Rich Vehicle Routing Problem: Survey

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The Vehicle Routing Problem (VRP) is a well-known research line in the optimization research community. Its different basic variants have been widely explored in the literature. Even though it has been studied for years, the research around it is still very active. The new tendency is mainly focused on applying this study case to real-life problems. Due to this trend, the Rich VRP arises: combining multiple constraints for tackling realistic problems. Nowadays, some studies have considered specific combinations of real-life constraints to define the emerging Rich VRP scopes. This work surveys the state of the art in the field, summarizing problem combinations, constraints defined, and approaches found.

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1. INTRODUCTION

Road transportation is the predominant way of transporting goods in Europe and other parts of the world. Direct costs associated with this type of transportation have increased significantly since 2000 and more so in recent years due to rising oil prices. Furthermore, road transportation is intrinsically associated with a good deal of indirect or external costs, which are usually easily observable: congestion, pollution, security- and safety-related costs, mobility, delay time costs, and the like. However, these costs are usually left unaccounted because of the difficulty of quantifying them [Sinha and Labi 2011]. For example, traffic jams in metropolitan areas constitute a serious challenge for the competitiveness of industry: for instance, according to some studies [EC 2008, 2011; Van Essen et al. 2011], external costs due to traffic jams could represent

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about 1–2% of the European GDP, a percentage that continues to increase. In addition to these easily observable costs, many others might be considered. In this scenario, it becomes evident that new methods must be developed to support the decision-making process so that optimal (or quasi-optimal) strategies can be chosen in road transportation. This need for optimizing road transportation affects both the public and private sectors and constitutes a major challenge for most industrialized regions.

Recent advances on Information and Communications Technologies (ICT)—such as the growing use of GPS and smartphone devices, Internet-scale (distributed) systems, and Internet computing technologies—open new possibilities for optimizing the planning process of road transportation [Orozco 2011]. In particular, when combined with advanced simulation and optimization techniques, Distributed- and Parallel-Computing Systems (DPCS) allow the practical development and implementation of new ICT-based solutions to support decision making in the Transportation and Logistics (T&L) arena. "Real-world applications, both in North America and in Europe, have widely shown that the use of computerised procedures generates substantial savings (generally from 5% to 20%) in the global transportation costs" [Toth and Vigo 2002]. Road transportation optimization (cost-saving) issues are especially critical in the case of Small and Medium Enterprises (SME) since they are rarely able to obtain the economic and human resources required to implement, maintain, and manage efficient routing-optimization methods. Similarly, those companies have difficulties in accessing the appropriate technologies, such as computer clusters and expensive commercial software, that would help them to improve their productivity level and reduce unnecessary costs and thus achieve a more sustainable business model.

1.1. Context and Motivation

In this context, the goal of the so-called Vehicle Routing Problem (VRP) is to optimize the routing design (distribution process from depots to customers) in such a way that customers' demand of goods is satisfied without violating any problem-specific constraint, such as route maximum distance or time-related restrictions [Golden et al. 2008]. The VRP has many variants depending on the parameters and constraints considered. Despite its apparent simplicity, in computational complexity theory, the classical version of VRP and its variants (for extension) are NP-hard (nondeterministic polynomial-time hard) [Lenstra and Rinnooy-Kan 1981]. This implies that, in practice, it will not be possible to guarantee the (mathematically) optimal solution; thus, the given problem cannot be solved by an algorithm in a finite number of steps [Garey and Johnson 1979]. NP-hard problems may be of any type: decision problems, search problems, or optimization problems. Some practical examples are found in data mining, scheduling, planning, decision support, etc. In recent years, due to the fast development of new and more efficient optimization and computing methods, the interest of academics and practitioners has been shifting toward realistic VRP variants, which are commonly known as Rich VRP (RVRP). These problems deal with realistic (and sometimes multiobjective) optimization functions, uncertainty (i.e., stochastic or fuzzy behaviors), and dynamism, along with a wide variety of real-life constraints related to time and distance factors, use of heterogeneous fleets, linkage with inventory and scheduling problems, integration with ICT, environmental and energy issues, and more.

After a number of VRP variants have appeared over the years, we found a need to classify those that can be part of the RVRP. As a matter of fact, there is no consensus on which problems can be described as rich ones and which are just a new variant of VRP. Thus, in this article, we describe the main variants of the VRP, analyze their constraints, and present the main techniques used to face them. This work allows us to create an extensive list of the main constraints that are applicable to an RVRP problem. Furthermore, we introduce a definition of RVRP summarizing all the previous information existing about the problem. Furthermore, classification of RVRP problems and a matrix table relating all the RVRP papers with the collected constraints are shown. Finally, future trends in both vehicle routing problems and tools used to solve them are summarized.

1.2. Structure

To begin, a definition of the basic Capacitated Vehicle Routing Problem (CVRP) is given in Section 2. Additionally, we introduce the basic formulation of the problem and the main classic variations of the VRP. In Section 3, the most common approaches for solving current variations of the VRP are introduced. In Section 4, a definition of the RVRP is given. A complete literature review is presented in Section 5. Section 6 presents a complete classification of all papers on the RVRP and an explanation of the different kinds of constraints found in them. In Section 7, some perspective on current and future trends regarding the RVRP is provided. Finally, we summarize the survey and present some conclusions in Section 8.

2. PROBLEM DEFINITION

In the CVRP, first defined by Dantzig and Ramser [1959], a homogeneous fleet of vehicles supplies customers using resources available from a depot or central node (see Figure 1). Each vehicle has the same capacity (homogeneous fleet) and each customer has a certain demand that must be satisfied. Additionally, there is a cost matrix that measures the costs associated with moving a vehicle from one node to another. These costs usually represent distances, traveling times, number of vehicles employed, or a combination of these factors.

More formally, we assume a set Ω of $n+1$ nodes, each of them representing a vehicle destination (depot node) or a visit (demanding node). The nodes are numbered from 0 to n, with node 0 being the depot and the remaining n nodes the visits to be performed ($\Omega* = \Omega - \{0\}$). A demand $q_i > 0$ of some commodity has been assigned to each nondepot node i, $i \in \Omega*$ (we assume $q_0 = 0$). On the other hand, $A = \{(i, j) : i, j \in \Omega; i < j\}$ represents the set of the $n \cdot (n+1)/2$ existing edges connecting the $n+1$ nodes. Each of these links has an associated aprioristic cost, $c_{ij} > 0$, which represents the cost of sending a vehicle from node i to node j. In this original version, these c_{ij} are assumed to be symmetric ($c_{ij} = c_{ji}, 0 \leq i, j \leq n$), and they are frequently expressed in terms of the Euclidean distance (d_{ij}) between the two nodes. The delivery process is to be carried out by a fleet of V vehicles ($V \geq 1$) with equal capacity, $Q >> \max\{q_i : i \in \Omega\}$. These V vehicles are responsible of R routes ($R \leq V$).

Some additional constraints associated with the CVRP are the following [Laporte et al. 2000]:

—Each nondepot node is supplied by a single vehicle.
—All vehicles begin and end their routes at the depot (node 0).
—A vehicle cannot stop twice at the same nondepot node.
—No vehicle can be loaded to exceed its maximum capacity.

The following generic formulation is based on the formulation proposed by Toth and Vigo [2002] and then used in Baldacci et al. [2008] for the heterogeneous fleet VRP variant. It is useful for both symmetrical and asymmetrical instances, as well as for both homogeneous and heterogeneous fleets. The vehicle fleet M is composed of m different vehicle types ($M = \{1, \ldots, m\}$). Each type $k \in M$ has m_k available vehicles at the depot, each having a capacity defined by Q_k. There is a three-index binary variable for each edge and possible vehicle type (Equation (8)). The variable x_{ij}^k indicates if the arc (i, j) $(i, j \in \Omega)$ is used or traveled by a vehicle of type k in the optimal solution. In

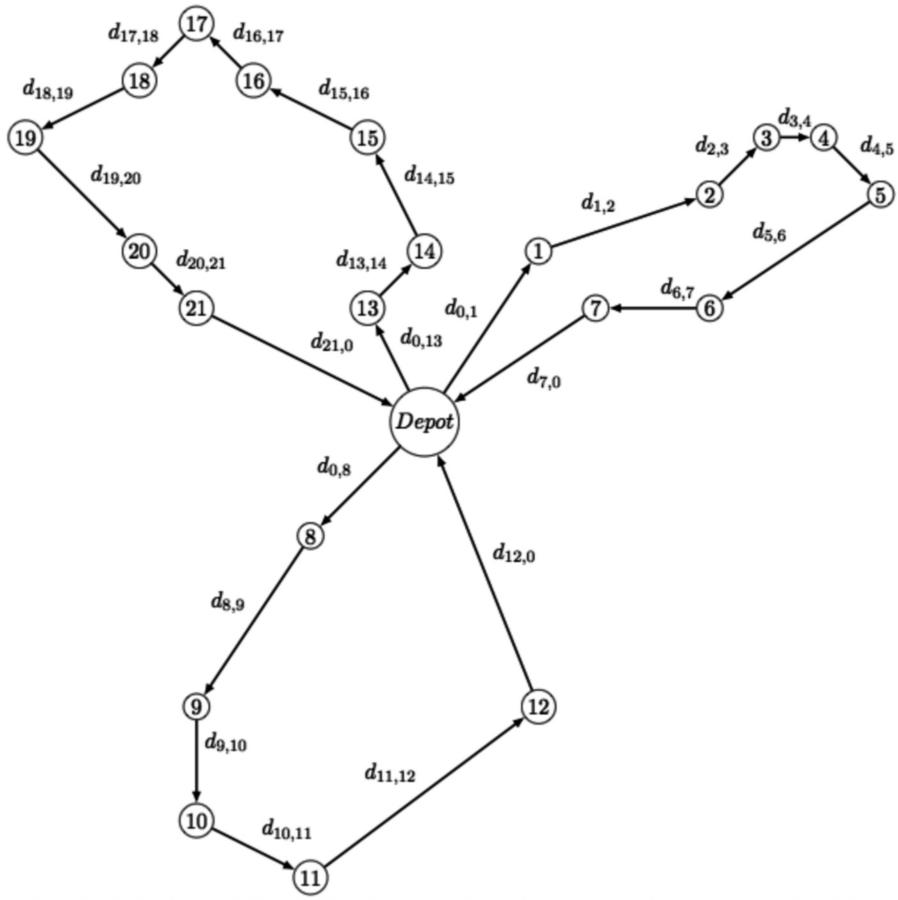

Fig. 1. Representation of a VRP example using a node design.

addition, flow variables y_{ij}^k represent the load in the vehicle servicing customer j after visiting customer i:

$$\min \sum_{k \in M} \sum_{\substack{i,j \in \Omega \\ i \neq j}} c_{ij}^k x_{ij}^k \tag{1}$$

subject to:

$$\sum_{j \in \Omega*} x_{0j}^k = \sum_{i \in \Omega*} x_{i0}^k \quad \forall k \in M \tag{2}$$

$$\sum_{k \in M} \sum_{i \in \Omega} x_{ij}^k = 1 \quad \forall j \in \Omega* \tag{3}$$

$$\sum_{i \in \Omega} x_{iu}^k = \sum_{j \in \Omega} x_{uj}^k \quad \forall u \in \Omega*, \forall k \in M \tag{4}$$

$$\sum_{j \in \Omega*} x_{0j}^k \leq m_k \quad \forall k \in M \tag{5}$$

$$\sum_{i\in\Omega} y_{ij}^k - \sum_{i\in\Omega} y_{ji}^k = q_j \sum_{i\in\Omega} x_{ij}^k \qquad \forall j \in \Omega*, \forall k \in M \tag{6}$$

$$0 \leq q_j x_{ij}^k \leq y_{ij}^k \leq (Q_k - q_i) x_{ij}^k \quad \forall i, j \in \Omega, i \neq j, \forall k \in M \tag{7}$$

$$x_{ij}^k \in \{0, 1\} \qquad \forall i, j \in \Omega, i \neq j, \forall k \in M \tag{8}$$

The objective function in Equation (1) minimizes the total distance cost required to service all customers. Equation (2) implies that the number of vehicles leaving the depot is the same as the number of vehicles returning to it. Equations (3) and (4) ensure that each customer is visited exactly once and that if a vehicle visits a customer, it must also depart from it. Equation (5) imposes that the number of used vehicles does not exceed the number of available vehicles for each vehicle type. Equation (6) states that the quantity of goods in the vehicle arriving at customer j, y_{ij}^k, minus the demand of that customer, equals the quantity of goods in the vehicle leaving it after the service has been completed. Equation (7) guarantees lower and upper bounds ensuring that the quantity of goods in the vehicle leaving customer i, y_{ij}^k, is equal to or greater than the demand of its next visit, q_j; and the total demand serviced by each vehicle of type k does not exceed its capacity Q_k.

2.1. Vehicle Routing Problem Variants

Different variants of the VRP have been target studies in the past 50 years [Laporte 2009]. In the literature, the variants of the VRP include a large family of specific optimization problems. For instance, the VRP with Time Windows (VRPTW) is one of the most popular families studied in the community [Bräysy and Gendreau 2005a, 2005b]. As main common feature, these studies are focused on considering one or few constraints in their mathematical models; this has created a huge set of separate branches of VRP research lines with long abbreviation names. Each research line has been identified by the acronym of the considered constraints or attributes used in the optimization problem. Many of these individual branches have been recombined to create new "basic" branches. The main variants of the VRP can be found in Toth and Vigo [2002] and Golden et al. [2008]. A relatively new variant that is not included in the aforementioned references is the Green VRP [Erdoğan and Miller-Hooks 2012; Kopfer et al. 2014]. So far, the most common current extensions studied in the literature are described here:

Asymmetric cost matrix VRP (AVRP). The cost for going from customer i to j is different from that for going from j to i.

Distance-Constrained VRP (DCVRP). The total length of the arcs in a route cannot exceed a maximum route length. This constraint can either replace the capacity constraint or supplement it.

Heterogeneous fleet VRP (HVRP). The company uses different kinds of vehicles, and the routes have to be designed according to the capacity of each vehicle. Some costs could be considered and the number of vehicles could be limited or not, thus creating different contexts. When the number of vehicles is unlimited, then it is called Fleet Size and Mix VRP (FSMVRP). If a specific type of vehicle cannot reach some clients for any reason, then the problem becomes a Site-Dependent VRP (SVRP). Also if a vehicle is allowed to perform more than one trip, then we are solving a HVRP with Multiple use of vehicles (HVRPM).

Multiple Depots VRP (MDVRP). A company has several depots from which it can serve its customers. Therefore, some routes will have different starting/ending points.

Open VRP (OVRP). The planned routes can end on several points distinct to the depot location.

Periodic delivery VRP (PVRP). The optimization is done over a set of days (while normally being planned daily). The customers may not have to be visited each day. Customers can have different delivery frequencies.

Pickup-and-delivery VRP (PDVRP). Each customer is associated with two quantities representing one demand to be delivered to the customer and another demand to be picked up and returned to the depot. In addition to the constraint that the total pickup and total delivery on a route cannot exceed the vehicle capacity, it also must ensure that this capacity is not exceeded at any point on the route. One variant of the pickup-and-delivery problem occurs when the pickup demand is not returned to the depot but should be delivered to another customer, as in the transport of people. In some cases, the vehicles must pick up and deliver items to the same customers in one visit (Simultaneous Pickup-and-delivery VRP), as when picking up and dropping off new and returned bottles. Notice that the other important variant is the $1 - M - 1$ ("one-to-many-to-one"); this means that all delivery demands are initially located at the depot, and all pickup demands are destined for the depot. Taken collectively, all delivery demands can be viewed as a single commodity, and all pickup demands can be viewed as a second commodity. This variant is generally referred to in the literature as Delivery and Collection [Gribkovskaia and Laporte 2008].

Split-delivery VRP (SDVRP). The same customer can be served by different vehicles if it will reduce the overall cost. This relaxation of the basic problem is important in those cases where a customer order can be as large as the capacity of the vehicle.

Stochastic VRP. In one realistic aspect of the routing problem, a random behavior is considered. This is typically the presence of a customer, its demand, its service time, or the travel time between customers. So far, this uncertainty aspect has emerged as a key aspect for future demanding developments [Juan et al. 2011].

VRP with Backhauls (VRPB). As in the PDVRP, customers are divided into two subsets. The first subset contains the linehaul customers, which are customers requiring a given quantity of product to be delivered. The second subset contains the backhaul customers, where a given quantity of inbound product must be picked up. Thus, all linehaul customers have to be visited before the backhaul customers in a route.

VRP with Time Windows (VRPTW). Each customer is associated with a time interval and can only be served within this interval. In this problem, the dimension of time is introduced, and one has to consider both travel time and service time at each customer location. A set of time windows for each customer could be also considered (VRP with Multiple Time Windows). And, these time windows could be flexible depending on some extra costs (VRP with Soft Time Windows).

Green VRP (GVRP). This variant of the VRP aims at including different environmental issues in the optimization process, such as greenhouse gas emissions [Ubeda et al. 2011], pollution, waste, noise [Bektaş and Laporte 2011], and the effects of using "greener" fleet configurations [Juan et al. 2014a], etc. An excellent and updated survey on GRVP can be found in Demir et al. [2014].

Several hybrid variants have been created in the literature from these basic variants, all inspired by real-life scenarios. A large number of VRP acronyms have been

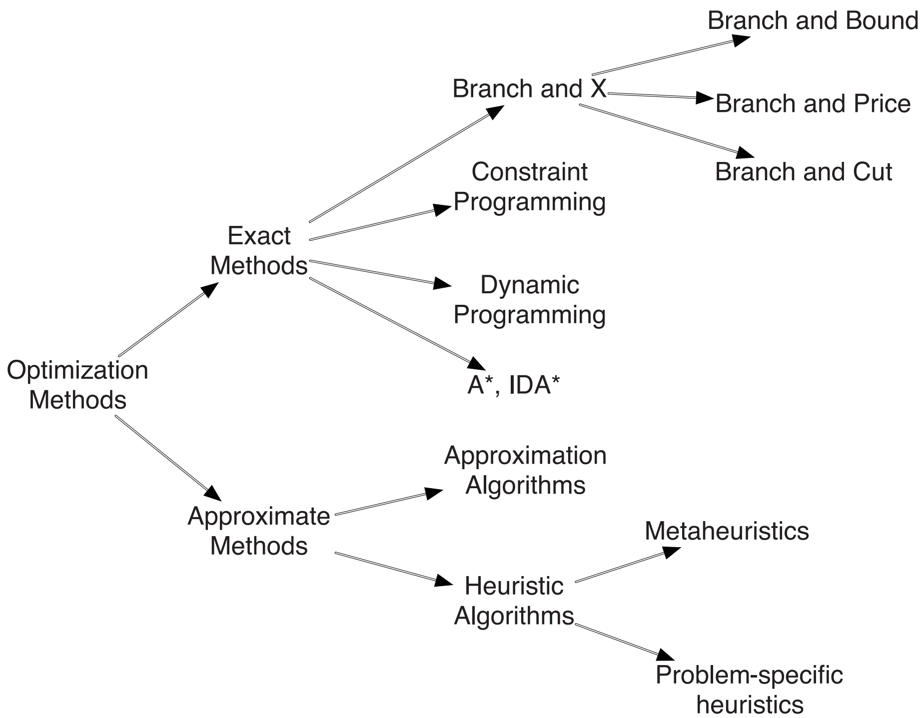

Fig. 2. Classification of the classical optimization methods.

developed to refer to these combinations of routing restrictions. However, all these new combinations can be encompassed in the larger family of RVRP, as we explain in Section 4.

3. VRP METHODOLOGIES

Different approaches to VRPs have been explored. These range from the use of pure optimization methods, such as mathematical programming to solve small- to medium-sized problems (up to about 75–100 customers) with relatively simple constraints, to the use of heuristics and metaheuristics that provide near-optimal solutions for medium- and large-sized problems with more complex constraints. Metaheuristics serve three main purposes: solving problems faster, solving larger problems, and obtaining more robust algorithms. This branch of optimization in computer science and applied mathematics is related to algorithms and computational complexity theory. Metaheuristics provide *acceptable* solutions in a reasonable time for solving hard and complex problems [Talbi 2009]. Even though the VRP has been studied for decades, and a large set of efficient optimization methods, heuristics, and metaheuristics have been developed [Golden et al. 2008; Laporte 2007], more realistic or RVRP problems—such as the VRP with Stochastic Demands or the Inventory VRP—are still in their infancy. Following the proposed division of Talbi [2009], this large family could be preliminarily summarized in the balanced tree presented in Figure 2.

3.1. Exact Methods

From Talbi [2009], “Exact methods obtain optimal solutions and guarantee their optimality.” This type of technique is often applied to small-sized instances. This family includes a broad set of methods, such as the family of Branch-and-*X* (where the *X* represent the different variants) methods used for solving Integer Linear Programming (ILP) and Mixed Integer Linear Programming problems (MILP), and also dynamic programming, which focuses on solving complex problems by breaking them down into simpler subproblems [Kok et al. 2010]. As well, column generation is a popular technique used for solving larger linear programming problems, which consists in splitting the given problem into two problems: the master problem and the subproblem [Desaulniers et al. 2005]. This allows one to simplify the original problem with only a subset of variables in the master problem. A new variable is created in the subproblem, which will be minimized in the objective function with respect to the current dual variables and constraints naturally associated with the new variable. Set Partitioning (SP) modeling is other binary variable formulation for each feasible route. This technique is quite general and can consider several constraints at a time [Subramanian et al. 2012; Subramanian 2012]. Constraint programming (CP) is a programming paradigm that uses constraints to define relations among variables [Van Hentenryck 1989]. It differs from other programming languages in that it is not necessary to specify a sequence of steps to execute to solve a problem, but rather its properties. Models in CP are based on three elements: variables, their corresponding domains, and constraints relating all the variables. The main mechanism for solving a problem using CP is called *constraint propagation*. It works by reducing variables domains, strengthening constraints, or generating new ones. This leads to a reduction of search space, making the problem easier to solve by means of search algorithms. Guimarans et al. [2011] presents a hybrid approach to solve the CVRP by applying Lagrange relaxation to each route and feasibility checking using a CP model. A* [Hart et al. 1968] is a computer algorithm commonly used in shortest paths and graph traversal problems; it uses the best-first search [Dechter and Pearl 1985] to find the most promising node to expand. In the same way, IDA* [Korf 1985] is a variant of the A* algorithm that uses less memory because it does not keep track of prior visits; instead, it uses iterative deepening. Some of these methods are also quite popular for solving basic CVRP: branch-and-bound, branch-and-cut, branch-and-price, branch-and-cut-and-price, Sbased, and dynamic programming. More details of these methods are reviewed [Baldacci et al. 2010, 2012; Laporte et al. 2013] for the CVRP and for some of its variants.

3.2. Approximate Methods

From Talbi [2009], "Heuristics find *good* solutions on large-size problem instances. They allow to obtain acceptable performance at acceptable costs in a wide range of problems. They do not have an approximation guarantee on the obtained solutions. They are tailored and designed to solve a specific problem or/and instance. Meta-heuristics are general-purpose algorithms that can be applied to solve almost any optimization problem. They may be viewed as upper level general methodologies that can be used as a guiding strategy in designing underlying heuristics." The author also proposes that two contradictory criteria must be taken into account: exploration of the search space (*diversification*) and exploitation of the best solutions found (*intensification*). Promising regions are determined by the obtained *good* solutions. In intensification, promising regions are explored more thoroughly in the hope of finding better solutions. In diversification, nonexplored regions must be visited to be sure that all regions of the search space are evenly explored and that the search is not confined to only a reduced number of regions.

There are many metaheuristics inspired by natural processes, such as the evolutionary algorithms (including Genetic Algorithms [GA]) and Ant Colony Optimization (ACO). For instance, the ACO metaheuristic is inspired by the communication and cooperation mechanisms found among real ants that allow them to find the shortest paths from their nest to food sources. The communication medium is a chemical compound (pheromone). The amount of pheromone is represented by a weight in the algorithm [Gendreau et al. 2008]. In ACO algorithms, the range [*min*, *MAX*] of a pheromone trail values can be controlled. This type of technique can also be classified as a population-based metaheuristic because it iteratively improves a population of solutions. Another member of this wide group is the deterministic strategy of scatter search, which recombines selected solutions from a known set to create new ones [Talbi 2009].

Other techniques are based on memory usage (short, medium, and long term). Tabu Search (TS) is a local search-based metaheuristic where, at each iteration, the best solution in the neighborhood of the current solution is selected as the new current solution, even if it leads to an increase in solution cost. A short-term memory (Tabu list) stores recently visited solutions (or attributes) to avoid short-term cycling [Gendreau et al. 2008]. This family can be considered as single-solution-based metaheuristics since all methods are focused on improving a single solution at a time. A common feature is that all include the definition of building an initial solution. Other promising techniques are Variable Neighborhood Search (VNS) and Greedy Randomized Adaptive Search Procedure (GRASP). VNS has been widely used in several problems. It is based on a successive exploration of a set of predefined neighborhoods to find a better solution at each step. Large Neighborhood Search (LNS) [Guimarans 2012] can be interpreted as a special case of VNS in which efficient procedures are designed to consider a high number of neighborhoods at the same time. Inside this branch, we can find one of the first techniques used for the traveling salesman problem: the nearest neighborhood method. Simulated Annealing (SA) [Nikolaev and Jacobson 2010] is another single-solution-based method based on the same physical principle used in the process of heating and then slowly cooling a substance to produce a strong crystalline structure. Thus, it is typical to include a temperature parameter to control the process.

Some approximate algorithms, called heuristics, are tailor-made to solve a specific problem. Systematically following a number of steps, they are used to find an acceptable solution. However, they do not guarantee finding the optimal solution. For instance, Savings (CWS) [Clarke and Wright 1964] is probably one of the most cited heuristics used to solve the CVRP. In the literature, there are several variants and improvements of the CWS [Golden et al. 1984]. The original version of CWS is based on the estimation of possible savings originating from merging routes, as for unidirectional or symmetric edges $Sav(i, j) = c_{i0} + c_{0j} - c_{ij}$. These savings are estimated between all nodes and then decreasingly sorted. Then, the bigger saving is always taken and is used to merge the two associated routes. As the authors propose, this procedure uses the concept of savings. In general, at each step of the solution construction process, the edge with the most savings is selected if and only if the two corresponding routes can feasibly be merged using the selected edge. The CWS algorithm usually provides relatively good solutions in less than a second, especially for small- and medium-sized problems. In addition, new algorithms have been proposed based on CWS. For instance, Juan et al. [2010] propose a multistart randomized approach, called Simulation in Routing via the Generalized Clarke and Wright Savings heuristic (SR-GCWS) that could be considered a metaheuristic in this general classification.

4. RICH VRP DEFINITION

A first attempt to define the RVRP was made by Toth and Vigo [2002]. The authors define the potential of extending the "vehicle flow formulations, particularly the more flexible three-index ones." The authors also state that models of symmetric and asymmetric CVRP "may be adapted to model some variants of the basic versions." Other authors have given a different description of this realistic problem. For the research community, the RVRP is a generalization or union of other independent problems. Goel and Gruhn [2005, 2006, 2008] deal with the General Vehicle Routing Problem

(GVRP) as "a combined load acceptance and routing problem which generalizes the well known Vehicle Routing Problem and Pickup and Delivery Problem. Furthermore, it amalgamates some extensions of the classical models which, up to now, have only been treated independently." In a Special Issue explicitly dedicated to Rich models, the editors [Hasle et al. 2006] summarize "non-idealised models that represent the application at hand in an adequate way by including all important optimization criteria, constraints, and preferences." In fact, Hasle and Kloster [2007] refer to this type of problem as an *industrial* or *applied* routing problem.

Pellegrini et al. [2007] state that "in recent years, moreover, thanks to the increasing efficiency of these methods and the availability of a larger computing power, the interest has been shifted to other variants identified as RVRP. The problems grouped under this denomination have in common the characteristics of including additional constraints, aiming a closer representation of real cases." Their case study is characterized by many different types of constraints, each of which is unanimously classified as "challenging even when considered alone." For instance, Crainic et al. [2009a, 2009b] introduce another term to refer to RVRP when dealing with multiattribute VRP-like *rich* problems. They also stated that "real-world problems are generally characterised by several interacting *attributes*, which describe their feasibility and optimality structures. Many problems also display a combinatorial nature and are, in most cases of interest, both formally difficult and dimensionally large. In the past, the general approach when tackling a combinatorial multi-attribute, rich problem was either to frontally attack it, to address a simplified version, or to solve in a pipeline manner a series of simpler problems." Therefore, the constraints may be known also as attributes of the RVRP (VRP with multiattributes).

More recently, Rieck and Zimmermann [2010] state that: "Hence, research has turned to more specific and rich variants of the CVRP. The family of these problems is identified as rich vehicle routing problems. In order to model RVRPs, the basic CVRP must be extended by considering additional constraints or different objective functions." The evolution of models can be appreciated when new needs concerning the models themselves emerge. In this respect, the authors stated that "rich vehicle routing problems are usually formulated as three-index vehicle-flow models with decision variables x_{ij}^k which indicate whether an arc $(i, j) : i, j \in \Omega$ is traversed by vehicle k ($k = 1, \dots, K$)." These models seem to be more flexible in incorporating additional constraints (e.g., different vehicle capacities). In their monograph, Toth and Vigo [2002] suggest that two-index vehicle-flow formulations "*generally are inadequate for more complex versions of vehicle routing problems*." Their arguments are based on the fact that "*these models are not suited for the cases where the cost or the feasibility of a circuit [each corresponding to a vehicle route] depends on the overall vertex sequence or on the type of vehicle allocated to the route*." The new models have been extended to include other features in the logistic or supply chain process. Furthermore, Schmid et al. [2013] have proposed six integrative models considering the classical version of the VRP and some important extensions in the context of supply chain management. These extensions are lotsizing, scheduling, packing, batching, inventory, and intermodality. The authors state that, as a benefit, their models consider an efficient use of resources as well as the inclusion of interdependencies among the subproblems. Lahyani et al. [2012] have pointed out the importance of stating a common and closed definition for RVRP scope, "in most papers devoted to RVRPs, definitions of rich problems are quite vague and not significantly different. There is no formal definition either criterion which leads to decide whether or not a VRP is rich. Such definition has to rely on a relevant taxonomy which can help to differentiate among numerous variants of the VRP." In fact, the authors conclude their study with a numerical proposal for a specific definition:

"a RVRP extends the academic variants of the VRP in the different decision levels by considering additional strategic and tactical aspects in the distribution system (4 or more) and including several daily restrictions related to the Problem Physical Characteristics (6 or more) [pure routing or operational]. Therefore, a RVRP is either a VRP that incorporates many strategic and tactical aspects and/or a VRP that reflects the complexities of the real-life context by various challenges revealed daily. The state of the art of RVRP has changed since 2006. Now studies incorporate *more complex* aspects of reality. Therefore, some variants described as rich by their authors in 2006 may not be considered as such anymore." So, depending on the considered paper, the RVRP definition evolves continuously.

In fact, some authors have stated that the taxonomy of VRPs is in constant evolution. The growing number of papers related to VRPs has created the necessity of classifying the context and different problems considered within it. Eksioglu et al. [2009] proposed a framework for classifying the literature of VRPs based on scenario characteristics. They tested their proposal with a disparate set of VRP details so that specific variants of VRP can be defined. However, this study does not mention emerging RVRPs. Recently, other realistic VRP variants have been promoted, and these new VRP applications are expanding the scope of RVRP. Thanks to technological advances, dynamic VRPs (so-called real-time VRP) can be also considered as part of the overall RVRP scope [Pillac et al. 2013]. This branch includes uncertainty over some variables (number of customers, travel times, and demands), and it explores the use of real-time communication of inputs (e.g., Global Positioning Systems). Therefore the target of this area is to generate "good" routing solutions applicable to any change in context and in a really rapid way for each data variation. In general, the border between RVRPs and other VRP fields is blurred, and other emerging VRP variants can be included within of RVRP for its current interest and future impact. One highlighted application is Green VRP [Lin et al. 2014; Erdoğan and Miller-Hooks 2012], in which sustainable transportation issues are involved through objective functions or variables related to environmental costs. Within this, a special branch has been developed: Pollution VRP [Bektaş and Laporte 2011; Demir et al. 2012a, 2012b]. Its main objective is to reduce emissions during transportation activities. A combination of previous VRP branches represents promising applications of RVRP, as was recently promoted in a special issue of Rich and Real-Life VRPs [Juan et al. 2014].

To summarize, we conclude that an RVRP reflects, as a model, most of the relevant attributes of a real-life vehicle-routing distribution system. These attributes might include dynamism, stochasticity, heterogeneity, multiperiodicity, integration with other related activities (e.g., vehicle packing, inventory management, etc.), diversity of users and policies, legal and contractual issues, environmental issues, and more. Thus, as a model, an RVRP is an accurate representation of a real-life distribution system and, therefore, the solutions obtained for the RVRP should be directly applicable to a real-life scenario.

As can be appreciated, the implications of the RVRP definition have evolved toward a more concise concept over time. The new demanding needs of enterprises have forced researchers to consider more complex approaches. There is also a clear trend toward creating generic and efficient approaches. Considering the large number of papers devoted to the VRP, only a few can be applied to the current RVRP context, and only a small number of papers have explicitly addressed the RVRP. This fact emphasizes the sparse nature of the literature as well as the opportunities that the academy sector has to collaborate with enterprises addressing real routing problems. The next section presents a literature review on some strategies aimed at solving RVRP instances with more than one constraint simultaneously.

5. LITERATURE REVIEW

In this section, we review the more than 50 papers selected because they are denominated *Rich* extensions of the original VRP or are related to other RVRPs, plus some few others that consider several VRP variants. All these papers have in common that they consider one or more variants of the classical VRP. The approaches presented in these papers solve separated VRP variants or different combinations of their constraints. One of the first explicitly RVRP cases is presented in Pellegrini [2005]. The author addresses a specific RVRP approach with the consideration of a heterogeneous fleet, multiple time windows, delivery restricted within some intervals of time, and a maximum time for a single tour. They proposed two heuristic algorithms based on the well-known Nearest Neighbor (NN) heuristic procedure [Solomon 1987] combined with a swap local search. In this article, a Deterministic version of a NN (DNN) algorithm as well as a Randomized NN (RNN) version is created. A random behavior in the selection of the next customer in the building process of a route is added to the procedure. The author showed encouraging results in a short computational time with generated instances of 50, 100, 150, and 200 customers. The RNN algorithm reaches better results than the DNN version, although the RNN version loses some efficiency as the number of customers increases.

On the other hand, Goel and Gruhn [2005, 2006] address capacity restrictions, time windows, and a heterogeneous fleet with different travel times, and also multiple pickup and delivery locations, travel costs, different start and end locations for vehicles, and other constraints. They propose iterative improvement approaches based on LNS. The authors created an instance generator of 50, 100, 250, and 500 orders to show the performance of their approach. In addition, Goel and Gruhn [2008] consider other sets of real-life requirements (e.g., time window restrictions, a heterogeneous vehicle fleet with different travel times, travel costs and capacity, multidimensional capacity constraints, order/vehicle compatibility constraints, orders with multiple pickup, delivery and service locations, different start and end locations for vehicles, and route restrictions for vehicles). The authors propose an iterative improvement approach. They use a reduced Variable Neighborhood Search (VNS) algorithm for exchanging elements between neighborhoods and also an LNS approach for using nested neighborhoods of different sizes. This combination helps avoid a local minimum.

Following the LNS research line, Ropke and Pisinger [2006a, 2006b] propose a heuristic based on LNS as proposed by Shaw [1998]. Furthermore, their approach is a unified heuristic with an adaptive layer. They are focused on the VRP with backhauls (VRPB) with time windows, pickup and delivery constraints, and multiple depots. They propose a model transformation of the VRPB to solve simultaneous pickups and deliveries. Nine datasets are used to test several configurations of the proposed heuristic in which more than 50% of best known solutions for those instances are improved. Later, the same authors developed an Adaptive LNS framework [Pisinger and Ropke 2007] for addressing capacitated deliveries, time windows, multiple depots, split deliveries, and open routes constraints. They use several sets of instances with up to 1,000 customers, and they improve 183 best-known solutions out of 486 benchmark tests.

Hasle et al. [2005] briefly describe four mechanisms to enhance scalability and present a generic route construction heuristic for RVRPs. The empirical investigation results based on standard test instances for several VRP variants show the effectiveness of this approach. In addition, Hasle and Kloster [2007] propose a generic approach to harness modeling flexibility. The authors present a generic solver based on a unified algorithmic approach that is a combined operation of the Variable Neighborhood Descent and a promising Iterated Local Search (ILS) [Lourenço et al. 2010]. An initial solution is generated using the parallel version of CWS. They address a capacitated constraint, distance limitation, pickup and delivery constraints, fleet size, and mix problems, as well as time windows. They present the possibility of extending their solution to multidepot and site-dependent problems. Classical benchmarks from Solomon [1987] and their modification of Li and Lim [2001] are also used. Their results are based on a range of customers between 50 and 1,000.

A wide classification of RVRP variants is presented in a special issue published by Hartl et al. [2006]. Seven papers were selected that cover different aspects and illustrate novel types of VRP applications. The editors state that "VRP research has often been criticized for being too focused on idealized models with non-realistic assumptions for practical applications." Several optimization methods are proposed for solving problems inspired by real applications of VRP knowledge. For instance, Reimann and Ulrich [2006] addressed the VRP with backhauls and time windows. Hoff and Løkketangen [2006] are focused in the traveling salesman problem with pickup and delivery. Ileri et al. [2006] work in pickup and delivery requests with time windows, a heterogeneous fleet, and some operational constraints over the driver routes. The authors use an SP technique and also CG to solve real-life instances. Fügenschuh [2006] proposes a metaheuristic for the VRP with coupling time windows. This method combines classical construction aspects with mixed-integer preprocessing techniques, and it is improved with a randomized search strategy. Several randomly generated instances are used, as well as a real-world case for public bus transportation considering school times in rural areas of Germany. Magalhães and Sousa [2006] present a real case adopting a system of variable routes that are dynamically designed. Sörensen [2006] shows a bi-objective case considering marketing and financial interests that is solved using a metaheuristic. Bolduc et al. [2006] addressed a multiple period horizon in an inventory context with a heterogeneous fleet, multiple trips, and capacity restrictions. The authors use heuristics to minimize the cost of distributing products to the retailers and the cost of maintaining inventory at the facility. Randomly generated instances were used to measure the performance of the approach with two sets of small and large cases.

Pellegrini et al. [2007] presented a case study characterized by multiple objectives, constraints concerning multiple time windows, a heterogeneous fleet of vehicles, a maximum duration of subtours, and periodic visits to the customers. They considered two versions of ACO: the Multiple Ant Colony System (M-ACS) first proposed by Dorigo and Gambardella [1997] and (b) the MAX-MIN Ant System (MMAS) based on Stützle and Hoos [1997]. The authors compared the results with a TS algorithm and an RNN heuristic (mentioned earlier). Both ACO algorithms perform significantly better than the TS and RNN approaches using an instance generator of 70–80 orders. Another ACO implementation is proposed by Rizzoli et al. [2007] that has been applied to real contexts addressing separately a heterogeneous fleet, time windows, pickup and delivery constraints, and time dependent deliveries. The authors tested four ACO algorithms using data from real distribution companies with between 15 and 600 customers.

In Hoff [2006], we find four papers [Hoff and Løkketangen 2006, 2007; Hoff et al. 2009, 2010] focused on the development of Lasso Solution Strategies using TS and heuristics for the VRP with pickup and delivery, time-dependent, and stochastic demands. Lasso Strategies consist of a path or spoke that is first followed by each vehicle to perform deliveries; the remaining customers assigned to this vehicle are then visited along a loop, and, finally, the spoke is followed in the reverse order to perform pickups. If the loop is empty, then the lasso reduces to a double-path; if the path is empty, then it reduces to a Hamiltonian cycle. The authors created instances with 7–262 nodes that are derived from classical benchmarks used in CVRP. A real-life problem from a Norwegian company is also considered. In Derigs and Döhmer [2005], the authors also addressed the pickup and delivery VRP with time windows. They proposed an indirect search procedure based on sequence/permutation of tasks, cheapest insertion of a visit, and a threshold-accepting local search metaheuristic. The proposed algorithm was implemented into a decision support system for a removal firm. They produce some promising preliminary results with randomly generated instances.

Irnich [2008] takes advantage of strong modeling capabilities and proposes a unified modeling and heuristic solution framework. The author highlights the potential of $k - edge$ exchange neighborhoods. This approach is intended to support efficient local search procedures for addressing all standard types of VRPs, such as capacitated and distance-constrained deliveries, multiple depots, time windows, simultaneous delivery and pickup, backhauling, pickup-and-delivery problems, periodic VRP, fleet mix and size, and site dependencies as well as mixtures and extensions of these. The author proposes integrating efficient search blocks into different metaheuristic. Some promising results are presented for VRPTW and MDVRPTW combining a VNS with LNS strategies and inspired by the work of Ropke and Pisinger [2006b].

A large number of studies use exact methods or combinations of them. In Wen [2010], we find three papers that address some variants of the RVRP inspired by real-life situations. The author proposes different strategies to solve (a) the VRP with cross-docking options through a TS-based heuristic tested over 200 pairs of suppliers and customers [Wen et al. 2008]; (b) the dynamic VRP with multiple objectives over a planning horizon that consists of multiple periods through MILP and a three-phase heuristic [Wen et al. 2010]; and (c) the VRP with multiperiod horizon, time windows for delivery, heterogeneous vehicles, drivers working regulations, and other constraints [Wen et al. 2011]. In the last work, the author proposes a MILP embedded in a multilevel VNS algorithm. Good quality solutions for solving up to 2,000 orders are generated using real case information. In this same research line, Rieck and Zimmermann [2010] propose a new MILP (two-index vehicle flow) model for a RVRP with docking constraints. They consider time windows, simultaneous delivery and pickup at customer locations, and multiple uses of vehicles. The test instances with 10–30 customers were generated from a classical set of VRP with time windows [Solomon 1987]. The proposed method solves small and medium problem instances efficiently. Another promising approach, as proposed by Doerner and Schmid [2010], consists in the combination of exact algorithms and metaheuristic search components. The authors present a survey of several hybrid techniques and also highlight some key aspects for future studies. Hybrid approaches allow researchers to conquer the obstacles observed when individual concepts are applied independently. These authors present three trends of hybridization schemes: set-covering-based approaches, local branching approaches, and decomposition techniques. They addressed the periodic VRP with time windows and multidepot VRP with time windows, but other variants are commented. An exact solution framework based on SP modeling is proposed by Baldacci et al. [2010, 2011a, 2011b] for individual types of VRPs. The results outperform all other exact methods published so far and also solve several previously unsolved test instances. The preliminary step to the proposed framework is presented in Baldacci and Mingozzi [2009], where a unified exact method based on SP is introduced for solving the well-known CVRP, HVRP, SVRP, and the MDVRP. Computational results assess the performance of their approach over main instances from the literature covering different variants of HVRP, SVRP, and MDVRP.

Several studies have developed CG-based methods as well. Oppen et al. [2010] consider a real scenario called the Livestock Collection Problem (LCP), which is considered an RVRP extended with inventory constraints. This context includes duration and capacity restrictions, heterogeneous fleets, time windows, multiple trips, and multiproduct issues. The authors addressed it through an exact solution method based on CG. The authors created instances with less than 30 customer orders inspired by the real world. The CG approach has helped to find optimal solutions in different scenarios, but the authors defined limitations for finding optimal solutions to LCP instances. Another CG heuristic is proposed by Goel [2010] for addressing a VRP with time windows, a heterogeneous vehicle fleet, multiple depots, and pickup and delivery constraints. Some small instances are randomly generated in order to test the heuristic performance. Ceselli et al. [2009] also propose the use of a CG combined with a dynamic programming algorithm to address simultaneously a heterogeneous fleet, different depots, time windows, route length variation, optionally opened routes, and pickup and delivery and several other constraints. The authors tested their approach with 46 randomly generated instances composed of 100 orders, and the results are compared with valid lower bounds. Under a similar restricted context, Ruinelli [2011] compared three methods on a master thesis: an ACS, a CG algorithm, and a general-purpose MILP solver. Computational results are presented using 14 real instances from a distribution company, where the CG outperforms the other two methods. Prescott-Gagnon et al. [2012] present a real-life case of an oil distribution company that presents a set of particular features. Some of the constraints addressed are the heterogeneous vehicle fleet, multiple depots, intraroute replenishment, time windows, driver shifts, and optional customers. The authors propose three metaheuristic solutions; namely, the TS algorithm, a LNS heuristic combined with TS, and another LNS based on a CG heuristic. Computational results indicate that both LNS methods outperform the TS heuristic. In fact, the LNS method based on CG tends to produce better quality solutions. Also Lannez et al. [2010] present an approach based on CG for a very particular extension of RVRP called the Rich Arc Routing Problem, in which the demand is located on the arcs and not in the nodes.

Other generic Rich solvers have emerged in the literature. Cordeau et al. [Cordeau et al. 1997, 2001; Cordeau and Laporte 2003; Cordeau et al. 2004] propose a unified TS approach for VRPs with time windows and multiperiod, multidepot, and site-dependent constraints. Several real and theoretical benchmarks were used to test the performance of this approach. Some ILS approaches are proposed [Ibaraki et al. 2005; Hashimoto et al. 2006, 2008]. In fact, Subramanian [2012] proposes a promising combination of ILS with integer programming aspects for several VRP variants. This work was extended to the Fleet Size and Mix (FSM) and HVRP research lines in Subramanian et al. [2012]. They developed a hybrid algorithm composed of an ILS- based heuristic and an SP formulation. The SP model is solved using a MIP solver that calls the ILS heuristic during its execution. Three benchmark instances with up to 360 customers were used to test the approach. For instance, Groër et al. [2010] implemented a library of seven local search heuristics for addressing several variants such as the CVRP, VRPTW, and MDVRP. Some classical heuristic are used (e.g., record-to-record, CWS). Their approach is based on easily removing and inserting customers from an existing solution (called neighborhood ejection). Several classical benchmarks are used to show the performance of their approach. In Battarra [2011] several exact and heuristic algorithms for routing problems are presented in individual RVRP cases [Baldacci et al. 2009; Battarra et al. 2009]. Some of the problems addressed are FSM and HVRP with multitrips and time windows.

Recently, Santillán et al. [2012] solved a routing-scheduling-loading problem using a heuristic-based system. As first step, the proposed system applies an ACS for the routing and scheduling problem, then a bin packing technique is used for the vehicle load problem. Some tests with Solomon [1987] instances are developed. Also, the authors use real information from the distribution of bottles provided by a Mexican company. Another hybrid approach is proposed by Vallejo et al. [2012]. They apply a three-phase heuristic that merges the use of a memory-based approach with clustering techniques. The authors present promising test results using between 100 and 2,000 customers, and they compare their approach to a GA. Next, two particular real cases are presented, inspired by Ropke and Pisinger [2006b]. First, Amorim et al. [2012] create a new Adaptive LNS for solving specific real instances of a heterogeneous fleet site- dependent vehicle routing problem with multiple time windows. This case is inspired by a food distribution company in Portugal. Second, Derigs et al. [2013] propose to combine the commented ALNS with local searches, with both controlled by two metaheuristic procedures (record-to-record travel and attribute-based hill climber) for addressing a particular real case, called a Rollon-Rolloff VRP (RRVRP), which occurred in sanitation/waste collection.

Vidal et al. [2013a] developed a study of more than 64 metaheuristics comparing their solutions to 15 classic variants of VRP with multiple attributes. They present a classification on the types of constraints as attributes and identify promising principles in algorithmic designfor Rich VRP. In conclusion, they state that the critical factor for efficient metaheuristics is the appropriate balance between intensification and diversification explorations in the solution space. The authors conclude that the combination of hybrid algorithms and parallel cooperative methods would create effective solutions. Later, the same authors proposed a unified solution framework called Unified Hybrid Genetic Search (UHGS) for several types of RVRP [Vidal et al. 2013c]. The framework uses efficient generic local search and genetic operators. This approach is also based on a giant-tour representation with a split procedure originally proposed by Prins [2004]. The authors present interesting computational results using 39 benchmarks over 26 different Rich VRPs. Furthermore, the authors apply their method combined with diversity management mechanisms to different large-scale instances of Rich Time-constrained VRPs [Vidal et al. 2013b]. The used instances involve up to 1,000 customers. The proposed framework outperforms all current state-of-the-art approaches and addresses any combination of periodic, multidepot, site-dependent, and duration-constrained VRPs with time windows.

Table I presents a summary of the cited state-of-the-art approaches developed for the RVRP by author, year of publication, type of proposed method, and maximum number of customers addressed in the study. The rows are sorted by type of method, year, and last name of first author. We applied a restrictive filter for approaches that can solve more than one RVRP. The asterisk (*) on the last column highlights the approaches that have been or can be tested with no restriction on the combination of constraints. The table is divided into two parts: complete methods first and incomplete later.

6. CLASSIFICATION OF RICH VRP PAPERS

Most of the routing constraints considered in the previous works were unified and classified. The next list presents the main distribution constraints considered in these papers. Table II shows the presence of each constraint in the cited papers. This is useful to appreciate the diversity of cataloged papers covering RVRP topics. And, finally, in Table III a classification of these routing constraints is done using the cited studies of Vidal et al. [2013c] and Lahyani et al. [2012]. In Vidal et al. [2013c], the routing constraints are related to their representation points inside the inner methodology process. For this, the authors propose three groups that represent the simple aspects that any solution must deal with: assignment of customers and routes to resources, sequence choices, and the evaluation of fixed sequences. The authors state that this “simple classification is intimately connected with the resolution methodology.” In Lahyani et al. [2012], constraints are associated with the company decision levels (operational, tactical, and strategic). The first level (strategic) includes decisions related to locations, the number of depots used, and the data type. The tactical level defines the order type and the visit frequencies of customers over a given time horizon. Finally, the operational considers vehicle and driver schedules so that constraints are related to distribution planning and specified for customers, vehicles, drivers, and roads. Additionally, we

Table I. State-of-the-Art of Rich VRP Methods

Authors	Year	Method	Maximum n	Several Rich VRPs
Ruinelli	[2011]	Column Generation	150	
Baldacci et al.	[2011a]	Exact Method	200	√*
Baldacci et al.	[2011b]	Exact Method	200	√*
Baldacci et al.	[2010]	Exact-Solution Framework	200	√*
Bettinelli et al.	[2011]	Branch-and-Cut-and-Price	144	
Doerner and Schmid	[2010]	MatHeuristics	-	
Goel	[2010]	Column Generation	250	
Oppen et al.	[2010]	Column Generation	27	
Rieck and Zimmermann	[2010]	Mixed-Integer Linear Programming	30	
Baldacci and Mingozzi	[2009]	Set Partitioning	100	√
Ceselli et al.	[2009]	Column Generation	100	
Fügenschuh	[2006]	Mixed-Integer Programming	404	
Derigs et al.	[2013]	LS/LNS-based metaheuristic	199	
Vidal et al.	[2013b]	Hybrid Genetic Search with Advanced Diversity Control	1,000	√*
Amorim et al.	[2012]	Adaptive Large Neighborhood Search Framework	366	
Santillán et al.	[2012]	Ant Colony System	100	
Subramanian et al.	[2012]	Iterated Local Search	360	
Vidal et al.	[2013c]	Unified local search and Hybrid Genetic Search	480	√*
Vallejo et al.	[2012]	3-phase heuristic using a memory-based and clustering techniques	2,000	
Battarra	[2011]	Exact and Heuristic algorithms	100	√
Groër et al.	[2010]	Local Search Heuristic	483	
Prescott-Gagnon et al.	[2012]	Tabu Search, LNS+TS heuristic, LNS+CG heuristic	750	
Wen et al.	[2010]	3-phase heuristic	80	
Goel and Gruhn	[2008]	Variable and Large Neighborhood Searches	40	
Irnich	[2008]	Heuristic Framework using Local Search-Based metaheuristic	1,000	√*
Wen et al.	[2008]	TS and Adaptive Memory Procedure	200	
Hasle and Kloster	[2007]	metaheuristic	199	√*
Pellegrini et al.	[2007]	Multiple Ant Colony Optimization	80	
Pisinger and Ropke	[2007]	LNS Heuristic	1,008	√*
Rizzoli et al.	[2007]	Ant Colony Optimization	600	√
Bolduc et al.	[2006]	Heuristics	75	
Goel and Gruhn	[2006]	Large Neighborhood Search	500	
Hoff and Løkketangen	[2006]	Tabu Search Heuristic	262	
Ileri et al.	[2006]	Set partitioning model	130	
Magalhães and Sousa	[2006]	Clustering Heuristic	450	
Reimann and Ulrich	[2006]	Ant Colony Optimization	100	

Continued

Table I. Continued

Authors	Year	Method	Maximum n	Several Rich VRPs
Ropke and Pisinger	[2006b]	LNS Heuristic	500	√
Ropke and Pisinger	[2006a]	LNS Heuristic	500	√
Sörensen	[2006]	Memetic algorithm with population management	199	
Derigs and Döhmer	[2005]	Local Search Algorithm	-	
Goel and Gruhn	[2005]	Large Neighborhood Search	500	
Pellegrini	[2005]	Nearest Neighbor	200	
Cordeau et al.	[2004]	Improved Unified Tabu Search heuristic	288	√
Cordeau et al.	[2001]	Unified Tabu Search heuristic	1,035	√
Cordeau et al.	[1997]	Tabu Search	288	

propose a second level of classification associated with the routing element involved (depot, customer, route, vehicle, and product) in order to help better understand the classification:

—Multiproducts (**CP**): Some vehicles can carry several types of products (fresh-cold, small-big, etc.).
—Multidimensional capacity (**CD**): The capacity of vehicles is considered in 2D or 3D.
—Vehicle Capacity (**C**): The capacity of vehicles is limited.
—Homogeneous Fleet of Vehicles (**FO**): All vehicles of the fleet have the same capacity.
—Heterogeneous Fleet of Vehicles (**FE**): Several type of vehicles (capacities) can be found in the fleet.
—Unfixed Fleet of Vehicles (**VU**): The number of vehicles considered is unlimited.
—Fixed Fleet of Vehicles (**VF**): The number of vehicles considered is limited.
—Fixed Cost per Vehicle (**FC**): To use a vehicle implies an extra cost.
—Variable Cost of Vehicle (**VC**): The real cost is represented by the product of the distance assigned to a vehicle and its price per distance unit.
—Multitrips (**MT**): All or some vehicles of the fleet can execute more than one trip (multiple uses of vehicles).
—Vehicle Site-Dependence (**DS**): Some vehicles cannot visit some nodes due to geographical, compatibility, or legal issues.
—Vehicle Road-Dependence (**DR**): Some vehicles can not pass through some edges of the network for some legal issues.
—Duration Constraints/Length (**L**): The duration of each route cannot exceeded a maximum value or cost, including service times on each visited client.
—Driver Shifts/Working Regulations (**D**): The design of routes include the number of legal working hours for drivers (stops, breaks, rest, etc).
—Balanced Routes (**BR**): The load of routes or vehicles must be balanced between all.
—Symmetric Cost Matrix (**CS**): The cost matrix has a symmetric nature.
—Asymmetric Cost Matrix (**CA**): The cost matrix has an asymmetric nature.
—Intraroute replenishments (**IR**): The vehicles must be reloaded at some point in the routes.
—Time-Dependent/Dynamic/Stochastic Times (**TD**): The target is minimizing time, and traveling times could vary during a day (hard or flexible). The location/distance of clients changes.
—Stochastic Demands/Dynamic (**S**): The demands of clients can change during the application of a routing solution.

Table II. Relation of Rich VRP Papers and Considered Constraints

Authors	CP	CD	C	FO	FE	VU	VF	FC	VC	MT	DS	DR	L	D	BR	CS	CA	IR	TD	S	TW	MW	PD	PS	B	MV	MP	I	CC	MD	WD	O	DA	DD	PC	MO
[Derigs et al. 2013]			√	√		√							√				√						√													
[Vidal et al. 2013b]			√	√	√	√					√					√					√	√					√			√						
[Amorim et al. 2012]			√		√	√		√	√		√		√			√						√														
[Santillán et al. 2012]			√		√	√		√	√	√	√	√				√				√	√	√				√				√		√				
[Subramanian 2012]			√	√	√	√	√	√	√							√	√						√	√						√		√				
[Subramanian et al. 2012]			√		√	√	√	√	√							√																				√
[Vidal et al. 2013c]			√	√	√	√	√	√	√		√		√	√		√			√		√	√		√	√		√			√		√				
[Vallejo et al. 2012]			√		√		√			√						√					√									√						
[Baldacci et al. 2011a]			√	√			√									√														√						√
[Baldacci et al. 2011b]			√	√			√									√											√									
[Battarra 2011]			√		√		√	√		√						√							√													
[Bettinelli et al. 2011]			√		√		√	√								√					√									√						
[Ruinelli 2011]			√		√			√			√		√	√								√	√									√		√		
[Yu and Zhen-Yang 2011]			√	√			√									√					√						√									
[Baldacci et al. 2010]			√		√	√	√	√	√		√					√					√		√				√			√						
[Doerner and Schmid 2010]			√	√			√									√					√						√			√						
[Groër et al. 2010]			√	√			√						√			√					√									√						
[Oppen et al. 2010]	√		√		√	√				√			√			√					√					√		√							√	
[Prescott-Gagnon et al. 2012]			√		√	√							√	√		√		√		√	√						√	√		√						
[Wen et al. 2010]			√	√			√									√				√							√									√
[Rieck and Zimmermann 2010]			√		√		√			√			√	√		√					√			√												
[Goel 2010]			√		√	√			√		√					√					√		√									√	√			
[Ceselli et al. 2009]			√		√	√		√	√		√		√	√		√					√					√				√		√				
[Baldacci and Mingozzi 2009]			√		√	√	√	√	√		√					√														√						
[Crainic et al. 2009a]			√	√			√						√			√					√						√			√						

Continued

Table II. Continued

Authors	CP	CD	C	FO	FE	VU	VF	FC	VC	MT	DS	DR	L	D	BR	CS	CA	IR	TD	S	TW	MW	PD	PS	B	MV	MP	I	CC	MD	WD	O	DA	DD	PC	MO
[Goel and Gruhn 2008]		√	√		√	√			√		√					√					√		√									√	√			
[Hashimoto et al. 2008]			√	√			√						√			√			√		√															
[Irnich 2008]			√	√	√	√	√				√		√			√					√		√	√	√		√			√				√	√	
[Wen et al. 2008]			√	√			√									√					√		√													
[Hasle and Kloster 2007]			√	√	√	√	√						√			√					√	√	√							√						
[Pellegrini et al. 2007]			√		√	√		√	√		√		√	√		√						√				√	√									√
[Pisinger and Ropke 2007]			√	√	√		√				√					√	√				√									√		√				
[Rizzoli et al. 2007]			√	√			√									√			√	√	√		√													
[Bolduc et al. 2006]			√		√		√									√											√	√								
[Fügenschuh 2006]			√	√			√									√					√	√														
[Hashimoto et al. 2006]			√	√			√									√					√	√														
[Goel and Gruhn 2006]		√	√		√		√		√		√		√	√		√				√	√		√									√	√			
[Hoff and Løkketangen 2006]			√	√			√									√							√													
[Ileri et al. 2006]			√		√		√	√	√							√					√		√													
[Magalhães and Sousa 2006]			√		√		√									√				√																
[Reimann and Ulrich 2006]			√	√			√									√					√				√											√
[Ropke and Pisinger 2006b]			√	√			√									√					√		√	√	√					√					√	
[Ropke and Pisinger 2006a]			√	√			√									√					√		√													
[Sörensen 2006]			√	√			√						√			√																				√
[Derigs and Döhmer 2005]			√	√		√									√							√	√													
[Goel and Gruhn 2005]		√	√		√		√		√		√					√				√	√		√									√	√			
[Ibaraki et al. 2005]			√	√			√						√			√					√							√								
[Pellegrini 2005]			√		√		√									√						√														√
[Cordeau et al. 2004]			√	√	√		√				√					√					√						√			√						
[Cordeau and Laporte 2003]			√	√			√						√			√					√		√													
[Cordeau et al. 2001]			√	√			√									√					√						√			√						
[Cordeau et al. 1997]			√	√			√									√											√			√						

Table III. Classification of Main Documented Rich VRP Constraints

Restriction	Code/Id	[Vidal et al. 2013c] Classification	[Lahyani et al. 2012] Classification	Our 2nd-Level Classification
Multiproducts	**CP**	Assign	Strategic	Veh-Prod
Multidimensional capacity	**CD**	Assign	Strategic	Veh-Prod
Vehicle Capacity	**C**	Assign	Operational	Veh
Homogeneous Fleet of Vehicles	**FO**	Assign	Operational	Veh
Heterogeneous Fleet of Vehicles	**FE**	Assign	Operational	Veh
Unfixed Fleet of Vehicles	**VU**	Evaluation	Operational	Veh
Fixed Fleet of Vehicles	**VF**	Assign	Operational	Veh
Fixed Cost per Vehicle	**FC**	Evaluation	Operational	Veh
Variable Cost of Vehicle	**VC**	Evaluation	Operational	Veh
Multitrips	**MT**	Sequence	Operational	Veh
Vehicle Site Dependence	**DS**	Assign	Operational	Veh-Cust
Vehicle Road Dependence	**DR**	Assign	Operational	Veh-Route
Duration Constraints/ Length	**L**	Evaluation	Operational	Route-Driver
Driver Shifts/Working Regulations	**D**	Evaluation	Operational	Route-Driver
Balanced Routes	**BR**	Assign	Operational	Route-Driver
Symmetric Cost Matrix	**CS**	Sequence	Operational	Route
Asymmetric Cost Matrix	**CA**	Sequence	Operational	Route
Intraroute Replenishments	**IR**	Assign	Tactical	Route
Time-Dependent/Dynamic/ Stochastic Times	**TD**	Evaluation	Tactical	Route
Stochastic Demands/ Dynamic	**S**	Evaluation	Tactical	Customer
Time Windows	**TW**	Evaluation	Tactical	Customer
Multiple Time Windows	**MW**	Evaluation	Tactical	Customer
Pickup & Delivery	**PD**	Sequence	Tactical	Customer
Simultaneous Pickup & Delivery	**PS**	Evaluation	Tactical	Customer
Backhauls	**B**	Sequence	Tactical	Customer
Multiple Visits/Split Deliveries	**MV**	Assign	Tactical	Customer
Multiperiod/Periodic	**MP**	Assign	Tactical	Customer
Inventory Levels Controls	**I**	Assign	Tactical	Customer
Customer Capacity	**CC**	Assign	Tactical	Customer
Multidepot	**MD**	Assign	Strategic	Depot
Time Windows for the Depot	**WD**	Evaluation	Strategic	Depot
Different End Locations/ Open Routes	**O**	Evaluation	Strategic	Depot
Different Start and End Locations	**DA**	Evaluation	Strategic	Depot
Departure from Different Locations	**DD**	Evaluation	Strategic	Depot
Precedence Constraints	**PC**	Sequence	Tactical	Depot
Multiobjectives	**MO**	Evaluation	Tactical	Depot

—Time Windows (**TW**): The clients cannot receive orders outside of a time window. Each client has a particular time window (hard or soft).
—Multiple Time Windows (**MW**): The clients can not receive the orders out of a set of time windows. Each client has a particular set of time windows.
—Pickup & Delivery (**PD**): The construction of routes must consider the picking up of products from some clients and the delivery to others in a sequential or separate way. The depot simply defines the starting/ending point for vehicles.
—Simultaneous Pickup & Delivery (**PS**): The construction of routes must consider the picking up and delivery of products/persons at the same time in all nodes by the same vehicle. The depot simply defines the starting/ending point for vehicles.
—Backhauls (**B**): The construction of routes must consider the picking up of products from some clients and delivery to others in a sequential or separate way. The critical assumption is that all deliveries must be made on each route before any pickups can be made (sometimes a client may require both a delivery and a pickup). The rearrangement of products may be expensive or unfeasible. The depot simply defines the starting/ending point for vehicles.
—Multiple Visits/Split Deliveries (**MV**): Clients are visited several times for delivering a summary of original orders. Each vehicle may deliver a fraction of a customer's demand.
—Multiperiod/Periodic (**MP**): The optimization is made over a set of days, considering several visits, and each client has a different frequency of visits.
—Inventory Levels Controls (**I**): The costs of stocks are also considered to be minimized with the routing costs while the levels of stock are controlled.
—Customer Capacity (**CC**): The capacity stock of clients is also considered.
—Multidepot (**MD**): There are more than one depots from whence vehicles leave and arrive.
—Time Windows for the Depot (**WD**): The depot is open during a period of time. Vehicles that need to do more than one trip need to consider this.
—Different End Locations/Open Routes (**O**): The routes start at the depot but finish on the last client. The return cost is not considered (optional).
—Different Start and End Locations (**DA**): The vehicles start and end in different locations.
—Departure from Different Locations (**DD**): The vehicles start in different locations.
—Precedence Constraints (**PC**): The visiting order of clients could be important for the loading and unloading of products. Its order could be important for health or security reasons.
—Multiobjectives (**MO**): The study considers more than one objective function or related costs at the same time.

7. INSIGHTS AND FUTURE TRENDS

From the previous sections, it is possible to extract some insight regarding the historical evolution of the VRP, both in terms of realism in the models that various authors consider and the methods employed to solve them. Thus, as shown in Figure 3, VRPs can be classified into three levels according to the degree of realism of their associated models.

At the lower level, we find the most theoretical (classical) VRPs, which are represented by mostly academic models (as opposed to real-life models). These lab models are, of course, of high interest in order to develop mathematical and computing-based approaches that are exact but also of heuristic nature. Thus, solution techniques can be tested in controlled environments to assess their performance before being used in solving more complex models. The CVRP, VRPTW, HVRP and AVRP, among others, constitute clear examples of this category.

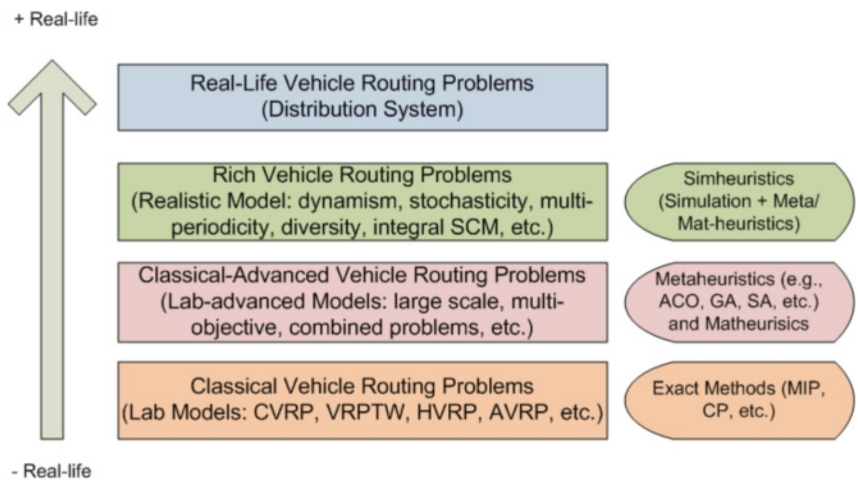

Fig. 3. Models classification and methodological future trends in RVRPs.

In a second level, the classical-advanced VRPs appear. These are models characterized by a higher level of realism: large-scale problems, multiobjective functions, combined routing and cross problems (e.g., VRPs combined with packing, allocation, or inventory management), and the like. More advanced and complex VRP variants are included in this category. Usually, these problems have been solved by metaheuristic approaches, such as GA, ACO, SA, GRASP, and the like.

Most of the existing work in the VRP literature so far deals with these two levels. Recently, however, and largely due to the maturity of existing exact and metaheuristic methods, researchers are able to go one step beyond and cope with RVRPs using a plethora of new hybrid methods that combine exact and metaheuristic approaches (matheuristics) [Doerner and Schmid 2010] or even simulation with metaheuristics (simheuristics). As discussed in Juan et al. [2014b], simheuristics allow a consideration of uncertainty in costs and constraints of the VRP model, thus making these models a more accurate representation of real-life routing distribution systems. These hybrid methods not only can deal with uncertainty (stochastic factors), but they can also consider aspects such as dynamism, diversity of vehicles and customers, multiperiodicity in the distribution activity, integration with other supply chain components, environmental issues, and more. As models and solving techniques are refined to tackle more realistic problems, a further increase of VRP variants considering complex constraints—and therefore included in the RVRP category—is to be expected.

8. CONCLUSION

The VRP is a classical combinatorial problem. Throughout history, many different variants have been studied. The main difference among them is either the kind of constraints or the cost function of the specific type. Nowadays, it is common to find increasingly complex problems that are closer to real world ones. These can be classified as RVRPs.

In this survey, we reviewed the evolution of studied problems in the RVRP arena. We present a variety of routing scenarios that can be found in the real world and the most common methods developed for addressing all types of RVRPs (i.e., exact, approximated, and their combinations). The RVRP domain appeared during the first decade of the 21st century, and it has shown itself to be a promising research area. There are many tailored approaches for specific cases of RVRP; however, in the past

10 years, general-purpose methods are slowly emerging that preserve previous quality features, but that can be applied to generic Rich VRP scenarios.

To organize the information about the RVRP, we analyzed the different constraints included in RVRP papers and tried to define how they can be characterized. Moreover, we collected all papers devoted to this area, classifying them according to the active constraints they addressed. Finally, we included a section that follows the evolution from the classical VRP to the so-called RVRP, and we speculate on future trends that this research line will face in the upcoming years.

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