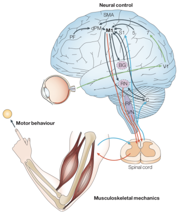
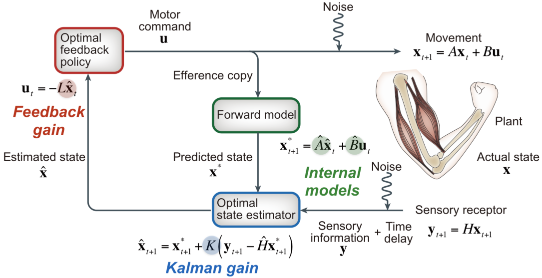
Neuromechanics of Human Motion

Cost Functions

Joshua Cashaback, PhD

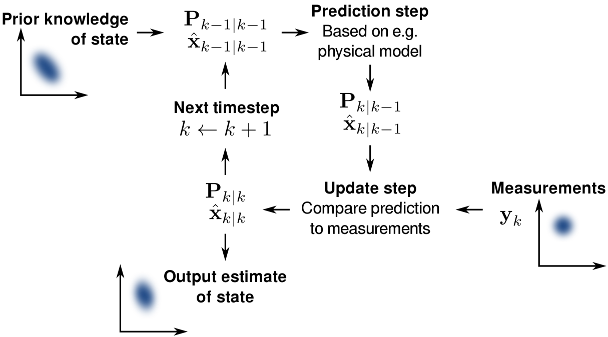
Recap — OFC


Neural control
SMA
dPM
M1
S1
PF
BG
RN
V1
Motor behaviour
Spinal cord
Musculoskeletal mechanics

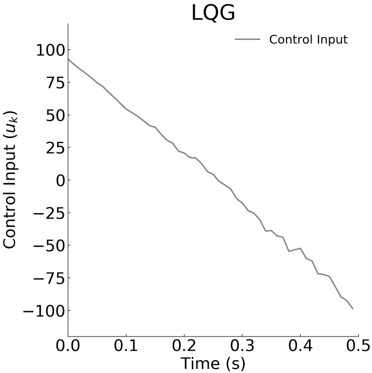

Optimal feedback policy
Motor command
$\mathbf{u}$
Noise
Movement
$\mathbf{x}_{t+1} = A\mathbf{x}_t + B\mathbf{u}_t$
Efference copy
$\mathbf{u}_t = -L\hat{\mathbf{x}}_t$
Feedback gain
Forward model
Plant
$\mathbf{x}^*_{t+1} = \hat{A}\hat{\mathbf{x}}_t + \hat{B}\mathbf{u}_t$
Estimated state
$\hat{\mathbf{x}}$
Predicted state
$\mathbf{x}^*$
Internal models
Noise
Actual state
$\mathbf{x}$
Optimal state estimator
Sensory receptor
Sensory information
$\mathbf{y}$
+
Time delay
$\mathbf{y}_{t+1} = H\mathbf{x}_{t+1}$
$\hat{\mathbf{x}}_{t+1} = \mathbf{x}^*_{t+1} + K\left(\mathbf{y}_{t+1} - \hat{H}\mathbf{x}^*_{t+1}\right)$
Kalman gain

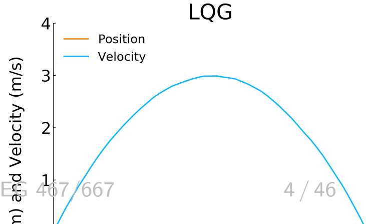
Recap — OFC

$$J = \sum_{k=0}^{N-1} \left(x_k^T Q x_k + u_k^T R u_k \right)$$



Recap — OFC


LQG
Control Input
100
75
50
25
0
−25
−50
−75
−100
Control Input (u_k)
0.0
0.1
0.2
0.3
0.4
0.5
Time (s)


LQG
Position
Velocity
4
3
2
1

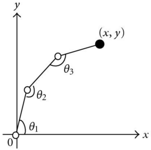
What Do Humans Optimize when Generating Movement?

Lecture Objectives — Cost Functions

1. Kinematics
2. Dynamics
   - joint-based or muscle-based
3. Neural Basis for Kinematics and Dynamics
4. Control Signals
5. Energy, Reward, Information
6. Multi-Objective Optimization
7. Multi-Agent Optimization
8. Survival and Evolution
9. A Common Currency?

Redundancy

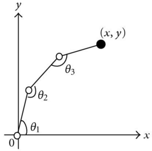
Redundancy: Degrees-of-Freedom (DOF) $>$ Task Space Dimensions



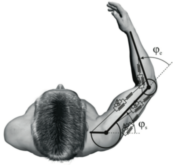
Joint Redundancy

Redundancy

Redundancy: Degrees-of-Freedom (DOF) > Task Space Dimensions

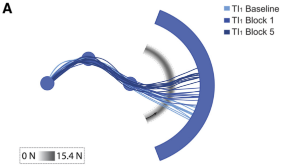


Joint Redundancy



Schematic of "full-blown" musculoskeletal model described in Kistemaker et al. (2010).

Muscle Redundancy



Work-Space Redundancy

The Curse of Redundancy

The Curse:

1. infinite ways to accomplish task goals
2. how does the brain decide which action to take???

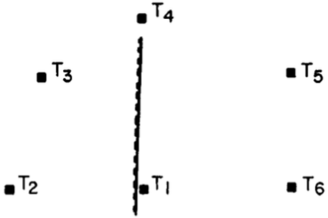
The Bliss of Redundancy

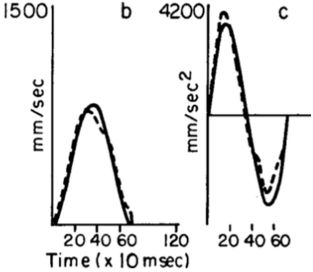
The Bliss:

1. infinite ways to accomplish task goals
2. neurological disorders, musculoskeletal disorders
   - may still function (less capacity)
3. allows us to consider many objectives (i.e., cost functions)

Kinematics

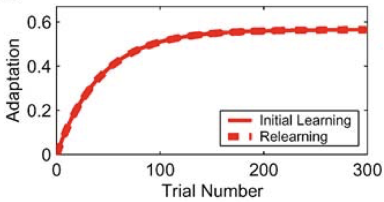
Kinematics — Jerk





Flash and Hogan (1985) — Minimum Jerk Trajectories
Journal of Neuroscience, 5(7), 1688-1703

Kinematics — Endpoint Error



Smith et al (2006) — Error-based Learning
PLoS biology, 4(6)

Joint Dynamics

Joint Moment Rate

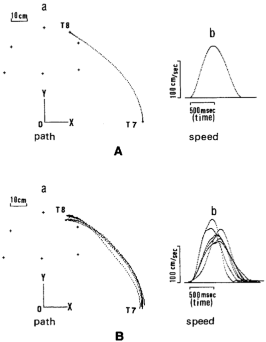
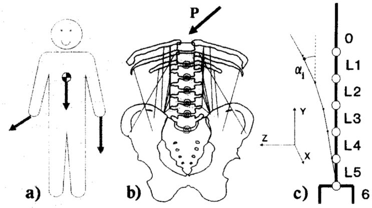


Fig. 4A and B. Large free movements between two targets ($T7 \rightarrow T8$); the starting posture is stretching an arm in the side direction and the end point is approximately in front of the body. **A** Hand trajectory predicted by the minimum torque-change model. **a** shows the path and **b** shows the corresponding speed profile. **B** Observed hand trajectories for the seven subjects. **a** shows the paths and **b** shows the corresponding speed profiles

Uno et al (1989) Biol. Cybern. 61, 1688-1703

Joint Stability



$$D=\det\left[\frac{\partial^2 V}{\partial Q_i \partial Q_j}\right]$$

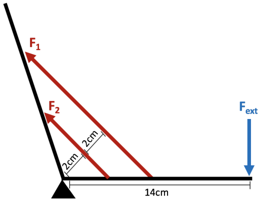
$$=\det\begin{bmatrix} \frac{\partial^2 V}{\partial Q_1^2} & \frac{\partial^2 V}{\partial Q_1 \partial Q_2} & \cdots & \frac{\partial^2 V}{\partial Q_1 \partial Q_n} \\ \frac{\partial^2 V}{\partial Q_2 \partial Q_1} & \frac{\partial^2 V}{\partial Q_2^2} & \cdots & \vdots \\ \vdots & \vdots & \cdots & \vdots \\ \frac{\partial^2 V}{\partial Q_n \partial Q_1} & \vdots & \cdots & \frac{\partial^2 V}{\partial Q_n^2} \end{bmatrix} > 0 \wedge D_{ij} > 0 \qquad (2)$$

Considering joint stability can account for muscle cocontraction

Cholewicki and McGill (1996) Clinical biomechanics, 11(1), 1-15.

Muscle Dynamics

Muscle Force



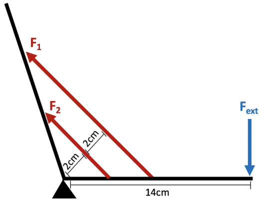
$$min\left(\sum_{i=1}^{n}(F_i)^2\right)$$

s.t.

$$0 \leq \gamma_i \leq 1$$

$$\sum M = 0 = F_1 \cdot 0.04 + F_2 \cdot 0.02 - F_{ext} \cdot 0.14$$

Muscle Force



$$min\left(\sum_{i=1}^{n}(F_i)^2\right)$$

s.t.

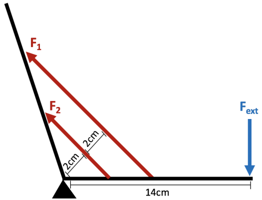
$$0 \leq \gamma_i \leq 1$$

$$\sum M = 0 = F_1 \cdot 0.04 + F_2 \cdot 0.02 - F_{ext} \cdot 0.14$$

$$\sum M = 0 = \gamma_1 * \cdot 35 \cdot 10 \cdot 0.04 + \gamma_2 \cdot 35 \cdot 5 \cdot 0.02 - 50 \cdot 0.14$$

muscle stress (N/cm^2), PCSA(cm^2), moment arm(m)

Muscle Force



$$min\left(\sum_{i=1}^{n}(F_i)^2\right)$$

s.t.

$$0 \leq \gamma_i \leq 1$$

$$\sum M = 0 = F_1 \cdot 0.04 + F_2 \cdot 0.02 - F_{ext} \cdot 0.14$$

$$\sum M = 0 = \gamma_1 * \cdot 35 \cdot 10 \cdot 0.04 + \gamma_2 \cdot 35 \cdot 5 \cdot 0.02 - 50 \cdot 0.14$$

muscle stress (N/cm^2), PCSA(cm^2), moment arm(m)

$$\gamma_1 = 0.4; \gamma_2 = 0.4$$

Min Muscle Force — Sample Python Code

```
import numpy as np
import scipy
from scipy import optimize

# elbow model, minimize muscle force (two muscles)

#cost function
def f2(x):
        return (x[0] * 35 * 10)**2 + (x[1] * 35 * 5)**2

#moment constraint
def con2(x):
        return x[0] * 35 * 10.0 * 0.04 + x[1] * 35 * 5.0 * 0.02 - .14 * 50

# predicted muscle activity
gamma = optimize.fmin_slsqp(f2, [0,0], eqcons = [con2], bounds = [(0,1),(0,1)])
gamma
```

Muscle Stress

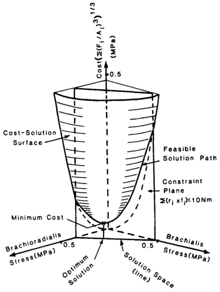
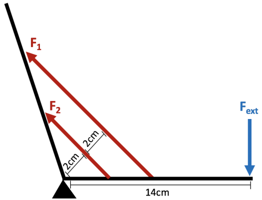


Fig. 4. The occurrence of an optimum solution to a model of two elbow flexors satisfying a constraint moment of 10 *N m*.

$$min\left(\sum_{i}^{n}(F_i/PCSA_i)^2\right)$$

This cost can reduce muscle fatigue and injury risk (Crowninshield, 1981)

Muscle Stress



$$min\left(\sum_{i}^{n}(F_i/PCSA_i)^2\right)$$

s.t.

$$0 \leq \gamma_i \leq 1$$

$$\sum M = 0 = F_1 \cdot 0.04 + F_2 \cdot 0.02 - F_{ext} \cdot 0.14$$

Find the solution

Neural Evidence

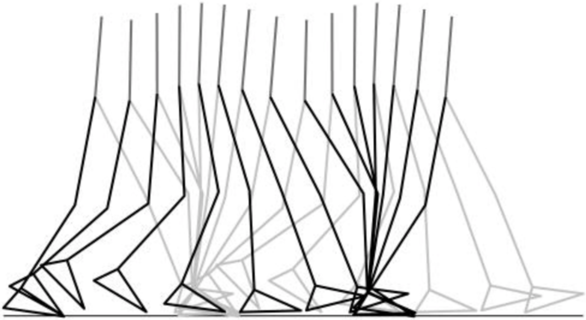
Neural Evidence — Kinematics & Dynamics

Primary motor cortex (M1) Neurons encode:

1. 1968 (Evarts) - muscle force to control individual joints
2. 1982 (Georgeopolis) - populations of neurons encode hand trajectory (multi-joint movements)
3. 1995 (Scott and Kalaska) - neurons more correlated to joint movement and muscle force than hand trajectory.
4. 1999 (Moran & Schwartz) - speed of the hand
5. 1999 (Strick et al.) - hand trajectory or muscle force
6. 2000 (Todorov) - muscle force: intrinsic properties of muscle (force-length and velocity) and limb dynamics.
7. Correlations to hand trajectory and velocity a bi-product of muscle and limb dynamics.

Control Signals

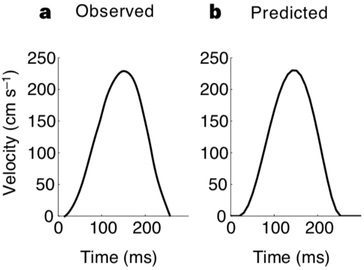
Muscle activity

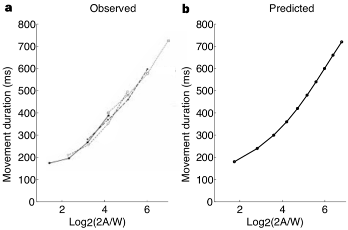


Ackerman & van den Bogert (2010) — JBiomech, 43(6), 1055-1060

Signal Dependent Noise

Is the Brain minimizing noise?



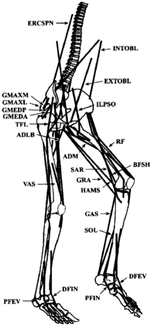


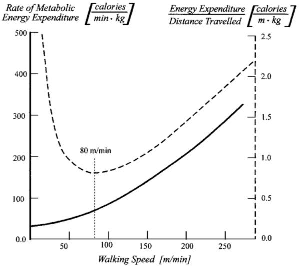
Predicts bell-shaped velocity and Fitt's Law

Harris and Wolpert (1998) — Nature, 394, 780-784

Other Notable Costs

Metabolic Energy





Predicted muscle activity, gait kinematics, preferred walking speed

Anderson and Pandy (2001) — J Biomech Eng 123, 381-390.

Postural Stability

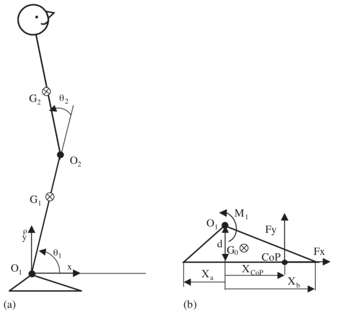
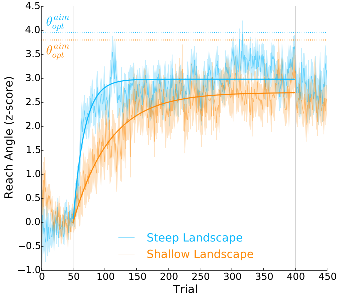


Fig. 1. Two-dimensional human body system (a) and mechanical characteristics of the feet (b).

Center of Mass within base of support

Martin et al (2006) — Journal of Biomechanics 39, 170-176.

Reward



Reward linked to dopamine release

Cashaback et al (2019) PLoS Comp Bio, 15 (3), e1006839.

Information

Exploratory behaviour as a goal

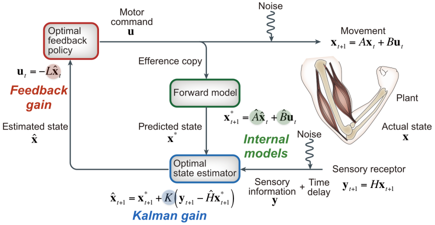
Language and Motor Babbling

Motor Learning

Songbird

Multi-Objective Optimization

OFC — control signal and states



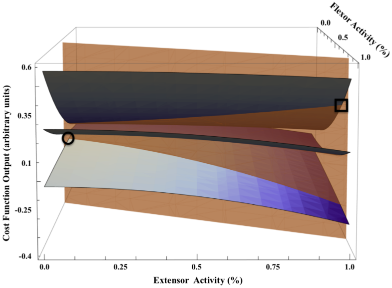
$$J = \sum_{k=0}^{N-1} \left(x_k^T Q x_k + u_k^T R u_k \right)$$

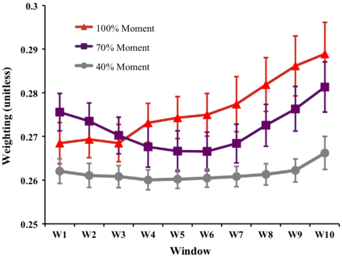
Cost on desired state (x) and neural input (u)

Feedback control adds a lot of richness matching human behaviour

(Previous costs are feedforward, open loop)

Muscle Fatigue and Stability





$$U = max\left(w\sum_{i=1}^{n}k(u)_i^2 - (1-w)\sum_{i=1}^{n}\sigma(u)_i^2\right)$$

The upper, middle, and lower blue surfaces: w = 0.2, 0.5, 0.8

Orange plane = moment constraint

Cashaback et al (2015) — Journal of Biomechanics 48, 621-626.

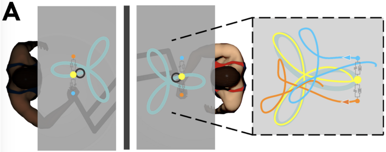
Energy and Reward

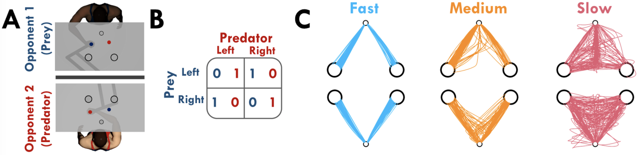
- Birds (Starlings) find optimal ratio of flying:walking to min(energy) when foraging (reward)
- Predator risk?
- Bautista et al (2001) PNAS, 98(3), 1089-1094.
- Humans finding optimal running:walking to min(energy) when travel some distance with a time constraint (task reward)
- Long & Srinivasan (2013) J R Soc Interface, 10(81), 20120980.

Notes of Caution with Multi-objective Optimization

1. Complex models may be confusing (high dimensionality) and overfit the data.
2. “With four parameters I can fit an elephant, and with five I can make him wiggle his trunk (John von Neumann)”
3. Think about whether additional parameters are justified physiologically

Multi-Agent Optimization

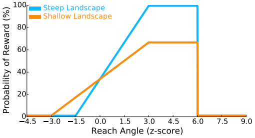


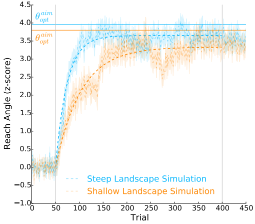


Click Me: Link

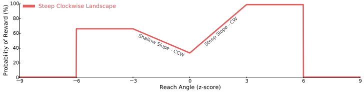
Suboptimality

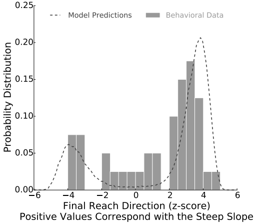
Experiment 1





Experiment 2

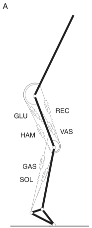




1. Greedy learning: Cashaback et al (2019) PLoS Comp Bio, 15 (3), e1006839.
2. “Good-enough” control: Loeb (2012) Biological Cybernatics 106, 757-765.
3. Noisy priors, likelihoods, and posteriors: Acerbi (2014) PLoS Comp Bio 10: e1003661, 2014.
4. Memory and forgetting: Smith et al (2006) PLoS biology, 4(6)

Survival — Genetics — Evolution

1. Every aspect of our lives depend on our ability to move.
2. Whether our genes get passed along depends on the success of our actions (i.e., survival and mating)
3. Genes code morphology (muscle), sensors (vision), brain evolved for movement. Daniel Wolpert TED talk (Click Me: Link)



Wong et al (2016) PloS One, 11(2) — genetic algorithms to optimize muscle volume distribution.

A Common Currency?

Energy???

Art Kuo, University of Calgary

Can convert many quantities (reward, error, muscle force, etc.,) into, or heavily correlated with, energy (joules).

A Common Currency?

Energy???

Art Kuo, University of Calgary

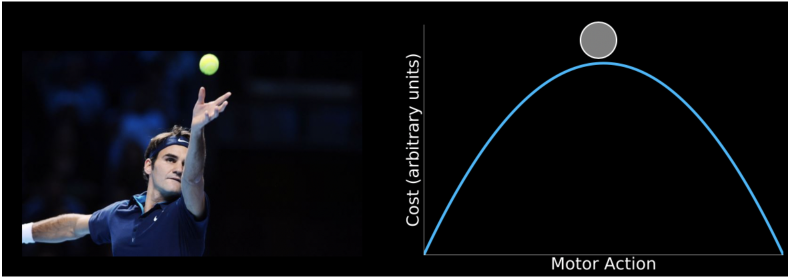
Can convert many quantities (reward, error, muscle force, etc.,) into, or heavily correlated with, energy (joules).

But, some evidence suggests we don't always minimize energy: Kistemaker et al (2010) J Neurophys, 104(6), 2985-2994.

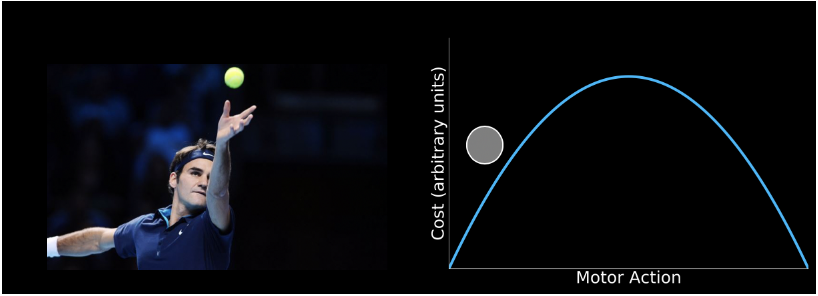
Some good news

Answer unknown (and we get to keep our day jobs!)

Learning to Become Optimal



Learning to Become Optimal



Summary

1. The nervous system needs to solve redundancy
2. Many possible (simultaneous) objectives
3. Think about Survival and Evolution
4. Common Currency?

Questions???

Homework

Minimize the sum of muscle force^2 with $F_{ext} = 75N$

Minimize the sum of muscle stress^2 with $F_{ext} = 75N$.

Next Class

Error-based Learning

1. one-state model
2. two-state model (Smith et al., 2006)
3. learning and forgetting
4. spontaneous recovery
5. savings

Other Considerations

1. Generalization
2. Variability
3. Learning vs. Adaptation

Acknowledgements

Jeremy Wong