

Computational Modeling of Chord Fingering for String Instruments

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Abstract

Fingering is a cognitive process that maps each note on a music score to a fingered position on some instrument. This paper presents a computational model for the fingering process with string instruments, based on a constraint satisfaction approach. The model is implemented in a computer program, which has been tested in an experiment in comparison with three human experts. The results have confirmed the predictions based on a set of constraints that encode the bio-mechanical aspects of the performer's hand in its interaction with the musical instrument.

Introduction

Music performance involves the transformation of symbolic representations of a score into physical gestures that can operate a music instrument. A model of music performance consists of the interpretation of the score and the application of the gestures to some sound synthesis device that represents the instrument. Gesture modeling is favorably coupled with a physical model of the instrument, since the latter permits a natural representation of the performer/instrument interaction.

Fingering is an essential part of gesture modeling, since it significantly affects the technical and expressive qualities of the sounds being produced (Traube *et al.*, 2003). Since fingering defines for each note in the score both the position on the keyboard/fingerboard and the (left hand) finger involved in playing, it sets the parameters that influence the final timbre of the sound during performance. In this paper, we address the modeling of the fingering process for the guitar. The case of guitar, and of string instruments in general, is particularly relevant, since the same note can be played in several positions on the fingerboard.

Fingering is a complex cognitive process of music production that relates a score, together with the technical and "idiomatic" aspects of the instrument for which that score has been conceived, and a performer, with all her/his knowledge upon the piece, the composer's intentions and the execution style (Clarke *et al.*, 1997; Parncutt *et al.*, 1997). So, fingering involves several competences: musical analysis, for the interpretation of the notes in input, physical constraints, posed by the instrument where the notes have to be played, bio-mechanical constraints, which characterize the possible figures of the hand.

Moreover, modeling the fingering process can contribute to the development of automatic performance environments (Parncutt, 1997). In fact, despite its central role in music performance, scores often lack of fingering indications, considered unnecessary (being common knowledge within a certain musical practice) or an execution choice (Gilardino, 1975a, 1975b). Therefore, a cognitive model may supply this fundamental information.

The fingering problem consists in determining for each note in the score, a *position* `<string, fret>` on the fingerboard and a finger of the left hand that presses it. The notion of position provides a unique identifier for the correspondence between the note and the fingerboard. A *fingered position* is the triple `<string, fret, finger>`, combining a position with one of the four available fingers. Provided that guitarists do use four fingers of the left hand (from the index to the little finger), n notes generate up to 4^n different fingerings in the worst case. Since the same note can be found on up to 4 positions (i.e., pairs `<string, fret>`) (Figure 1), this number might grow up to 16^n .

From the temporal point of view, fingering can be decomposed into two subproblems: the fingering of *melodies*, where the notes to be played form a sequence, and the fingering of *chords*, where multiple notes (2 to 6 in the case of guitar) have to be played simultaneously. The two subproblems are addressed differently. In melody fingering, the fingered position of one note depends on the application of constraints over a sequence of notes, and the model must account for the consistency of subsequent positions; in chord fingering, the fingered position of one note depends on the other notes of the chord, and all the positions must satisfy the constraints applied at the same time. We must also consider that a whole account of fingering includes both melody and chord fingerings, since a melody can consist of both chords and individual notes.

In previous papers we have addressed the melody fingering problem in the reduced case of individual notes (Radicioni *et al.*, 2004); in this paper we approach the chord fingering problem by introducing a novel model based on the *constraint satisfaction problem* (CSP) framework. CSP framework has been successful in modeling several problems, like map coloring, vision, robotics, job-shop scheduling, diagnosis, spatiotemporal reasoning, *etc.* (Dechter, 1998). A constraint satisfac-

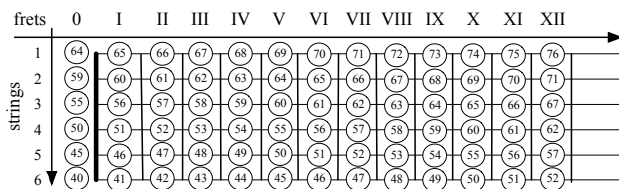


Figure 1: Outline of the notes (indicated as MIDI numbers) on a guitar fingerboard, showing that the same tone can be found on up to 4 different positions, i.e., the case of E treble (MIDI number 64), that lies at $\langle 1, 0 \rangle$, $\langle 2, 5 \rangle$, $\langle 3, 9 \rangle$, $\langle 4, 14 \rangle$ (the figure displays only the first XII frets). The fret labelled 0 indicates an open string: i.e., the note produced by plucking the string without pressing any fret.

tion problem consists in assigning *values* to *variables*, which satisfy a set of *constraints*. Here we cast the chord fingering problem by representing each note that composes a chord as a variable; each variable may then be assigned up to 16 values given by the possible fingered positions (triples $\langle \text{string}, \text{fret}, \text{finger} \rangle$), and physical and bio-mechanical constraints restrict the viable instantiations. To the best of our knowledge, this is the first modeling attempt that addresses specifically the chord fingering problem.

The paper is organized as follows: we first review the existing approaches to general fingering; then we introduce the CSP methodology and we show that the fingering problem can be cast in CSP terms together with the search strategy. Finally, we present and discuss the results of an experimental test in which we compare the output of the implemented model on 34 chords with the fingerings provided by 3 expert guitarists.

Related work

The interdisciplinary nature of the fingering problem has been addressed from several points of view, both in the music practice and the scientific research. In the music practice, the importance of fingering is acknowledged in music performance (Gilardino, 1975a; 1975b), composition (see e.g., Aguado, 1843), education (Gellrich & Parncutt, 1998) and theory (Duarte, 1966). In the scientific research, motor behavior in the fingering process has been investigated in (Palmer & Meyer, 2000), sometimes with a focus on keyboard and fingerboard instruments (Jacobs, 2001; Heijink & Meulenbroek, 2002). Physical parameters that result from fingering have been taken into account in the physical model of the classical guitar by Cuzzucoli and Lombardo (1999), while others have inferred some aspects of the instrumental gesture, including fingering, by analyzing the signal that results from the sound production process (Traube *et al.*, 2003). The cognitive issues involved in fingering have been raised in the introductory work of Sloboda (1985), refined by Clarke *et al.* (1997) for the case of keyboard fingering, and modeled with a computational approach by Parncutt (1997) and Parncutt *et al.* (1997).

The models in (Sayegh, 1989), (Parncutt *et al.*, 1997) and (Radicioni *et al.*, 2004) are fully functional computational models. In this case the models have been implemented in systems that are able to take in input a score and return a sequence of fingered positions (consider, in fact, that both models addressed the melody fingering problem). These models share the principle of penalizing difficulties: fingering is represented like a search for a combination of positions that pursues an overall effort-saving behavior. The model in (Parncutt *et al.*, 1997) addresses the main ergonomic constraints that pianists meet while playing melodies. The fingering is computed in two steps, with a “generate and test” approach: *enumeration* of all possible fingerings, and *weighting* of fingerings according to the degree of difficulty. Weighting is conducted on the basis of a set of 12 rules, each one determining the contribution of some source to an overall difficulty score (e.g., *rule 6* slightly discourages the use of ring and little fingers by penalizing their use with a heavier score). In such setting, the lowest rated fingering is “the one that [...] will be used most often in performance” (Parncutt *et al.*, 1997).

The algorithmic approach proposed by Sayegh (1989) exploits a graph-based representation, with the vertices grouped in layers; for each note, the possible fingered positions, each encoded by a vertex, form a layer; each vertex of a layer is connected to all the vertices of the following layer. Weights on the edges represent the difficulties of a transition from a fingered position to the next. The problem of finding a suitable fingering is represented as the problem of finding a path in the graph, such that the difficulties are minimized. Unfortunately, neither the sources of difficulty (in the form of constraints on bio-mechanical, ergonomic or cognitive factors) nor experimental results on the viability of this approach have been provided by Sayegh. Recent evidence support the fact that performers pursue an overall effort-saving behavior (Parncutt *et al.*, 1997; Parlitz *et al.*, 1998; Heijink & Meulenbroek, 2002). The graph-based Sayegh’s approach has been recently improved by Radicioni *et al.* (2004) by introducing the relevant notion of segmentation of a music score and a characterization of the biomechanical constraints (see also below).

The present work proposes a model which attempts to grasp the main physical and bio-mechanical difficulties implied in chord fingering. The overall approach is similar to Parncutt & colleagues’ for the case of keyboard instruments in melody fingering. The novelties of our approach are the framework in which the problem is encoded, namely the CSP framework, the application to the chord fingering problem and an experimental validation. We now introduce the CSP framework and the encoding of the fingering problem.

Chord fingering modeled as a constraint satisfaction problem

The fingering model takes in input chords described by a score in the traditional western music notation (called *common practice notation* – CPN) and returns in output all the fingerings that satisfy a set of constraints that

Table 1: The set of bio-mechanical constraints.

Set of constraints				
<i>One_note_per_string</i> : on each string it is possible to play at most one note at a time				
<i>NoOverlaps</i> : higher fingers press higher frets				
<i>Max_span_over_finger_pairs</i> : for each finger pair, there exists a maximum span that can never be exceeded:				
maxSpan	index	middle	ring	little
little	4	3	1	-
ring	3	0	-	-
middle	2	-	-	-
index	-	-	-	-

Barré_index: all the positions of the barré are on the same fret and all the other positions in the chord are in higher-numbered frets

are derived from the instrument shape and the anatomy of the hand. The problem is cast in CSP terms, where the variables are the notes indicated in the score, the domains of the variables are the fingered positions (i.e. position on the fingerboard plus a finger of the left hand) and the set of constraints that restrict the possible combinations of fingered positions. In the case several fingerings satisfy such a set of constraints, the model also provides a ranking on comfort accounts. We now introduce the problem cast in CSP and the search strategy that computes the suitable fingerings.

Problem representation

Given that CSP variables are the notes and the domains are the fingered positions, we need to define a suitable set of constraints. Based on guitar handbooks from historical composers (Aguado, 1843) and contemporary teachers (Duarte, 1966), on a review of some of the didactic opuses of the early 19th century guitar composers (Giuliani, 1812; Sor, 1815) and on our musical experience, we devised a set of bio-mechanical constraints that express which combinations of fingered positions could actually be played by a human performer (see Table 1). *One note per string* expresses the constraint that it is possible to play at most one note at a time on each string, so any position prevents from other placements on the same string; *no overlaps* ensures that higher-numbered fingers press higher-numbered frets¹; *max spans over finger pairs* expresses a maximum span of frets for each pair of fingers, that can never be exceeded². Finally, since a chord can be composed by 2 to 6 fingered positions and the available fingers are only four, we can apply the barré technique, in which a single finger can press more than one position simultaneously. We restrict the use of barré to the index finger: the constraint *Barré_index* states

¹Recall that we start numbering fingers from index, numbered 1, to little finger, numbered 4, and frets from I to XVII (see Figure 1).

²There also exists a minimum span, which is not reported here for space reasons.

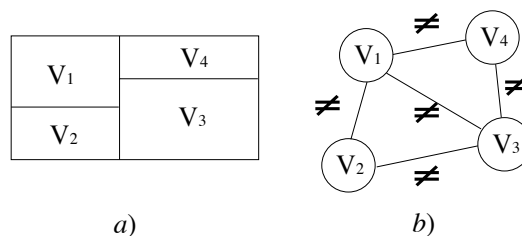


Figure 2: Frame a): The map-coloring problem, consisting of assigning a color (from a set of colors) to each region of the map, such that no two adjacent regions have the same color. Frame b): its equivalent graph.

that all the positions of the barré are on the same fret and all the other positions in the chord are in higher-numbered frets.

Given such a set of constraints, the fingering problem can be encoded as a CSP: a CSP consists of the assignment of *values* to *variables*, which satisfy a set of *constraints*. A constraint satisfaction problem $\mathcal{P} = \{\mathcal{X}, \mathcal{D}, \mathcal{C}\}$ is defined by a set of variables, X_1, X_2, \dots, X_n , and a set of constraints, C_1, C_2, \dots, C_m . Each variable X_i has a domain D_i of possible values. Each constraint C_i involves a subset of the variables and specifies the allowed combinations of values for the current subset. An assignment that satisfies all the constraints is a *consistent* assignment, and a *solution* to a CSP is an assignment to all the variables so that all constraints are satisfied (Kumar, 1992). In order to represent graphically a CSP, one can build the equivalent *constraint graph*, in which each vertex represents a variable, and each arc represents a constraint between variables standing at the end points of the arc. A typical example of CSP is the *map-coloring problem*, depicted in Figure 2: four regions on a map have to be colored in such a way that no two adjacent regions have the same color. In the CSP formulation, we have a variable for each region and its domain is the whole set of colors. For each pair of variables corresponding to adjacent regions there is a binary constraint that prevents from assigning the same value to adjacent vertices.

Our formulation of the fingering problem for chords can be stated as follows: we are given 1) a *set of variables*: the notes composing the chord; 2) a *finite and discrete domain* for each variable: from 1 up to 16 fingered positions (that is, 4 *positions* combined with the 4 available fingers) per note; 3) a *set of constraints* defined over each pair of the original set of variables plus the barré specific constraint. The goal is to find one assignment to the variables such that the assignment satisfies all the constraints; yet, if more than a unique assignment arises, the goal is to rank them in order to predict which will be preferred by human performers.

The graph in Figure 3 represents a chord fingering problem. For example, the note F2, corresponding to the variable x in the graph, could be played on $\langle 6, 1, 1 \rangle$ i.e. on the 6th string, 1st fret, by index finger; on $\langle 6, 1, 2 \rangle$, by middle finger; and so forth. Each vertex is connected to

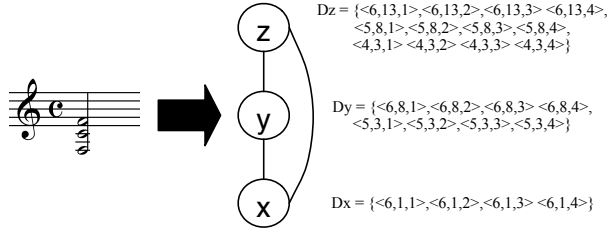


Figure 3: A chord in input and the domains related to each note of the chord. Each triple in the note domains indicates $\langle \text{string}, \text{fret}, \text{finger} \rangle$.

all the other vertices: an edge connecting two vertices, e.g. x and y , indicates that while pressing a fingered position related to y , the performer is constrained by pressing a fingered position related to x .

The search strategy

Given a representation in CSP terms, a search strategy is applied to find the assignments to variables that satisfy the constraints. A CSP search strategy propagates constraints statically to yield a simpler problem and then proceeds with a standard depth-first algorithm to ground the variables. In case the algorithm reaches a dead end, backtracking resorts to the last instantiated variable that has alternatives open.

Constraint propagation reduces the size of the variable domains while not affecting the final set of solutions. The graph of the simplified problem satisfies the properties of *arc-consistency* and *path-consistency*. Arc-consistency means that given an arc (V_x, V_y) —i.e., connecting the variable x to the variable y —, it is *arc consistent* if for every value i in D_x , there is some value j in D_y such that the instantiations $V_x = i$ and $V_y = j$ are permitted by the binary constraint between V_x and V_y . Path-consistency (or *k-consistency*) means that it is possible to instantiate up to k -variables consistently. Since path-consistency has an exponential complexity, it is customary to trade-off between the consistency degree and the computational effort via the introduction of the *directional arc-consistency* and *directional path-consistency* (Dechter, 2003). Directionality limits consistency to apply only along a given order on variables: in the case of directional path-consistency that is adopted in this paper, given the order $\langle x_1, x_2, \dots, x_n \rangle$, the requirement is that for all $i, j \leq k$ we have that $\{x_i, x_j\}$ is path-consistent relatively to x_k . Directional path-consistency is useful in the modeling of the fingering process since the search intuitively starts from one fingered position and then proceeds through the order given by the increasing pitch. Increasing pitch provides a *natural order* over the notes of a chord; the whole western classical harmony theory is grounded on this principle. The application of directional path-consistency to the chord fingering problem implies that the performer considers only a subset of the cartesian product of the fingered positions available for each note of the chord as s/he proceeds in order of increasing pitch.

In particular, the algorithm that checks the directional path-consistency scans the variables in order and retrieves, for each note, the relative domain. Once reached the k -th variable, the algorithm restricts the domains of the lower variables on the basis of the constraints that involve any $x_i \leq x_k$ and x_k ; the algorithm then iterates the restriction taking as reference the variable x_{k-1} and restricting the domains of $x_i \leq x_{k-1}$; and so on. The same iterative process is executed on pairs of linked variables, thus restricting the combined domains on the basis of the constraints with respect to a higher variable. So, the combined domains of x_i and x_j are restricted with respect to the domain of some variable $x_k \geq x_i$ and x_j . Once the graph has been made directional path-consistent, a *depth-first* search with backtracking occurs. The search follows the natural order, and starts by combining pairs of fingered positions from higher strings (namely, the basses: the sixth and fifth string in Fig. 1), lower frets and lower fingers. Underpinned by the didactic guitar literature (Aguado, 1843; Duarte, 1966), we assume that playing at the lower frets is more comfortable; yet, the first fingers (index, middle) are stronger, and then they are expected to press the strings with more ease (see also Parncutt *et al.*, 1997): the first fingerings that the search finds are expected to be more comfortable than the last ones.

So the search strategy implements a preference for comfortable fingerings, given the evidence that, in absence of higher cognitive constraints like phrasing, performers choose the bio-mechanically easiest solutions (Heijink & Meulenbroek, 2002). This is immediately applicable to cases of chord fingering on spot (out of any musical context), like in pedagogical situations. These cases are considered for the experimental validation described below.

Example

Let us consider the chord presented in Figure 3. The order of variables is $\{x, y, z\}$. The directional path-consistency algorithm performs two steps: *a*) the domain of each variable $i < z$ is revised (that is, restricted) relatively to z ; *b*) each binary constraint (combining the allowed pairs of values from i and j) R_{ij} , $i, j \leq z$ is further revised relative to z . The combinations that are still available after running the algorithm are in Table 2. **Step a:** DPC revises (that is, restricts the domain of) y relatively to z , yielding $D_y = \{ \langle 6, 8, 1 \rangle; \langle 6, 8, 2 \rangle; \langle 6, 8, 3 \rangle; \langle 5, 3, 1 \rangle; \langle 5, 3, 2 \rangle; \langle 5, 3, 3 \rangle \}$; then it revises x relative to z and x relative to y , yielding $D_x = \{ \langle 6, 1, 1 \rangle; \langle 6, 1, 2 \rangle; \langle 6, 1, 3 \rangle \}$. **Step b:** inconsistencies are deleted from R_{xy} , obtaining $R_{xy} = \{ [\langle 6, 1, 1 \rangle, \langle 5, 3, 2 \rangle]; [\langle 6, 1, 1 \rangle, \langle 5, 3, 3 \rangle] \}$. When DPC terminates, the graph is directional path-consistent relatively to the order, and the combinations of fingered positions which satisfy the constraints are presented in Table 2: on this new and simpler problem we perform the search.

The depth-first search takes the first entry of the fingered positions in R_{xy} , and searches in R_{yz} for a combination such that $y \in R_{xy} = y \in R_{yz}$. Then it is

Table 2: "Legal" combinations after DPC algorithm has terminated. The search is then performed on this simplified problem.

R_{xy}	R_{yz}	R_{xz}
$\langle 6, 1, 1 \rangle, \langle 5, 3, 2 \rangle$	$\langle 5, 3, 1 \rangle, \langle 4, 3, 1 \rangle$	$\langle 6, 1, 1 \rangle, \langle 4, 3, 2 \rangle$
$\langle 6, 1, 1 \rangle, \langle 5, 3, 3 \rangle$	$\langle 5, 3, 1 \rangle, \langle 4, 3, 2 \rangle$	$\langle 6, 1, 1 \rangle, \langle 4, 3, 3 \rangle$
	$\langle 5, 3, 2 \rangle, \langle 4, 3, 1 \rangle$	$\langle 6, 1, 1 \rangle, \langle 4, 3, 4 \rangle$
	$\langle 5, 3, 2 \rangle, \langle 4, 3, 3 \rangle$	$\langle 6, 1, 2 \rangle, \langle 4, 3, 4 \rangle$
	$\langle 5, 3, 3 \rangle, \langle 4, 3, 4 \rangle$	
	$\langle 6, 8, 1 \rangle, \langle 5, 8, 1 \rangle$	
	$\langle 6, 8, 1 \rangle, \langle 5, 8, 2 \rangle$	
	$\langle 6, 8, 2 \rangle, \langle 5, 8, 2 \rangle$	
	$\langle 6, 8, 2 \rangle, \langle 5, 8, 3 \rangle$	
	$\langle 6, 8, 3 \rangle, \langle 5, 8, 4 \rangle$	

checked whether the tuple $\{x, z\} \in R_{xz}$, (it is a solution) or not (it is a dead-end). Whenever the search reaches a dead end, it performs backtracking to the last instantiated variable that still has alternatives available. In the case exemplified, the search leads to the solutions $\{\langle 6, 1, 1 \rangle, \langle 5, 3, 2 \rangle, \langle 4, 3, 3 \rangle\}$ and $\{\langle 6, 1, 1 \rangle, \langle 5, 3, 3 \rangle, \langle 4, 3, 4 \rangle\}$.

In general, the higher the connectivity of the problem, the smaller the number of solutions, thus more notes compose the chord, the more the performer is constrained: e.g., if the entire F Major chord (F2-C3-F3-A3-C4-F4) spanning over the 6 strings was examined, a single solution would have been found, despite a widely larger number of possible combinations.

Experimental validation

To the ends of providing a first experimental validation of the model described, we have developed a computer program to evaluate the set of constraints (What is the degree of predictive power of the set of constraints? Does it actually allow to find all the fingering(s) that human performers provide? What is the relation between the model and the experts' results?) and the control strategy (Does the pruning strategy miss any solution given by the human performers? Does the control strategy consider fingered positions in the same order as the human performers?)

Material. Written fingerings of 3 guitarists, bachelor in guitar performance, were considered. Thirty-four chords composed by 3 to 6 notes were selected: they all admit at least two different fingerings. To avoid ambiguity, each fingered position had to be expressed in the notation $\langle \text{string}, \text{fret}, \text{finger} \rangle$.

Procedure. Human performers were requested to write, when possible, three different fingerings for each chord, in the order of preference. The same set of chords was given in input to the implemented model. After the guitarists ended their task, they were requested to indicate whether any fingering computed by the model was not practicable.

Predictions. On the bases of previous literature, we make the following predictions. We expect that 1) all

the fingerings computed by the model are recognized to be practicable by the experts (the set of constraints is adequate); 2) the fingerings provided by human experts (that had to indicate only three fingerings) must be a subset of those computed by the model (human performers have further constraints that are not related to biomechanical issues). 3) A weak prediction concerns the order of the control strategy in terms of strings, frets and fingers, which is consistent with the instrumental practice. In this case we predict that the highest ranked fingerings of the model and the experts coincide.

Results. 1) All the fingerings computed by the model have been found viable (100% precision), and 2) include those provided by the experts, except for overall 6 cases (over 218 fingerings computed in total by the model, so 2.75% missing – 97.25% recall); 3) on average over the three experts, the 66.6% of the highest ranked fingerings computed by the model matched the highest ranked by the experts (67.6% of the first expert, 70.6% of the second, 61.7% of the third); making the comparison over the first three fingerings, on average over the three experts 48% of the triplet found by the model matched exactly the triplet provided by the experts (47%, 47%, 50%).

Discussion and conclusion

The analysis of the results informs that the set of constraints, given the restricted conditions of the experiment that does not take into account any musical context, performs well. The 6 missed fingerings are due to a heavy restriction of the *Max.span* constraint, which forbids large stretches between the middle and ring fingers although these seem realizable by human experts. In fact, the current model does not take into account the diminishing width of frets towards the body end of the fingerboard, thus making the difficulty associated with stretching constant all along.

The control strategy only makes a weak prediction. However, this is more controversial even in the human experts data. In fact, there is a poor homogeneity of the first fingerings provided by the human performers (only in the 67% of the cases they agree on the preferred fingering). If we neglect the finger component of the triple $\langle \text{string}, \text{fret}, \text{finger} \rangle$, we can compare the results on the basis of the pairs $\langle \text{string}, \text{fret} \rangle$. Now we find that they agree in the 97% of the cases, and the model agreement raises to the 92.1%. So, the model and the human experts tend to choose the same places on the fingerboard, whereas not always they use the same fingers. Similarly, if we restrict the comparison between experts and model to the cases where the experts exactly agreed on the triplets, the success ratio of the model raises to 75%. A further analysis over the difficult cases of fingering, namely those that have received only two fingerings (instead of three) by at least one of the experts (14 over 34), reveals that the model produces exactly the same data in the 71% of cases (against the 48% above).

Several complexity factors were disregarded, which may complete the assessment of the results and help explaining the limitations of the model and suggesting a

guideline for future work. First, chords have been considered as spots, without a context, while we know that fingering is also an expressive mean (Gilardino, 1975a, 1975b; Parncutt *et al.*, 1997; Traube *et al.*, 2003): the experiment has tackled exactly this condition, but the model would perform worse in realistic conditions. Also, *tempo* plays an important role: fast tempi may determine situations of high task load, which increases the demand for economic fingerings, thus compelling performers towards 'ingrained finger habits' (Clarke *et al.*, 1997); slow tempi -and even more the case considered- would make easier achieving an intended expressive or timbral effect. Familiar fingerings may be adopted for new chords (e.g., we refer to the practice of the *transposition*, which on fretted instruments is a widespread habit). Chords may also be part of musical patterns together with melodic fragments, and such patterns may be learned, stored and retrieved as a block (Drake & Palmer, 2000). This may help explaining some differences, where fingerings provided by performers under the exclusive bio-mechanical aspect are not always preferable. Lastly, it is reasonable to suppose that skilled performers are able to distinguish between more/less salient constraints; for the present, a preference for some sorts of fingerings is implemented by the search strategy.

Although in this paper we report experimental data for the chord fingering subproblem, we are confident that the CSP approach can be successful on modeling the whole of the fingering problem. Future work will address the merging of chord and melody fingering, which leads to consider real pieces from the classical repertoire, and to explore the automatic expressive performance, where the modeling of instrument/performer interactions plays a central role.

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