ASSIGNMENT 1 — DYNAMICAL SYSTEMS AND MUSCLE SPINDLES

For all questions below, show your work (e.g., use an equation editor), as well as providing all programming code and plots in the report. Total marks (undergrad 30 | graduate 40).

1. $dy/dt = y - t^2 + 1$; Initial conditions (IC): $y_0 = 0.5, t_0 = 0$; $h = 0.5$; (6 marks)

   a. Solve this equation for 3 time steps using Euler integration by hand and show with a table (e.g., $n|t_n|y_n|f_n|h \cdot f_n|y_{n+1}$). (3 marks)

   b. Solve this equation for 3 time steps using RK4 integration by hand and show with a table (e.g., $n|t_n|y_n|k1t|k1y|...|k4y|y_{n+1}$). (3 marks)

2. The following set of coupled differential equation are known as the Lotka-Volterra equations, which can be used to model predator-prey relationships in nature:

$$dx/dt = \alpha x - \beta xy$$

$$dy/dt = \delta xy - \gamma y$$

x and y represent the population of the prey and predator, respectively. Use $\alpha = 1.0$, $\beta = 0.5$, $\delta = 0.5$, and $\gamma = 1.0$, length of time (60 days), step size (0.001 days), and initial conditions for your prey ($x_0 = 20$) and predator ($y_0 = 10$). (6 marks | 9 marks)

   a. Find the solution to this set of equations with Euler integration (**i.e., do not use the built in integrator**). Plot the states over time. Include a title, x-label, y-label, and legend in your plot. Remember to include the code for Euler integration in your report. (5 marks)

   b. Describe what you observe in terms of the predator-prey relationship over time. (1 mark)

   c. Describe what each term in the equation represents. **(Graduates only)** (1 mark)

   d. What are potential limitations of the model? **(Graduates only)** (1 mark)

   e. Increase the value of α to 2.0. What happens and why? **(Graduates only)** (1 mark)

3. For the following 2nd order differential equation: $3\ddot{x} - 4\dot{x} + x = 0$ (3 marks)

   a. Convert into a system of 1st order, ODEs. (2 marks)

   b. Express in matrix form. (1 mark)

4. For the following mass-damper-spring differential equation: $m\ddot{x} = -b\dot{x} - kx + mg$. Start off by using $k = 1.5$, $b = 0.5$, and $m = 1.5$, length of time of 50 seconds, step size of 0.01 seconds, and initial conditions for position and velocity ($x = 0.0, \dot{x} = 0.0$). (7 marks | 10 marks)

a. Convert into a system of 1st order, ODEs. (2 mark)

b. Solve using a built-in numerical integrator (e.g., odeint in Python). Plot the states over time and state-space plots. Include a title, x-label, y-label, and legend in your plot. (5 marks)

c. What is the undamped angular frequency, $w_0 = \sqrt{\frac{k}{m}}$, of your system? **(Graduates only)**. (1 mark)

d. Calculate, $\zeta = \frac{b}{2\sqrt{mk}}$, to find out whether your system is overdamped ($\zeta > 1$), critically damped ($\zeta = 1$), or underdamped ($\zeta < 1$). **(Graduates only)** (1 mark)

e. Change the b in your system such that it becomes critically damped and replot your states over time and state-space plots. What do you notice? **(Graduates only)**. (1 mark)

5. Perform Euler integration to solve a muscle spindle model. (5 marks | 7 marks)

a. Use the constant parameter values from the lecture slides. Initial Conditions: $t_0, x_0, \mu_0 = 0.0, 0.0374, -5.0$

b. Integrate the following muscle velocity to get muscle length, where dx/dt: 0 to 0.5 s = 0 mm/s, 0.5 to 5.5 s = 1 mm/s, 5.5 to 7.5 s = 0mm/s. Plot muscle length over time. (1 mark).

c. Integrate $d\mu/dt$, calculate r, and then plot both over time. (4 marks)

d. Is this muscle spindle sensitive to velocity. Explain. **(Graduates only)**. (1 mark)

e. Is this muscle spindle sensitive to position. Explain. **(Graduates only)**. (1 mark)

6. Additional questions to be answered independently (3 marks | 5 marks)

a. Why do colorblind people interpret color differently? 1 mark.

b. Can we determine the direction of gravity from the otoliths? 1 mark.

c. How might the stretch reflex and reciprocal inhibition be useful to prevent injury? 1 marks.

d. Name three states that a musculotendon unit provides feedback of? **Graduate Only.** 1 marks.

e. Which skin receptors would be useful for reading braille? **Graduate Only.** 1 mark.