Getting inaccurate simulation results due to matlab struggling to evaluate the normal CDF at high quantiles. Thus, useful to have an upper bound on the truncated normal p-value. Based on "Bounding Standard Gaussian Tail Probabilities," for Φ and ϕ the standard normal CDF and pdf, respectively,

$$\frac{2\phi\left(x\right)}{\sqrt{4+x^{2}}+x}\leq1-\Phi\left(x\right)\leq\frac{2\phi\left(x\right)}{\sqrt{2+x^{2}}+x}$$

for $x\in\mathbb{R}^{+}$.

Thus, since the normal p-value is

$$T=1-\frac{\Phi\left(\left(X_{i}-\theta_{0}\right)/\sqrt{\sigma_{ii}}\right)-\Phi\left(\left(\mathcal{V}_{i}^{lo}\left(X\right)-\theta_{0}\right)/\sqrt{\sigma_{ii}}\right)}{\Phi\left(\left(\mathcal{V}_{i}^{up}\left(X\right)-\theta_{0}\right)/\sqrt{\sigma_{ii}}\right)-\Phi\left(\left(\mathcal{V}_{i}^{lo}\left(X\right)-\theta_{0}\right)/\sqrt{\sigma_{ii}}\right)},$$

if we define

$$t_{i}=\left(X_{i}-\theta_{0}\right)/\sqrt{\sigma_{ii}}$$

$$l_{i}=\left(\mathcal{V}_{i}^{lo}\left(X\right)-\theta_{0}\right)/\sqrt{\sigma_{ii}}$$

$$u_{i}=\left(\mathcal{V}_{i}^{up}\left(X\right)-\theta_{0}\right)/\sqrt{\sigma_{ii}}$$

then

$$1-T=\frac{\Phi\left(t_{i}\right)-\Phi\left(t_{i}^{lo}\right)}{\Phi\left(t_{i}^{up}\right)-\Phi\left(t_{i}^{lo}\right)}=\frac{\left(1-\Phi\left(t_{i}^{lo}\right)\right)-\left(1-\Phi\left(t_{i}\right)\right)}{\left(1-\Phi\left(t_{i}^{lo}\right)\right)-\left(1-\Phi\left(t_{i}^{up}\right)\right)}.$$

Since both the numerator and denominator are positive, we know that this expression is increasing in the numerator and decreasing in the denomiator, so

$$1-T_{i}\geq\frac{\frac{2\phi(l_{i})}{\sqrt{4+l_{i}^{2}}+l_{i}}-\frac{2\phi(t_{i})}{\sqrt{2+t_{i}^{2}}+t_{i}}}{\frac{2\phi(l_{i})}{\sqrt{2+l_{i}^{2}}+l_{i}}-\frac{2\phi(u_{i})}{\sqrt{4+u_{i}^{2}}+u_{i}}}$$

provided t_i, u_i, and l_i are all positive. We can re-write the RHS as

$$\frac{\left(\sqrt{4+l_{i}^{2}}+l_{i}\right)^{-1}e^{-\frac{1}{2}l_{i}^{2}}-\left(\sqrt{2+t_{i}^{2}}+t_{i}\right)^{-1}e^{-\frac{1}{2}t_{i}^{2}}}{\left(\sqrt{2+l_{i}^{2}}+l_{i}\right)^{-1}e^{-\frac{1}{2}l_{i}^{2}}-\left(\sqrt{4+u_{i}^{2}}+u_{i}\right)^{-1}e^{-\frac{1}{2}u_{i}^{2}}}$$

$$=\frac{\left(\sqrt{4+l_{i}^{2}}+l_{i}\right)^{-1}-\left(\sqrt{2+t_{i}^{2}}+t_{i}\right)^{-1}e^{-\frac{1}{2}\left(t_{i}^{2}-l_{i}^{2}\right)}}{\left(\sqrt{2+l_{i}^{2}}+l_{i}\right)^{-1}-\left(\sqrt{4+u_{i}^{2}}+u_{i}\right)^{-1}e^{-\frac{1}{2}\left(u_{i}^{2}-l_{i}^{2}\right)}}.$$

Thus,

$$T_i \leq 1 - \frac{\left(\sqrt{4+l_i^2}+l_i\right)^{-1} - \left(\sqrt{2+t_i^2}+t_i\right)^{-1} e^{-\frac{1}{2}\left(t_i^2-l_i^2\right)}}{\left(\sqrt{2+l_i^2}+l_i\right)^{-1} - \left(\sqrt{4+u_i^2}+u_i\right)^{-1} e^{-\frac{1}{2}\left(u_i^2-l_i^2\right)}}.$$