ASSIGNMENT 5 — BRAIN, BEHAVIOUR, AND OPTIMAL FEEDBACK CONTROL

For all questions below, provide all programming code and plots in the report. Total marks (undergrad 22 | graduate 41).

1. Here the goal is to run a forward model that generates a minimum jerk hand trajectory in the horizontal plane. Lets assume the following constants = $m_1(2.1), m_2(1.65), \mathcal{I}_1(0.025), \mathcal{I}_2(0.075), l_1(0.34), l_2(0.46), r_1(0.1692), r_2(0.2277), g(0.0)$. Let's also assume the initial hand coordinates are $H_x = -0.103923$, $H_y = 0.4$ and we are reaching to a target such that our hand coordinate will be at $H_x = -0.103923$, $H_y = 0.65$. Use a movement time of $0.5sec$ and a step size of $0.00001sec$. [i.e., 49999 data points]. (2 marks | 7 marks)

   a Plot the position, velocity, accelerate, and jerk of the hand along the y-axis. 2 mark

   b Plot the angular position, velocity, and acceleration for the shoulder and elbow. **(Graduates Only)**. 3 marks

   c Plot the moment (Q) at the shoulder and elbow require to produce these kinematics. **(Graduates Only)**. 2 marks

2. Questions to be independently answered. (4 marks | 7 marks).

   a. A mono-articular elbow extensor muscle contracts, which leads to a shoulder _____ moment. 1 mark

   b. What evidence supports the idea of internal models. 1 marks

   c. Ideally, what does a reinforcement learning model accomplish? 1 mark

   d. When searching for Waldo, how many bits of information are in an 8x8 grid? 1 mark

   e. Why is Reinforcement Learning such a powerful framework to explain many aspects of human behaviour? **(Graduates Only)**. 1 mark

   f. Lets pretend then when a rat pulls a lever it is rewarded with a food pellet 75% of the time. On a particular trial, the rat receives a pellet. This would represent a _____ reward prediction error. On another trial the rat does not receive a pellet. This would represent a _____ reward prediction error. **Graduates Only**. 1 mark

   g. For each model—Fitt's Law, Min(Jerk), Min(Moment Rate), and Min(signal dependent noise)—name the phenomena they captured. **(Graduates Only)**. 1 mark

Please refer to slides for LQG equations. Initial Conditions and Constants

- Simulation time from 0.0 to 0.5s $(51$ time steps) using a step size (h) of $0.01s$.
- $m(4.0), b(1.0), k(0.25)$
- $C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
- $R_k = \begin{bmatrix} 0.0000001 \end{bmatrix}$
- $Q_k = \begin{cases} \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, & \text{if } k \neq N \\ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, & \text{if } k = N \end{cases}$
- $v_0 = v_k = \mathcal{N}\left(\begin{bmatrix} 0.0 \\ 0.0 \end{bmatrix}, \begin{bmatrix} 0.005 & 0.0 \\ 0.0 & 0.005 \end{bmatrix}\right)$
- $w_0 = w_k = \mathcal{N}\left(\begin{bmatrix} 0.0 \\ 0.0 \end{bmatrix}, \begin{bmatrix} 0.01 & 0.0 \\ 0.0 & 0.01 \end{bmatrix}\right)$
- $W = \begin{bmatrix} 0.01 & 0.0 \\ 0.0 & 0.01 \end{bmatrix}$
- $V = \begin{bmatrix} 0.005 & 0.0 \\ 0.0 & 0.005 \end{bmatrix}$
- $P_0^{prior} = W$
- $x_0 = \begin{bmatrix} -1.0 \\ 0.0 \end{bmatrix}$
- $y_0 = x_0 + w_0$
- $\hat{x}_0^{post} = y_0$

LQR

3. Convert the following continuous, 2^{nd} order differential equation into 2 coupled ODEs:

   a. $m\ddot{x} = -b\dot{x} - kx + F$, such that $u = F$. 1 mark.

   b. express your answer found in **(a)** into matrix form. 1 mark.

   c. convert the above continuous system found in **(b)** into a discrete system (i.e., A_d, B_d). 1 mark.

4. Calculate the optimal feedback gains (F_k) based on the following (backwards) recursive equations:

   . Initial Condition: $P_{k+1} = Q_N$

   . $F_k = (R_k + B_d^T P_{k+1} B_d)^{-1}(B_d^T P_{k+1} A_d)$

   . $P_k = A_d^T P_{k+1} A_d - (A_d^T P_{k+1} B_d)\left(R_k + B_d^T P_{k+1} B_d\right)^{-1}(B_d^T P_{k+1} A_d) + Q_k$

   a. Plot the optimal feedback gains (F_k). 3 marks.

5. Run an LQR controller **without** noise based on the following equations:

   . Initial conditions: x_0

   . $u_k = -F_k x_k$

   . $x_{k+1} = A_d x_k + B_d u_k$

   a. Plot the states, x_k (position and velocity), and input signal, u_k, over time. 3 marks.

6. Run an LQR controller **with** noise based on the following equations:

   . Initial conditions: x_0

   . $u_k = -F_k x_k$

   . $x_{k+1} = A_d x_k + B_d u_k + v_k$

   a. Plot the states, x_k (position and velocity), and input signal, u_k, over time. 1 mark.

LQG

7. Compute the Kalman Gain based on the following equations (**Graduate Only**):

   - Initial Condition: $P_0^{prior} = W$
   - $S_k = CP_k^{prior}C^T + W$
   - $K_k = P_k^{prior}C^T S_k^{-1}$
   - $P_k^{post} = (I - K_kC)\, P_k^{prior}$
   - $P_{k+1}^{prior} = A_d P_k^{post} A_d^T + V$

   a. No plotting here, just show code (marks included in question 6, below)

8. Run an LQG controller **without** noise based on the following equations (**Graduate Only**):

   - Initial Conditions: x_0 ; w_0 ; $y_0 = x_0 + w_0$; $\hat{x}_0^{post} = y_0$
   - $u_k = -F_k \hat{x}_k^{post}$
   - $x_{k+1} = A_d x_k + B_d u_k$
   - $y_{k+1} = Cx_{k+1}$
   - $\hat{x}_{k+1}^{prior} = A_d \hat{x}_k^{post} + B_d u_k$
   - $\tilde{y}_{k+1} = y_{k+1} - C\hat{x}_{k+1}^{prior}$
   - $\hat{x}_{k+1}^{post} = \hat{x}_{k+1}^{prior} + K_{k+1}\tilde{y}_{k+1}$

   a. Plot the states, x_k (position and velocity), and input signal, u_k, over time. 6 marks.

9. Run an LQG controller **with** noise based on the following equations (**Graduate Only**):

   - Initial Conditions: x_0 ; w_0 ; $y_0 = x_0 + w_0$; $\hat{x}_0^{post} = y_0$
   - $u_k = -F_k \hat{x}_k^{post}$
   - $x_{k+1} = A_d x_k + B_d u_k + v_k$
   - $y_{k+1} = Cx_{k+1} + w_{k+1}$
   - $\hat{x}_{k+1}^{prior} = A_d \hat{x}_k^{post} + B_d u_k$
   - $\tilde{y}_{k+1} = y_{k+1} - C\hat{x}_{k+1}^{prior}$
   - $\hat{x}_{k+1}^{post} = \hat{x}_{k+1}^{prior} + K_{k+1}\tilde{y}_{k+1}$

   a. Plot the states, x_k (position and velocity), and input signal, u_k, over time. 1 mark.

10. OFC questions to be independently answered. (6 marks | 10 marks)

a. Use the LQR (Undergraduates) or LQG (Graduates) model you developed for your assignment:
Set $Q_N = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ and $R = 0.0000001$, plot the states, control input and interpret. 2 marks

b. Set $Q_N = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ and $R = 0.0000001$, plot the states, control input, and interpret. **Graduates Only**. 2 marks

c. Describe how a position feedback gain leads to hand movement. 1 mark.

d. Following a mechanical perturbation, at what time period do we see the first physiological evidence of an intelligent feedback gain? 1 mark

e. Why does an optimal feedback controller not correct for either task-irrelevant noise or task-irrelevant perturbations? 1 mark.

f. When trying to understand human behaviour, what should you also consider? 1 mark

g. Why is an efference copy important to human movement? **(Graduates Only)**1 mark

h. In optimal feedback control, which area of the brain would handle the forward model? **Graduates Only**. 1 mark