Neuromechanics of Human Motion

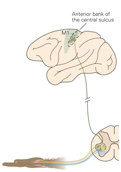
Muscle Models

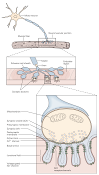
Joshua Cashaback, PhD

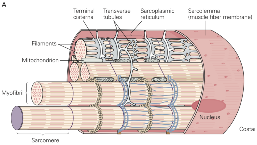
Recap — Motor Units

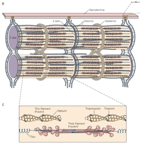
Motor Unit

A motor neuron and the skeletal muscle fibers innervated by that motor neuron's axonal terminals.

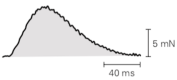


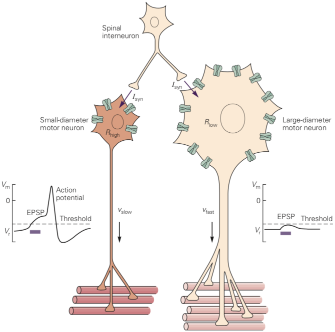


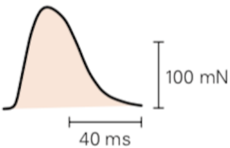




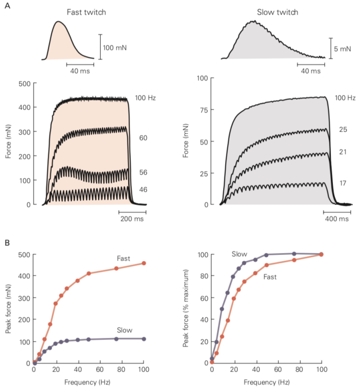
Recap — Recruitment

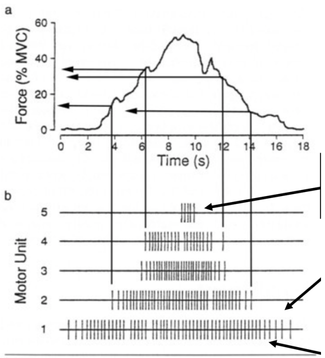






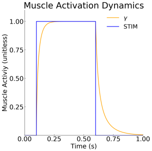
Recap — Discharge Rate

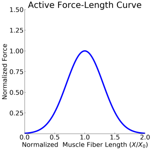


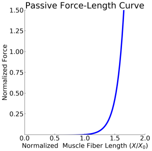


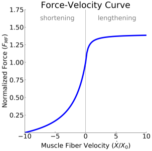
UNIVERSITY OF DELAWARE

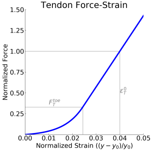
Recap — Muscle Dynamics

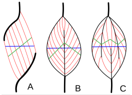












Lecture Objectives — Muscle Models

1. Hill-Type Model
   a. phenomenological
2. Distribution Moment Approximation Model
   a. mechanistic
   b. more states
   c. emergent behaviour

Hill-Type Models

. Thelen, D. G. (2003). J Biomech Eng, 125(1), 70-77.

. Zajac, F. E. (1989). Crit. Rev. Biomed. Eng., 17(4), 359-411.

. Buchanan, T. S., et al. (2004). Jbiomech, 20(4), 367-395.

Hill Models — ODEs

$$\dot{\gamma} = \frac{(STIM - \gamma)}{\tau} \tag{1}$$

$$\frac{\dot{X}}{X_0} = (0.25 + 0.75\gamma) V_{MF}^{max} \frac{F_{MF} - \gamma\alpha}{b} \tag{2}$$

Hill Models — ODEs

$$\dot{\gamma} = \frac{(STIM - \gamma)}{\tau} \tag{1}$$

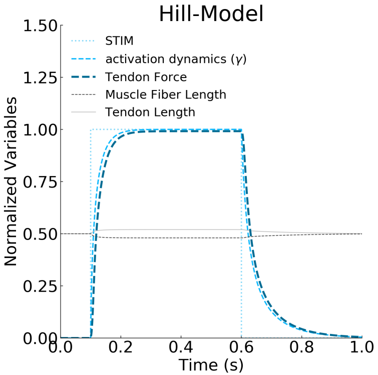
$$\frac{\dot{X}}{X_0} = (0.25 + 0.75\gamma) V_{MF}^{max} \frac{F_{MF} - \gamma\alpha}{b} \tag{2}$$

1. Inputs: STIM, Muscle-Tendon Length
2. Outputs: Tendon Force
3. IC: $X/X0$, γ
4. Unknown: F_{MF}

Hill Models — Steps

1. Find α (Active Force Length Curve)
2. Find Y (use $L_{MT} = Y + Xcos(\theta)$; since X and L_{MT} known)
3. Use Y to calculate F_T (Tendon Strain-Force Relationship)
4. Use X to calculate F_{PE} (Parallel Elastic Curve)
5. Find F_{MF} ($F_T = F_{MF} cos\theta + F_{PE} cos\theta$; F_T and F_{PE} known)
6. Now you can solve equations 1 and 2 in slide above!
7. Tip: keep track of X vs $X/X0$ and make $STIM > 0.0001$

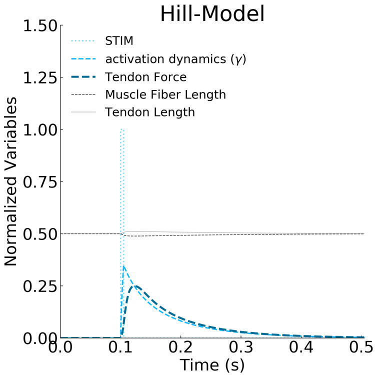
Hill Models — Max Stim



$X = 0.5, (X0 = 0.5), Y = 0.5(Y0 = 0.5), L_{MT} = 1.0$

$\gamma 0 = 0.0001, \theta = 0$

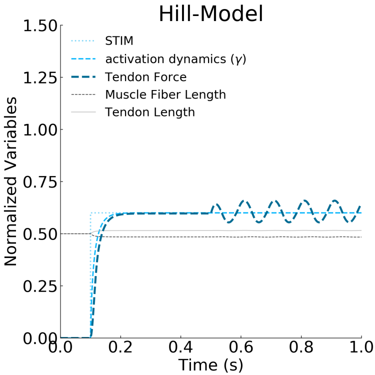
Hill Models — Twitch Force



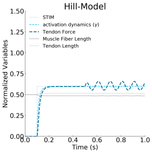
$X = 0.5, (X0 = 0.5), Y = 0.5(Y0 = 0.5), L_{MT} = 1.0$

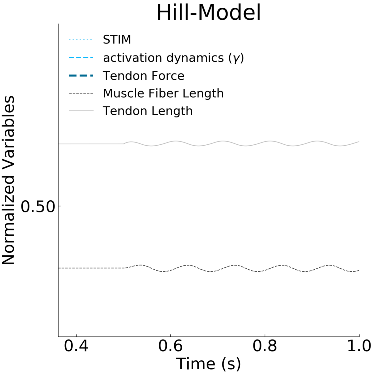
$\gamma 0 = 0.0001, \theta = 0, STIM = 5ms$ unit pulse

Hill Models — Oscillating MT unit



Hill Models — Oscillating MT unit





Distribution Moment Approximation Model

. Zahalak, G. I. (1981). Mathematical biosciences, 55(1-2), 89-114.

. Zahalak GI. (1986) Journal of Biomechanical Engineering, 108:131-140.

. Ma, S., & Zahalak, G. I. (1991). Journal of biomechanics, 24(1), 21-35.

Why a Cross-Bridge Model?

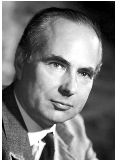
More Macroscopic Variables of Interest

- Length
- Stiffness
- Force
- Energy
- Heat Generation

Emergent Phenomena

- Nonexistence of a Unique Force-Velocity Relationship
- Decrease in Force During Oscillation
- Yeilding

Andrew F. Huxley's Cross-Bridge Theory



$$\left(\frac{\partial n}{\partial t}\right)_x - v(t)\left(\frac{\partial n}{\partial x}\right)_t$$

- cross-bridges either bound or unbound
- cross-bridge force $\propto$ to displacement x (i.e., spring)
- $n(x,t)$ fraction of bound CBs with displacement x at time t.
- $v(t)$ velocity of half sacromere

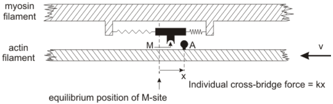
Cross-Bridge Theory — Rate Parameters

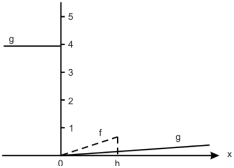
$$\left(\frac{\partial n}{\partial t}\right)_x - v(t)\left(\frac{\partial n}{\partial x}\right)_t = f(x) - [f(x) + g(x)]\, n$$

f(x) = binding rate (e.g., Ca2+ attaches to troponin)

g(x) = unbinding rate (e.g, ATP attaches to myosin)

Cross-Bridge Theory — Rate Parameters



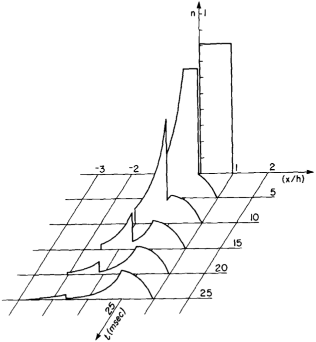


h = ideal length (fastest binding); $+x$ = long ; $-x$ = short

force velocity curve?

a. lengthening from 0 ($+x$ direction) $= f > g_1$ (more force!)
b. shorting from h ($-x$ direction) $= f : g_1$ ratio decrease, then g_2 dominates = (less force!)

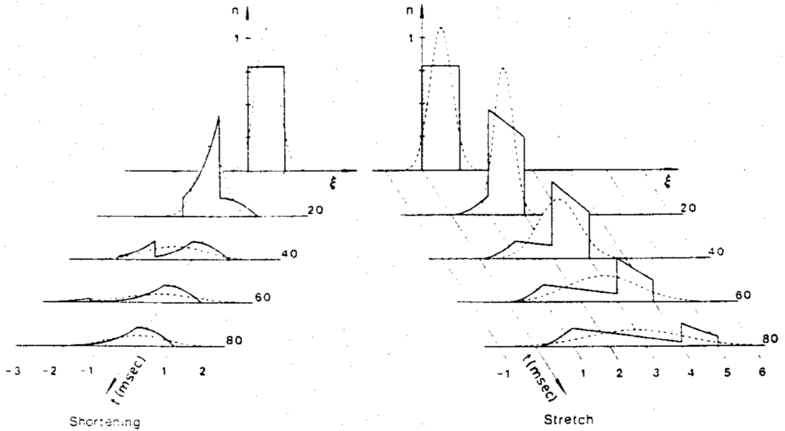
$n(x, t)$ — Huxley Model



1. start simulation with most bound cross-bridges
2. muscle shortens with time (moves leftward)
3. number of bound cross bridges decreases

University of Delaware

$n(x,t)$ — Normal Approximation



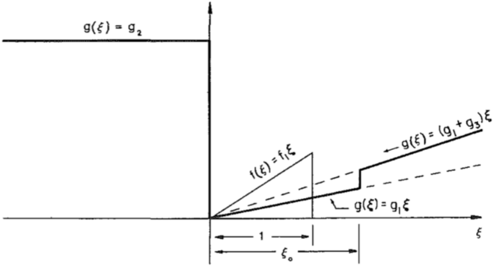
Method of Moments — Q

$$Q_\lambda = \int_{-\infty}^{\infty} x^\lambda n(x,t)dx$$

- $Q_0 =$ stiffness, zero-moment (area, bound cross bridges; $-k$)
- $Q_1 =$ force, first-moment (mean; $-kx$)
- $Q_2 =$ energy, second-moment (variance; $1/2kx^2$)

If we know the number of bound CBs and their linear stiffness, we can estimate Q_0, Q_1, and Q_2 of any muscle

Additional Rate Parameter



Zahalak (1981) — ODE

$$\dot{Q}_0 = \beta_0 - \phi_0(Q_0, Q_1, Q_2)$$
$$\dot{Q}_1 = \beta_1 - \phi_1(Q_0, Q_1, Q_2) - v(t)Q_0$$
$$\dot{Q}_2 = \beta_2 - \phi_2(Q_0, Q_1, Q_2) - 2v(t)Q_1$$

See paper for full (long) derivation

Zahalak (1981)

$$p(Q_0, Q_1) = \frac{Q_1}{Q_0} \quad \text{and} \quad q(Q_0, Q_1, Q_2) = \sqrt{\frac{Q_2}{Q_0} - \left(\frac{Q_1}{Q_0}\right)^2}.$$

$$\beta_0 = \frac{f_1}{2.0}$$

$$\beta_1 = \frac{f_1}{3.0}$$

$$\beta_2 = \frac{f_1}{4.0}$$

Zahalak (1981) — mind your p's and q's

$$\Phi(\tau) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\tau} e^{-\zeta/2} d\zeta = \text{error function}$$

probability x will occur within some range

$\tau = (1 - p)/q$ OR $\tau = -p/q$ (see next slide)

Error Function in Python:

```
tau_a = (1 - p) / q
tau_b = -p / q
Phi_a = 0.5 * (1 + math.erf((tau_a) / (2**0.5)))
Phi_b = 0.5 * (1 + math.erf((tau_b) / (2**0.5)))
```

Zahalak (1981)

$$J_0(\tau)=\Phi(\tau)$$

$$J_1(\tau)=p\Phi(\tau)-q\frac{e^{-\tau^2/2}}{\sqrt{2\pi}}$$

$$J_2(\tau)=p^2\Phi(\tau)-2pq\frac{e^{-\tau^2/2}}{\sqrt{2\pi}}+q^2\left\{\Phi(\tau)-\frac{\tau e^{-\tau^2/2}}{\sqrt{2\pi}}\right\} \tag{A-11}$$

$$J_3(\tau)=p^3\Phi(\tau)-3p^2q\frac{e^{-\tau^2/2}}{\sqrt{2\pi}}+3pq^2\left\{\Phi(\tau)-\frac{\tau e^{-\tau^2/2}}{\sqrt{2\pi}}\right\}-q^3(2+\tau^2)\frac{e^{-\tau^2/2}}{\sqrt{2\pi}}$$

Zahalak (1981)

$$
\begin{aligned}
\frac{\phi_0}{Q_0} &= g_2 J_0\left(-\frac{p}{q}\right) + (f_1+g_1)\left[J_1\left(\frac{1-p}{q}\right) - J_1\left(-\frac{p}{q}\right)\right] \\
&\quad + g_1\left[p - J_1\left(\frac{1-p}{q}\right)\right] + g_3\left[p - J_1\left(\frac{1-p}{q}\right) - 1 + J_0\left(\frac{1-p}{q}\right)\right], \\
\frac{\phi_1}{Q_0} &= g_2 J_1\left(-\frac{p}{q}\right) + (f_1+g_1)\left[J_2\left(\frac{1-p}{q}\right) - J_2\left(-\frac{p}{q}\right)\right] \\
&\quad + g_1\left[p^2+q^2 - J_2\left(\frac{1-p}{q}\right)\right] + g_3\left[p^2+q^2 - J_2\left(\frac{1-p}{q}\right) - p + J_1\left(\frac{1-p}{q}\right)\right], \\
\frac{\phi_2}{Q_0} &= g_2 J_2\left(-\frac{p}{q}\right) + (f_1+g_1)\left[J_3\left(\frac{1-p}{q}\right) - J_3\left(-\frac{p}{q}\right)\right] \qquad \text{(A-12)} \\
&\quad + g_1\left[p^3+3pq^2 - J_3\left(\frac{1-p}{q}\right)\right] \\
&\quad + g_3\left[p^3+3pq^2 - J_3\left(\frac{1-p}{q}\right) - (p^2+q^2) + J_2\left(\frac{1-p}{q}\right)\right].
\end{aligned}
$$

Remember, J's are a function of τ (not multiplication!)

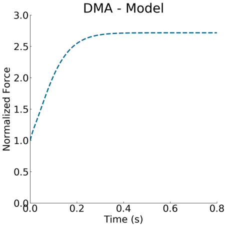
Don't forget to place Q_0 on other side of equation

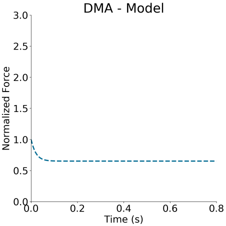
Zahalak (1981) — ODE

$$\begin{aligned}
\dot{Q}_0 &= \beta_0 - \phi_0(Q_0, Q_1, Q_2) \\
\dot{Q}_1 &= \beta_1 - \phi_1(Q_0, Q_1, Q_2) - v(t)Q_0 \\
\dot{Q}_2 &= \beta_2 - \phi_2(Q_0, Q_1, Q_2) - 2v(t)Q_1
\end{aligned}$$

See paper for full (long) derivation

Zahalak (1981) — Constant Velocity





$f1(43.4), g1(10.0), g2(209.0), g3(0.0)$

$IC: Q_0^0(0.684), Q_1^0(0.438), Q_2^0(0.322)$

left figure, $v(t) = -10$ (lengthening)

right figure, $v(t) = 10$ (shortening)

normalize outputs by initial condition (e.g., $Q_1(t)/Q_1^0$)

Zahalak (1986) — Non-Unique FV curve

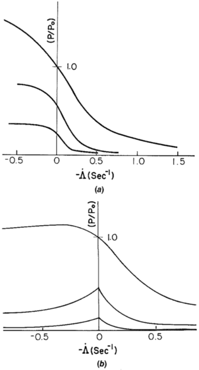
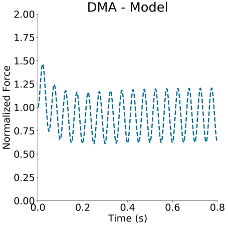
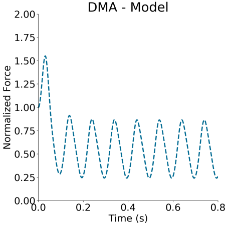
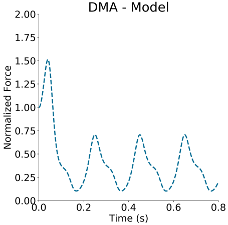


Fig. 3 "Force-velocity relations" in isotonic vs. isovelocity experiments, according to the D-M model with length independent parameters. In all cases $g_1 = 7\ s^{-1}$, $g_2 = 200\ s^{-1}$, $g_3 = 30\ s^{-1}$, $\xi_o = 1.1$, $\alpha = 1.0$, $\gamma = 0.028$, and $\kappa = 0.040$. (*a*) Isotonic experiment: the velocity was measured 15 ms after an imposed constant increase or decrease in the muscle force. Each curve is for a constant value of f_1: upper—$f_1 = 35\ s^{-1}$, middle—$f_1 = 7\ s^{-1}$, lower—$f_1 = 2\ s^{-1}$. (*b*) Isovelocity experiment: the force was measured after it had attained its steady-state value when the muscle was shortened or lengthened at various constant velocities. Each curve is for a constant value of f_1: upper—$f_1 = 35\ s^{-1}$, middle—$f_1 = 5\ s^{-1}$, lower—$f_1 = 1\ s^{-1}$.

Zahalak (1981) — Oscillating Muscle

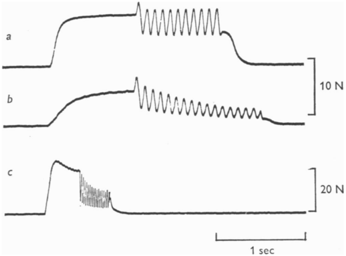






$f1(1.0), g1(10.0), g2(210.0), g3(100.0)$
$IC : Q_0^0(0.0712), Q_1^0(0.0436), Q_2^0(0.0295)$
$v(t) = -25 sin(2\pi t/T)$, where t = current time in seconds
T is equal to 0.05, 0.1, 0.2 for the left, center, right figures, respectively

Rack & Westbury (1974) v Zahalak (1986)



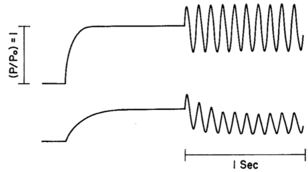


Fig. 4 Force response of the D-M model, with length-independent parameters, to small-amplitude sinusoidal length perturbations. In both cases $g_1 = 7\ s^{-1}$, $g_2 = 2000\ s^{-1}$, $g_3 = 30\ s^{-1}$, $\xi_o = 1.1$, $\alpha = 1.0$, $\gamma = 0.028$, and $\kappa = 0.04$. The muscle was relaxed with $f_1 = 0$ at the beginning of each "experiment", and then f_1 instantaneously changed to a constant value, $-35\ s^{-1}$ for the upper curve and $7\ s^{-1}$ for the lower curve. The muscle was allowed to contract isometrically for one second, after which a small-amplitude sinusoidal length oscillation was imposed on the muscle, of the form $\delta\Lambda(t) = 0.016 \sin(2\pi t/T)$ with $T = 0.1$ s.

Ma and Zahalak (1991) — ODE

$$\dot{Q}_0 = r\alpha(X)\beta_0 - r\phi_{10}(Q_0, Q_1, Q_2) - \phi_{20}(Q_0, Q_1, Q_2) \tag{1a}$$

$$\dot{Q}_1 = r\alpha(X)\beta_1 - r\phi_{11}(Q_0, Q_1, Q_2) - \phi_{21}(Q_0, Q_1, Q_2) - u(t)Q_0(t) \tag{1b}$$

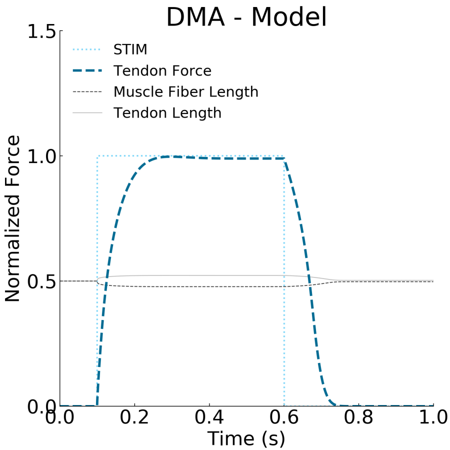
$$\dot{Q}_2 = r\alpha(X)\beta_2 - r\phi_{11}(Q_0, Q_1, Q_2) - \phi_{22}(Q_0, Q_1, Q_2) - 2u(t)Q_1(t) \tag{1c}$$

$$\dot{\Lambda} = \dot{Y} + \dot{X} = \frac{\kappa(Q_1(t))}{L_0}\dot{Q}_1 - \frac{2h}{l_{s0}} \cdot \frac{X_0}{L_0}u(t) \tag{1d}$$

Accounts for more states during cross-bridge cycling (thus additional ϕ terms)

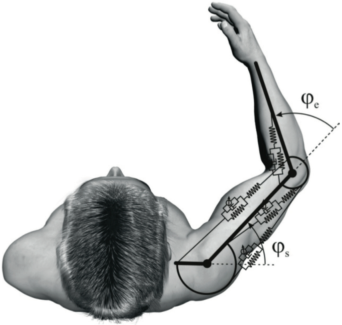
$u(t)$, velocity of a half sarcomere, can be solved by substituting $1b$ and $1d$ into one another

DMA — Max Stim



DMA - Model
Normalized Force
Time (s)
STIM
Tendon Force
Muscle Fiber Length
Tendon Length
1.5
1.0
0.5
0.0
0.0
0.2
0.4
0.6
0.8
1.0

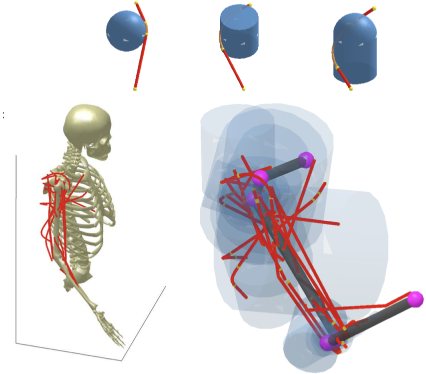
2D Arm Model



Schematic of "full-blown" musculoskeletal model described in Kistemaker et al. (2010).

$$\frac{\partial L_{MT}}{\partial \varphi} = ma$$

3D Arm Model



Summary

1. Hill Models
    a. combine equations to solve muscle force (F_{MF})
2. Cross-Bridge Models
    a. More macroscopic variables
    b. Emergent phenomena

Questions???

Assignment 3

see handout

Next Class

1. Kinematics
   - two-link arm
   - forward kinematics
   - inverse kinematics
   - Jacobians

Acknowledgements

Dinant Kistemaker