$$h_x X(x,t) = \alpha_x X(x,t) - B(x) \sum_{s=1}^{m} K_s \int_D \psi_s^*(y) X(y,t)\, dD,$$

$$y \in D,\ x \in \partial D. \quad (10)$$

The new eigenvalue problem

$$A_x \phi_j(x) = \rho_j \phi_j(x),\ x \in D, \quad (11)$$

$$j = 1,2,\cdots,m$$

$$h_x \phi_j(x) = 0,\ x \in \partial D \quad (12)$$

is defined by the operators A_x, h_x, and the prescribed eigenvalues ρ_j. These m equations can be used to determine the gains K_s. Multiplication of (11) with $\psi_k^*(x)$ and the application of Green's identity yield

$$(\rho_j - \lambda_k) q_{jk} + \mu_k \sum_{s=1}^{m} K_s q_{js} = 0, \qquad j = 1,2,\cdots,m,\ k = 1,2,\cdots,\infty. \quad (13)$$

The first m of these equations can be written in vector form

$$(\rho_j I - A) q_j = 0, \qquad q_j = [q_{j1}, q_{j2}, \cdots, q_{jm}]^T \quad (14)$$

and since $q_j \neq 0$, the same result as in (8) can be deduced from this equation.

IV. Example

The example of Tzafestas

$$\frac{\partial}{\partial t} X(x,t) = \alpha^2 \frac{\partial^2}{\partial x^2} X(x,t), x \in [0,l],\ t \geq 0,\ X(0,t) = U(t),\ X(l,t) = 0$$

is considered to calculate the eigenfunctions $\phi_j(x)$ of the closed loop. Shifting the eigenvalues λ_1 and λ_3 to $\rho_1 = -8(\alpha\pi/l)^2$ and $\rho_3 = -12(\alpha\pi/l)^2$, the boundary value problem reads

$$\alpha^2 \frac{d^2}{dx^2} \phi_j(x) = \rho_j \phi_j(x), \qquad x \in [0,l],\ t \geq 0$$

$$j = 1,3$$

$$\phi_j(0) = K_1 q_{j1} + K_3 q_{j3}, \quad \phi_j(l) = 0$$

which has the solution

$$\phi_j(x) = \phi_j(0) \frac{\sin \sqrt{-\rho_j/\alpha^2}\,(l-x)}{\sin \sqrt{-\rho_j/\alpha^2}\, l}, \qquad j = 1,3.$$

The constants $\phi_j(0)$ contain the coefficients q_{j1}, q_{j3}, and the gains $K_1 = -77\pi/8\sqrt{2l}$, $K_3 = -\pi/8\sqrt{2l}$, which are different in sign and the factor $\sqrt{l}$ to those calculated by Tzafestas. Note that the new eigenfunctions $\phi_j(x)$ of the closed loop are not orthogonal due to the nonhomogeneous boundary conditions and the prescribed values ρ_j.

V. Conclusions

In this correspondence it has been shown that in Tzafestas' note, the assignment of the eigenvalues $\lambda_k (k = 1,\cdots,m)$ is based on equations which are only correct under an additional assumption about the uncontrollability of the modes $\xi_k (k > m)$. However the gains $K_s (s = 1,2,\cdots,m)$ of the control law can be obtained from the general equations without the restriction $\mu_k = 0 (k > m)$ by treating them in a way similar to lumped parameter systems [3]. Moreover, the boundary value problem for the closed loop has been formulated and the new eigenfunctions calculated for an example. The second method of calculating the gains K_s can be regarded as an extension of the eigenvalue assignment proposed by Bradshaw and Porter [1], [2]. This extension to boundary control problems as well as the method investigated by Tzafestas are both applicable to spatially n-dimensional distributed parameter systems.

References

[1] B. Porter and A. Bradshaw, "Modal control of a class of distributed-parameter systems," *Int. J. Contr.*, vol. 15, no. 4, pp. 673–681, 1972.
[2] ——, "Modal control of a class of distributed-parameter systems: Multi-eigenvalue assignment," *Int. J. Contr.*, vol. 16, no. 2, pp. 277–285, 1972.
[3] J. D. Simon and S. K. Mitter, "A theory of modal control," *Inform. Contr.*, vol. 13, pp. 316–353, 1968.

Author's Reply[2]

SPYROS G. TZAFESTAS

I thank the authors for their interest in my technical note.[1] I really agree with their comments. Actually, the misleading point in my note is the transition from (6) to (8)–(9), which should be written as $d\zeta_i/dt = \lambda_i \zeta_i - \mu_i K \zeta_k$, $i = 1,2,\cdots,\infty$, or in matrix form $d\zeta/dt = A\zeta$, where

$$A = \left[\begin{array}{c|c|c} \Lambda_{k-1} & -\mu_1 K & O \\ \hline & \vdots & \\ 0\cdots 0 & \lambda_k - \mu_k K & 0 \cdots 0 \\ \hline O & -\mu_{k+1} K & \Lambda_{k+1} \\ & \vdots & \end{array}\right]$$

and $\Lambda_{k-1} = \mathrm{diag}(\lambda_1,\cdots,\lambda_{k-1})$, $\Lambda_{k+1} = \mathrm{diag}(\lambda_{k+1}, \lambda_{k+2}, \cdots)$. Clearly,

$$\det(\rho I - A) = [\rho - (\lambda_k - \mu_k K)] \prod_{\substack{i=1 \\ i \neq k}}^{\infty} (\rho - \lambda_i)$$

and so the eigenvalues of the closed-loop are the same with those of the open-loop system, except the kth eigenvalue, which is changed to $\rho_k = \lambda_k - \mu_k K$. Equations (12) and (13) were written by a direct generalization of (8) and (9). However, as the authors point out, the main result, i.e., the eigenvalue controller gain (16) provides the solution of the boundary control problem without the need of any additional assumption. An interesting open problem is to study the effect of feedback on systems with continuous spectrum.

[2]Manuscript received October 30, 1974.
The author is with the Department of Electrical Engineering, University of Patras, Patras, Greece.

Correction to "Canonical Forms for the Identification of Multivariable Linear Systems"

M. J. DENHAM

In the above paper,[1] the following omission in the statement of Lemma 3 should be corrected. The definition of the set $\mathfrak{L}$ should read

$$\mathfrak{L} \triangleq \{L : L \in \theta \text{ and } L \text{ is lower triangular,}$$

$$\text{with positive diagonal elements}\}.$$

The author of a subsequent paper [1] has asked me to point out that this error has been reproduced there as a result of the original omission.

References

[1] D. Q. Mayne, "Canonical forms for identification," in *New Directions in Signal Processing in Communication and Control*, NATO Advanced Study Institute Series. Boston, Mass.: Reidel, 1974.

Manuscript received December 4, 1974.
The author is with the Department of Computing and Control, Imperial College of Science and Technology, London, England.
[1]M. J. Denham, *IEEE Trans. Automat. Contr.*, vol. AC-19, pp. 646–656, Dec. 1974.