

- Describe entropy/ratchet/balance
- quantify deletion bias
- quantify entropic force
- quantify selective pressures
- implement evolutionary simulation to check above \uparrow .
- How does equilibrium network size depend on the parameters $p_{\text{mut}}, p_{\text{del}}, \alpha = p_{\text{new}}/p_{\text{del}}, \sigma_{\text{mut}}, N$?

parameter	minimum	default	maximum
p_{mut}	10^{-4}	10^{-3}	10^{-1}
p_{del}	10^{-6}	10^{-4}	10^{-1}
α	10^{-2}	10^{-1}	1
σ_{mut}	10^{-4}		10^{-1}
N		1000	

- different systems
- change code to make p_{add} and p_{del} per gene instead of per system.
- possible experiment: compare equilibrium sizes of systems with a range of large real and negative real eigenvalues. Do systems with large real and positive eigenvalues tend to be smaller due to numerical instability?

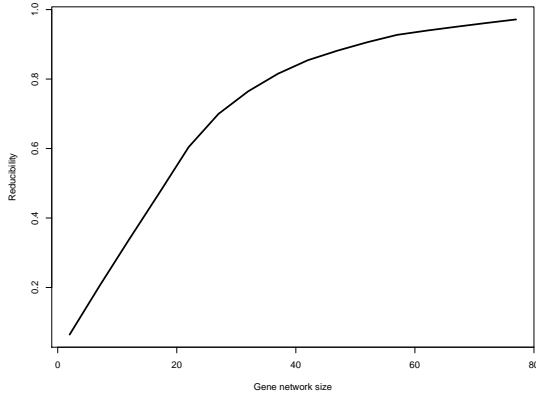


Figure 1: Network reducibility. Where α between $(0,1)$ on the y-axis, determines equilibrium system size for deletion bias. For instance, at $\alpha = 0.5$, we'd expect system sizes to be around $n = 20$.

Deletion bias $\beta :=$ probability a deletion is neutral-ish.

At equilibrium: rate of deletion \approx the rate of new genes.

$$p_{\text{del}}\beta \approx p_{\text{new}} \quad \text{i.e. } \beta \approx \alpha.$$

Load System size n

mutation cost s

Strong mutation: $s \approx 1$

Weak selection: mean offspring fitness relative to parent $= sp_{\text{mut}}n^2 = \delta_{\text{add}}$

Where δ_{add} is the cost of adding a dimension to a system.

Analytical description Can we find a function that, given the current mean size n_T of a population of systems, describes what is the expected size in the next generation/offspring n_{T+1} ?

Offspring size	%	Mean fitness relative to parent
$n - 1$	np_{del}	$1 - s_n^{\text{DEL}}$
n	$1 - (p_{\text{del}} + p_{\text{add}})n$	$1 - p_{\text{mut}}n^2s_n$
$n + 1$	np_{add}	$1 - p_{\text{mut}}(n + 1)^2s_{n+1}$

- Mean fitness of offspring:

$$\begin{aligned}
&= (1 - p_{\text{mut}}n^2s_n) + np_{\text{del}}(1 - s_n^{\text{DEL}} - (1 - p_{\text{mut}}n^2s_n)) + np_{\text{add}}(1 - p_{\text{mut}}(n + 1)^2s_{n+1}) - (1 - p_{\text{mut}}n^2s_n) \\
&= 1 - p_{\text{mut}}n^2s_n - np_{\text{del}}(s_n^{\text{DEL}} - p_{\text{mut}}n^2s_n) - np_{\text{add}}p_{\text{mut}}((n + 1)^2s_{n+1} - n^2s_n)
\end{aligned}$$

- Mean network size in offspring

$$\begin{aligned}
&= \frac{\sum_{\text{sizes}} (\% \text{size}) \times (\text{fitness at size}) \times \text{size}}{\sum_{\text{sizes}} (\% \text{size}) \times (\text{fitness at size})} \\
&= n + \frac{np_{\text{add}}(1 - p_{\text{mut}}(n + 1)^2s_{n+1}) - np_{\text{del}}(1 - s_n^{\text{DEL}})}{\bar{w}} \\
&= \frac{(n - 1)a_{n-1} + na_n + (n + 1)a_{n+1}}{a_{n-1} + a_n + a_{n+1}} = n + \frac{a_{n+1} - a_{n-1}}{a_{n+1} + a_n + a_{n-1}}
\end{aligned}$$

- So: network size will go up if this is $> n$,

that is if

$$np_{\text{add}}(1 - p_{\text{mut}}(n + 1)^2s_{n+1}) > np_{\text{del}}(1 - s_n^{\text{del}})$$

that is,

$$\begin{aligned}
\alpha = \frac{p_{\text{add}}}{p_{\text{del}}} &> \frac{1 - s_n^{\text{DEL}}}{1 - p_{\text{mut}}(n - 1)^2s_{n+1}} \\
&\approx 1 + p_{\text{mut}}(n + 1)^2s_{n+1} - s_n^{\text{DEL}}
\end{aligned}$$

To be more exact:

$$\begin{aligned}
\alpha &> \frac{1 - s_n^{\text{DEL}}}{1 - p_{\text{mut}}((n + 1)^2s_{n+1} - (n - 1)^2s_{n-1})} \\
&\approx \frac{1 - s_n^{\text{DEL}}}{1 - p_{\text{mut}}4ns_n}
\end{aligned}$$

$$\mathbb{E}[n_{t+1}] \approx n_t \left(1 + \frac{p_{\text{add}}(1 - p_{\text{mut}}4ns_n) - p_{\text{del}}(1 - s_n^{\text{DEL}})}{\bar{w}} \right) \quad (1)$$

For any type of system (e.g. oscillator), we need to find empirically, s_n^{DEL} and s_n . We also need to know how this changes with σ_{mut} .