HIDDEN MARKOV MODELS FOR MOVEMENT ECOLOGY

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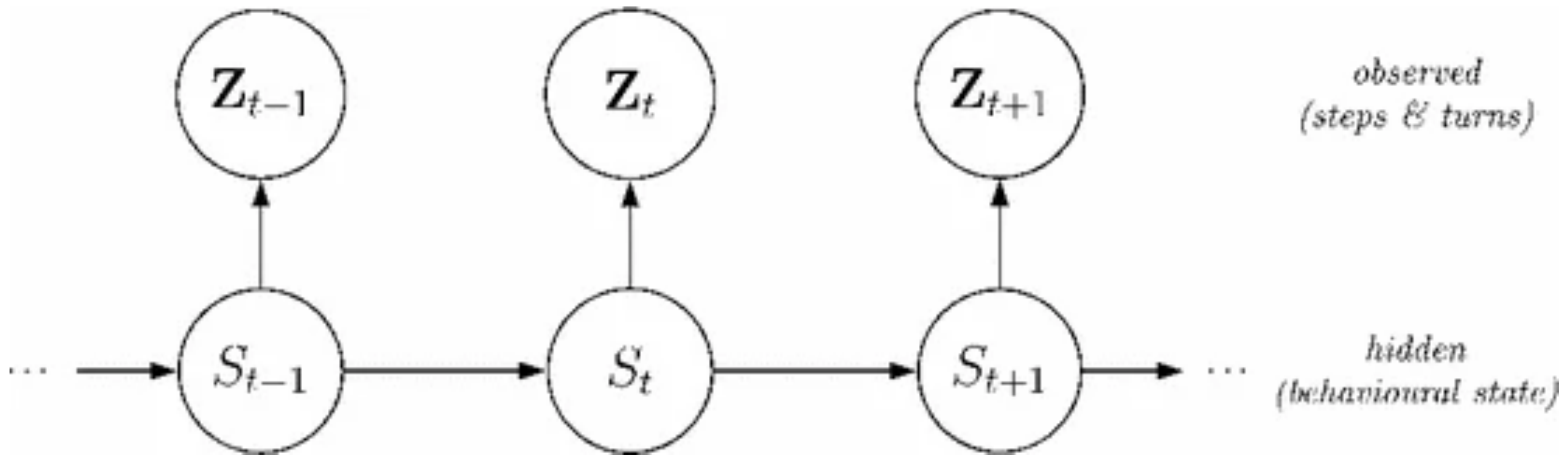
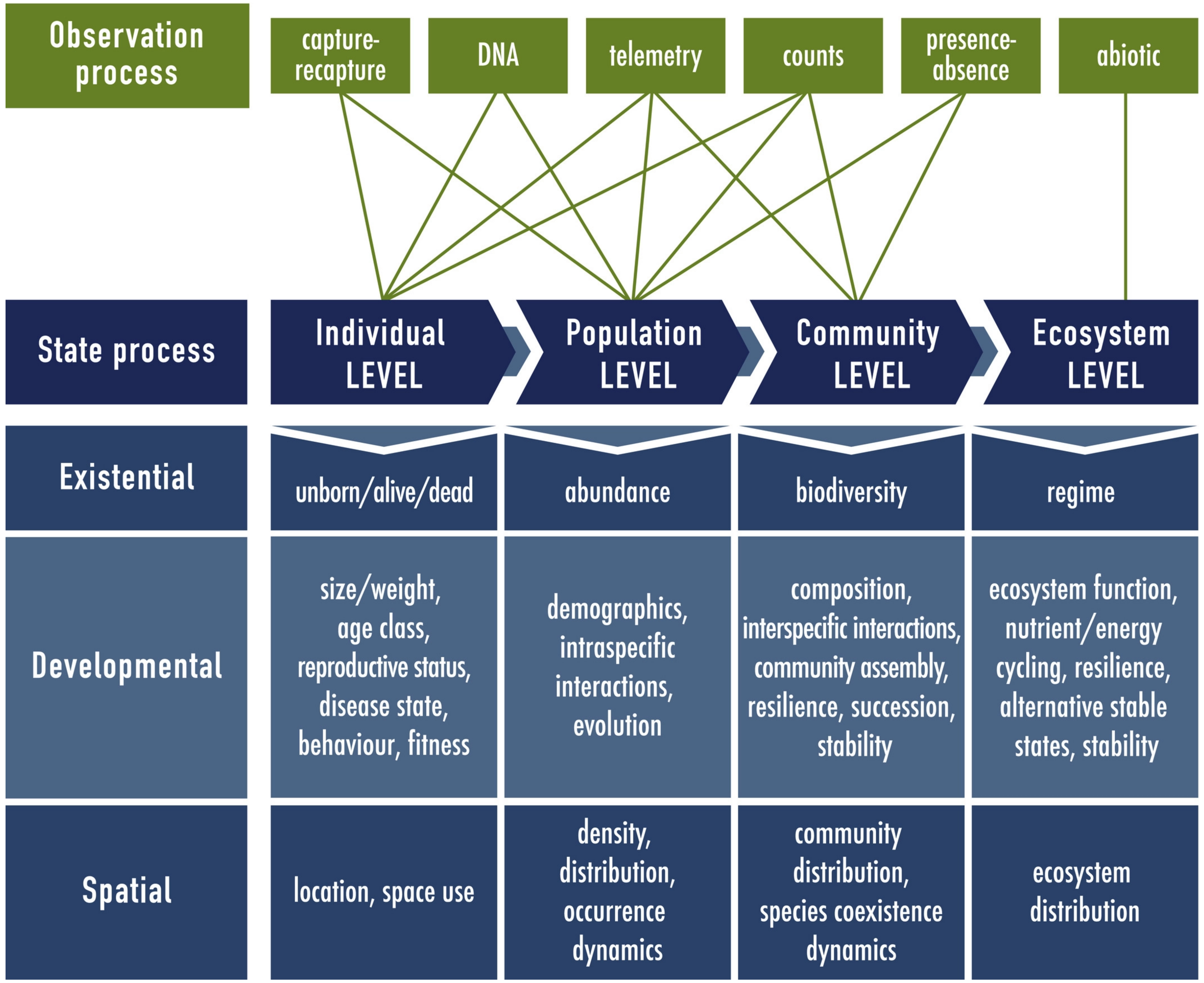
What is a hidden Markov model (HMM)?

- **McClintock et al. (2020)**: "A special class of state-space model with a finite number of hidden states that typically assumes some form of Markov property and the conditional independence property."
- **Langrock et al. (2012)**: "Discrete-time hidden Markov models (HMMs) achieve considerable computational gains by focusing on observations that are regularly spaced in time, and for which the measurement error is negligible. These conditions are often met, in particular for data related to terrestrial animals, so that a likelihood-based HMM approach is feasible."
- **Patterson et al. (2009)**: "…HMMs provide a statistically rigorous framework for incorporating covariates, for allowing for the autocorrelation commonly encountered in [animal telemetry] data, and for making inferences about behavioural states."
- **Patterson et al. (2017)**: For HMMs, "one typically considers bivariate time series comprising step lengths and turning angles, regularly spaced in time and assumed to be observed with no or only negligible error."



Patterson et al. 2017

What is a hidden Markov model (HMM)?



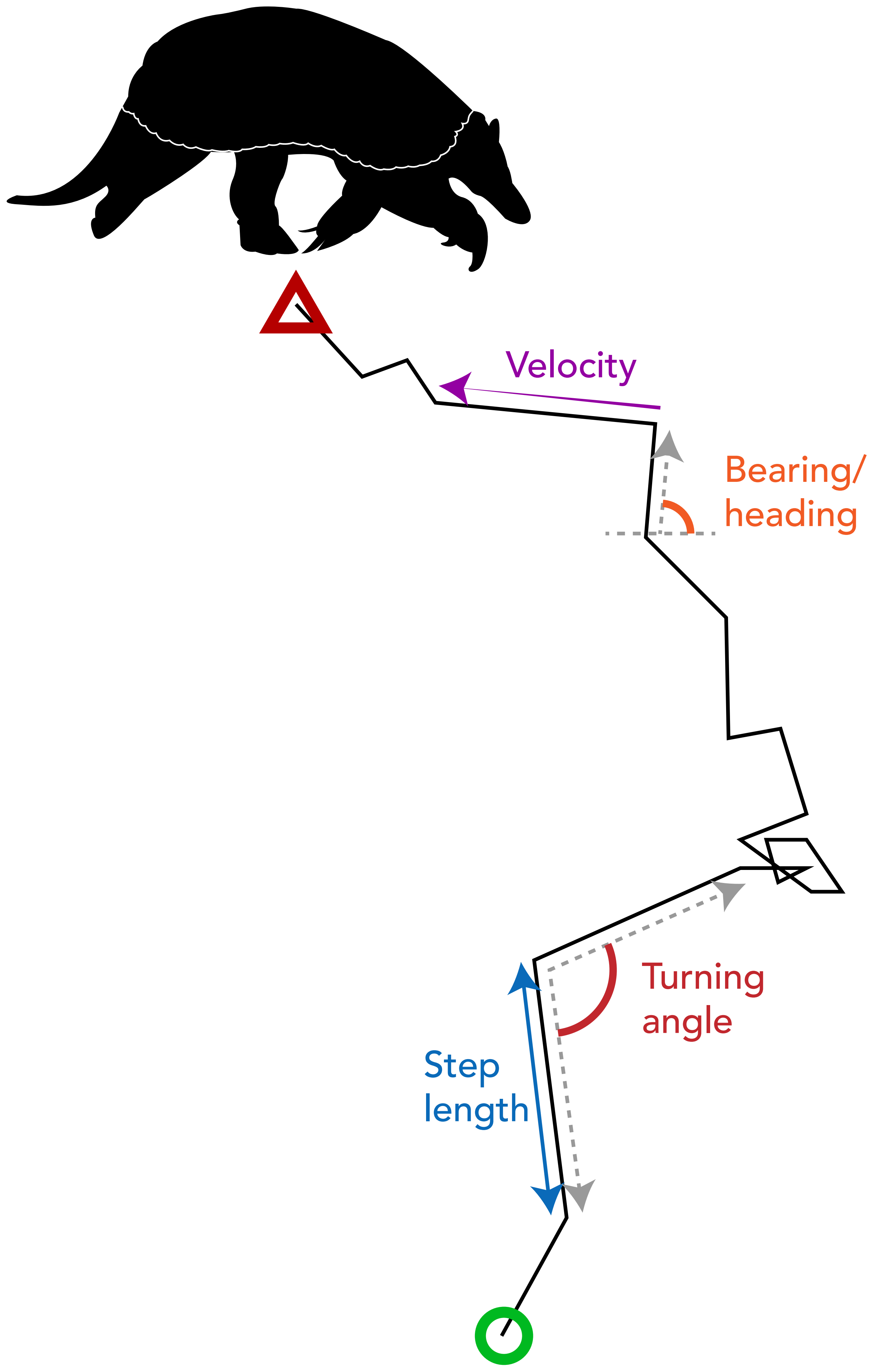
Latent variable models

	State space	
	Continuous	**Discrete**
Temporal dependence	State-space model	Hidden Markov model
Temporal independence	Continuous mixture model	Finite mixture model

McClintock et al. 2020

Movement metrics analyzed by HMMs

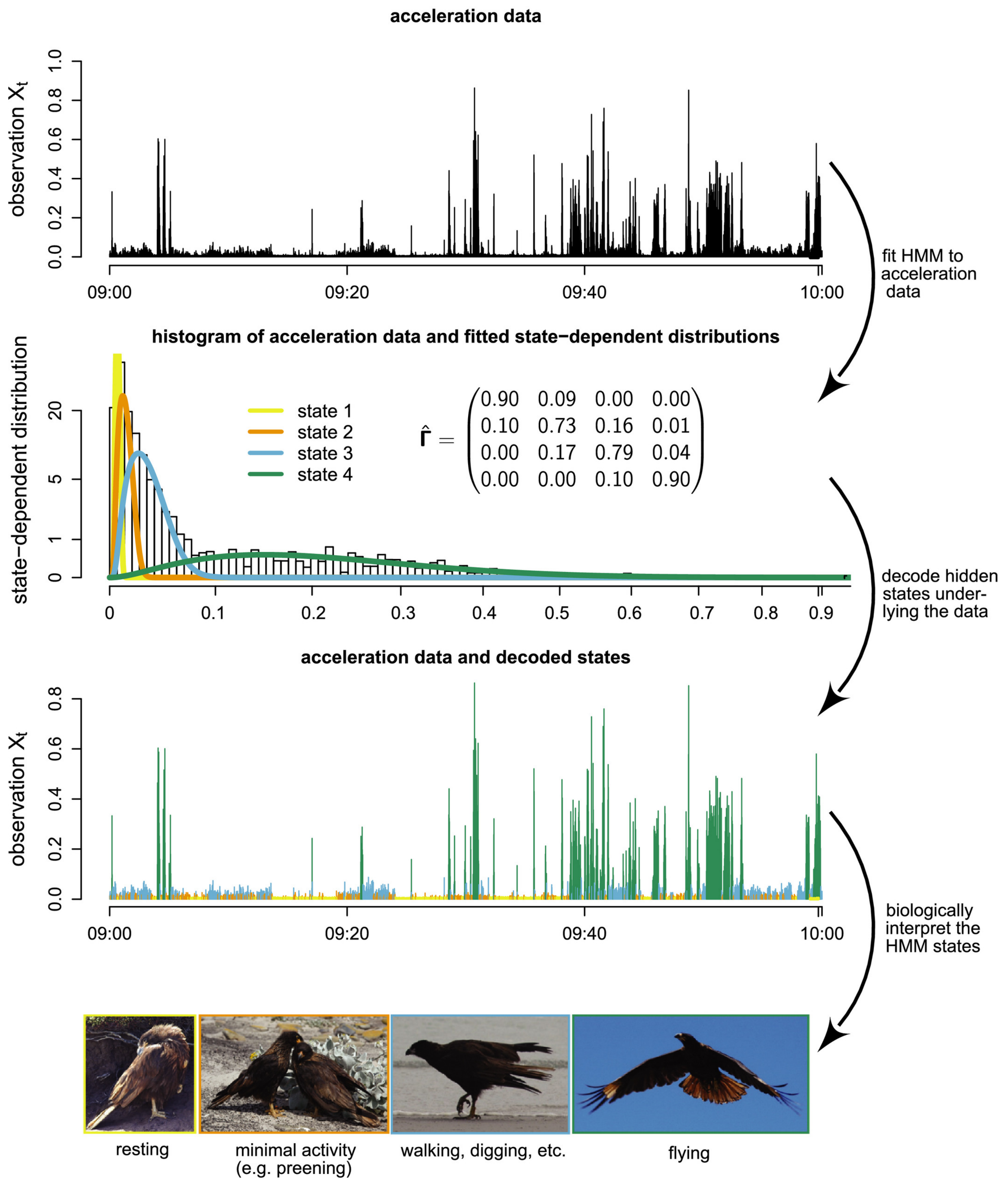
- Bivariate positions or their increments
- Distance between successive observations
- Compass direction (i.e., heading/ bearing)
- Changes in direction between successive relocations



Patterson et al. 2017

Which process models are used for HMMs?

- Almost always use discrete-time models w/ CRW
- Some options starting to become available for biased RW (BRW), biased-CRW (BCRW), and continuous-time
    - But infrequently used



McClintock et al. 2020

Components of an HMM

1. Initial state distribution (δ)
   - Defines the starting probabilities for each of the N states

2. State transition probability matrix (t.p.m.; $\mathbf{\Gamma}$)
   - Defines the probabilities of remaining in (or changing) behavioral states

3. State-dependent distributions ($f_i(\mathbf{z_t})$)
   - Defines the distributions of the data streams that characterize each state

Model parameters

Realizations of state-dependent process for $t \in [1,T]$

Probability of being in behavioral state s at time t

$$
\begin{aligned}
\mathscr{L}^{\text{HMM}}(\boldsymbol{\theta}) &= f(\mathbf{z}_1, \ldots, \mathbf{z}_T) \\
&= \sum_{s_1=1}^{N} \cdots \sum_{s_T=1}^{N} f(\mathbf{z}_1, \ldots, \mathbf{z}_T \mid s_1, \ldots, s_T) f(s_1, \ldots, s_T) \\
&= \sum_{s_1=1}^{N} \cdots \sum_{s_T=1}^{N} \delta_{s_1} \prod_{t=1}^{T} f(l_t \mid s_t) f(\phi_t \mid s_t) \prod_{t=2}^{T} \gamma_{s_{t-1}, s_t},
\end{aligned}
$$

Patterson et al. 2017

McClintock et al. 2020

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Example of 2-state HMM

Behavioral state at time $t = 1$

$$\delta = [Pr(S_1 = 1), Pr(S_1 = 2)]$$

Behavioral state index for all N states

$$\sum_{i=1}^{N} \delta_i = 1$$

Probabilities must sum to 1

McClintock et al. 2020

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Example of 2-state HMM

Probability of switching from state j to i

Stay in state 1

Switch from state 2 to 1

$$\mathbf{\Gamma} = (\gamma_{ij}) = \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{bmatrix}$$

Switch from state 1 to 2

Stay in state 2

$$\mathbf{\Gamma}^{(t)} = (\gamma_{ij}^{(t)}) = \frac{exp(\eta_{ij})}{\sum_{k=1}^{N} exp(\eta_{ik})}$$

Multinomial logit link function

η_{ij} = linear model

Incorporate effect of covariates on state switching probability

McClintock et al. 2020

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Example of 2-state HMM

(Including only step lengths and turning angles)

Realization of state-dependent process at time t

State-dependent turning angle

$$\mathbf{z_t} = (l_t, \phi_t)$$

State-dependent step length

Symbol for probability density function (PDF) in state i

$$f_i(\mathbf{z_t}) = f(\mathbf{z_t} \mid S_t = i) = f((l_t, \phi_t) \mid S_t = i) = f(l_t \mid S_t = i) f(\phi_t \mid S_t = i)$$

State-dependent step length <u>distribution</u>

State-dependent turning angle <u>distribution</u>

McClintock et al. 2020

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Patterson et al. 2017

$$\mathscr{L}^{\text{HMM}}(\boldsymbol{\theta}) = f(\mathbf{z}_1, \ldots, \mathbf{z}_T)$$
$$= \sum_{s_1=1}^{N} \cdots \sum_{s_T=1}^{N} f(\mathbf{z}_1, \ldots, \mathbf{z}_T \mid s_1, \ldots, s_T) f(s_1, \ldots, s_T)$$
$$= \sum_{s_1=1}^{N} \cdots \sum_{s_T=1}^{N} \underbrace{\delta_{s_1}}_{\text{Initial state distribution}} \prod_{t=1}^{T} \underbrace{f(l_t \mid s_t)}_{\text{State-dependent step length distribution}} \underbrace{f(\phi_t \mid s_t)}_{\text{State-dependent turning angle distribution}} \prod_{t=2}^{T} \underbrace{\gamma_{s_{t-1}, s_t}}_{\text{Probability of transition from state } j \text{ to } i},$$

McClintock et al. 2020

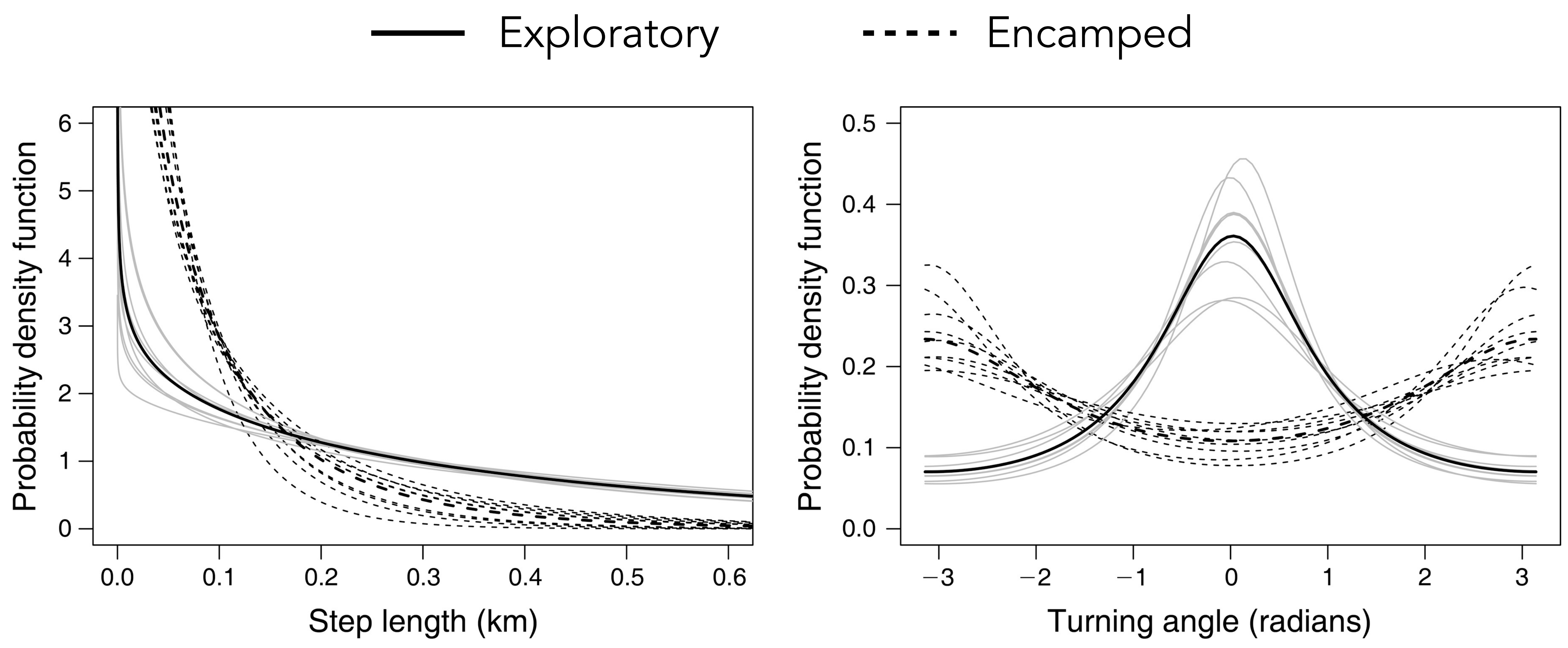
Common PDFs for movement variables

<u>Step length</u>

- Real, positive values that often exhibit a right-skewed distribution
- Gamma, Weibull

<u>Turning angle</u>

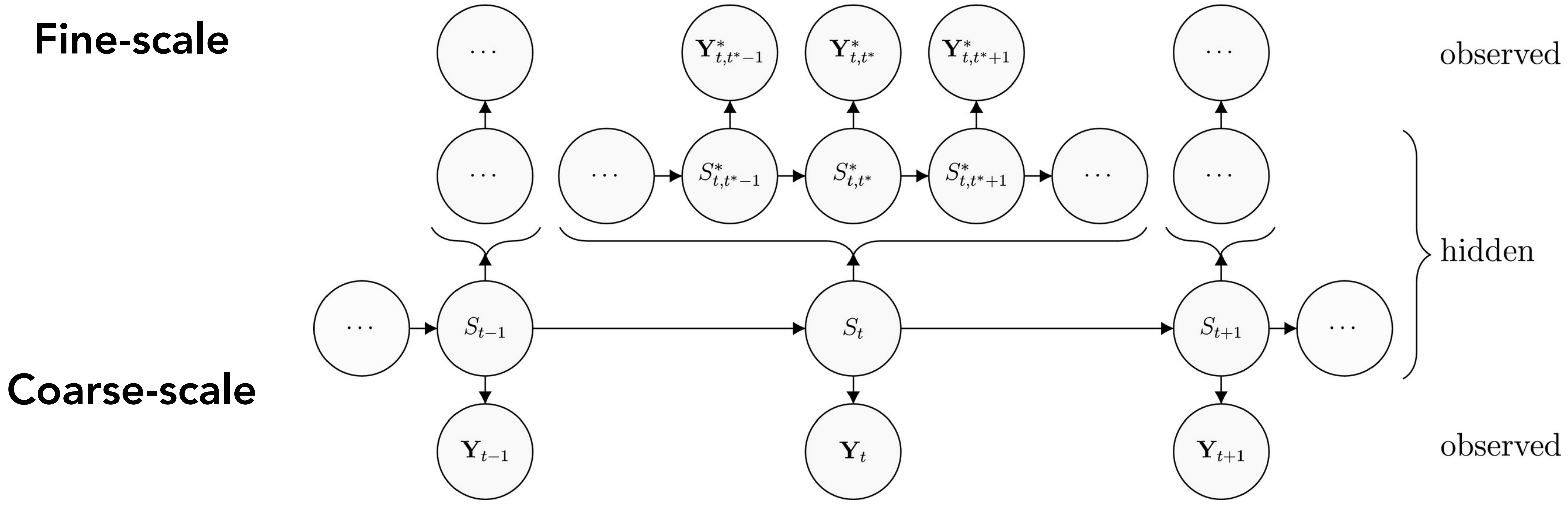
- Real values constrained from $-\pi$ to $+\pi$
- von Mises, wrapped Cauchy

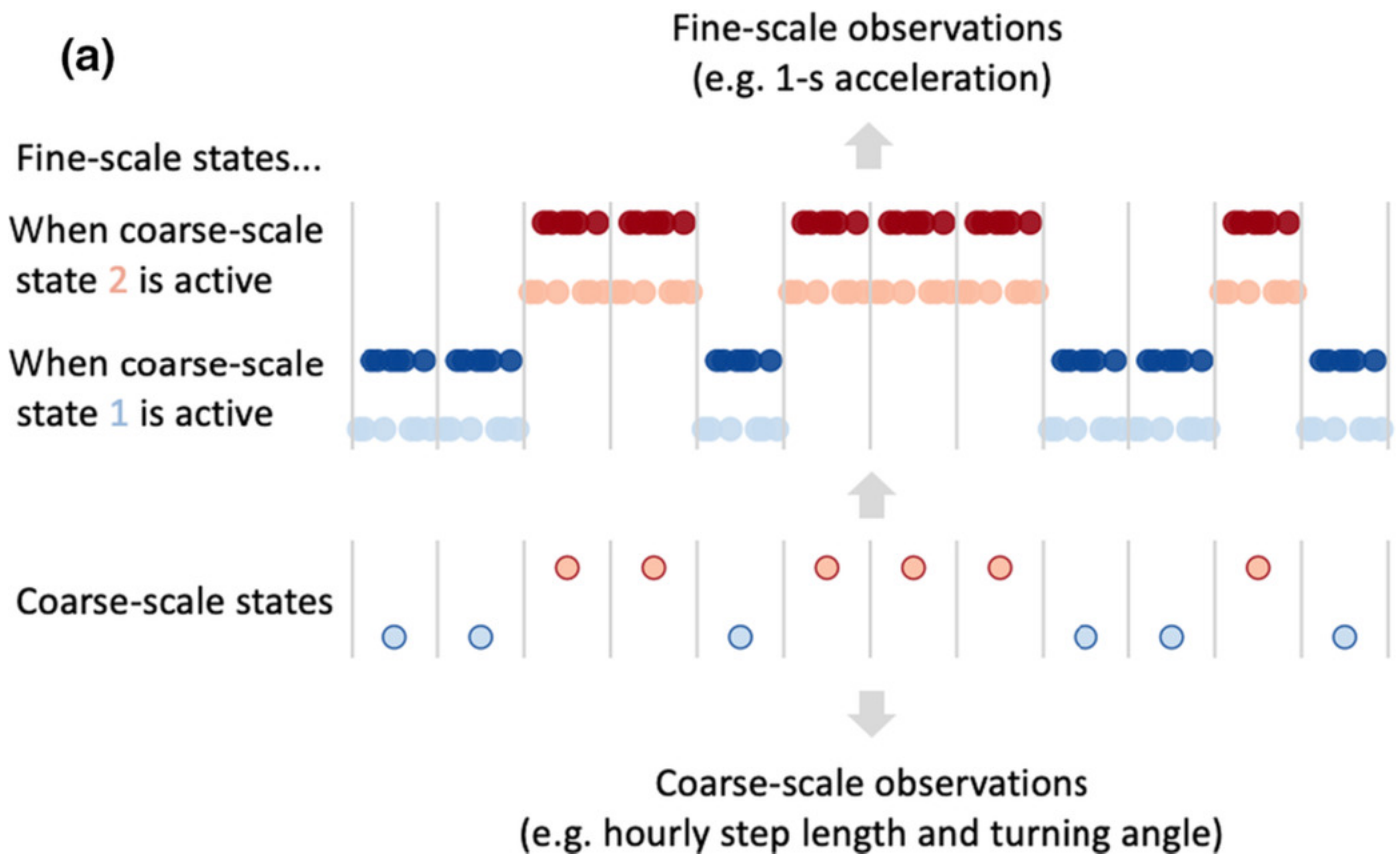


Langrock et al. 2012

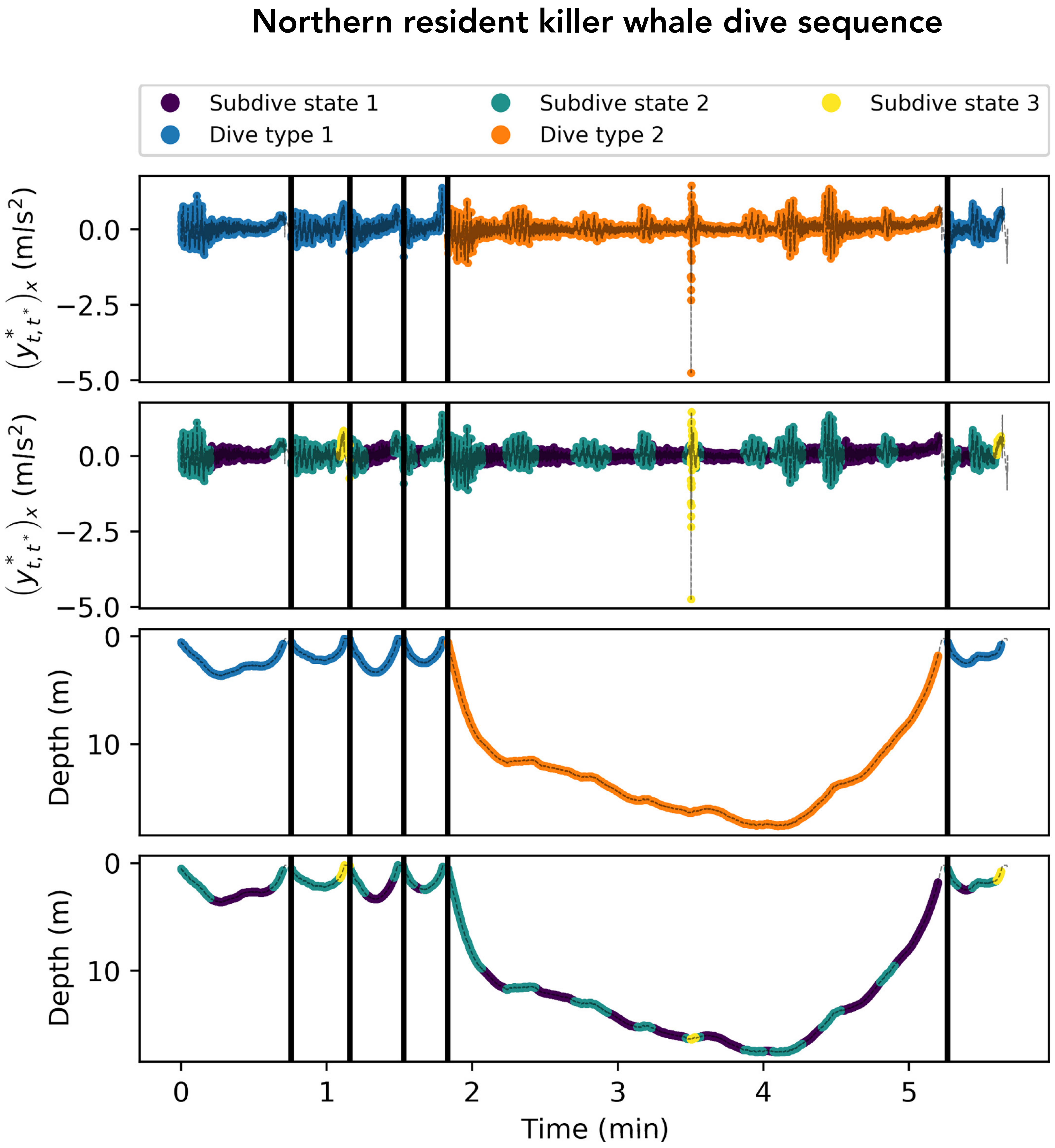
Hierarchical HMMs

Adam et al. 2019





Glennie et al. 2022



Sidrow et al. 2022

Methods to fit HMM

- Maximum likelihood estimation (MLE) is most common (for behavioral state estimation)

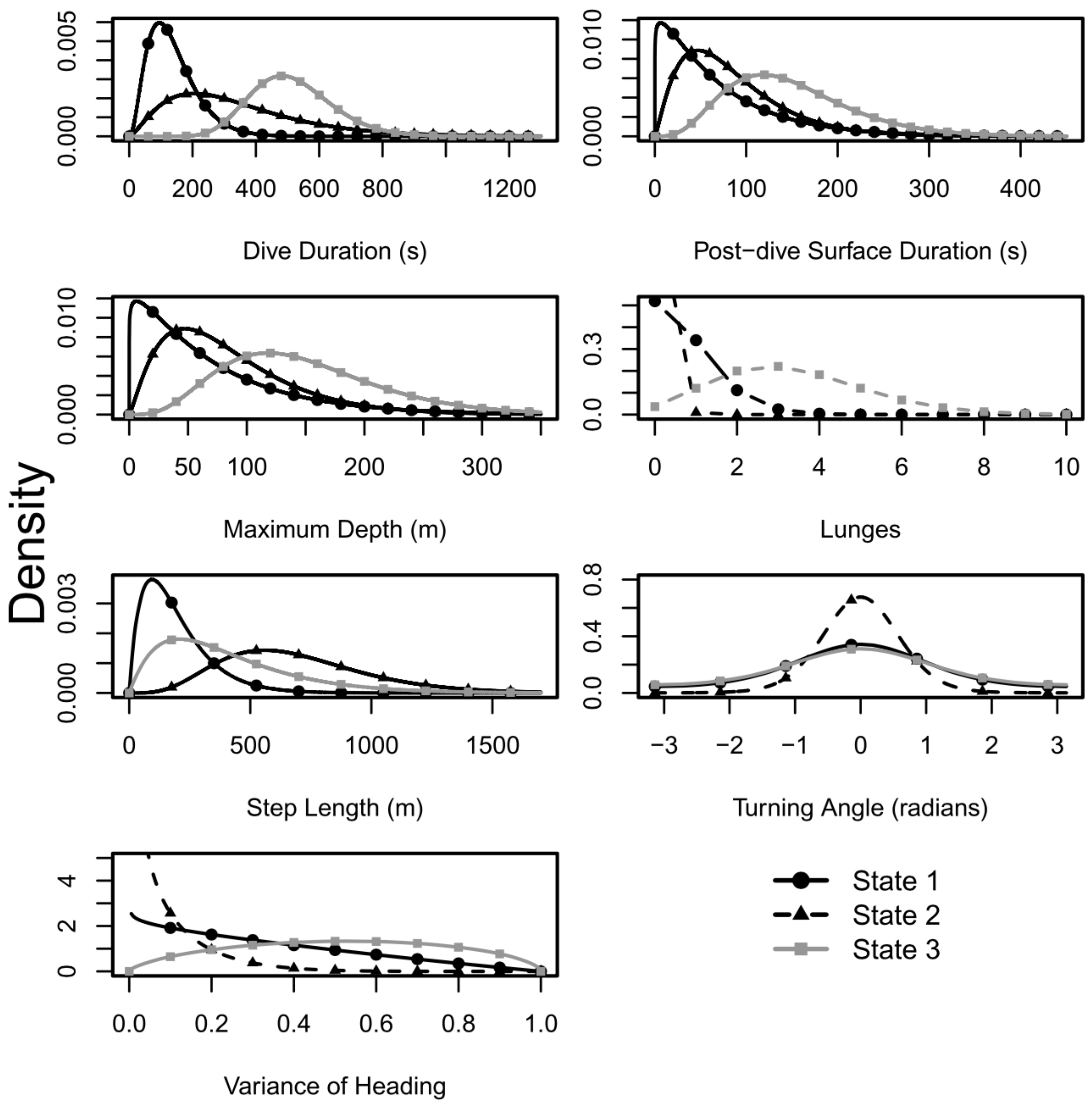
- Bayesian methods (via MCMC, HMC, etc) also possible, but less common

- Readily available functions to fit animal movement models using the `moveHMM` or `momentuHMM` R packages
  - `hmmTMB` is very new package that uses Laplace approximation via `TMB` package (similar to `foieGras`)

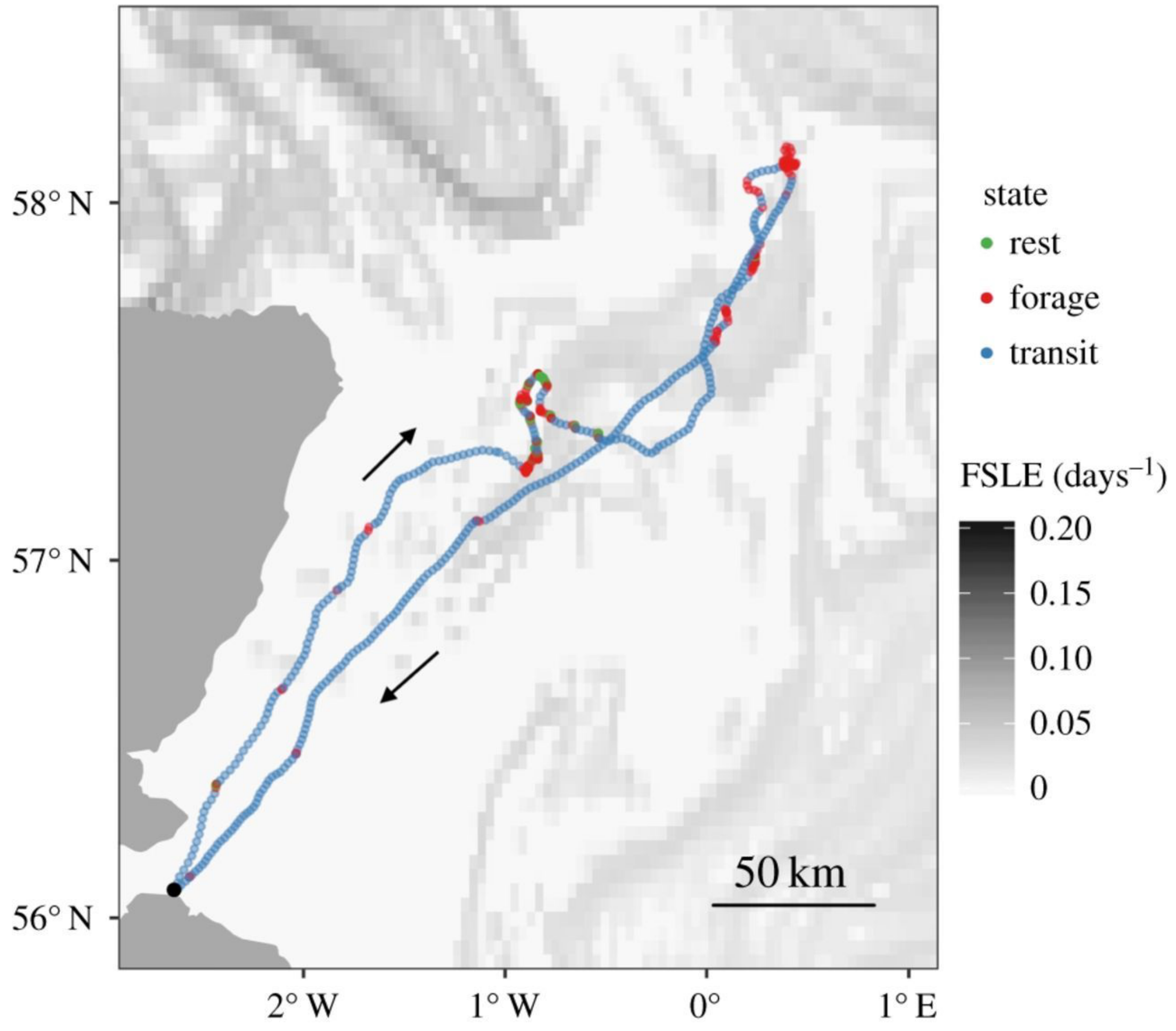
- If interested in comparison among available R packages, look at Table 2 of McClintock et al. (2020) for a comparison

Motivating examples

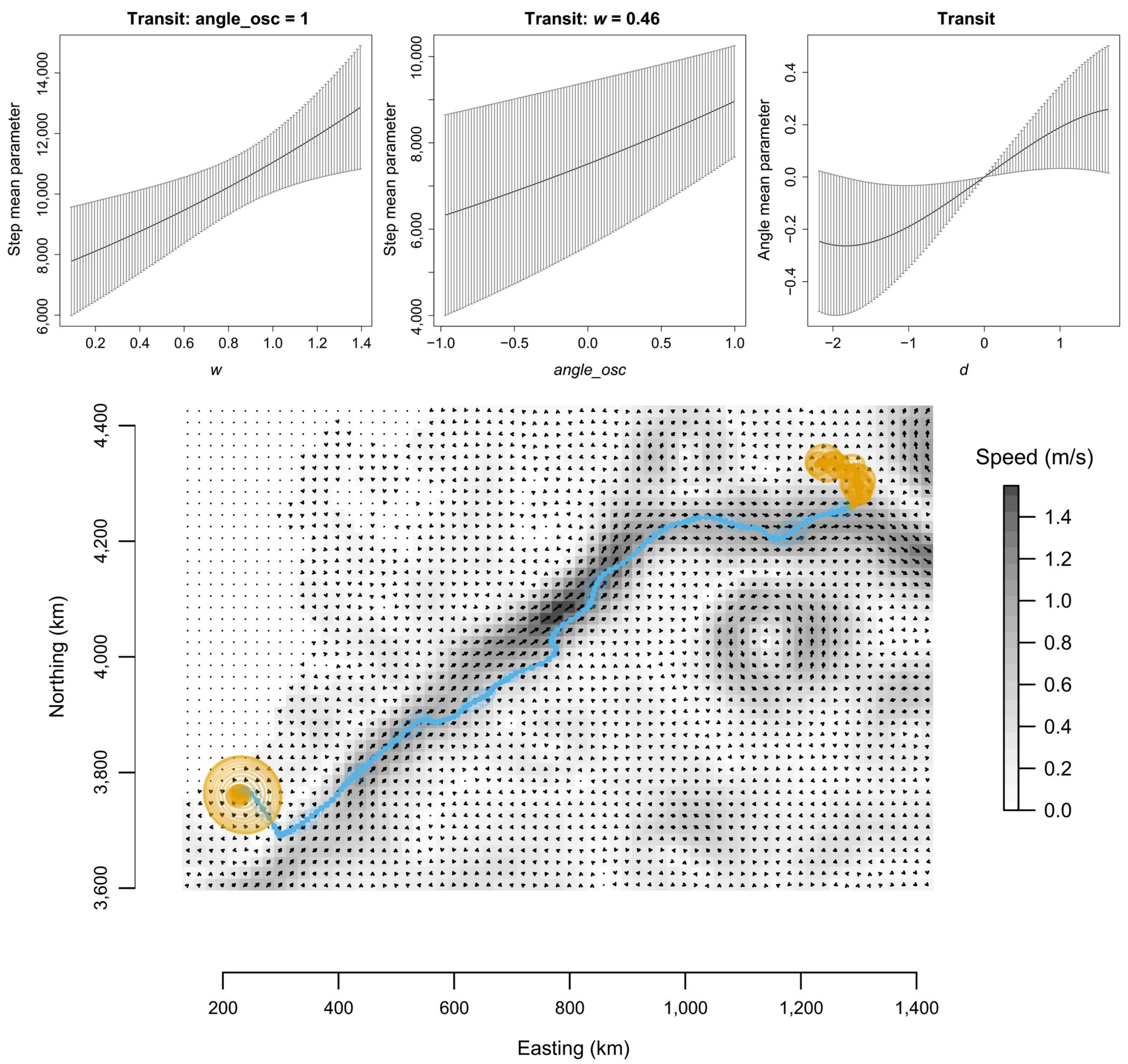
McClintock and Michelot 2018

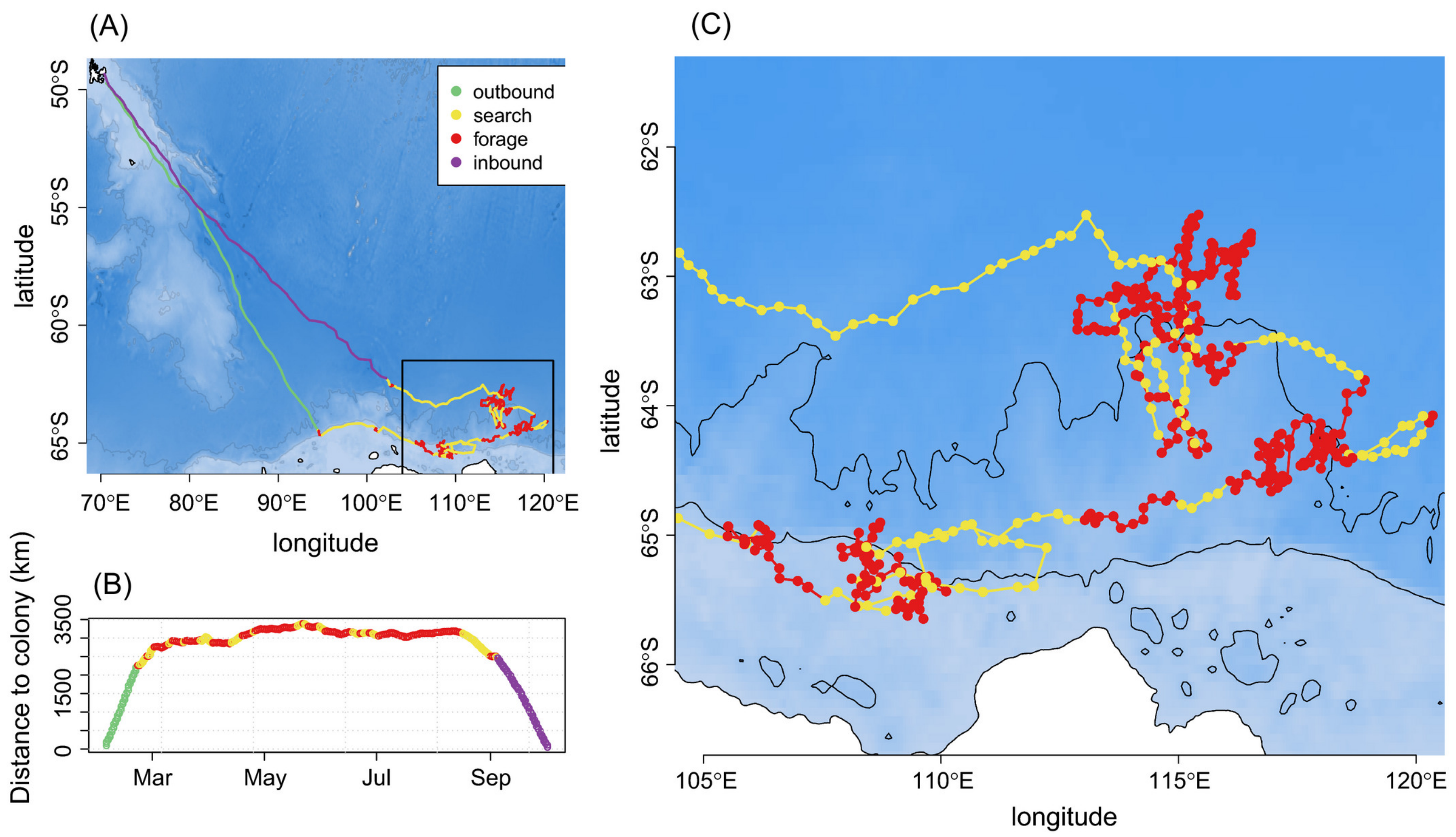


Deruiter et al. 2017



Grecian et al. 2018





Michelot et al. 2017

Let’s do some modeling!

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