- Describe entropy/ratchet/balance
- quantify deletion bias
- quantify entropic force
- quantify selective pressures
- implement evolutionary simulation to check above ↑.
- How does equilibrium network size depend on the parameters p_{mut}, p_{del}, $\alpha = p_{\text{new}}/p_{\text{del}}$, σ_{mut}, N?

parameter	minimum	default	maximum
p_{mut}	10^{-4}	10^{-3}	10^{-1}
p_{del}	10^{-6}	10^{-4}	10^{-1}
α	10^{-2}	10^{-1}	1
σ_{mut}	10^{-4}		10^{-1}
N		1000	

- different systems
- change code to make p_{add} and p_{del} per gene instead of per system.
- possible experiment: compare equilibrium sizes of systems with a range of large real and negative real eigenvalues. Do systems with large real and positive eigenvalues tend to be smaller due to numerical instability?

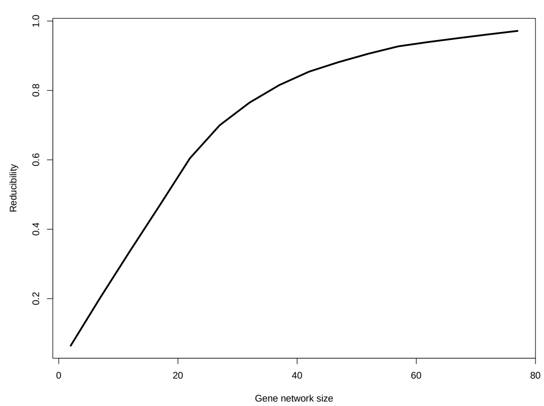

Figure 1: Network reducibility. Where α between $(0, 1)$ on the y-axis, determines equilibrium system size for deletion bias. For instance, at $\alpha = 0.5$, we'd expect system sizes to be around $n = 20$.

Deletion bias $\beta :=$ probability a deletion is neutral-ish.
At equilibrium: rate of deletion $\approx$ the rate of new genes.
$p_{\text{del}}\beta \approx p_{\text{new}}$ $\quad$ *i.e.*$\beta \approx \alpha$.

Load System size n
mutation cost s
Strong mutation: $s \approx 1$
Weak selection: mean offspring fitness relative to parent $= s p_{\text{mut}} n^2 = \delta_{\text{add}}$
Where δ_{add} is the cost of adding a dimension to a system.

Analytical description Can we find a function that, given the current mean size n_T of a population of systems, describes what is the expected size in the next generation/offspring n_{T+1}?

Offspring size	%	Mean fitness relative to parent
$n-1$	np_{del}	$1-s_n^{\text{DEL}}$
n	$1-(p_{\text{del}}+p_{\text{add}})n$	$1-p_{\text{mut}}n^2 s_n$
$n+1$	np_{add}	$1-p_{\text{mut}}(n+1)^2 s_{n+1}$

- Mean fitness of offspring:
  $= (1-p_{\text{mut}}n^2 s_n) + np_{\text{del}}(1-s_n^{\text{DEL}} - (1-p_{\text{mut}}n^2 s_n)) + np_{\text{add}}(1-p_{\text{mut}}(n+1)^2 s_{n+1}) - (1-p_{\text{mut}}n^2 s_n)$
  $= 1-p_{\text{mut}}n^2 s_n - np_{\text{del}}(s_n^{\text{DEL}} - p_{\text{mut}}n^2 s_n) - np_{\text{add}}p_{\text{mut}}((n+1)^2 s_{n+1} - n^2 s_n)$
- Mean network size in offspring
  $= \frac{\sum_{\text{sizes}}(\%\text{size})\times(\text{fitness at size})\times\text{size})}{\sum_{\text{sizes}}(\%\text{size})\times(\text{fitness at size})}$
  $= n + \frac{np_{\text{add}}(1-p_{\text{mut}}(n+1)^2 s_{n+1}) - np_{\text{del}}(1-s_n^{\text{DEL}})}{\bar{w}}$
  $= \frac{(n-1)a_{n-1}+na_n+(n+1)a_{n+1}}{a_{n-1}+a_n+a_{n+1}} = n + \frac{a_{n+1}-a_{n-1}}{a_{n+1}+a_n+a_{n-1}}$
- So: network size will go up if this is $> n$,
  that is if
  $\cancel{n}p_{\text{add}}(1-p_{\text{mut}}(n+1)^2 s_{n+1}) > \cancel{n}p_{\text{del}}(1-s_n^{\text{del}})$
  that is,
  $\alpha = \frac{p_{\text{add}}}{p_{\text{del}}} > \frac{1-s_n^{\text{DEL}}}{1-p_{\text{mut}}(n-1)^2 s_{n+1}}$
  $\approx 1 + p_{\text{mut}}(n+1)^2 s_{n+1} - s_n^{\text{DEL}}$
  To be more exact:
  $\alpha > \frac{1-s_n^{\text{DEL}}}{1-p_{\text{mut}}((n+1)^2 s_{n+1} - (n-1)^2 s_{n-1})}$
  $\approx \frac{1-s_n^{DEL}}{1-p_{\text{mut}}4ns_n}$

$$\mathbb{E}\left[n_{t+1}\right] \approx n_t \left(1 + \frac{p_{\text{add}}\left(1-p_{\text{mut}}4ns_n\right) - p_{\text{del}}\left(1-s_n^{\text{DEL}}\right)}{\bar{w}}\right) \tag{1}$$

For any type of system (e.g. oscillator), we need to find empirically, s_n^{DEL} and s_n. We also need to know how this changes with σ_{mut}.

What is the math for mutational meltdown?