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Circular-Fourier–radial-Mellin transform descriptors for pattern recognition

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Image descriptors based on a circular-Fourier–radial-Mellin transform are proposed. They are invariant with respect to rotation, translation, and change of scale. They represent a generalized approach to specific descriptors using a circular-harmonic expansion, a Mellin transform, or moment invariants. The possibility of computing the Fourier–Mellin descriptors by using an optical processor in real time is discussed.

1. INTRODUCTION

The definition of invariant image features is of interest in the context of pattern recognition. Moment invariants[1–3] do not depend on image translation, magnification, and rotation: Hu's moments[1] were derived using the theory of algebraic invariants in rectangular coordinates; similar results were obtained using Zernike circular polynomials.[4] As complex combinations of moments in rectangular coordinates, the physical meaning of moment invariants is not clear, except in some low-order cases. As far as rotation invariance is concerned, an obvious alternative is to use polar coordinates.[5–7] The circular-Fourier–radial-Mellin transform takes advantage of the properties of the Fourier and Mellin[8] transforms in order to define a new set of image invariants called Fourier–Mellin descriptors (FMD's). In this paper we generalize the work performed by Casasent[7] in two ways. First, a general Mellin transform with complex parameters is introduced. Second, invariant image descriptors of general rank are used instead of second-order descriptors.

2. DEFINITION OF FOURIER-MELLIN DESCRIPTORS

Let $g(r, \theta)$ be the irradiance distribution in a two-dimensional image, expressed in polar coordinates. Different image transformations can be applied that will permit a further definition of invariant image descriptors:

Circular harmonic-expansion[5,6]

$$g_M(r) = \int_0^{2\pi} g(r, \theta)\exp(-jM\theta)\mathrm{d}\theta, \tag{1}$$

which leads to rotation- and translation-invariant descriptors.

Mellin–Fourier transform of imaginary order[7,8]

$$M(\omega_\rho, \omega_\theta) = \int_{-\pi}^{\pi}\int_0^{\infty} g'(\rho, \theta)\exp[-j(\rho\omega_\rho + \theta\omega_\theta)]\mathrm{d}\rho\mathrm{d}\theta, \tag{2}$$

with

$$\rho = \ln(r),$$

where $g'(\rho, \theta)$ denotes the image function $g(r, \theta)$ scaled by a radial logarithmic transformation. Rotation and scale invariance are involved.

Radial and angular moments[9]

$$\psi(k, p, q, g) = \int_0^{\infty}\int_{-\pi}^{\pi} r^k \cos^p(\theta)\sin^q(\theta)g(r, \theta)\mathrm{d}r\mathrm{d}\theta, \tag{3}$$

which permits the definition of rotation, translation, and scale invariants.

These three specific definitions can be generalized using a single transformation. We propose FMD's, $M_{s,l}$, expressed as

$$M_{s,l} = \int_0^{\infty}\int_{-\pi}^{\pi} r^{s-1}g(r, \theta)\exp(-jl\theta)\mathrm{d}r\mathrm{d}\theta, \tag{4}$$

where s denotes a constant (possibly complex) and l is the angular frequency of the image. This expression shows an explicit radial-Mellin transform with parameter s and an explicit circular-Fourier transform with parameter l. The selection of the rank of the descriptors $M_{s,l}$ can therefore be discussed in terms of the spectral content of the images. Moreover, a possible optical implementation of the transform is suggested by its analytic form. Obviously descriptors invariant with respect to rotation, translation, and change of scale are also derived.

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3. PROPERTIES

Rotation Invariance

According to the properties of the circular harmonic expansion[5] with respect to the variable θ, a rotation of the image by an angle ϕ induces a phase shift $\exp(-jl\phi)$ of the FMD's. The rotation invariance can be achieved by taking the moduli of the images' FMD's.

Scale Invariance

Let $g(\alpha r, \theta)$ be the image expanded by the factor α. The radial-Mellin transform gives new FMD's that are multiplied by α^{-s}. This term does not depend on the angular variable: any FMD that does not depend on this variable will help to define a scale-normalization factor. The total energy of the image, expressed as

$$\int_0^\infty \int_{-\pi}^{\pi} rg(r, \theta)\mathrm{d}r\mathrm{d}\theta, \tag{5}$$

is multiplied by α^{-2} when the expansion factor is applied. This energy is equal to $M_{2,0}$; consequently this FMD contains the information for scale normalization.

Translation Invariance

Translation invariance is conventionally achieved by putting the origin of coordinates at the image centroid.[1,9] The practical implementation of translation invariance, using an optical system, is discussed below.

Normalized Fourier-Mellin Descriptors

According to the above properties, FMD's can be made invariant to rotation, translation, and change of scale by

$$\phi_{s,l} = |M_{s,l}|^2/|M_{2,0}^s|. \tag{6}$$

Table 1 illustrates the efficiency of the normalized FMD's with the example of a digitized black-and-white square and its rotated, scaled, and translated versions.

Choosing the Radial and Circular Orders

When defining the FMD's, the exponent s of the Mellin transform can be an arbitrary complex number. Casasent[8] used $s = -j\omega$. Reddi[9] proposed integer or real values for s. From a physical point of view it can be added that the successive integer values of s correspond to the energy ($s = 1$), the centroid abscissa ($s = 2$), and the gyration radius ($s = 3$) of the radial image distribution with respect to the image energy centroid. The weight of the peripheral regions of the image is determined by the value of s ($s \geqslant 1$). Therefore no general rule for the value of s can be derived, the image structure being relevant.

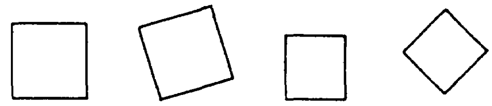
The angular frequency l can be discussed after integrating the radial variable r. In this case the FMD's are nothing but the Fourier transform of

$$G(s, \theta) = \int_0^\infty r^{s-1}g(r, \theta)\mathrm{d}r, \tag{7}$$

which are angular distributions with period 2π. One deals in fact with a discrete Fourier expansion, and the angular frequency takes discrete values. In many applications the

Table 1. Invariance of the Normalized FMD's of the Same Image under Rotation, Translation, and Change of Scale

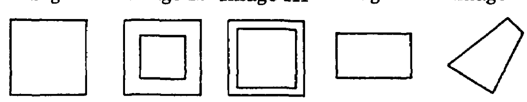
	Image I (original)	Image II (rotated)	Image III (scaled, shifted)	Image IV (scaled, rotated)



FMD	Image Number I	II	III	IV
$\Phi_{1,0}$	12.2950	12.3010	12.2640	12.2570
$\Phi_{1,1}$	0.0002	0.0002	0.0003	0.0003
$\Phi_{1,2}$	0.0000	0.0002	0.0000	0.0000
$\Phi_{1,3}$	0.0000	0.0000	0.0000	0.0000
$\Phi_{1,4}$	0.0600	0.0555	0.0600	0.0601
$\Phi_{1,5}$	0.0000	0.0000	0.0000	0.0000
$\Phi_{2,0}$	1.0000	1.0000	1.0000	1.0000
$\Phi_{2,1}$	0.0000	0.0000	0.0000	0.0000
$\Phi_{2,2}$	0.0000	0.0000	0.0000	0.0000
$\Phi_{2,3}$	0.0000	0.0000	0.0000	0.0000
$\Phi_{2,4}$	0.0199	0.0185	0.0198	0.0202
$\Phi_{2,5}$	0.0000	0.0000	0.0000	0.0000
FMD	I	II	III	IV
$\Phi_{3,0}$	0.1463	0.1461	0.1462	0.1465
$\Phi_{3,1}$	0.0000	0.0000	0.0000	0.0000
$\Phi_{3,2}$	0.0000	0.0000	0.0000	0.0000
$\Phi_{3,3}$	0.0000	0.0000	0.0000	0.0000
$\Phi_{3,4}$	0.0067	0.0062	0.0066	0.0068
$\Phi_{3,5}$	0.0000	0.0000	0.0000	0.0000
$\Phi_{4,0}$	0.0277	0.0276	0.0277	0.0278
$\Phi_{4,1}$	0.0000	0.0000	0.0000	0.0000
$\Phi_{4,2}$	0.0000	0.0000	0.0000	0.0000
$\Phi_{4,3}$	0.0000	0.0000	0.0000	0.0000
$\Phi_{4,4}$	0.0023	0.0021	0.0023	0.0023
$\Phi_{4,5}$	0.0000	0.0000	0.0000	0.0000

Table 2. Normalized FMD's of Five Images[a]

	Image I	Image II	Image III	Image IV	Image V



FMD	Image Number I	II	III	IV	V
$\Phi_{1,0}$	12.2140	3.0591	1.3649	11.590	11.5030
$\Phi_{1,1}$	0.0003	0.0000	0.0000	0.0054	0.0417
$\Phi_{1,2}$	0.0000	0.0000	0.0000	0.2276	0.1629
$\Phi_{1,3}$	0.0000	0.0000	0.0000	0.0007	0.1443
$\Phi_{1,4}$	0.0583	0.0150	0.0068	0.0062	0.0134
$\Phi_{1,5}$	0.0001	0.0001	0.0000	0.0002	0.0220
$\Phi_{1,6}$	0.0000	0.0000	0.0000	0.0179	0.0046
$\Phi_{1,7}$	0.0000	0.0000	0.0000	0.0001	0.0038
$\Phi_{1,8}$	0.0063	0.0015	0.0007	0.0024	0.0028
$\Phi_{1,9}$	0.0000	0.0000	0.0000	0.0001	0.0000
$\Phi_{1,10}$	0.0000	0.0000	0.0000	0.0009	0.0001

[a] The first three FMD sets present the same shapes as those of the corresponding images. Nonzero FMD's appear as the angular image shape departs from that of a square. The $\Phi_{1,0}$ is associated with the thickness of the image contour.

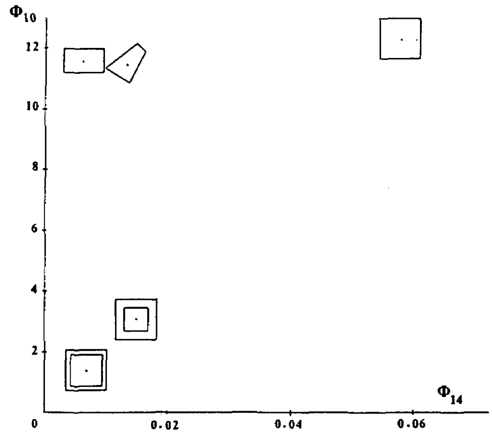


Fig. 1. Mapping of the five images of Table 2 in the plane of two of their FMD's. They are clustered according to their shape and thickness.

directional content and the angular periodicities are used for image classification.[10–12] They can be assessed from the image autocorrelation function.[13] The radial-Mellin transform does not affect the angular symmetry about the image centroid. When the $G(s, \theta)$ function is a multimodal angular distribution[14] defined in the interval $(0, 2\pi/n)$, where n denotes the number of modes, its angular Fourier coefficients take the discrete values $l = np$ $(p = 0, 1, 2 \ldots)$. The selection of dominant angular frequencies leads to the choice of significant FMD's. An illustration is given Table 2, which shows the FMD's of a square, two square frames, a rectangle, and a quadrilateral. In the first three cases the angular shapes of the images are similar; the corresponding FMD sets present the same variations. The last two images are characterized by different FMD sets. These five images are well classified and clustered in the feature space spanned by the $(\phi_{1,0}, \phi_{1,4})$ FMD's as shown Fig. 1.

Relationships with Other Invariant Descriptors

FMD's are related to Hu's moment invariants,[1] which consists of seven functions of the moments m_{pq} of an image, computed in Cartesian coordinates:

$$m_{pq} = \int\int_{-\infty}^{\infty} x^p y^q g(x, y) \mathrm{d}x \mathrm{d}y. \quad (8)$$

A simple calculation shows that Hu's invariants appear as particular $\phi_{s,l}$ (for $s = 4, 5$ and $l = 0, 1, 2, 3$). For a given image the lowest (nonzero) Hu moments begin with a weighting function of the form r^2, while nonzero FMD's begin with r^0, inducing a low computing error.[4]

When the image is a contour (i.e., a plane closed curve) the $M_{1,l}$ FMD's are

$$M_{1,l} = \int_{-\pi}^{\pi} r(\theta) \exp(jl\theta) \mathrm{d}\theta, \quad (9)$$

where $r(\theta)$ represents the contour. These FMD's are identical to the Fourier descriptors for plane closed curves defined in Refs. 10 and 15. Their moduli were shown to be invariant to rotation and change of scale, which is in agreement with the general properties of the FMD's.

4. PRINCIPLE OF OPTICAL GENERATION OF FOURIER-MELLIN DESCRIPTORS

The FMD's of an image can be computed by digital computer, but the computational load is heavy and the result is not obtained in real time. As to the specific descriptors mentioned in Section 2, several optical methods for moment generation with coherent light have been proposed.[16–18] They apply only to the case of rectangular coordinates. Reference 7 described how a geometrical transformation from Cartesian coordinates (x, y) to other rectangular coordinates $[\ln(r), \theta]$ was implemented by electronic processing, giving a FMD of imaginary order $(s = -j\omega)$. We have described a multichannel incoherent optical processor[19] that computes the Hu moments of a TV image in real time. This device is also suited to the generation of FMD's: A video system permits a real-time input of the image and a real-time preprocessing that determines the image centroid. The input image is imaged onto a set of masks whose transparencies code the real and imaginary parts of the kernel of the FMD transform. The FMD's are available after integration of the light flux passing the masks.

5. CONCLUSION

We have proposed new image descriptors based on the circular-Fourier–radial-Mellin transform. They are independent of image translation, change of scale, and rotation. Some image invariants previously described in the literature have been noted as particular cases. The new descriptors permit images to be classified according to their shapes. They can be generated with a video optical processor using optical masks.

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