutations of snub tilings: 3

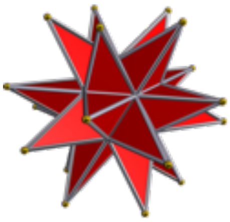
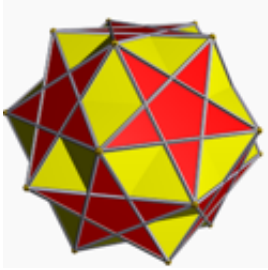
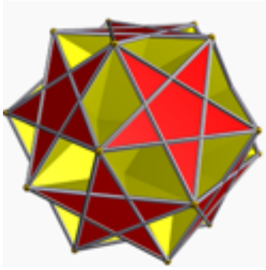
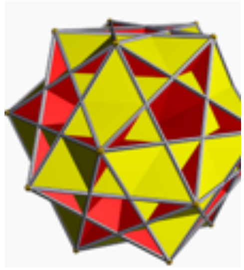
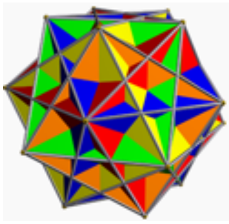
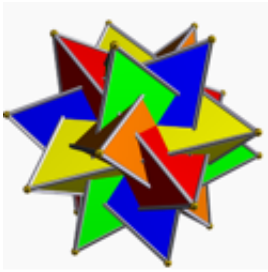
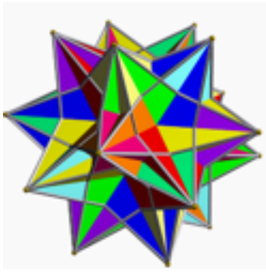
- v
- t
- e

Symmetry n32	Spherical				Euclidean
	232	332	432	532	632
Snub figures					
Config.	3.3.3.3.2	3.3.3.3.3	3.3.3.3.4	3.3.3.3.5	3.3.3.3.6
Gyro figures					
Config.	V3.3.3.3.2	V3.3.3.3.3	V3.3.3.3.4	V3.3.3.3.5	V3.3.3.3.6

Vertex arrangement

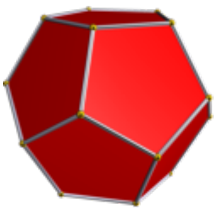
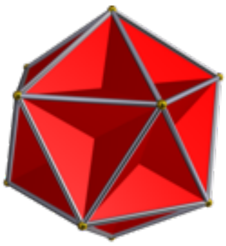
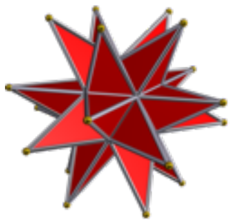
The regular dodecahedron shares its vertex arrangement with four nonconvex uniform polyhedra and three uniform polyhedron compounds.

Five cubes fit within, with their edges as diagonals of the regular dodecahedron's faces, and together these make up the regular polyhedral compound of five cubes. Since two tetrahedra can fit on alternate cube vertices, five and ten tetrahedra can also fit in a regular dodecahedron.

 Great stellated dodecahedron	 Small ditrigonal icosidodecahedron	 Ditrigonal dodecadodecahedron	 Great ditrigonal icosidodecahedr
 Compound of five cubes	 Compound of five tetrahedra	 Compound of ten tetrahedra	

Stellations

The 3 stellations of the regular dodecahedron are all regular (nonconvex) polyhedra: (Kepler–Poinsot polyhedra)

	0	1	2	3
Stellation	 *Regular dodecahedron*	 Small stellated dodecahedron	 Great dodecahedron	 Great stellated dodecahedron

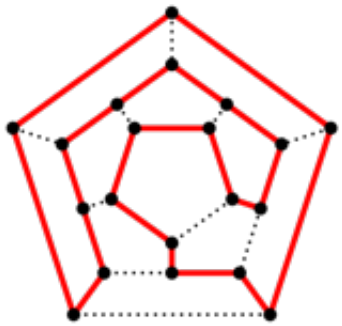
Facet diagram				

Dodecahedral graph

The skeleton of the dodecahedron (the vertices and edges) form a graph. It is one of 5 Platonic graphs, each a skeleton of its Platonic solid.

This graph can also be constructed as the generalized Petersen graph $G(10,2)$. The high degree of symmetry of the polygon is replicated in the properties of this graph, which is distance-transitive, distance-regular, and symmetric. The automorphism group has order 120. The vertices can be colored with 3 colors, as can the edges, and the diameter is 5.[16]

Regular dodecahedron graph



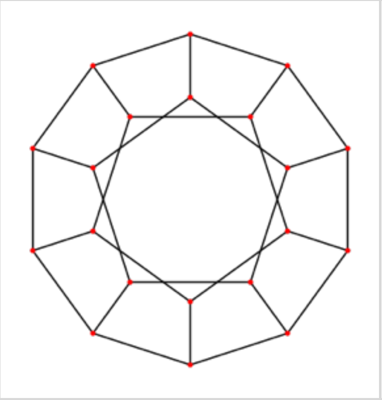
A Hamiltonian cycle in a dodecahedron.

Vertices	20
Edges	30
Radius	5
Diameter	5
Girth	5
Automorphisms	120 ($A_5 \times Z_2$)[15]
Chromatic number	3
Properties	Hamiltonian, regular, symmetric, distance-regular, distance-transitive, 3-vertex-connected, planar graph

Table of graphs and parameters

The dodecahedral graph is Hamiltonian – there is a cycle containing all the vertices. Indeed, this name derives from a mathematical game invented in 1857 by William Rowan Hamilton, the icosian game. The game's object was to find a Hamiltonian cycle along the edges of a dodecahedron.

Orthogonal projection



See also

- 120-cell, a regular polychoron (4D polytope whose surface consists of 120 dodecahedral cells)
- Dodecahedrane (molecule)
- Pentakis dodecahedron
- Snub dodecahedron
- Truncated dodecahedron

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External links

	Wikimedia Commons has media related to ***Dodecahedron***.

- *Weisstein, Eric W. "Regular Dodecahedron". MathWorld.*
- *Klitzing, Richard. "3D convex uniform polyhedra o3o5x – doe".*
- Editable printable net of a dodecahedron with interactive 3D view
- The Uniform Polyhedra
- Origami Polyhedra – Models made with Modular Origami
- Dodecahedron – 3-d model that works in your browser
- Virtual Reality Polyhedra The Encyclopedia of Polyhedra
  - VRML#Regular dodecahedron
- K.J.M. MacLean, A Geometric Analysis of the Five Platonic Solids and Other Semi-Regular Polyhedra
- Dodecahedron 3D Visualization
- Stella: Polyhedron Navigator: Software used to create some of the images on this page.
- How to make a dodecahedron from a Styrofoam cube
- The Greek, Indian, and Chinese Elements – Seven Element Theory

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Polyhedra

Listed by number of faces

- Monohedron
- Dihedron
- Trihedron

1–10 faces	• Tetrahedron • Pentahedron • Hexahedron • Heptahedron • Octahedron • Enneahedron • Decahedron
11–20 faces	• Hendecahedron • Dodecahedron • Tridecahedron • Tetradecahedron • Pentadecahedron • Hexadecahedron • Heptadecahedron • Octadecahedron • Enneadecahedron • Icosahedron
Others	• Triacontahedron (30) • Hexecontahedron (60) • Enneacontahedron (90) • Hectotriadiohedron • Apeirohedron

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Convex polyhedra

Platonic solids (regular)	• tetrahedron • cube • octahedron • **dodecahedron** • icosahedron
	• truncated tetrahedron • cuboctahedron

Archimedean solids (**semiregular** or **uniform**)	- truncated cube - truncated octahedron - rhombicuboctahedron - truncated cuboctahedron - snub cube - icosidodecahedron - truncated dodecahedron - truncated icosahedron - rhombicosidodecahedron - truncated icosidodecahedron - snub dodecahedron
Catalan solids (**duals of Archimedean**)	- triakis tetrahedron - rhombic dodecahedron - triakis octahedron - tetrakis hexahedron - deltoidal icositetrahedron - disdyakis dodecahedron - pentagonal icositetrahedron - rhombic triacontahedron - triakis icosahedron - pentakis dodecahedron - deltoidal hexecontahedron - disdyakis triacontahedron - pentagonal hexecontahedron
Dihedral regular	*dihedron* *hosohedron*
Dihedral uniform	- prisms - antiprisms **duals:** - bipyramids - trapezohedra
	- pyramids - truncated trapezohedra - gyroelongated bipyramid - cupola - bicupola

Dihedral others	• pyramidal frusta • bifrustum • rotunda • birotunda • prismatoid • scutoid
Degenerate polyhedra are in *italics*.	

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Fundamental convex regular and uniform polytopes in dimensions 2

Family	A_n	B_n	$I_2(p)$ / D_n	E_6 / E_7 / E_8 / F_4 / G_2	H_n
Regular polygon	Triangle	Square	p-gon	Hexagon	Pentag
Uniform polyhedron	Tetrahedron	Octahedron • Cube	Demicube		**Dodec** • Icosa
Uniform 4-polytope	5-cell	16-cell • Tesseract	Demitesseract	24-cell	120-ce cell
Uniform 5-polytope	5-simplex	5-orthoplex • 5-cube	5-demicube		
Uniform 6-polytope	6-simplex	6-orthoplex • 6-cube	6-demicube	1_{22} • 2_{21}	
Uniform 7-polytope	7-simplex	7-orthoplex • 7-cube	7-demicube	1_{32} • 2_{31} • 3_{21}	
Uniform 8-polytope	8-simplex	8-orthoplex • 8-cube	8-demicube	1_{42} • 2_{41} • 4_{21}	
Uniform 9-polytope	9-simplex	9-orthoplex • 9-cube	9-demicube		

Uniform 10-polytope	10-simplex	10-orthoplex • 10-cube	10-demicube		
Uniform *n*-polytope	*n*-simplex	*n*-orthoplex • *n*-cube	*n*-demicube	1_{k2} • 2_{k1} • k_{21}	*n*-pent polyto
Topics: Polytope families • Regular polytope • List of regular polytope compounds					