Multithread Monte Carlo Counterfactual Regret Minimization for Texas Heads Up No Limit Poker

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1 Introduction

This paper serves as a journal for my experiences in improving the Monte Carlo Counterfactual Regret Minimization (MCCFR) algorithm, by implementing multithreading and exploration decay, with the goal of creating a strategy that approximates Nash equilibrium for Texas Heads Up No Limit (HUNL) Poker. Essentially, I wanted to create an unbeatable poker bot. My contribution improves MCCFR's performance and efficiency. This paper is meant to be read casually, as indicated by the very causal diction. Please reach out of there is any part of the paper that I could better clarify. I plan to release the source code of this project sometime in the future. Hopefully you can learn something, whether that's about game theory, reinforcement learning, incomplete information games, etc, that you can use in your own work.

It is expected that the reader is familiar with Texas No Limit Holdem and have a high level understanding of reinforcement learning, deep learning, multithreading, and data structures. The Nash equilibrium strategy and the bot will be used hand-in-hand, because the bot is simply executing the strategy.

This project, like the reason for starting all my computer science projects, started because I was bored and I wanted to solve a problem in my life. At the time of beginning this project, I became extremely interested in poker. Given my passion for computer science, it's only natural for me to combine these interests and try to create a poker bot of my own. As I was creating the bot, I implemented some of my own optimizations. Though my main contribution is creating a multithread version of MCCFR and exploration decay, this paper will focus more on the entire process of creating a poker bot. Also, please reach out if there are any factually incorrect statements.

2 Basic Game Theory

Game theory is the study of decision making in social situations involving competing players. Game theory is used in business, finance, politics, economics, psychology, and many more situations[1]. There is a type of game called zero-sum games. A player's benefit in a zero-sum game is exactly the other person's loss. So if all the gains and losses are added together at the end of a zero-sum game, the sum would be zero. Nash equilibrium, in the context of this paper, will be defined as a set of game theory optimal (GTO) strategies[2]. This Nash equilibrium will be where neither player can improve their expected payout by changing their strategy. No matter the situation, a Nash equilibrium strategy is guaranteed to not lose money over many games. This does not mean that you will make money, just that you will not lose money. Nash equilibrium and GTO will be synonymous in this paper.

A strategy is a probability distribution of actions in a given situation. All GTO strategies cannot be altered to increase expected payout, assuming we know nothing about the opponents' strategy. This differs from exploitative play, which is when one changes their strategies to specifically counter the opponent's weakness. For example, one can play extremely tight if the other person is a calling machine. Of course, one can play exploitatively if they know information about how the other player plays, but we'll focus on GTO strategies because that's more generalizable, you won't always know what type of player the other players are, your assumption(s) about others may not always be correct, and I find GTO more interesting.

Let's use the classic game of rock, paper, scissors to demonstrate the difference between GTO and exploitative play. Given that you know nothing about the opponent, the best strategy you can use is to simply play $\frac{1}{3}$ rock, $\frac{1}{3}$ paper, and $\frac{1}{3}$ scissors. This is GTO. Now let's say you know the opponent plays 34% rock, 33% paper, and 33% scissors. If you were playing exploitatively, it is in your best interest to

play 100% paper. But this is risky because your observations may have been from just noise instead of from an underlying pattern (in other words, the other person had an equal chance of all 3 hands, but it just happened to be that they played more rock than the other hands due to randomness) and/or the other player may change their play style to exploit your changed play style. These concepts are generalizable to poker.

Poker, without a rake, is a zero-sum game because any chips a player gains is a direct result of other players' loss of chips. In heads up poker, virtually all poker bots attempt to reach Nash equilibrium. Lastly, poker is known as an incomplete information game. This means that the different players have access to different information, namely their cards. So each player has to make the best decision in a given situation without knowing all the information. You have to use information, such as past betting history, general priors, psychology, and other tells to make an optimal decision. The benefits of being able to make good decisions with incomplete information extends beyond poker. Many decisions in life are made with incomplete information.

3 Counterfactual Regret Minimization

My bot, and most of today's best performing poker bots, use counterfactual regert minimization (CFR) to optimize their strategy. Essentially, CFR works by the model playing against itself and improving itself based on the outcome.

I recommend that you read the original CFR paper, first published by the University of Alberta in 2009, for more formal notation, but I will include some important definitions here[3]. These definitions will be in the context of poker, but generalizing these concepts should not be too difficult.

History contains everything that defines a given situation in poker: the hands, the betting street, the action history, and the cards on the board. An *infoset* is the part of history only accessibly to a given player and is unique for each player; it essentially describes the situation the player is in. The main difference between a history and an infoset is that the infoset does not contain the cards of the other player. The *utility* of an action is how much money that action would have made, and the utility is solely measured by how many chips you gained or lost. *Regret* is the difference between the utility of a certain action and the action that was chosen. Lastly, a *strategy* is the probability distribution of the events that a player would do in a given situation. For example, the strategy of the small blind's first move after the flop, given that they hit top pair, could be:

Action	Probability
Fold	0
Check	0.2
Raise 0.33x pot	0.2
Raise 0.5x pot	0.2
Raise 0.67x pot	0.15
Raise 1x pot	0.1
Raise 1.5x pot	0.1
Raise 2x pot	0.05

Vanilla CFR plays many games against itself and improves based on the results. Counterfactual regret is the regret weighed by the probability of the opponent reaching that state. This makes it so that events that may have a large impact on behavior, but are extremely rare, do not have a large effect on the strategy. For example, if the opponent puts you all in, you have ace high, there was a lot of preflop action, there are a lot of high value cards on the board, the board is extremely connected, and the board is mostly monotone (so there is flush potential), you may win a lot of money by calling with ace high. But the probability of you being ahead is so low that the Nash equilibrium strategy would put a low percentage on calling and a high percentage on folding in this situation.

For each training iteration in vanilla CFR, the algorithm traverses the entire game tree. A game tree is a tree representation of all the possible game states. This is extremely inefficient and not practical in large scale games because it takes too long for the algorithm to converge to Nash equilibrium. So we utilize MCCFR. MCCFR essentially utilizes random sampling in the learning part of the agent. Although more iterations are required to reach Nash for MCCFR, it takes a lot less time per iteration. MCCFR is less precise than vanilla CFR, but it takes a lot less time total for MCCFR to approximate Nash than vanialla CFR. So instead of multiplying each regret by the probability of it happening, we directly use the fact that random sampling represents the probability that the other player will reach the current state to put more weight towards events that happen more often and vice versa for less common events. Similar to vanilla CFR, MCCFR improves its strategy by

playing against itself.

4 Bucketing

It is not possible to reach all unique infosets in a feasible amount of time. In fact, there are more infosets available in poker than atoms in the universe. Therefore, a concept called "bucketing" is required for large games, such as poker. Bucketing is putting similar infosets, that would have similar strategies, in "buckets". An example of a bucket is top pair and good kicker. You would not play AcTc with the board Ts5s2h much differently than AcJc with the board Js5s2h. In both situations, you have top pair, a good kicker, and virtually no draws. So the training task becomes figuring out the strategies for all the buckets. And when the bot plays real games, it categorizes the infoset into the appropriate bucket, then uses the bucket's strategy. Therefore, bucketing decreasing the number of necessary strategies by many orders of magnitude, making approaching Nash faster. I'm also training on my laptop (2021 Macbook Pro with M1 Max), so minimizing training time is crucial.

Unfortunately, I could not find any existing effective bucketing algorithms on the internet; therefore, I had to create my own. It is lossless for preflop and lossy for postflop. But the way different hands in the same bucket should be played are approximately the same for postflop and exactly the same for preflop.

The preflop bucketing algorithm is simply <the rank of card #1><the rank of card #2><suited or not suited>. Hands, such as AhKh and AcKc are technically different, but they will both be bucketed as AKs and played the exact same way.

Figure 1 displays my bucketing algorithm for postflop situations. The postflop bucketing algorithm takes into account the board texture (if the board is double paired, number of suited cards, etc), the made hand, potential draws, and the number of previous betting streets that each player raised. Board texture describes different arrangements of the board. For example, a "monotone" or "flushy" board texture means a board of many cards of the same suit. A made hand is the hand you currently have. For example, pairs, trips, and flushes are made hands. Draws are hands that are incomplete and requires additional cards to become valuable. For example, if your hand and the board creates 2345, then it is currently worthless (high-card). But any ace or six that comes out will turn your hand into a straight. My bucketing algorithm only considers flush and straight draws because they are the most common draws people consider. Lastly, taking into account raising history is important for concepts like gauging villain hand strength and making a bluffing story more believable. For example, if the other player did not raise for all the betting streets and raises large on an inconsequential river card, it is likely they are bluffing.

5 My Contributions

5.1 Multithreading

I implemented multithreading in the MCCFR algorithm. For those unfamiliar, multithreading is where computer can (essentially) do multiple things at the same time. I created a thread lock for every pair of infoset and matching regret sum, strategy, and strategy sum so different threads can alter the data of different infosets simultaneously. I altered the original MCCFR program to create a new thread each time the external_cfr function, the function that runs MCCFR for a given game state, is called if there are more than 3 possible actions and it is preflop or flop. The specific hyperparameter of the minimum number of actions and rounds left required to create a thread was determined by a hyperparameter sweep. You cannot create a thread for every possible recursive call because it is expensive to create a thread and the diminishing positive gain from each additional thread. Essentially, you want to balance creating a thread for new game tree branches, but ensuring that there are enough calculations to be done in that game subtree for adding a thread to be worth it. For example, the bot would not have many regret calculations left if it was on the river round. So, sacrificing computing resources to create the thread and clog up all the other threads only to barely do any work is wasteful. Also, the locks are the main bottle neck of the multithreading algorithm anyway.

Contrary to this section's introduction, a computer can actually still only do one thing at a time. Multithreading gives the illusion that the computer is doing multiple things at a time by constantly doing a little bit one one task and then switching to another one. The amount of time a processor spends on each thread is dependent on that system, but the time is usually extremely small. To put it nontechnically, each task is put on a thread. But the computer cannot instantly switch between threads. It takes time and resources for the computer to switch be-

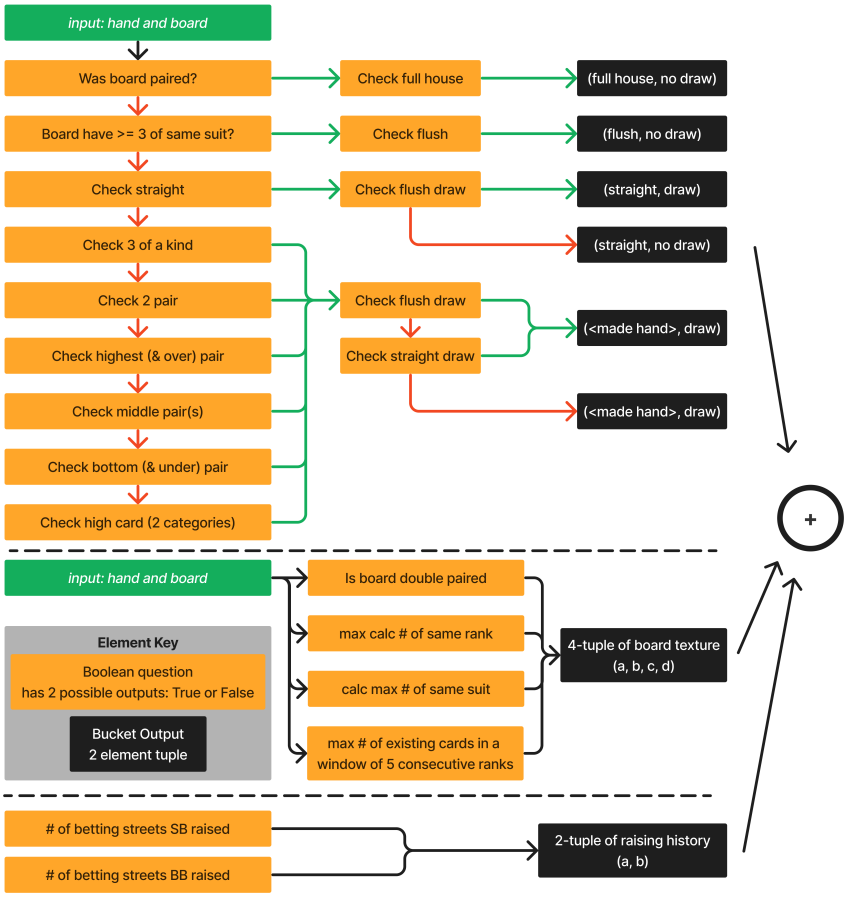

Figure 1: Postflop Bucketing Algorithm. A green arrow represents "yes", a red arrow represents "no", and a black arrow signifies to proceed. The three sections - made hand, board texture, and raising history - are concatenated to represent a bucket.

tween threads and to create a thread. That is why we cannot simply create a new thread for each subtree in the game tree. Instead, we have to maximize the number of threads up until adding more threads actually slows down the learning.

The multithreading optimizations decreased the time it takes for one iteration to run from ≈ 30 seconds to $\approx \frac{1}{2}$ seconds, a 60x improvement.

5.2 Exploration Decay

As we all know, poker is a game with high variance. The MCCFR algorithm can be sensitive to extreme situations, especially in the beginning, possibly throwing the learning algorithm off balance. Thus, we must sacrifice accuracy in the beginning for the sake of exploration. This is essentially a form of regularization, a term taken from machine learning, that decreases as training time increases.

We want the algorithm to explore different game paths in the beginning, because a few uncommon situations in the beginning of training has the potential to completely throw of the weights for a given infoset. This is similar to you learning a new game. If you've only played the new game a couple of times, it would make sense for you to continue trying different actions to see if what happened to you in a given situation was simply due to randomness or an underlying pattern you can exploit. The idea behind prioritizing game tree exploration is the same idea. The algorithm wants to explore many different actions in the beginning, even if these actions may seem unfavorable according to the simulated games the algorithm has trained on. Messed up (non-GTO) weights in the beginning can also have a positive reinforcement loop, because the bot plays against itself to learn. Let's

say that the bot learns early on to go all-in when the board is dry (rainbow, low value cards, disconnected suits, etc). Then the bot may learn to call a lot in that situation, even when it's not supposed to, because itself would frequently shove with nothing. Therefore, this is an effective way to integrate regularization in the learning algorithm.

We keep this exploration open by setting a minimum probability value for a given action. In the beginning, since not many games have been played, we would naturally want to set it relatively high. I personally set it to 5%. Then as the number of simulated games increases, you would want to decrease this probability, because it is likely that the observed data is more accurate. After around $100,000$ games, I set the minimum to 1%. And after $150,000$ games, I remove the minimum. So, actions that the algorithm deems should be played 0% of the time will actually never be played (such as folding pocket aces preflop) by the end.

Feel free to skip this paragraph if you are *not* familiar with MCCFR. Every time the strategy weights are called for an infoset, I set the minimum probability to the given minimum chance at that moment. And then when the strategy is added to the strategy sum data, the altered strategy is added. But theoretically, as time goes on and the minimum probability decreases, the average strategy should still converge to equilibrium. This method simply decreases variance in the beginning of training.

6 Initial Weights

Typically in CFR, initial weights are uniform. So the probability of doing any given action is equal to the probability of doing any other action. Minimizing training time is crucial, so the starting weights for an unseen infoset are not uniform.

First, I simulated tens of millions of games of poker. With each game, I kept track of the win, tie, and loss rates of each bucket. Then, I created a preassigned probability distribution for actions based on ranges of win rates. For example, if the win rate of a given infoset is below 20%, then the starting probability of folding/checking is extremely high, and the starting chance of going all-in is extremely low. Theoretically, this method should decrease the required training time for the algorithm to reach Nash, because the initial weights should be closer to GTO than strictly uniform strategies. I used existing knowledge I already knew about poker, called "domain knowledge" in general terms, to create a process for choosing the initial weights.

7 Exploitability

Exploitative poker is a playing style that is dependent on your opponent. For example, if you know that your opponent plays extremely tight and only plays extremely good hands, then you know to fold anytime that person raises. Being unexploitable means that even if the opponent knew your exact strategy, there is nothing they can do to win more chips from you. A strategy that reaches Nash equillibrium is unexploitable.

I measure exploitability by playing the bot against different versions of itself. Theoretically, if the bot plays GTO, then it should be profitable against any variant of itself. The strategies for these bot variations are created by taking the strategy profile of the original bot and modifying it appropriately for the given play style. If the bot is able to consistently be profitable against all the following play styles, the bot is unlikely to be exploitable. Obviously, exploitability is more detailed and nuanced than described, and there are more complex poker strategies than the ones I listed. But, my evaluation of exploitability is adequate for the time being.

- Aggressive: The bot folds less and raises larger amounts more often.
- Tight: The bot folds more and raises larger amounts less often.
- Random: All strategies in the strategy profile are uniform. All possible actions in a given situation are equally likely to be selected.
- Calling Machine: The bot is more likely to call than any other action.
- Tight Aggressive: The bot plays tight if the win rate of the current situation is less than a certain value and plays aggressive if the win rate of the current situation is more than a certain value. This is essentially an unbalanced player - their actions perfectly reflect their hand value.

As you can see by Figure 2, the bot is profitable against every version of itself except for tight. This means that you can beat the bot by playing tightly - the bot has not reached Nash. I will need to do

some further work to figure out why the bot is playing overaggressively, and this problem is mentioned later in this paper.

8 Modifications to Game

I noticed that after the first set of training the bot, the bot played too aggressively. It folded less than it should have, and the bot would overbet more frequently than it should have. So, you could consistently beat the bot if you simply played tight. This problem can be seen in figure 2. The bot is consistently profitable against all play styles except for a tight version of itself. I determined the root of the problem to be that the pot would balloon to unrealistic sizes in some cases. For example, you could win around 5000 BB if there was a lot of action preflop and many reraises during postflop betting streets. Generally in poker, you play with 50-300 BBs. So if the bot typically won 50 BB per hand, then it would only have to win that hand that it played overaggressively in $\frac{1}{101} \approx 1\%$ of the time. But given the previous constraint of pot sizes, this method of thinking is not reasonable.

9 Deep Learning

A problem with utilizing CFR for such a large scale game, such as HUNL Poker, is that the average strategy takes a long time to converge to equilibrium. A possible solution to this is to utilize deep learning to output the strategy for a given infoset because deep learning has been proven to be effective in generalizing complex patterns. Noam Brown, et al. have successfully created a version of CFR that utilizes deep learning [4].

10 Game Modifications

The first Artificial Intelligence (AI) that definitively beat humans in poker was Libratus, created by a Carnegie Mellon University research team. Libratus used a similar algorithm to MCCFR for part of its solving process, but this bot took 15 million core hours to train on the Pittsburgh Supercomputing Center's Bridges computer, amounting to 2.5 petabytes of data[5]. The computing power and storage required to train a less abstracted version of the game is not feasible with my resources. Therefore, I had to make several changes to the game that still preserve the core ideas of poker, could easily be reversed, and would barely affect one's game strategy.

Both players start with 100 BB in every game, instead of starting with the number of chips left over from the previous hand. The stack size affects gameplay. So, my software would have to train on every single combination of starting stack pairs from 1 BB to 100 BB if this modification was not made. Even with this modification, my bot generated 1 gigabit of data. I chose 100 BB because that is the standard buy in amount.

There is a limited number of raise sizes. Preflop, a player can only raise with the following number of BB: $1, 2, 4, 8, 16$. These selection of raise values were chosen because each raise value after a given raise value are valid. For example, if a player raised to 2 BB, all values after 2, namely $4, 8, 16$, are valid raise values that the next player could raise to. Postflop, a player can raise $0.3x, 0.5x, 0.7x, 1.0x, 1.5x, 2.0x$ pot. The validity of the raise logic is similar to preflop raise values, and these values are typical raise values postflop. Restricting the raise values also removes the need for an action abstraction algorithm, such as the problem if an opponent's raise of 3 should be categorized as 2 or 4 in the bot's decision making. There is a wide enough range of raise values to cover most actions, yet granular enough for the raise values to distinguish between intended strategies.

11 Results

The bot is showing promising results. It has learned how to bluff, semi-bluff, and value bet. The program has also learned strategies such as keeping aggression up across betting streets if it is bluffing.

The viability of the bot is still being tested against experienced players to assess whether the set of strategies produced by the algorithm is profitable.

Currently, the bot is too aggressive - it is going all-in in positions it shouldn't and calling all-ins when it shouldn't.

12 Future Plans

There is still a lot of work to be done on this project. Here is a list of things I plan to do in the near future regarding this project:

- Train multiple models and average the result. So for a given infoset, I run the infoset through x different models and then sample randomly

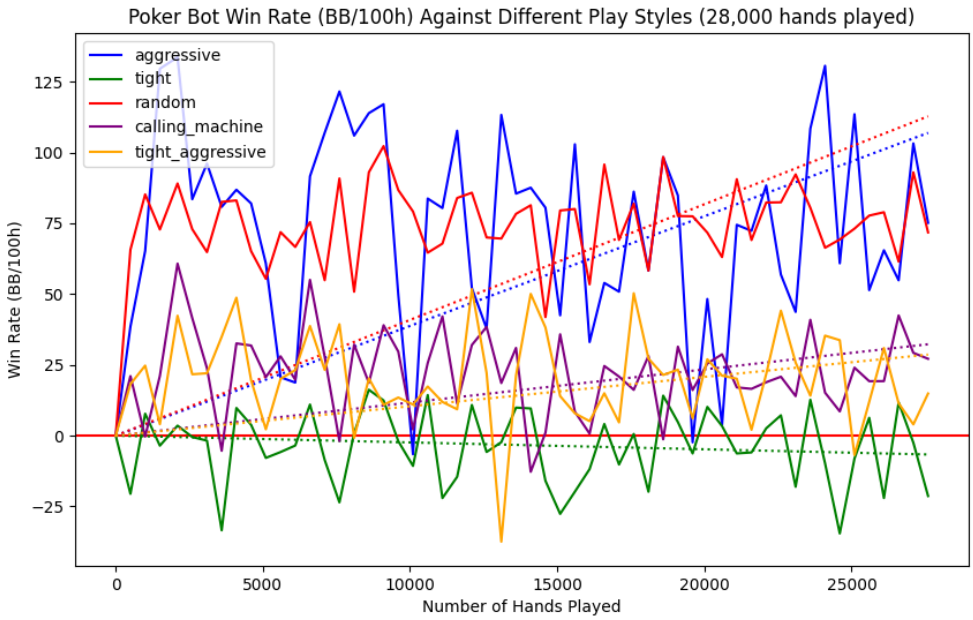

Figure 2: Exploitability of bot v1, measured by the win rate (BB/100h) of the bot against variants of itself.

from the outputs. I want to try this because the performance of the poker bot has extremely high variance, and the bot's weights could easily spiral away from GTO if it gets many improbable poker game simulations. This method of averaging results obtained from independently trained models is known as *ensembling* in machine learning.

- Continue playing with deep learning and try to implement material I learned in CS155 (a theoretical machine learning course at Caltech) and see if performance improves.
- Create a poker bot for multi-way tables of Texas No-Limit Holdem Poker, though this will be far more complex than heads up and only be pursued if this bot proves to be extremely effective.

13 Additional Resources

Here are some resources, in order of decreasing importance, I found helpful in teaching myself the concepts behind creating this bot. I also recommend reading through the cited references at the end of this paper for a formal framing of CFR, game theory, and adjacent topics.

1. A YouTube video that helps to visualize vanilla CFR: link
2. A video lecture by Noam Brown, one of the leaders of the CMU team that created Libratus, about Libratus, game theory, and various forms of CFR: link
3. A YouTube video that gives an idea on what coding CFR may look like: link

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