# Prob HW2_5

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5. (a)

$f_X(x) = \lambda e^{-\lambda x} \qquad F_X(t) = 1 - e^{-\lambda t}$

$F_Y(t) = P(Y \le t) = P(aX + b \le t) \overset{a>0}{=} P(X \le \frac{t-b}{a}) = F_X(\frac{t-b}{a}) = 1 - e^{-\lambda(\frac{t-b}{a})}$

$\overset{a<0}{=} P(X \le \frac{b-t}{a}) = F_X(\frac{b-t}{a}) = 1 - e^{-\lambda(\frac{b-t}{a})}$

$f_Y(t) = F_Y'(t) \overset{a>0}{=} \frac{\lambda}{a} e^{-\frac{\lambda}{a}(t-b)}$

$\overset{a<0}{=} \frac{\lambda}{a} e^{-\frac{\lambda}{a}(b-t)}$

If $Y$ is an exponential random variable, $t = t - b \Rightarrow b = 0$

$Y \sim \text{Exp}(\frac{\lambda}{a}) \qquad \frac{\lambda}{a} > 0 \Rightarrow a > 0$

(b) $f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$

$E[X] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x e^{-\frac{x^2}{2}} dx = \frac{1}{\sqrt{2\pi}} \left[ -e^{-\frac{x^2}{2}} \right]_{-\infty}^{\infty} = \frac{1}{\sqrt{2\pi}}(-1+1) = 0$

$E[X^2] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x^2 e^{-\frac{x^2}{2}} dx = \int_{-\infty}^{\infty} x \cdot x e^{-\frac{x^2}{2}} dx = x e^{-\frac{x^2}{2}} - \int_{-\infty}^{\infty} 1 \cdot e^{-\frac{x^2}{2}} dx$

$= \frac{1}{\sqrt{2\pi}} \left( \lim_{t \to -\infty} t e^{-\frac{t^2}{2}} - \lim_{t \to \infty} t e^{-\frac{t^2}{2}} + \int_{-\infty}^{\infty} 1 \cdot e^{-\frac{x^2}{2}} dx \right)$

$= \int_{-\infty}^{\infty} \underbrace{\frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx}_{\text{pdf of } x}$

$= 1 \qquad \lim_{t \to \pm\infty} t e^{-\frac{t^2}{2}} = \lim_{t \to \pm\infty} \frac{t}{\frac{1}{e^{-\frac{t^2}{2}}}} \overset{\text{l'H}}{=} \lim_{t \to \pm\infty} \frac{e^{-\frac{t^2}{2}}}{-t} = 0$

$\text{Var}(X) = E[X^2] - E[X]^2 = 1 - 0 = 1$ *