

## ASSIGNMENT 2 — ACTION POTENTIALS AND MULTISENSORY INTEGRATION

For all questions below, provide all programming code and plots in the report. Total marks (undergrads 23 marks | graduates 43 marks).

### ACTION POTENTIAL

#### 1. Program the Hodgkin-Huxley Model (11 marks):

- Perform Euler integration to solve the HH model (i.e., the  $Na$  and  $K$  channels, not  $Ca$  or  $K_{(Ca)}$ ) using the parameter values in the lecture slides.
- Use the following Initial Conditions:  $E_{soma0}, m0, h0, n0, t0 = -70 \cdot 10^{-03}, 0, 1, 0, 0$ .
- Solve for 0.2s with a time-step of 0.00001s, with  $I_{ext} = 1.0 \cdot 10^{-10}$ .
- Plot the membrane potential and each of the channels ( $K, Na_m, Na_h$ ) over time. 10 marks.
- Show on separate plots, phase space for each possible pairing of states (e.g.,  $E_{soma}$  vs.  $K$ ,  $K$  vs.  $Na_m$ , etc.). 1 mark

#### 2. Program the Ekeberg Model (**Graduates Only**, 12 marks):

- Perform Euler integration to solve the Ekeberg model (i.e., the  $Na, K, Ca, CA_{AP}$ ) using the parameter values in the lecture slides.
- Use the following Initial Conditions:  
 $E_{soma0}, m0, h0, n0, q0, CaAP0, t0 = -70 \cdot 10^{-03}, 0, 1, 0, 0, 0, 0$ .
- Solve for 0.2s with a time-step of 0.00001s, with  $I_{ext} = 2.0 \cdot 10^{-9}$ .
- Plot the membrane potential and  $K, Na_m, Na_h, Ca, CA_{AP}$  over time. 6 marks
- What is the minimum  $I_{ext}$  needed to elicit an action potential within 0.2s? why? 1 mark
- At what value does the external current get high enough where the action potentials are no longer repeatedly generated within 0.2s? why? 1 marks
- Why does  $CA_{AP}$  become greater than 1, unlike the other differential equations (e.g.,  $K, Na_m$ )? 1 mark
- Find and report the time (in seconds) of each membrane potential peak. 1 mark
- Calculate and report each inter-spike interval (isi) in seconds (the time between each peak). 1 mark
- Calculate the nerve firing rate in Hz for each isi ( $Hz = 1/s$ ). 1 mark

3. Nerve Conduction questions to be answered on independently (3 marks | 5 marks)

- Describe  $Na^+$  permeability and  $K^+$  permeability during depolarization, repolarization, and hyperpolarization. 2 marks.
- Why do action potentials propagate in one direction? 1 mark.
- When the  $Na^+$  voltage gates open,  $Na^+$  rushes into the cell. At this moment, which way do the chemical gradient and electrical gradients for  $Na^+$  and  $K^+$  point? **Graduate Only.** 2 marks.

## MULTISENSORY INTEGRATION

4. Point Probability 1 (5 marks):

- As in the lecture example, let's say we are recording from a monkey and there is some probability that the electrode will spike when the monkey's hand is open. Initial conditions:  $p(open) = 0.5$ ,  $p(spike|open) = 0.6$ ,  $p(spike|closed) = 0.35$ . You observe 10 spikes in row. List and plot probability that the hand is open for each observed spike. 1 mark
- Change your initial prior to 0.01. List and plot probability that the hand is open for each observed spike. What happens and why? 1 mark
- Change your initial prior to 0.0. List and plot probability that the hand is open for each observed spike. What happens and why? 1 mark
- Once again, let's assume  $p(open) = 0.5$ ,  $p(spike|open) = 0.6$ ,  $p(spike|closed) = 0.35$ . This time, estimate the probability  $p(open|no\ spike)$  given you observe ten trials in a row with no spikes. Tip,  $p(no\ spike|open)$  is the complement of  $p(spike|open)$ , and  $p(no\ spike|closed)$  is the complement of  $p(spike|closed)$ . 1 mark
- Again assuming  $p(open) = 0.5$ ,  $p(spike|open) = 0.6$ ,  $p(spike|closed) = 0.35$ . Let's say that over 10 trials sometimes you observe spikes and other times you do not observe spikes according to: [1,0,0,1,0,1,1,0,1,1], such that 1 = spike, 0 = no spike. List and plot the probability that the hand is open for each observation. 1 mark

5. Point Probability 2 (**Graduate Only**, 4 marks):

- Let's pretend we are recording from a neuron in a mouse, but a label has fallen off one of the electrodes and we don't know which type of neuron the electrode is recording. Luckily, we have some information: a) We know that a Type 1 Neuron will spike at a probability of 90.4% [ $p(spike|Type\ 1) = .904$ ] when the mouse moves its right paw, but Type 1 neurons only represent 20% of the neurons we typically record from [ $p(Type\ 1) = 0.2$ ]. b) A Type 2 Neuron will spike at a probability of 75.8% when the mouse moves its right paw, and represents 30% of the neurons we typically record. c) Finally, a Type 3 Neuron will spike

at a probability of 52.7% when the mice moves its right paw, and represents 50% of the neurons we typically record. The mouse moves its right paw and we observe a spike. What is the probability that it is a Type 1 Neuron [ $p(\text{Type1}|\text{spike})$ ]? Tip: for the marginal probability you will need to also consider the remaining possible types. 1 mark

- b. What is the probability that it is a Type 2 neuron? 1 mark
- c. What is the probability that it is a Type 3 neuron? 1 mark
- d. Lets say the mouse moves its right paw two more times and each time we observe a spike. List the probability that the electrode is recording Type 1, Type 2, or Type 3 neurons for each of the three movements. Interpret these findings. 1 mark

6. Continuous Probability 1 (4 marks):

- a. Similar to lecture, lets assume we are lifting a box a single trial ( $n = 1$ ). Looking at the box, you originally assumed the box weighs about 5N but are uncertain about your visual estimate (i.e., there is some amount of  $\sigma_0$ ). When you lift the box once, you feel that it weighs about 7.0N with some level of uncertainty ( $\sigma_1 = 1.0$ ). We want to find the posterior,  $p(x|\mu, \sigma^2)$ , where  $x$  is probability of different possible box weights. Use hyperparameters to calculate the posterior. Assume: prior =  $\mathcal{N}(5.0, 100^2)$ , likelihood =  $\mathcal{N}(7.0, 1.0^2)$ . Plot the prior, likelihood, and posterior. 1 mark
- b. This time, lets assume a different prior. Again, use hyperparameters to calculate the posterior. Assume: prior =  $\mathcal{N}(5, 1.5^2)$ , likelihood =  $\mathcal{N}(7.0, 1.0^2)$ . Plot the prior, likelihood, and posterior. How does this result differ from the question above and why? 1 mark
- c. Lets iterate through four trials, **one at a time (n = 1 for each lift)**. Lets use the initial prior =  $\mathcal{N}(5.0, 1.5^2)$ . Lets assume the box is always the same weight (7N) and we have the same level of uncertainty of its weight on each lift ( $\sigma = 1.0$ ). On each new iteration, update the prior with the newly found posterior. Plot the prior, likelihood, and posterior for each lift. What is the most likely weight of the box after each lift? 1 mark
- d. Use the same initial prior and likelihood as above (prior =  $\mathcal{N}(5.0, 1.5^2)$ ; likelihood =  $\mathcal{N}(7.0, 1.0^2)$ ), but do not iterate through each trial. Instead, use the hyperparameters to calculate the the posterior after 4 lifts all at once (i.e.,  $n = 4$ ). Plot the prior, likelihood, and posterior. 1 mark

7. Continuous Probability 2 (**Graduate Only**, 2 marks):

- a. Repeat question **3c** above (same initial prior and likelihood), but do it by numerically solving the posterior by discretizing the prior and likelihood along the x-axis. Use 101 data points for each Normal curve and keep the range of the x-axis from 0 to 10. Plot the prior, likelihood, and posterior. What are the advantages of numerical estimates. What are the weaknesses? 1 mark

- b. Pretend you are running an experiment where a participant is facing several speakers and several lights that are placed in a semicircle ( $\pm 90^\circ$ ) around them. As the experimenter, you can manipulate the average location and noise of auditory and visual feedback by controlling which lights and speakers will turn on. On a particular trial, you control a few lights in a general area to light up, but it is hard to pinpoint exactly where ( $\mu_v = -10.0^\circ$ ,  $\sigma_v = 5^\circ$ ). Simultaneously, you set a few speakers in a general area to produce a tone ( $\mu_a = 25.0^\circ$ ,  $\sigma_a = 15^\circ$ ). The participant has to point towards a single location that best captures the source of the light and sound. Assuming no prior knowledge of the source location, where should they point? 1 mark