Preliminary Power Simulations for Moment Inequality Tests

September 27, 2015

To get a sense of the relative power of different tests for moment inequalities, I simulate draws of a k-dimensional normal random vector

$$X \sim N(\mu, \Sigma)$$

and consider the performance for several of different size $1-\alpha$ tests for $H_0 : \mu \leq 0$. All the tests considered reject for large values of $\max_i X_i$. The difference among these tests is in their approach to computing critical values. In particular, all tests reject when

$$\max_i X_i > c_\alpha(X)$$

for $c_\alpha(X)$ a potentially data-dependent critical value. The approaches I consider are:

1. Least Favorable: This takes $c(X) = c_{\alpha,LF}$ to be the $1-\alpha$ quantile of $\max_i X_i$ under $\mu = 0$.

2. Romano Shaikh and Wolf (2014): The least favorable critical value c_{LF} corresponds to taking the distribution of $\max_i X_i$ as large as possible under the null. Unless many of the moment inequalities are binding (i.e. μ is close to zero), however, this will generally be overly conservative. To get a test with better power when some of the moment inequalities are slack (i.e. when $\mu_i \ll 0$ for some i), Romano Shaikh and Wolf (2014) instead calculate an initial confidence set for

μ (with coverage β) and calculate critical values by making the distribution of $\max_i X_i$ as large as possible for μ in this initial confidence set. Ultimately, their procedure amounts to calculating a vector λ^* with

$$\lambda_i^* = \min\left\{X_i + c_{\beta,LF}, 0\right\}$$

and setting $c_{\alpha,RSW}(X)$ equal to max of the $1-\alpha+\beta$ quantile of $\max_i X_i$ under $\mu = \lambda^*$ (their test also does not reject if $\lambda_i^* < 0$). Following Romano Shaikh and Wolf (2014) I take $\beta = 0.005$ in all simulations.

3. Conditional: This is the approach outlined in the previous note, "t-test for maximum of normal means," which for notation defined there leads to the conditional critical value

$$c_{\alpha,C}(X) = \Phi^{-1}\left((1-\alpha)\zeta_{i_{\max}}^{up} + \alpha\zeta_{i_{\max}}^{lo}\right).$$

4. Two-Step: As shown below, the conditional testing approach can have poor power when the second largest element of μ is nearly as large as the largest element. To address this case, it is helpful to also reject when all elements of μ are large. Hence, I consider a two-step test which rejects if $\max_i X_i$ exceeds either the level $\alpha - \beta$ conditional critical value or the level β least-favorable critical value. This corresponds to using the two-step critical value

$$c_{\alpha,T} = \min\left\{c_{\alpha-\beta,C}(X), c_{\beta,LF}\right\}.$$

In all simulations, I take $\beta = 0.005$.

Simulation Design

For the simulations reported in this note, I let Σ equal the identity matrix. I consider simulation designs with $k = 2$, $k = 5$, and $k = 10$ moments. In each case I consider values for μ_1 and μ_2 in a grid

$$(\mu_1, \mu_2) \in \{-10, -9.9, ..., 10\}^2. \tag{1}$$

I set the other elements of μ to be equal $\mu_3 = ... = \mu_k$, and consider a range of values for μ_k. All results are reported for 5% tests and 1000 simulation draws.

Simulation Results: $k = 2$

Figure 1 plots contours of the rejection probability for the four tests discussed above in the calibration with two instruments. The simulated size of the RSW and conditioning-based tests is slightly higher than 5% due to simulation noise, and one can see that the RSW, conditioning, and Two-step tests have slightly better power than the least-favorable test when one of (μ_1, μ_2) is much less than zero, though the differences are small. On the other hand, the conditioning test has substantially less power than the least favorable and RSW tests when both of (μ_1, μ_2) are large. The two-step test greatly reduces the extent of this problem, but still has somewhat lower power than the RSW test when both moment inequalities are binding.

Since precise visual comparisons across panels in Figure 1 are difficult, Table 1 reports maximum and mean differences is rejection probability across the different tests. From this, we see that the rejection probability of the conditional test sometimes falls short of that of the LF and RSW tests by nearly 50%, while the Two-step test reduces these shortfalls to about 30%. By contrast, the maximal shortfall of the LF and RSW tests relative to the conditional and two-step tests is only about 10%. Likewise, the average rejection probabilities of the two-step and conditional tests are smaller than that of the RSW test (though the average rejection probability of the two-step test is higher than that of the LF test. Overall, I'd say the RSW test looks more appealing (in terms of power) than the conditional and two-step tests in this calibration.

Simulation Results: $k = 5$

Tables 2 and 3 report simulations for the $k = 5$ calibrations, where I consider

$$\mu_3 = \mu_4 = \mu_5 \in \{-7, -5, -3, -2, -1, 0\}.$$

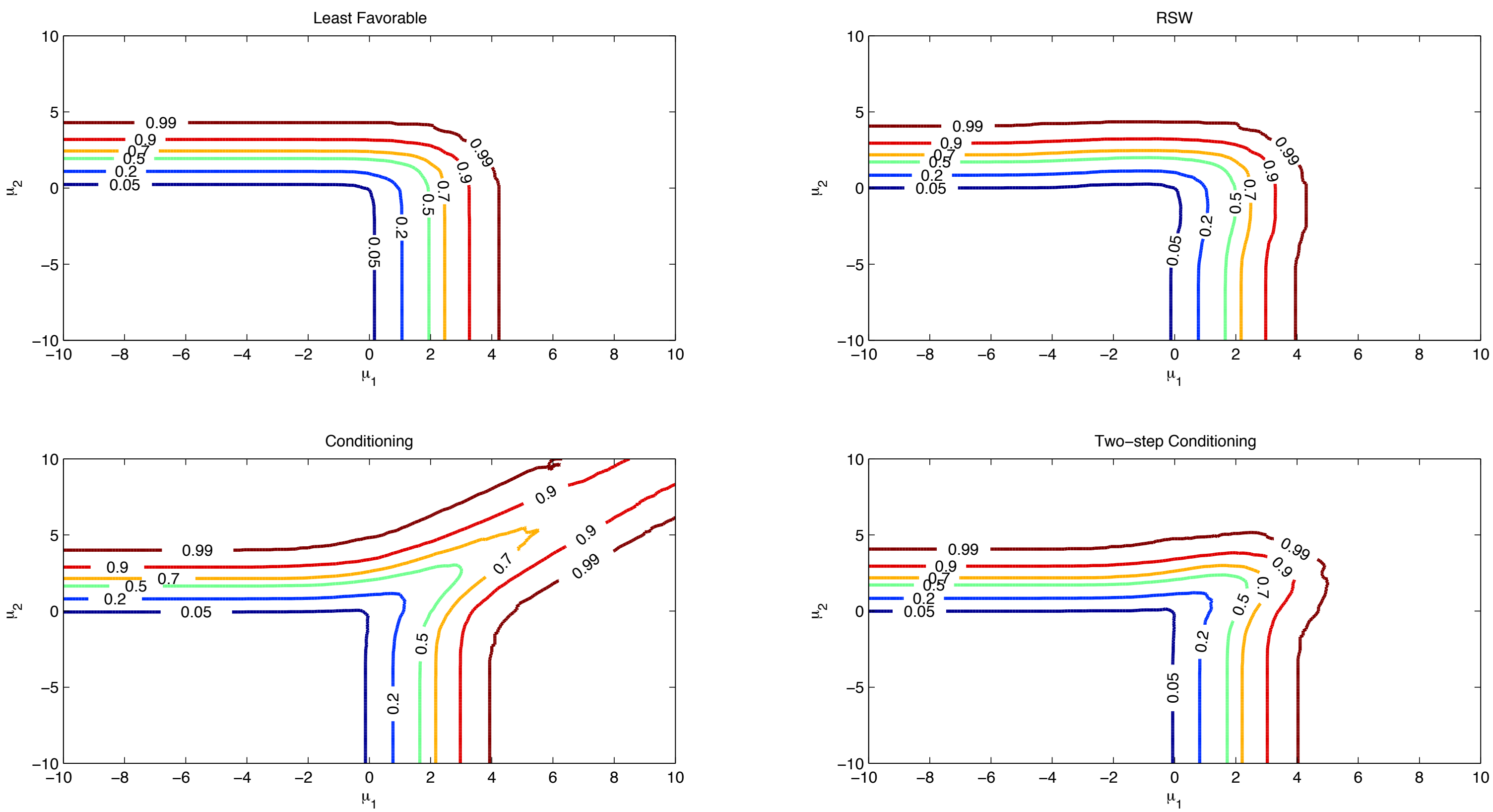

Figure 1: Contours of rejection probability for moment inequality tests

Maximal Rejection Difference (row-column)						Mean Rejection Difference (row-column)				
	LF	RSW	C	T			LF	RSW	C	T
LF	0.00%	3.40%	48.40%	33.50%		LF	0.00%	-0.75%	1.13%	-0.40%
RSW	11.60%	0.00%	47.60%	32.00%		RSW	0.75%	0.00%	1.88%	0.35%
C	13.00%	11.90%	0.00%	3.10%		C	-1.13%	-1.88%	0.00%	-1.52%
T	10.40%	9.80%	38.20%	0.00%		T	0.40%	-0.35%	1.52%	0.00%

Table 1: Maximal and mean differences across tests in rejection probability for calibrations with $k = 2$. In particular, the entry in the RSW row, LF column of the max table reports the largest difference between the rejection probability of the RSW test and LF test over (1), while the corresponding entry of the mean table reports the difference in rejection probabilities averaged over μ in (1).

For $\mu_5 = 0$ and $\mu_5 = -7$ the RSW test continues to look more appealing that the conditional and two-step tests, but there is a range of intermediate values, particularly $\mu_5 = -2$ and $\mu_5 = -3$, where the the conditional and two-step tests look more appealing than the RSW test in terms of mean rejection probability (although the RSW test still looks more appealing in terms of the the maximal difference of rejection probabilities).

Simulation Results: $k = 10$

Table 4 and 5 report simulations for the $k = 10$ calibrations, where I again consider

$$\mu_3 = ... = \mu_{10} \in \{-7, -5, -3, -2, -1, 0\}.$$

The results are broadly similar to those in the $k = 5$ calibrations. In particular, while the RSW test looks more appealing at the extremes, there is a range of intermediate values where the conditioning and two-step tests look (at least potentially) more appealing. In contrast to the $k = 5$ calibations, for $\mu_{10} = -2$ and $\mu_{10} = -3$, the conditioning and two-step tests now win on both the maximal power difference and mean power difference criteria.

Conclusion

Overall, these simulations suggest that there are some regions of the parameter space where tests based on the conditioning approach we've been discussing look more appealing than the RSW test, in particular when many moments are 2 to 3 standard deviations away from binding. At the same time, when most moments are either fairly close to binding or clearly non-binding, the RSW approach looks more appealing than the conditional or two-step tests.

Maximal Rejection Difference (row-column)						Mean Rejection Difference (row-column)			
$\mu_3 = \mu_4 = \mu_5 = 0$									
	LF	RSW	C	T		LF	RSW	C	T
LF	0.00%	3.30%	49.00%	34.10%	LF	0.00%	-0.33%	2.59%	1.13%
RSW	3.60%	0.00%	48.10%	32.40%	RSW	0.33%	0.00%	2.93%	1.46%
C	1.90%	2.00%	0.00%	2.50%	C	-2.59%	-2.93%	0.00%	-1.46%
T	1.70%	1.80%	38.90%	0.00%	T	-1.13%	-1.46%	1.46%	0.00%
$\mu_3 = \mu_4 = \mu_5 = -1$									
	LF	RSW	C	T		LF	RSW	C	T
LF	0.00%	2.60%	43.30%	25.30%	LF	0.00%	-0.08%	0.36%	-1.07%
RSW	2.90%	0.00%	43.00%	24.00%	RSW	0.08%	0.00%	0.44%	-0.99%
C	14.80%	15.70%	0.00%	2.60%	C	-0.36%	-0.44%	0.00%	-1.43%
T	12.60%	13.60%	37.80%	0.00%	T	1.07%	0.99%	1.43%	0.00%
$\mu_3 = \mu_4 = \mu_5 = -2$									
	LF	RSW	C	T		LF	RSW	C	T
LF	0.00%	1.90%	44.90%	26.60%	LF	0.00%	-0.18%	-0.62%	-2.04%
RSW	3.60%	0.00%	44.20%	26.40%	RSW	0.18%	0.00%	-0.44%	-1.86%
C	24.20%	23.60%	0.00%	2.90%	C	0.62%	0.44%	0.00%	-1.42%
T	22.30%	21.70%	37.50%	0.00%	T	2.04%	1.86%	1.42%	0.00%

Table 2: Maximal and mean differences across tests in rejection probability for calibrations with $k = 5$. In particular, the entry in the RSW row, LF column of the max table reports the largest difference between the rejection probability of the RSW test and LF test over (1), while the corresponding entry of the mean table reports the difference in rejection probabilities averaged over μ in (1).

Maximal Rejection Difference (row-column)						Mean Rejection Difference (row-column)			
$\mu_3 = \mu_4 = \mu_5 = -3$									
	LF	RSW	C	T		LF	RSW	C	T
LF	0.00%	0.20%	42.40%	23.50%	LF	0.00%	-0.72%	-1.04%	-2.48%
RSW	8.40%	0.00%	43.30%	25.20%	RSW	0.72%	0.00%	-0.32%	-1.76%
C	26.40%	22.40%	0.00%	3.00%	C	1.04%	0.32%	0.00%	-1.44%
T	24.40%	20.70%	35.30%	0.00%	T	2.48%	1.76%	1.44%	0.00%
$\mu_3 = \mu_4 = \mu_5 = -5$									
	LF	RSW	C	T		LF	RSW	C	T
LF	0.00%	0.00%	44.50%	24.10%	LF	0.00%	-2.55%	-1.19%	-2.61%
RSW	23.50%	0.00%	48.20%	35.30%	RSW	2.55%	0.00%	1.37%	-0.05%
C	27.80%	14.30%	0.00%	3.00%	C	1.19%	-1.37%	0.00%	-1.42%
T	25.60%	12.90%	37.50%	0.00%	T	2.61%	0.05%	1.42%	0.00%
$\mu_3 = \mu_4 = \mu_5 = -7$									
	LF	RSW	C	T		LF	RSW	C	T
LF	0.00%	0.00%	44.50%	23.10%	LF	0.00%	-2.88%	-1.11%	-2.57%
RSW	24.60%	0.00%	48.40%	34.10%	RSW	2.88%	0.00%	1.77%	0.30%
C	27.60%	11.80%	0.00%	3.50%	C	1.11%	-1.77%	0.00%	-1.46%
T	25.20%	9.90%	38.10%	0.00%	T	2.57%	-0.30%	1.46%	0.00%

Table 3: Maximal and mean differences across tests in rejection probability for calibrations with $k = 5$. In particular, the entry in the RSW row, LF column of the max table reports the largest difference between the rejection probability of the RSW test and LF test over (1), while the corresponding entry of the mean table reports the difference in rejection probabilities averaged over μ in (1).

Maximal Rejection Difference (row-column)						Mean Rejection Difference (row-column)				
$\mu_3 = \mu_4 = \mu_5 = 0$										
	LF	RSW	C	T			LF	RSW	C	T
LF	0.00%	2.90%	41.90%	26.90%		LF	0.00%	0.07%	2.49%	1.22%
RSW	0.60%	0.00%	41.20%	25.30%		RSW	-0.07%	0.00%	2.42%	1.15%
C	1.20%	1.60%	0.00%	1.80%		C	-2.49%	-2.42%	0.00%	-1.27%
T	0.50%	0.90%	34.30%	0.00%		T	-1.22%	-1.15%	1.27%	0.00%
$\mu_3 = \mu_4 = \mu_5 = -1$										
	LF	RSW	C	T			LF	RSW	C	T
LF	0.00%	3.00%	40.70%	23.40%		LF	0.00%	0.11%	-0.23%	-1.57%
RSW	0.30%	0.00%	40.40%	22.20%		RSW	-0.11%	0.00%	-0.34%	-1.67%
C	16.90%	17.80%	0.00%	2.60%		C	0.23%	0.34%	0.00%	-1.33%
T	15.10%	16.10%	34.40%	0.00%		T	1.57%	1.67%	1.33%	0.00%
$\mu_3 = \mu_4 = \mu_5 = -2$										
	LF	RSW	C	T			LF	RSW	C	T
LF	0.00%	2.10%	43.30%	22.70%		LF	0.00%	-0.03%	-1.89%	-3.22%
RSW	1.50%	0.00%	43.00%	22.10%		RSW	0.03%	0.00%	-1.86%	-3.20%
C	32.70%	33.00%	0.00%	2.90%		C	1.89%	1.86%	0.00%	-1.33%
T	30.50%	31.00%	37.40%	0.00%		T	3.22%	3.20%	1.33%	0.00%

Table 4: Maximal and mean differences across tests in rejection probability for calibrations with $k = 10$. In particular, the entry in the RSW row, LF column of the max table reports the largest difference between the rejection probability of the RSW test and LF test over (1), while the corresponding entry of the mean table reports the difference in rejection probabilities averaged over μ in (1).

	Maximal Rejection Difference (row-column)					Mean Rejection Difference (row-column)			
	$\mu_3 = \mu_4 = \mu_5 = -3$								
	LF	RSW	C	T		LF	RSW	C	T
LF	0.00%	0.10%	42.80%	23.70%	LF	0.00%	-0.41%	-2.41%	-3.71%
RSW	4.20%	0.00%	43.20%	25.60%	RSW	0.41%	0.00%	-1.99%	-3.29%
C	34.60%	32.00%	0.00%	3.10%	C	2.41%	1.99%	0.00%	-1.30%
T	32.40%	30.50%	36.40%	0.00%	T	3.71%	3.29%	1.30%	0.00%
	$\mu_3 = \mu_4 = \mu_5 = -5$								
	LF	RSW	C	T		LF	RSW	C	T
LF	0.00%	0.00%	40.60%	22.90%	LF	0.00%	-3.28%	-2.69%	-3.96%
RSW	25.00%	0.00%	45.80%	36.60%	RSW	3.28%	0.00%	0.59%	-0.68%
C	35.80%	18.00%	0.00%	2.90%	C	2.69%	-0.59%	0.00%	-1.28%
T	34.00%	16.60%	34.30%	0.00%	T	3.96%	0.68%	1.28%	0.00%
	$\mu_3 = \mu_4 = \mu_5 = -7$								
	LF	RSW	C	T		LF	RSW	C	T
LF	0.00%	0.00%	42.00%	23.40%	LF	0.00%	-4.56%	-2.80%	-4.07%
RSW	37.40%	0.00%	48.70%	42.70%	RSW	4.56%	0.00%	1.76%	0.48%
C	39.00%	12.40%	0.00%	3.00%	C	2.80%	-1.76%	0.00%	-1.27%
T	37.20%	10.50%	35.50%	0.00%	T	4.07%	-0.48%	1.27%	0.00%

Table 5: Maximal and mean differences across tests in rejection probability for calibrations with $k = 10$. In particular, the entry in the RSW row, LF column of the max table reports the largest difference between the rejection probability of the RSW test and LF test over (1), while the corresponding entry of the mean table reports the difference in rejection probabilities averaged over μ in (1).