ASSIGNMENT 4 — KINEMATICS AND DYNAMICS

For all questions below, provide all programming code and plots in the report. (undergrads 12 marks | graduates 25 marks)

KINEMATICS

1. For a 1-link arm, assume $l = 0.46(m)$, $\theta = 60°$, $\dot{\theta} = 2(rads/s)$, $\ddot{\theta} = 1(rads/s^2)$. Use Python (or other program) to calculate (3 marks):

   a. Hand Position. 1 mark

   b. Hand Velocity. 1 mark

   c. Hand Acceleration. 1 mark

   d. Note: make sure you work through taking the derivates by hand to find $J(\theta)$ and $J(\dot{\theta})$

2. Using your calculations from 1a, 1b, and 1c, use inverse kinematics to retrieve (3 marks):

   a. Joint Angle (deg). 1 mark

   b. Angular Velocity (rad/s). 1 mark

   c. Angular Acceleration (rad/s^2). 1 mark

3. For a 2-link arm, assume $l_1 = 0.34$, $l_2 = 0.46$, $H_x = 0.36$, $H_y = 0.65$, $\dot{H}_x = -3.89$, $\dot{H}_y = 1.30$, $\ddot{H}_x = -7.79$, $\ddot{H}_y = -26.18$. Use Python (or other program) to calculate (**Graduate Only**) (4 marks):

   a. Joint Angle (deg). 2 marks

   b. Angular Velocity (rad/s). 1 marks

   c. Angular Acceleration (rad/s^2). 1 mark

   d. Note: make sure you work through taking the derivates by hand to find $J(\theta)$ and $J(\dot{\theta})$

4. Kinematic questions to be independently answered. (**Graduate Only**, 2 marks)

   a. You are driving your grandmother to the airport and you don't want her to spill her Earl Grey tea. What is one way to accomplish this? **Graduate Only**. 1 mark.

   b. You are playing a game of horseshoes. What is one strategy that may increase your accuracy? **Graduate Only**. 1 mark.

DYNAMICS

5. For a 1-link arm (2 marks):

   a. Derive the equations of motion by hand using Lagrange mechanics and show your work. 1 mark

   b. Convert this second-order system into two, first-order ODEs and show your work. 1 mark.

6. For your derived 1-link arm model (4 marks):

   a. Run a forward simulations using Euler integration and plot the states over time. Simulation time = 10 seconds, time step (h) = 0.0001s. Constants = $m(1.65kg), r(0.5m), g(9.81), \mathcal{I}(0.025)$; Initial Conditions = $\theta = 90; \dot{\theta} = 0$. 1 mark

   b. Run a forward simulation using odeint (or ode45) and plot the states over time. Same constants and initial conditions as above, but with h = 0.001s. 1 mark

   c. Plot the linear kinetic energy (T^{lin}), rotational kinetic energy (T^{rot}), potential energy (U), and the total energy ($T + U$) over time for the simulation above. 1 mark.

   d. What do you notice with the energy terms? Why would this be? 1 mark.

7. For a 2-link arm (**Graduate Only**) (4 marks):

   a. Run a forward simulation a two-link arm using Euler integration, using the equations of motion defined in class, and plot the states over time. Sim time = 2 s, time step (h) = 0.00001s.
   Constants = $m_1(2.1), m_2(1.65), \mathcal{I}_1(0.025), \mathcal{I}_2(0.075), l_1(0.3384), l_2(0.4554),$
   $r_1(0.1692), r_2(0.2277), g(9.81), Q_1(0), Q_2(0)$
   Initial Conditions = $\theta_1(180°), \theta_2(1°), \dot{\theta}_1(0^{°/s}), \dot{\theta}_2(0^{°/s})$. 3 mark.

   b. Plot the kinetic energy (i.e., T, sum of the kinetic energy terms), potential energy (U, sum of the potential energy terms), and total energy ($T + U$) over time for the simulation above. 1 mark.

8. Dynamic questions to be independently answered. (**Graduate Only**, 3 marks)

   a. What is a great way to check if the derived equations of motion are correct? 1 mark.

   b. If Q was nonzero (e.g., muscles were creating moments about the joint) would the energy in the system remain constant? Why? 1 mark.

   c. There is both muscle redundancy and joint redundancy. Is there a potential advantage of considering a dynamic cost function over a kinematic cost function? 1 mark.