STATE-SPACE MODELS FOR MOVEMENT ECOLOGY

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What **is** a state-space model (SSM)?

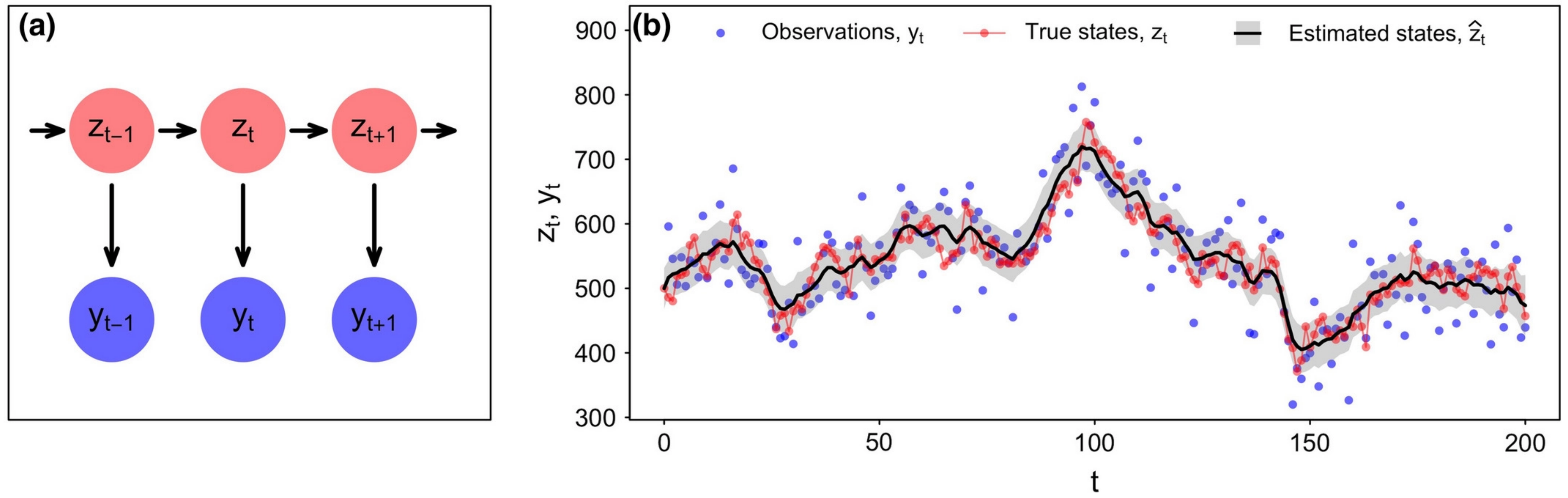
- **Jonsen et al. (2003)**: "…time-series models that allow unobservable, true states to be inferred from observed data by accounting for errors arising from imprecise observations and from stochasticity in the process being studied"

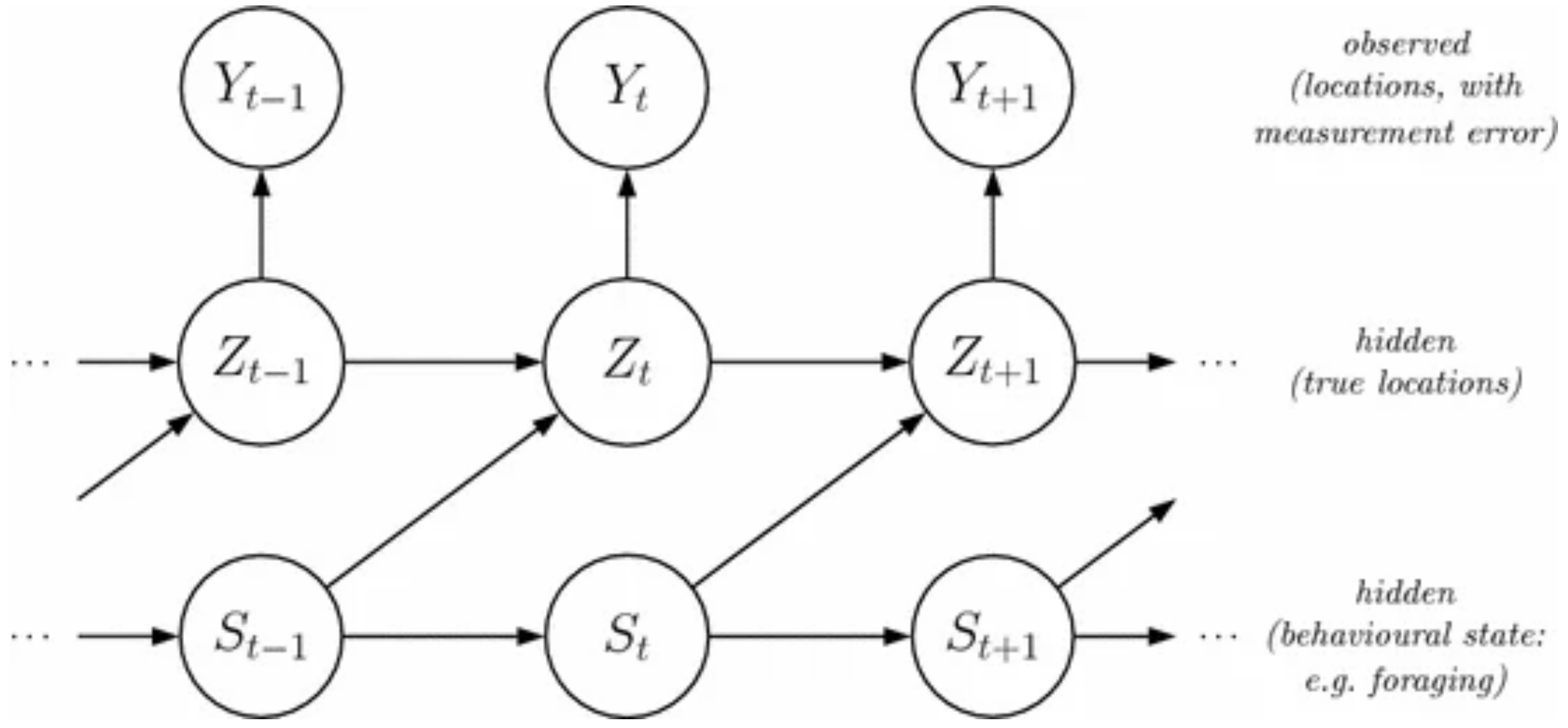
- **Patterson et al. (2008)**: "A time-series model that predicts the future state of a system from its previous states probabilistically, via a process model. The SSM describes mathematically how observations of the state of the system are generated via an observation model "

- **Schick et al. (2008)**: "The model can be thought of as two 'time series running in parallel' (Newman 1998) – one for the process and one for the observations."

- **Jonsen et al. (2013)**: "…a stochastic, model-based approach that allows mechanistic models of the movement process to be fit directly to telemetry data, while accounting for measurement error when appropriate…"

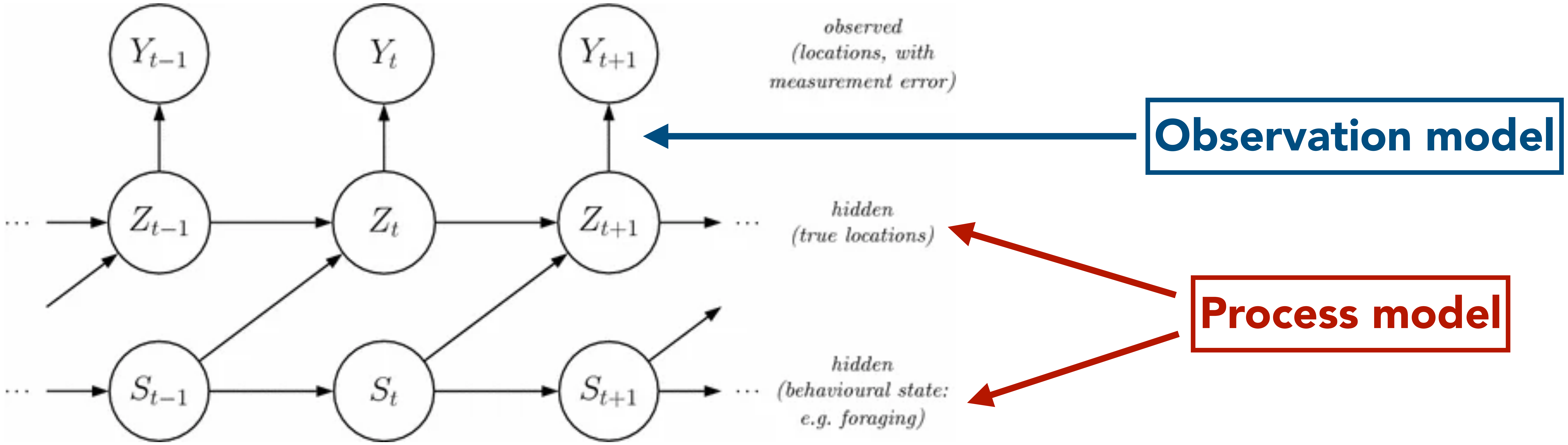
Auger-Methe et al. 2021





Patterson et al. 2017

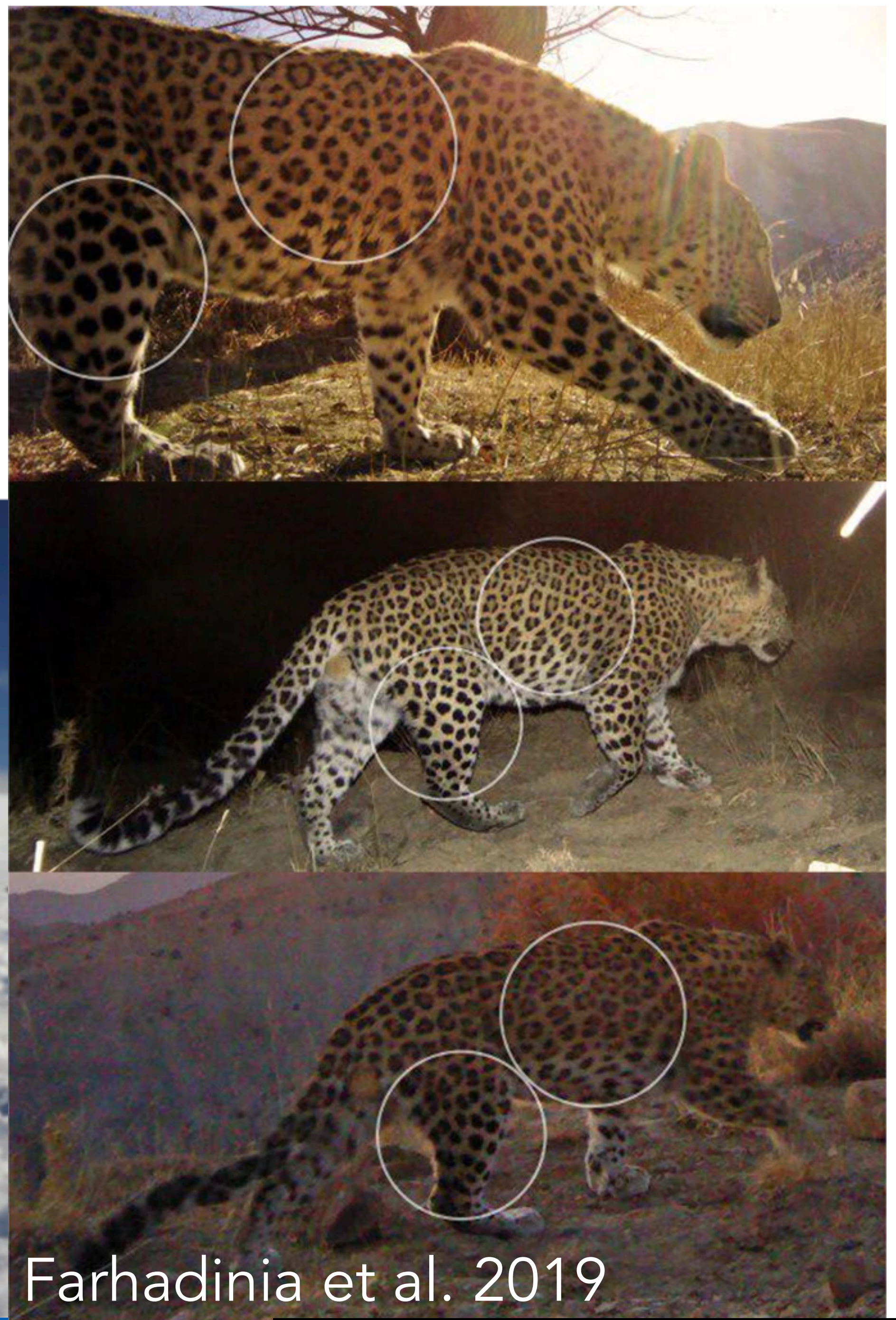
What <u>is</u> a state-space model (SSM)?



Patterson et al. 2017

How else are SSMs used?

- Animal movement
- Fisheries stock assessments
- Population dynamics
- Analysis of capture-recapture data
- Biodiversity assessments
- Econometrics
- Engineering



Costa et al. 2012

Farhadinia et al. 2019

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What does this look like for animal telemetry data?

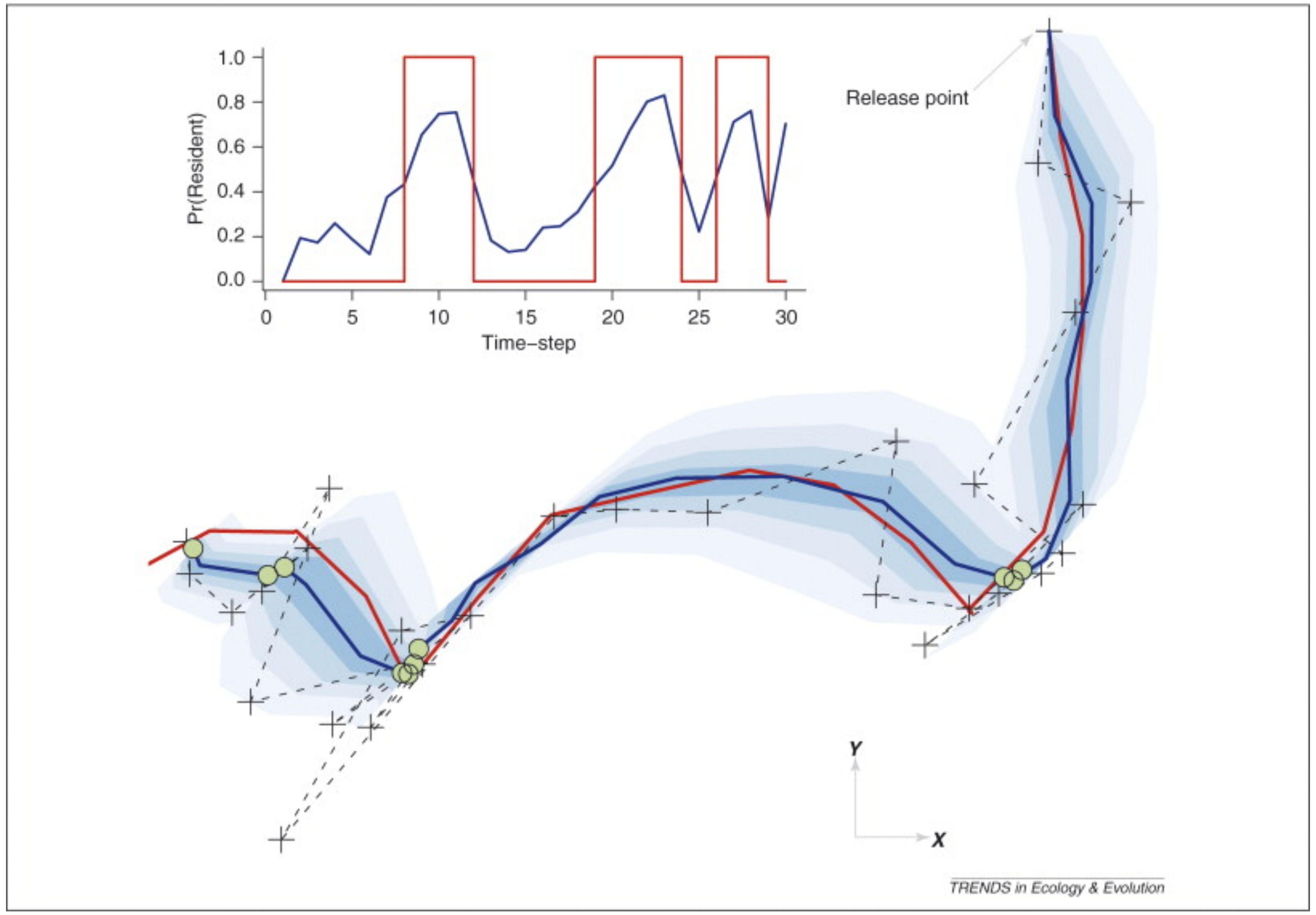
It depends! But here are some common process models that are used:

Discrete-time

- First-difference correlated random walk (DCRW)
- DCRW with behavioral state switching (DCRWS)
- CRW with switching
- Hierarchical forms of the above models

Continuous-time

- RW (Brownian motion) velocity model
- CRW (Ornstein-Uhlenbeck) velocity model



Patterson et al. 2008

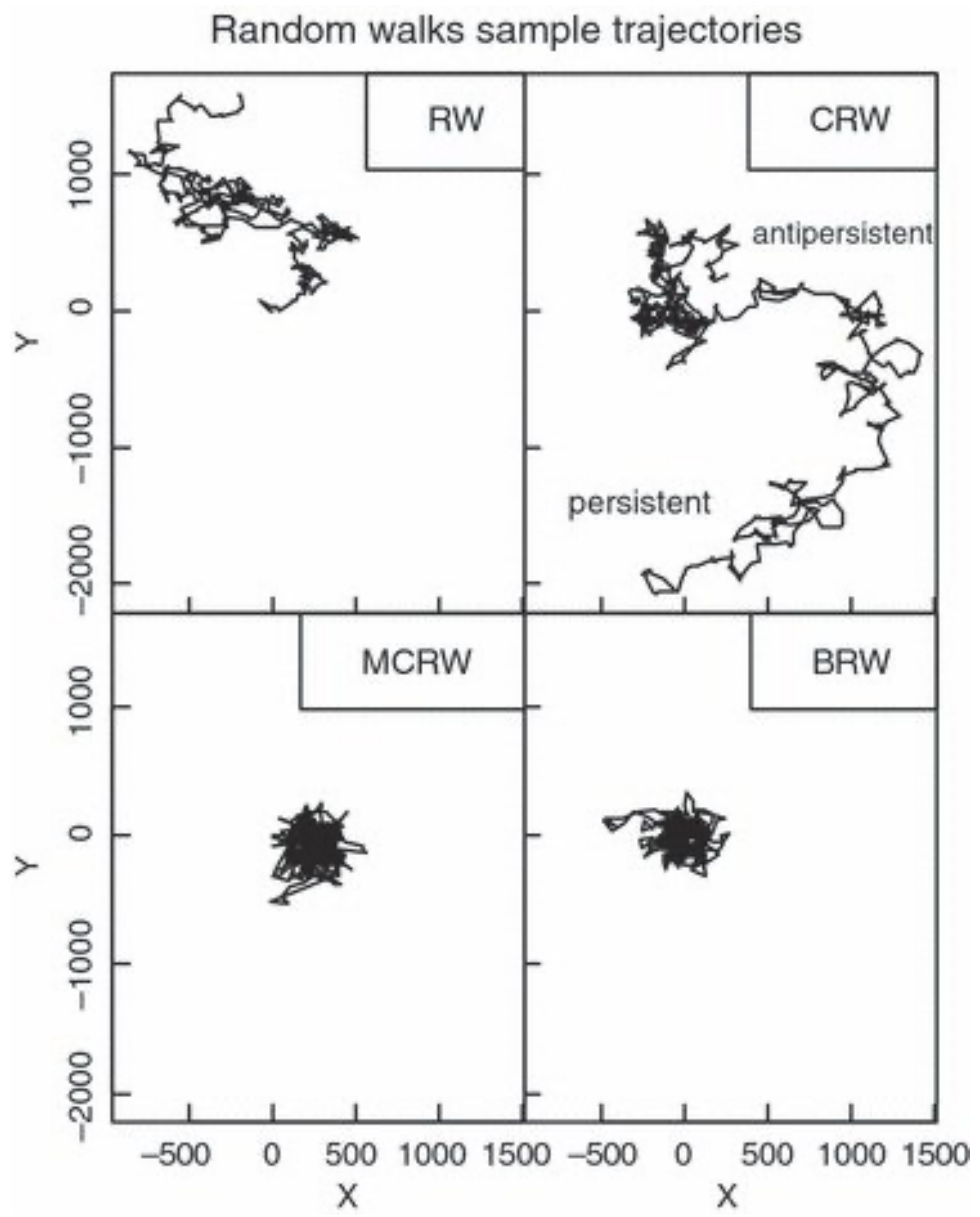
Random walk vs correlated random walk

RW more reflective of general diffusive process through space

- Better for longer time intervals where there is limited directional persistence or autocorrelation in behavior

CRW directly accounts for persistent movements

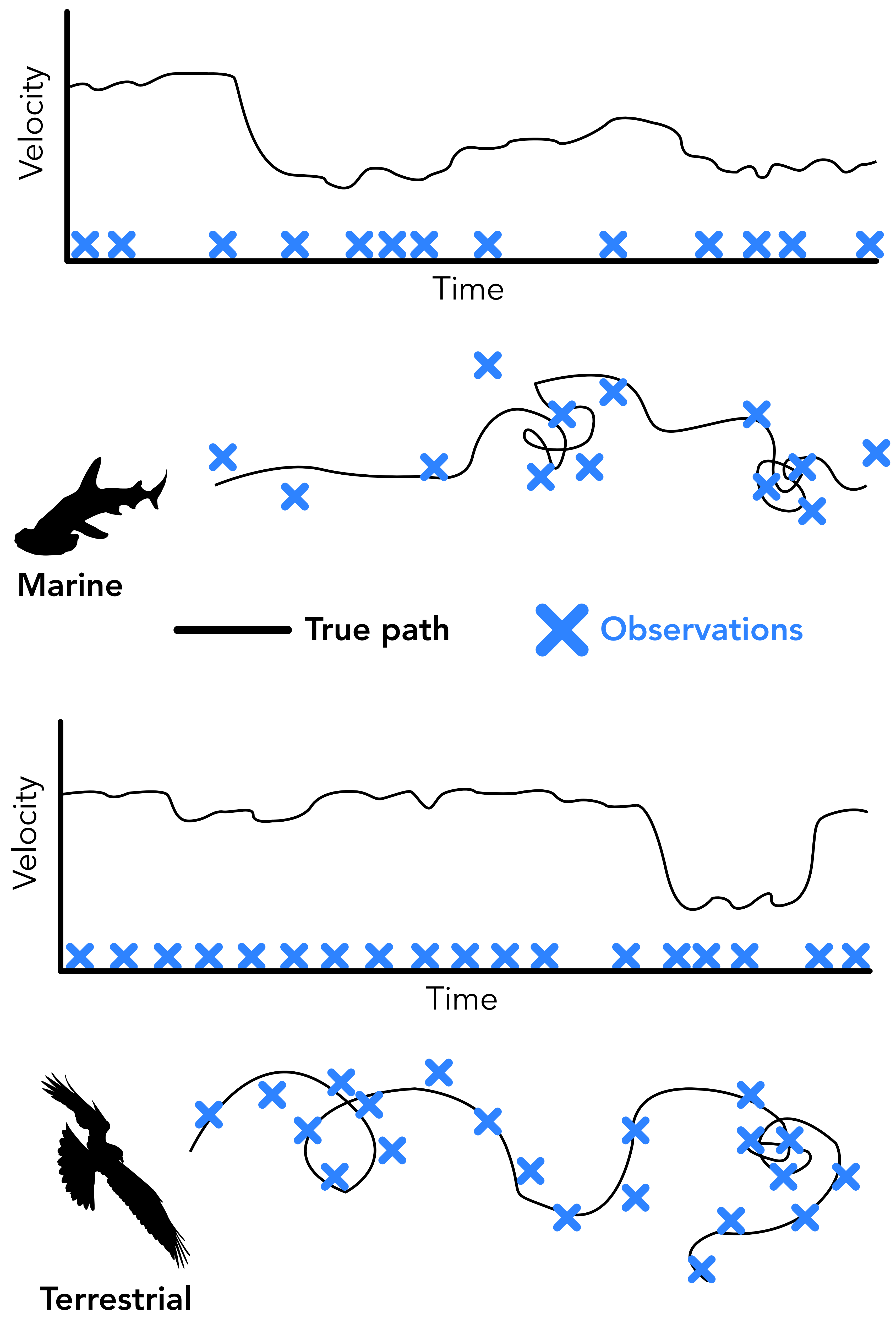
- Better for shorter time intervals with high autocorrelation in movement processes



Börger et al. 2008

Discrete vs continuous time

- Discrete-time models were implemented first, so more widely used
- Discrete-time model breaks up a trajectory into discrete steps
- Continuous-time model can evaluate the “true” continuous movement process by estimating parameters at any point along entire length
- **Discrete-time model more appropriate for tracks w/ regular time interval**
- **Continuous-time more naturally handles observations transmitted over irregular time intervals**



First-difference correlated random walk (DCRW)

Process model

Auger-Methe et al. 2021

Directional persistence

Transition matrix of mean turning angle θ

Stochastic uncertainty in true position at time *t*

$$\mathbf{z_t} = \mathbf{z_{t-1}} + \gamma \mathbf{T}(\theta)(\mathbf{z_{t-1}} - \mathbf{z_{t-2}}) + \epsilon_t$$

True position at time *t*

True position at time *t-1*

True position at time *t-2*

Mean turning angle

$$\mathbf{T}(\theta) = \begin{bmatrix} cos\ \theta & -sin\ \theta \\ sin\ \theta & cos\ \theta \end{bmatrix}$$

Controls rotation of CRW

Where:

$$\mathbf{z_t} = \begin{bmatrix} z_{t,lon} \\ z_{t,lat} \end{bmatrix}$$

True longitude

True latitude

Vector w/ mean of 0

$$\epsilon_t \sim N_2(\mathbf{0}, \mathbf{\Sigma})$$

Variance-covariance matrix

Bivariate Normal distribution

'Regular' time step *t*

Max time step

$$1 \leq t \leq T$$

Variance in longitude

Covariance in longitude and latitude

$$\mathbf{\Sigma} = \begin{bmatrix} \sigma^2_{\epsilon,lon} & \rho\sigma_{\epsilon,lon}\sigma_{\epsilon,lat} \\ \rho\sigma_{\epsilon,lat}\sigma_{\epsilon,lon} & \sigma^2_{\epsilon,lat} \end{bmatrix}$$

Variance in latitude

No correlation w/ previous step $\longrightarrow 0 \leq \gamma \leq 1 \longleftarrow$ Strong correlation w/ previous step

First-difference correlated random walk (DCRW)

Observation model

Auger-Methe et al. 2021

Observed position at time *i*

True position at time *t-1*

True position at time *t*

$$\mathbf{y_i} = (1 - j_i)\mathbf{z_{t-1}} + j_i\mathbf{z_t} + \eta_\mathbf{i}$$

Uncertainty in observed position at time *i*

Proportion of regular time interval between *t-1* and *t*

Correction factor

Scale parameter

$$\eta_\mathbf{i} \sim t_2(0,\ \mathbf{\Psi} \odot \mathbf{S_i},\ \mathbf{D_i})$$

Bivariate *t* distribution

Degrees of freedom

Where:

$$\mathbf{y_i} = \begin{bmatrix} y_{i,lon} \\ y_{i,lat} \end{bmatrix}$$

Observed longitude

Observed latitude

'Irregular' time step *i*

Number of observed locations

$$1 \leq i \leq N$$

$$\mathbf{S_i} = \begin{bmatrix} S_{lon,q_i} \\ S_{lat,q_i} \end{bmatrix}$$

Longitude scaling parameter associated w/ each *q* Argos LC

Latitude scaling parameter associated w/ each *q* Argos LC

$$\mathbf{\Psi} = \begin{bmatrix} \psi_{lon} \\ \psi_{lat} \end{bmatrix}$$

Longitude correction factor

Latitude correction factor

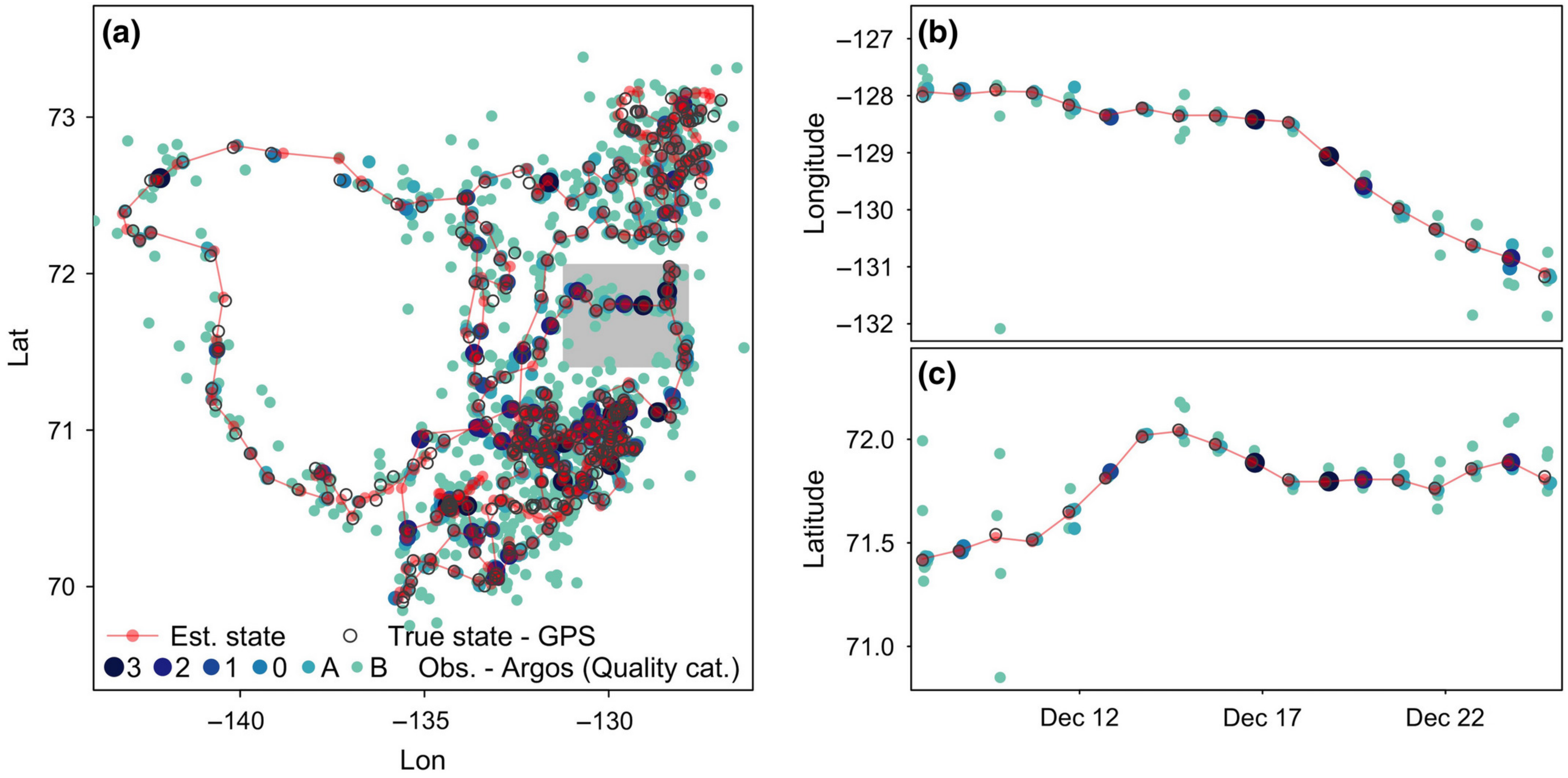
$$\mathbf{D_i} = \begin{bmatrix} df_{lon,q_i} \\ df_{lat,q_i} \end{bmatrix}$$

Longitude df for each *q* Argos LC

Latitude df for each *q* Argos LC

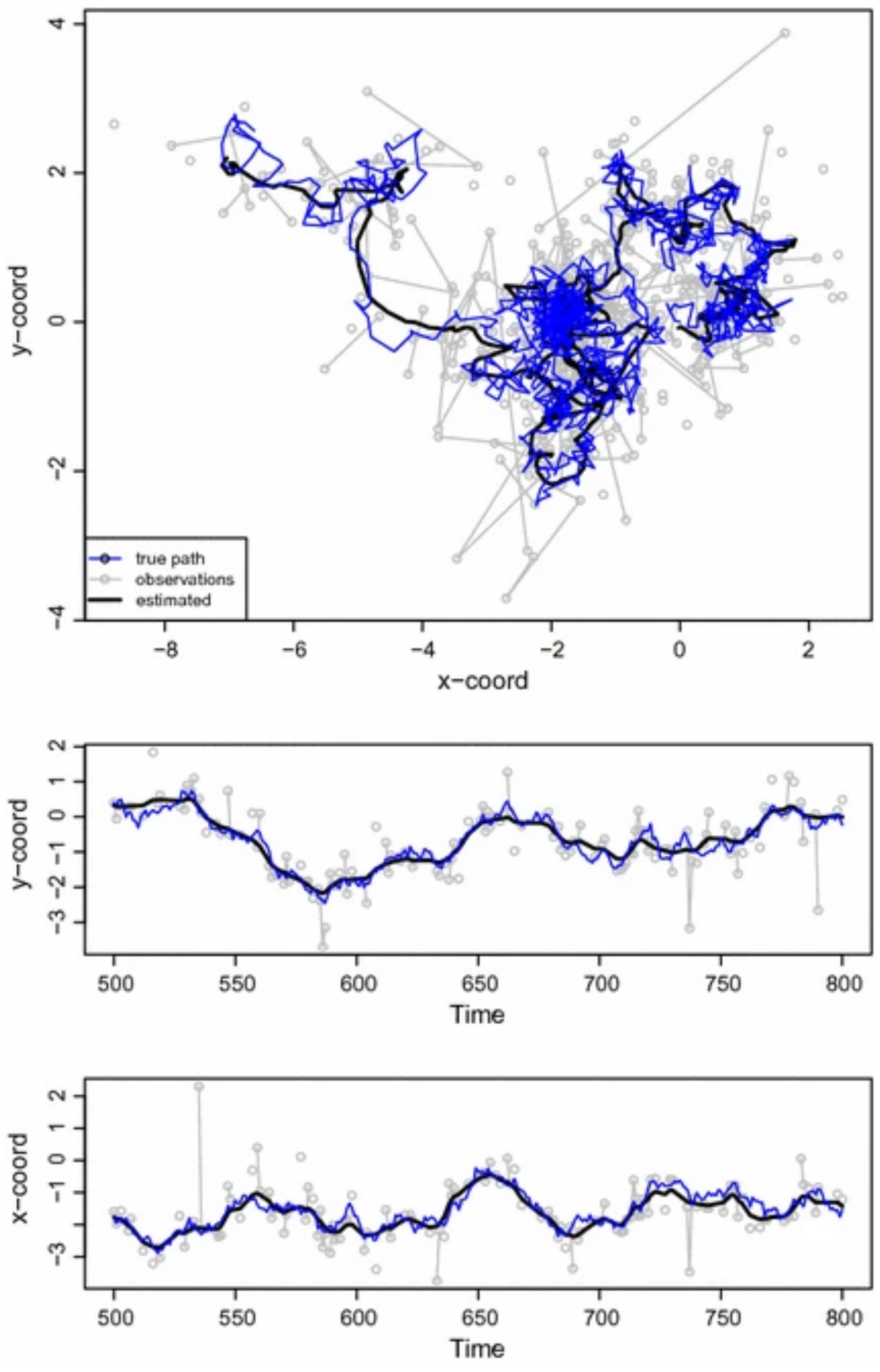
First-difference correlated random walk (DCRW)

Polar bear tracks fitted to DCRW SSM



Auger-Methe et al. 2021

First-difference random walk



Simulated example

Patterson et al. 2017

DCRW with behavioral state switching (DCRWS)

Jonsen et al. 2013

Nearly identical to DCRW, but with slight difference

Directional persistence

Transition matrix of mean turning angle θ

Stochastic uncertainty in true position at time t

$$\mathbf{z_t} = \mathbf{z_{t-1}} + \gamma_{b_{t-1}} \mathbf{T}(\theta_{b_{t-1}})(\mathbf{z_{t-1}} - \mathbf{z_{t-2}}) + \epsilon_t$$

True position at time t

True position at time t-1

True position at time t-2

$$Pr(b_t = i \,|\, b_{t-1} = j) = \alpha_{ij}$$

Probability of being in behavioral state i for time t given that they were in state j for time t-1

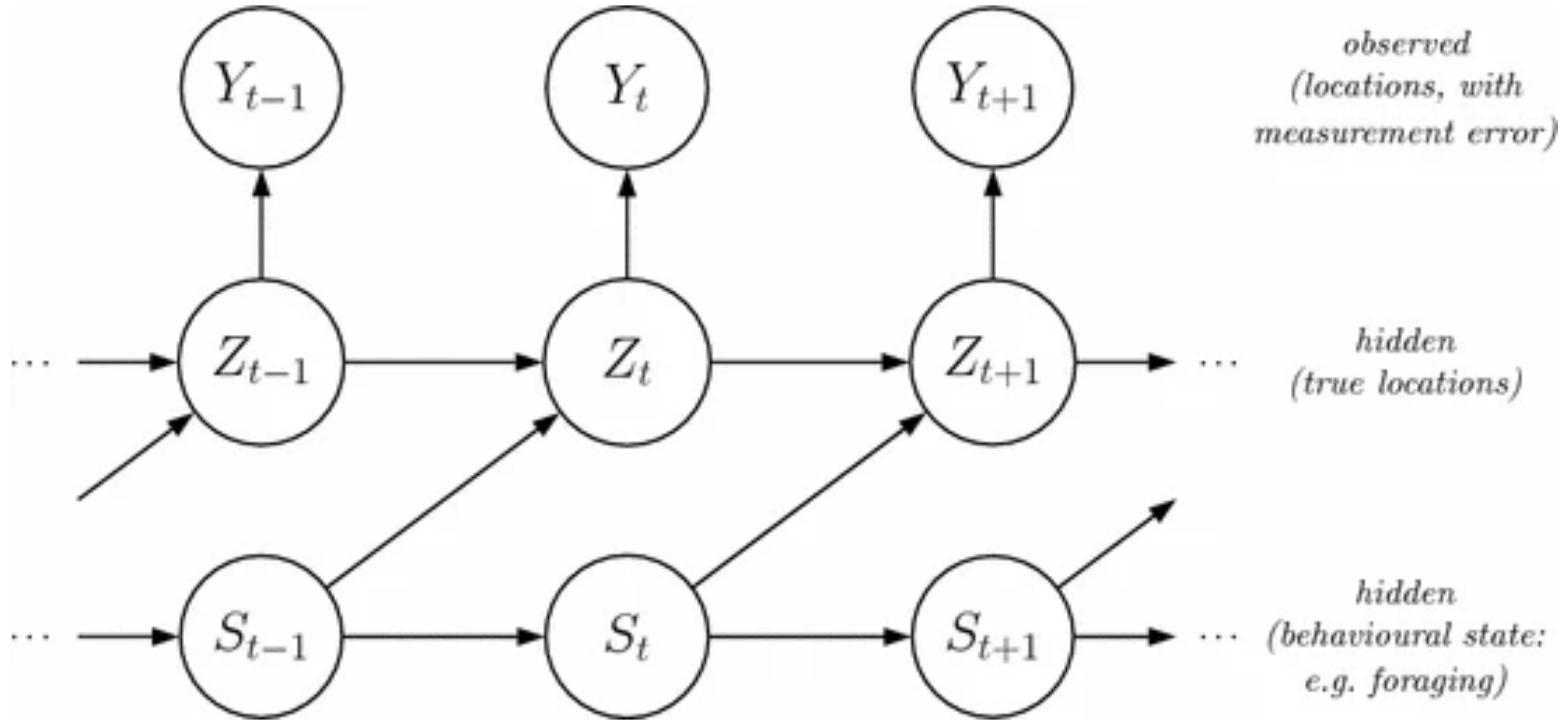
Stay in state 1

$$\alpha = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix}$$

Change from state 2 to 1

Stay in state 2

Change from state 1 to 2



Patterson et al. 2017

Continuous-time CRW (or RW) models

- Won't go into details since it involves some integrals and we're time limited
- Instead of operating on the difference between a pair of consecutive locations, focus is on velocity and its relationships w/ locations
- Also accounts for temporal autocorrelation in velocity process

Methods to fit SSM

TABLE 2. Comparison of the fitting methods discussed in *Fitting SSMs.*

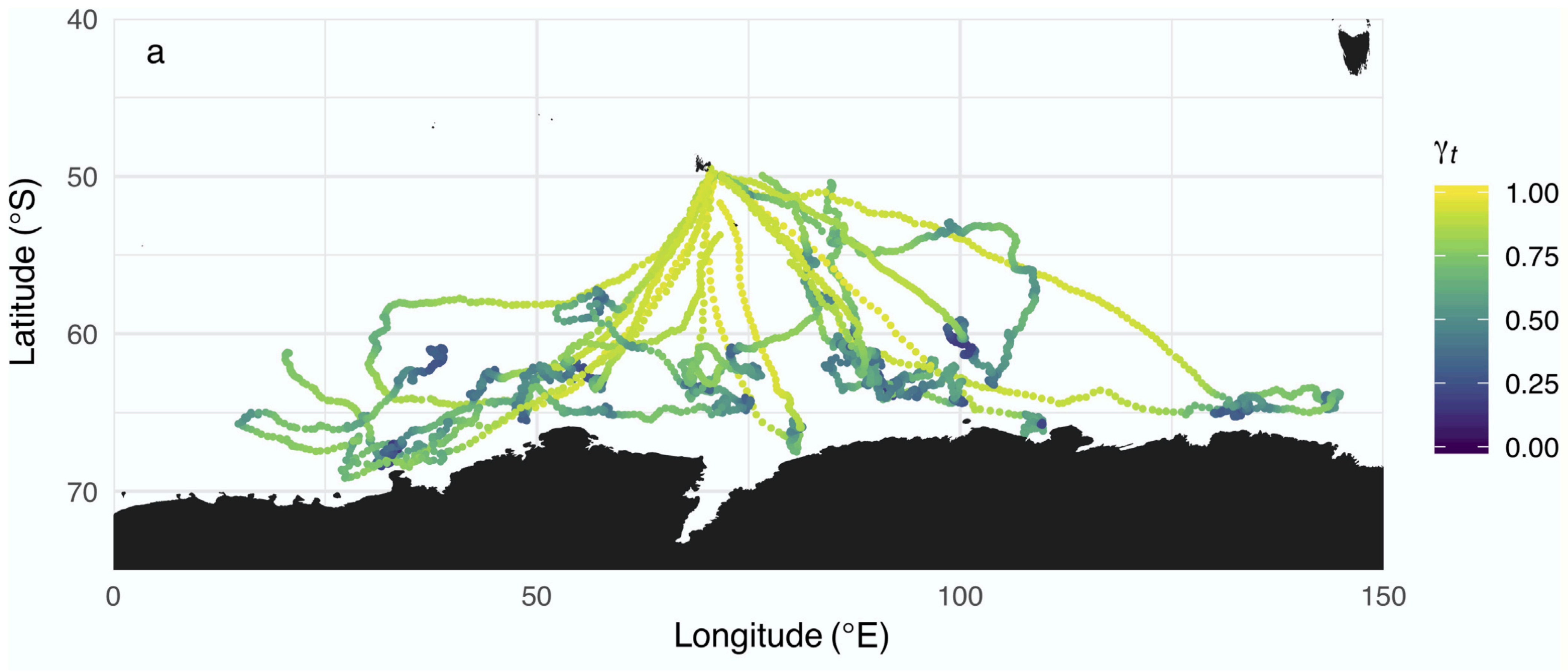
Method	Framework	Pros	Cons	R package	
Kalman filter and MLE	Frequentist	Efficient and exact	Only applicable to linear Gaussian SSMs	dlm, MARSS	
Laplace approximation	Frequentist	Efficient and flexible	States need to be approximable with a continuous unimodal distribution (e.g., no discrete states)	TMB	foieGras
Particle filter and iterated filtering	Frequentist	Flexible	Can be slow and sensitive to starting values	pomp, nimble	
MCMC-MH	Bayesian	Flexible	Can be slow and sensitive to convergence problems	rjags, nimble, R2WinBUGS, BRugs	bsam
MCMC-HMC	Bayesian	Efficient and flexible	Require continuous parameters and states or marginalization	rstan	
Information reduction	Bayesian	Flexible and fewer estimation problems	Can be slow and imprecise	EasyABC	

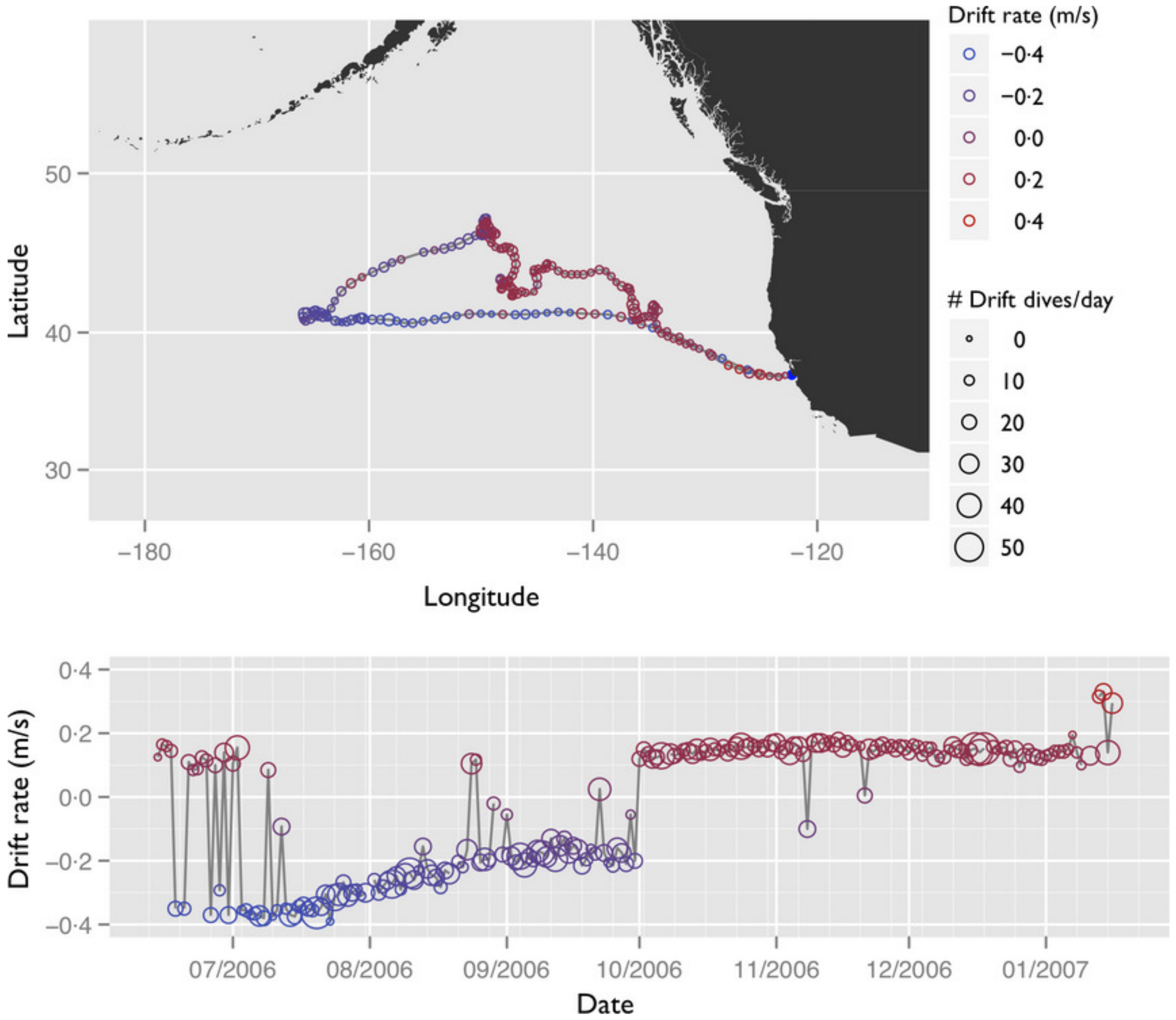
Notes: HMC, Hamiltonian Monte Carlo; MCMC, Markov chain Monte Carlo; MH, Metropolis–Hastings; MLE, maximum-likelihood estimate.

Auger-Methe et al. 2021

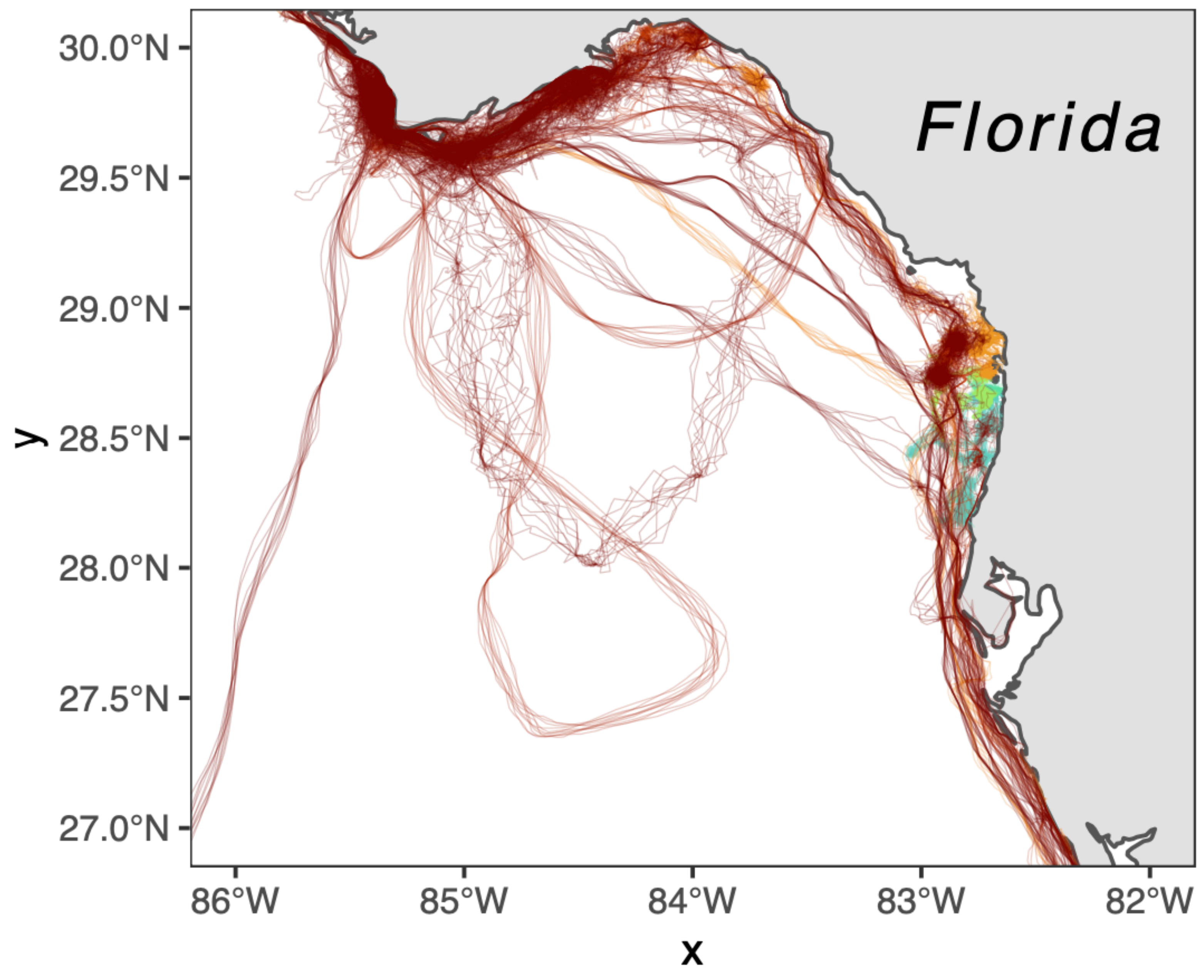
Motivating examples

Jonsen et al. 2019





Schick et al. 2013



Cullen et al. in prep

Let’s do some modeling!

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