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A new method for identification of nonlinear systems using MISO model with Swept-Sine technique: Application to loudspeaker analysis

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ABSTRACT

This work presents a Multiple Input Single Output (MISO) nonlinear model in combination with sine-sweep signals as a method for nonlinear system identification. The method is used for identification of loudspeaker nonlinearities and can be applied to nonlinearities of any audio components. It extends the method based on nonlinear convolution presented by Farina, providing a nonlinear model that allows to simulate the identified nonlinear system. The MISO model consists of a parallel combination of nonlinear branches containing linear filters and memory-less power-law distortion functions. Once the harmonic distortion components are identified by the method of Farina, the linear filters of the MISO model can be derived. The practical application of the method is demonstrated on a loudspeaker.

1. INTRODUCTION

There are so many nonlinearities or nonlinear devices in electro-acoustics that almost all the systems used for audio applications are more or less nonlinear. On one hand, nonlinear systems have been studied for a long time and as the demand for high audio-quality increases, the cancelation of nonlinearities is one of the most important task in electro-acoustics. On the other hand, the usual parameters used for description of nonlinear systems, such as harmonic or intermodulation distortions, are not sufficient and do not show good correlation with listening tests [1].

The justification for studying the nonlinear systems may be detailed in two cases. First, lot of systems are used as if they are linear, even if they are in fact nonlinear, or at least weakly nonlinear, such as loudspeakers, amplifiers and most electro-acoustics devices. Second, there are nonlinear systems that are known to be nonlinear, but their nonlinear behavior is not known. In both cases nonlinear identification can help; in the first case to find a way to get rid of the undesirable nonlinearities, in the second case to analyze the nonlinear system under test for better understanding of its physical behavior.

2. NONLINEAR MODEL

The main purpose of this paper is to present a method combining two techniques for nonlinear system analysis: the swept-sine technique proposed by Farina [2, 3] and the MISO model originally described by Bendat [4]. The aim of this work is to propose a nonlinear model of the measured system that allows for example to reconstruct any output signal of the identified nonlinear system as a function of the input signal. The MISO model consists of a parallel combination of nonlinear branches containing linear filters $G_n(f)$ and memoryless nonlinearities. If the memoryless nonlinearities are represented by the power-law distortion functions, the MISO model is like the Volterra subclass [5, 6]. The main reason for using the power-law MISO nonlinear model instead of Volterra model is to provide a simpler model than the one given by Volterra. The MISO model with power-law functions, used in this work, is schematically shown in Figure 1. The output signal of the nonlinear MISO model is

$$y(t) = \sum_{n=1}^{N} \int_0^{\infty} x^n(t-\tau) g_n(\tau) d\tau, \tag{1}$$

where N is the number of the nonlinear branches and $g_n(t)$ represents the impulse response of the linear filters [4].

3. Swept-Sine TECHNIQUE

The swept-sine technique described by Farina is one of the most robust methods used in nonlinear system analysis according to the robustness even for low signal-to-noise ratios. Combining these two methods (the MISO nonlinear modeling and the swept-sine technique) leads to a simple and effective method in nonlinear system identification as described in section 4.

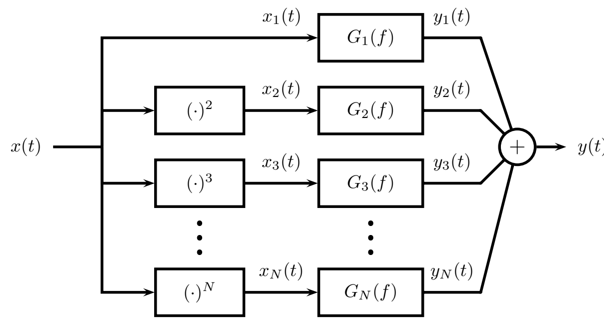

Fig. 1: Multiple Input Single Output MISO model with power-law functions.

The so-called swept-sine technique is based on swept sine excitation of the system under test and a nonlinear convolution process with an inverse filter. The input signal $s(t)$ is a swept sine with start frequency f_1 and end frequency f_2, defined as

$$s(t) = \sin\left[2k\pi\left(\exp\left\{\frac{f_1}{k}t\right\} - 1\right)\right], \tag{2}$$

where the constant k is defined as

$$k = \text{round}\left(\frac{\tilde{T} f_1}{\ln\left(\frac{f_2}{f_1}\right)}\right). \tag{3}$$

The variable $\tilde{T}$ is an approximative time length that is adjusted using the rounding of k. The true length of the signal is then

$$T = \frac{k \ln\left(\frac{f_2}{f_1}\right)}{f_1}. \tag{4}$$

This definition differs to the one used in [3]. The reason of using the rounded k is to synchronize all the partial frequency responses (section 4), that is necessary for the output signal reconstruction. Otherwise, the filtered power-law products $y_n(t)$ of the MISO model would not match in phase. If k is rounded, the instantaneous frequencies of the signal $s(t)$ satisfies the property depicted in Figure 2.

The swept-sine method, based on a nonlinear convolution [3], uses an inverse filter $f_i(t)$, derived from the input excitation signal

$$s(t) * f_i(t) = \delta(t). \tag{5}$$

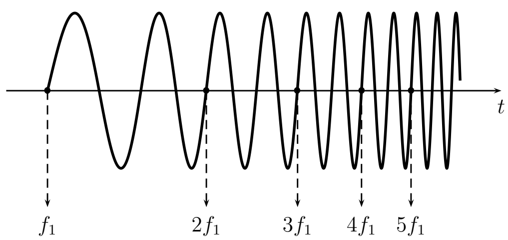

Fig. 2: Swept-Sine signal in time domain, with the length chosen according to instantaneous frequency.

The so-called sequence $h(t)$ of impulse responses can be easily obtained by convolving the measured output signal $y(t)$ with the inverse filter $f_i(t)$ [3]

$$h(t) = f_i(t) * y(t). \tag{6}$$

The results are the partial impulse responses $h_n(t)$, which can be expressed in frequency domain as partial frequency responses $H_n(f)$. These responses correspond to the higher harmonic components of the nonlinear system. The sequence $h(t)$ consists of the partial impulse responses $h_n(t)$, as

$$h(t) = \sum_{n=1}^{N} h_n(t + \Delta t_n), \tag{7}$$

where N is number of higher harmonic components taken into account, or equivalently the number of the nonlinear branches used for the nonlinear model as seen in Figure 1. The time lag Δt_n between two partial responses corresponds to the frequency intervals between f_1 and nf_1 in Figure 2 and is defined as [2]

$$\Delta t_n = T \frac{\ln(n)}{\ln\left(\frac{f_2}{f_1}\right)}. \tag{8}$$

Usually the inverse filter $f_i(t)$ is simply the time-reversal of the excitation signal $s(t)$, multiplied by exponential function decreasing by 6dB/oct as used in [3]. Nevertheless, in that case, the frequency range of the inverse filter is the same as of the input excitation signal. The harmonic components higher than the maximum frequency f_2, generated by the nonlinear system, can not be estimated after the convolution process (6). In other words, all the partial frequency responses $H_n(f)$ end at the same frequency f_2 instead of nf_2. For that reason, the inverse filter $f_i(t)$ used in this paper is derived from a signal $\hat{s}(t)$, that starts at f_1 and ends at $f_3 = Nf2$. The signal $\hat{s}(t)$ can be defined using equations (2-4), with $f_3 = Nf_2$ replacing.

4. FROM SWEPT-SINE TECHNIQUE TO THE MULTIPLE INPUT - SINGLE OUTPUT MODEL

Using the technique described in 3, the set of partial frequency responses $H_n(f)$ may be estimated. In order to express the components $H_n(f)$ as a function of the linear filters $G_n(f)$ (the nonlinear branches of the MISO model), the relation between the higher harmonic signals $\sin(nx)$ and the n-th powers of harmonic signal $\sin^n(x)$ may be used [7],

$$\sin^{2n+1} x = \frac{(-1)^n}{4^n} \sum_{i=0}^{n} (-1)^i \binom{2n+1}{i} \sin\left[(2n+1-2i)x\right], \tag{9}$$

$$\begin{aligned} \sin^{2n} x \quad &= \quad \frac{(-1)^n}{2^{2n-1}} \sum_{i=0}^{n-1} (-1)^i \binom{2n}{i} \sin\left[2(n-i)x + \frac{\pi}{2}\right] \\ &\quad + \frac{1}{2^{2n}} \binom{2n}{n}. \end{aligned} \tag{10}$$

These equations can be rewritten into the matrix form (), where $\mathbf{A}$ and $\mathbf{B}$ represent the coefficients from equations (9-10),

$$\begin{pmatrix} \sin x \\ \sin^2 x \\ \sin^3 x \\ \vdots \end{pmatrix} = \mathbf{A} \begin{pmatrix} \sin x \\ \sin 2x \\ \sin 3x \\ \vdots \end{pmatrix} + \mathbf{B} \tag{11}$$

The matrix $\mathbf{B}$ is a one-column matrix and represents the constant values of the even power low products. These products reflect only the mean value of the output signal. The matrix $\mathbf{A}$ is a square matrix and is used as a linear transformation matrix between the frequency dependent nonlinear distorted products $H_n(f)$ and the linear filters from the nonlinear MISO model $G_n(f)$,

$$\begin{pmatrix} G_1(f) \\ G_2(f) \\ G_3(f) \\ \vdots \end{pmatrix} = \left(\mathbf{A}^T\right)^{-1} \begin{pmatrix} H_1(f) \\ H_2(f) \\ H_3(f) \\ \vdots \end{pmatrix}. \tag{12}$$

5. APPLICATION TO A LOUDSPEAKER

Identifying the nonlinearities of a loudspeaker may lead at least to two points of view.

- The loudspeaker, considered to be a nonlinear system, can be measured in its whole, as a black box. In that case, the loudspeaker is generally placed in an anechoic room and the reproduced audio signal is measured via a microphone.
- The nonlinearities of the loudspeaker are studied as the physical causes for signal distortion. Part by part, the electrical impedance of the moving coil, the force factor Bl and mechanical impedance of the membrane are measured as the nonlinear parameters of an electrodynamic loudspeaker. [8].

The first view is considered in this paper. Consequently, the measured nonlinear system includes the microphone, the amplifiers and the A/D and D/A converters needed to transfer the signals. Nevertheless, the nonlinearities of all these other parts can be neglected in comparison with the nonlinearities of the loudspeaker. The block diagram of the set-up is shown in Figure 3.

The characteristics of the measured electrodynamic loudspeaker are the following: diaphragm diameter $7cm$, power handling capacity $100W$, SPL max (continuous) $110dB$, usable frequency range $50 - 4000Hz$, rated impedance 8Ω, resonance frequency $146Hz$. The loudspeaker is baffled in a CEI normalized screen. An electrostatic microphone (G.R.A.S. 1/4" pressure microphone) is set and at a distance of 2 meters pointing towards the front of the speaker (on-axis).

Only one measurement is necessary for the MISO model estimation. The swept-sine signal is generated with start frequency $f_1 = 500Hz$ and end frequency $f_2 = 5kHz$, in respect to the frequency range of the loudspeaker. The level of the signal at the loudspeaker input is $10V_{pp}$. The swept-sine signal passes through the nonlinear system depicted in Figure 3 and the output signal is processed using the nonlinear convolution [3]. The linear filters $G_n(f)$ of the MISO model (see Figure 1) are estimated using equation (12).

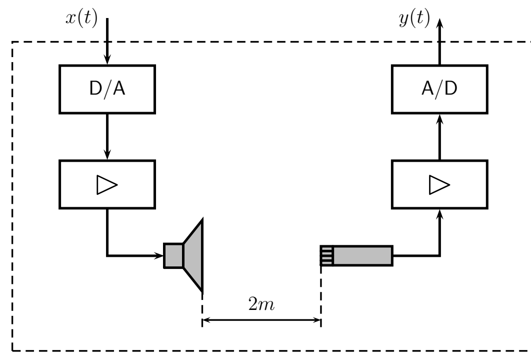

Fig. 3: Block diagram of the system under test

In order to test the MISO model, the following procedure is setup. Firstly, the linear filters $H_n(f)$ of Figure 1 are estimated. Secondly, a signal is generated, made up of the sum of two harmonic components with different frequencies f_a and f_b. Then, this signal is chosen as excitation of the MISO model and an output signal is generated. Lastly, the very same input signal is used for exciting the actual system and both output signals (actual and teh MISO-based generated) are compared in the Fourier domain as seen in Figures 4 and 5. For each figure three different levels are proposed, $1V_{pp}$, $5V_{pp}$ and $10V_{pp}$. Two different cases has been chosen to test the intermodulation distortion. First, with f_a far from f_b (Figure 4) and second, with f_a near the f_b (Figure 5).

The results confirm that the output of the MISO model corresponds to the output of the system under test with only some small differences. The model corresponds even for the levels less than the level of the swept-sine signal used for the identification.

6. CONCLUSION

In this paper, a technique for the identification of nonlinear system has been presented. The excitation signal used for the identification is the swept-sine signal, which is commonly used for linear transfer function measurement, thanks to its ability of complete separation of harmonic distortion components. Moreover, using this signal and the nonlinear convolution process, one can get a set of nonlinear system impulse responses, which correspond to the harmonic distortion products over the entire frequency range, used for distortion analysis.

The method described in this paper allows to derive a nonlinear MISO model. Such a model is able to reconstruct a nonlinear response to any input signal defined by frequency and amplitude range of the original swept-sine signal used for the identification. In other words, the response of the nonlinear system to the signal with all the input levels, less than or equal to the amplitude of the excitation swept-sine signal, can be also inferred. One of the advantages of this method is that it allows to obtain the nonlinear system model by taking only one measurement.

The method has been tested on a loudspeaker. After the identification, a signal made of the sum of two harmonic components with different frequencies has been used as the excitation signal. Both outputs the loudspeaker and the MISO model have been successfully compared in the Fourier domain.

ACKNOWLEDGEMENTS

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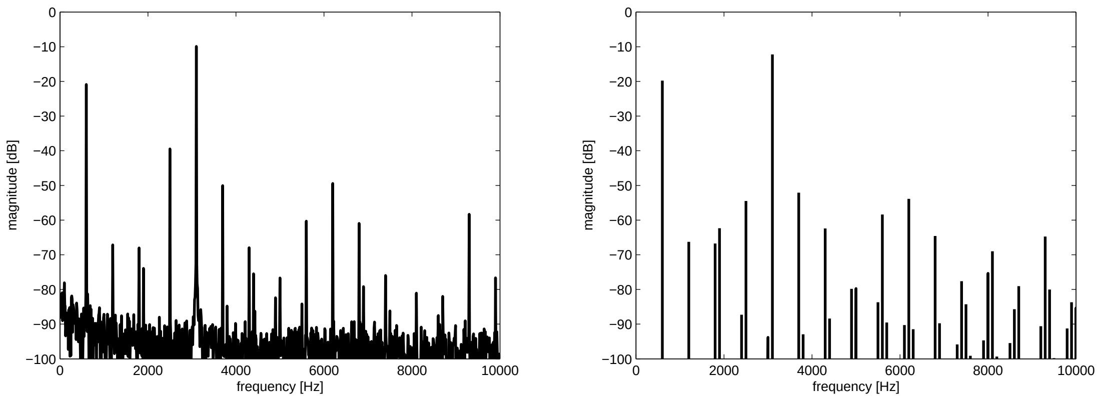
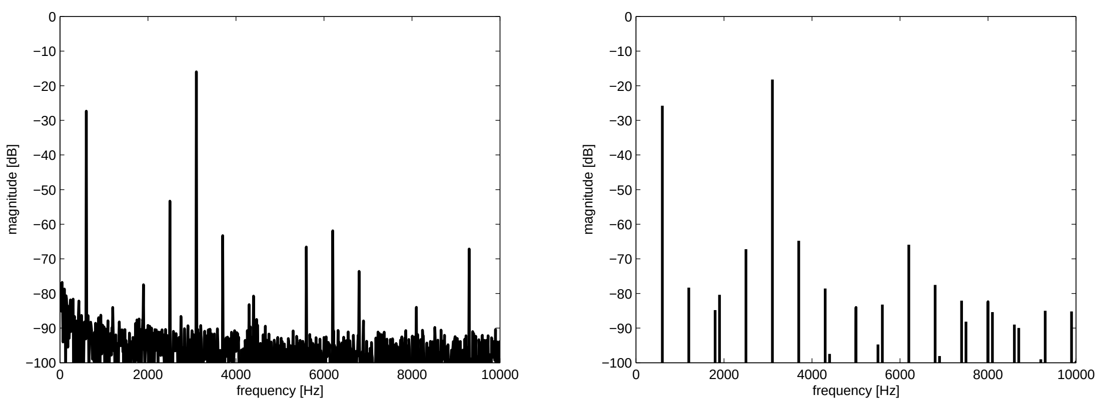
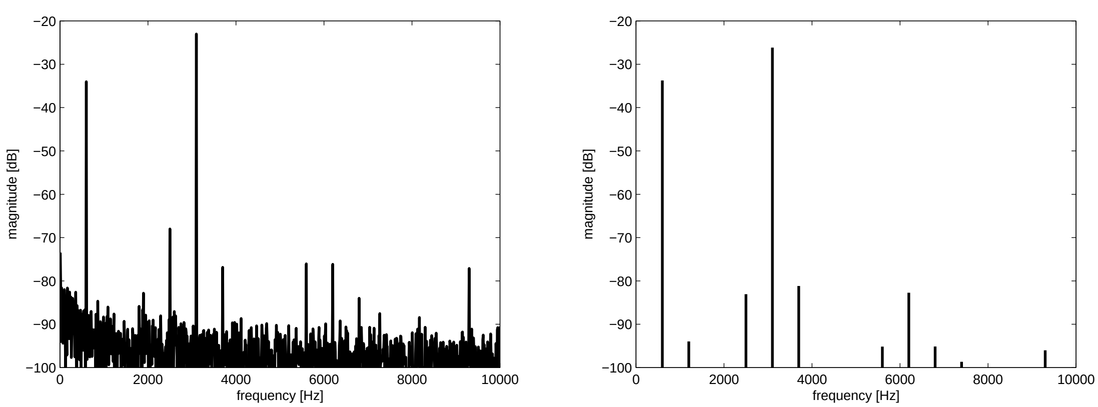


(a) amlitude of the input signal 10 V_{pp}

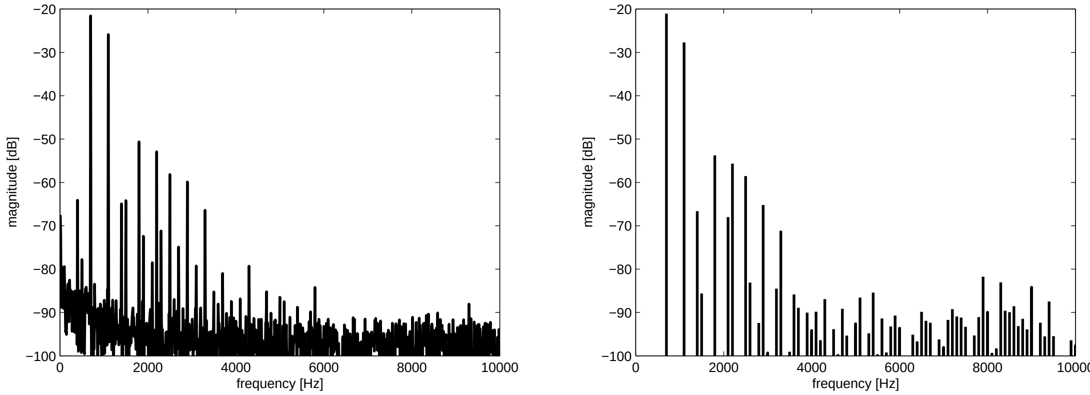


(b) amlitude of the input signal 5 V_{pp}

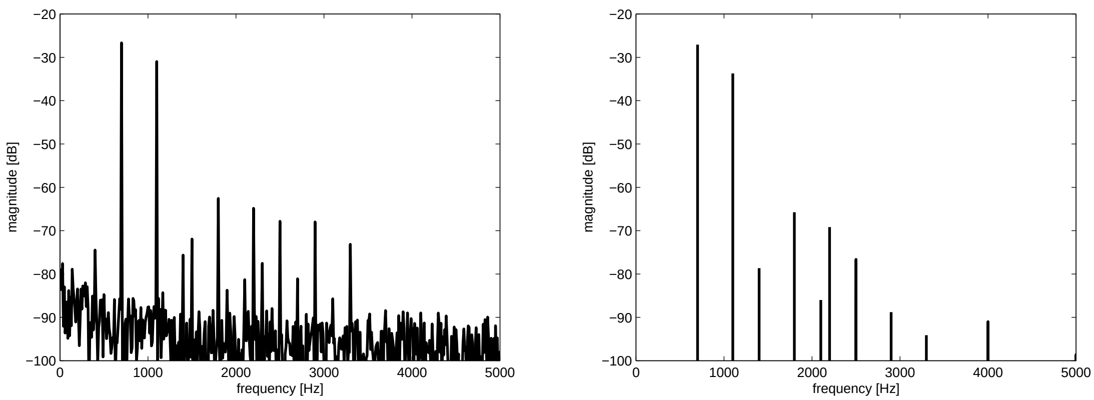


(c) amlitude of the input signal 1 V_{pp}

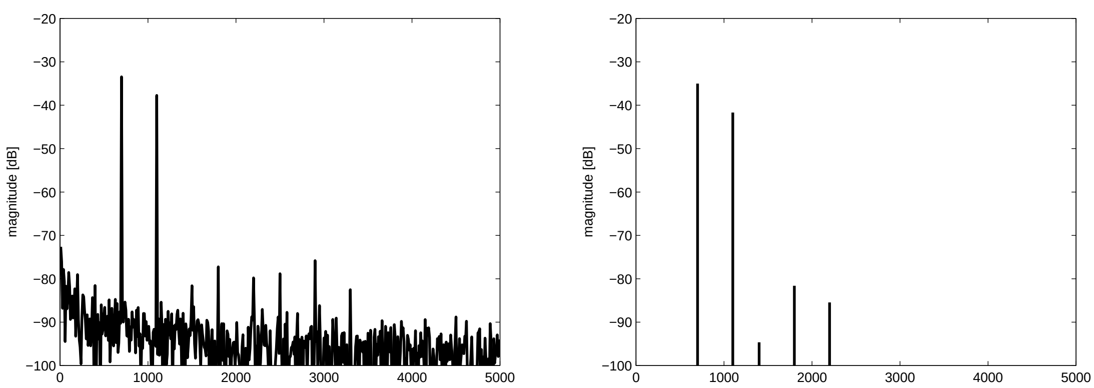
Fig. 4: Spectrum of the responses to the two-harmonic signals with frequencies 600 Hz and 3100 Hz: comparison between the real systems output (left) and the MISO models output (right).



(a) amlitude of the input signal 10 V_{pp}



(b) amlitude of the input signal 5 V_{pp}



(c) amlitude of the input signal 1 V_{pp}

Fig. 5: Spectrum of the responses to the two-harmonic signals with frequencies 700 Hz and 1100 Hz: comparison between the real systems output (left) and the MISO models output (right).