Inventing Modern Sequence Models as a Music 320 Project

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Abstract

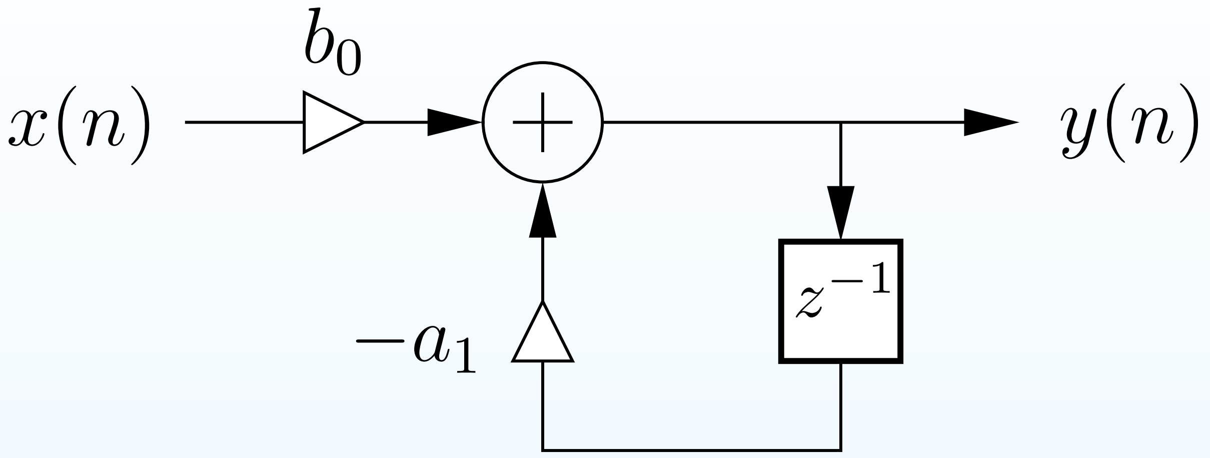
Today's sequence models (such as large language models) in machine learning (AI) arose from a blend of principle-based design and empirical discovery, spanning several fields. This talk describes how the ideas could have emerged from an elementary signal-processing approach. This viewpoint offers some features:

1. Signal processing folks can quickly learn what is happening in a motivated way
2. Machine-learning experts might benefit from signal-processing insights
3. Obvious suggestions for things to try next naturally arise

[Open House Schedule]

Music 320 Project Idea

One Pole Recursive Digital Filter



Pole at $z = -a_1$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{b_0}{1 + a_1\, z^{-1}}$$

Idea: Let's Make an Associative Memory!

- $x(n)$ can be a *long vector* $\underline{x}(n) \in \mathbb{R}^N$ representing *anything we want* — any "label"
- Set $\underline{b}_0 = 1$ and $\underline{a}_1 = -1$ to make $\underline{y}(n)$ a *sum of all input vectors* ("integrator")
- Choose the dimension N so large that *vectors in the sum are mostly orthogonal*
- Retrieve similar vectors using a *matched inner product* $\underline{w}^T \underline{x} > b$,
  for some suitable threshold b (Hey! That's one simulated neuron! ("Perceptron"))

Vector Retrieval by Inner Product

Given the sum of vectors

$$\underline{y}(n) = \sum_{m=0}^{n} \underline{x}(m)$$

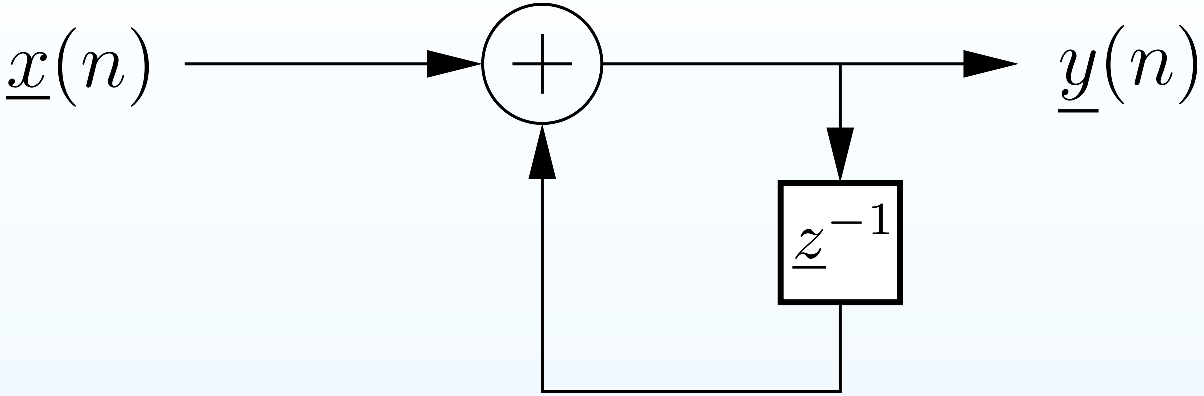
and a "query vector" $\underline{w} = \underline{x}(k)$,
find the query in the sum using an *inner product:*

$$\underline{w}^T \underline{y}(n) \;=\; \sum_{m=0}^{n} \underline{w}^T \underline{x}(m) \;\approx\; \underline{x}^T(k)\, \underline{x}(k) \;=\; \|\underline{x}(k)\|^2 \;>\; b(k)$$

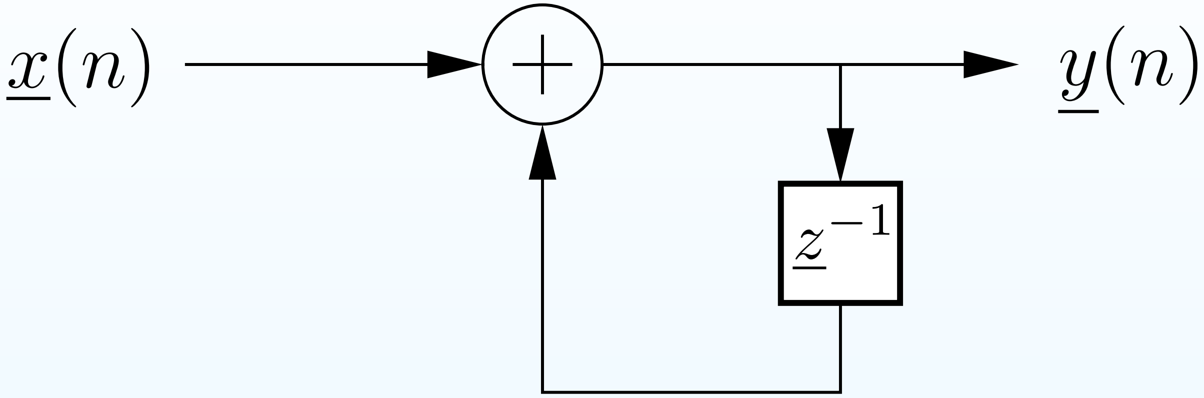
where $b(k)$ is the *detection threshold* for $\underline{x}(k)$

- This works because the spatial dimension is so large that $\underline{x}^T(j)\,\underline{x}(k) \approx 0$ for $j \neq k$
- Retrieval threshold $b(k)$ depends on $\|\underline{x}(k)\|^2$
  $\Rightarrow$ *reserve the radial dimension for similarity scoring*
- *I.e., only populate the* ***hypersphere*** in $\mathbb{R}^N$: $\|\, \underline{x}(k)\, \| = 1,\ \forall k$
- We just invented `RMSNorm`, used extensively in neural networks (not `LayerNorm`)

Cumulative Vector Memory



Gated Vector Memory



Input Vector Summer

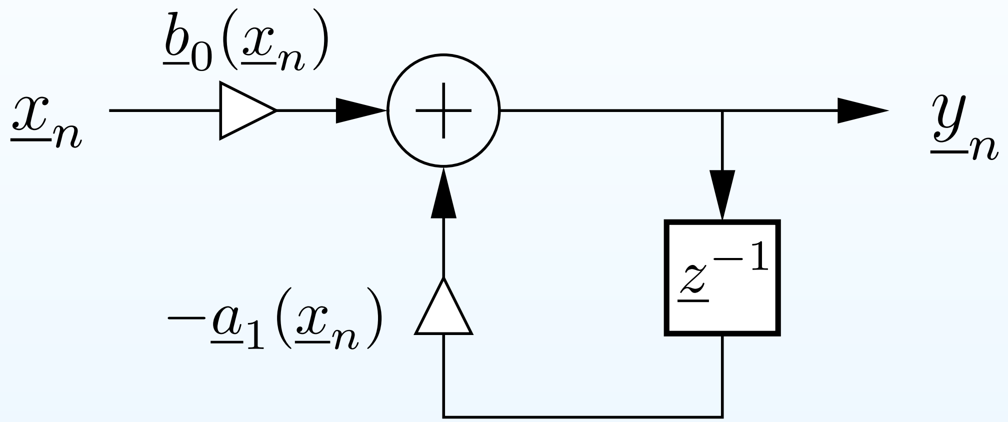
- **Problem:** Need a *memory reset*
- **Solution:** Set *feedback gain to zero* for one step to clear the memory

- **Problem:** Need an *input gate* to suppress unimportant inputs
- **Solution:** Set *input gain to zero* for unimportant inputs

- We just invented **gating**, used extensively in neural sequence models

Gated Recurrent Network

Idea: *Learn* the input and feedback gates as functions of $\underline{x}_n$ based on many input-output examples $(\underline{x}_n, \underline{y}_n)$ ("training data"):



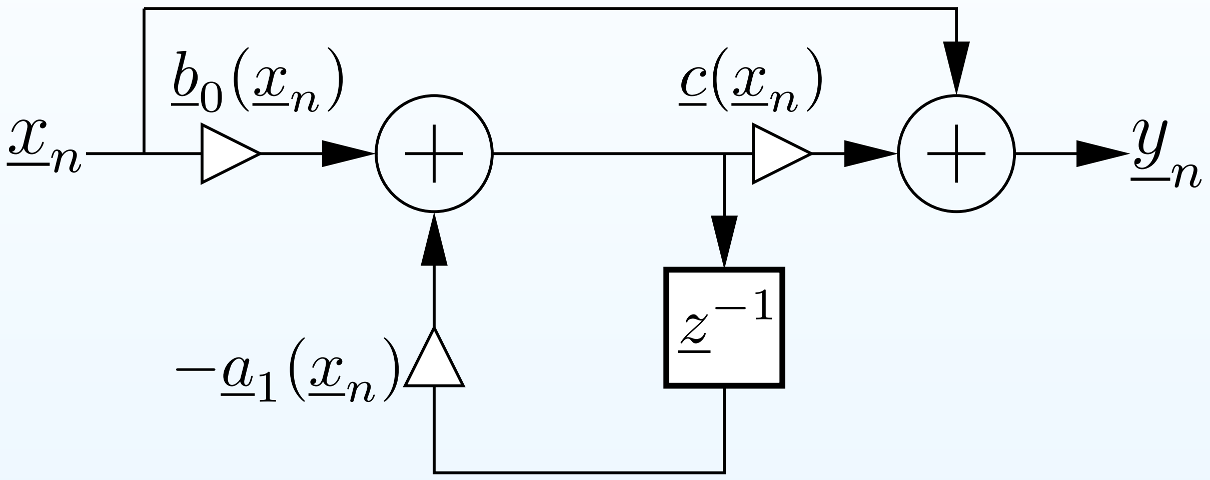
Vector Memory with Learned Input and Feedback Gates

"Obvious" Training Considerations:

- Initialize $-\underline{a}_1$ for desired initial memory duration (exponentially fading)
- Learn $-\underline{a}_1(\underline{x}_n)$ as $\mathbf{I} \cdot e^{-\Delta} \approx \mathbf{I} - \mathbf{I}\Delta$, and $\underline{\beta}_0(\underline{x}_n)$ as Linear$(\underline{x}_n, \underline{y}_n) \cdot \Delta$, where $\Delta =$ softPlus(parameter$(\underline{x}_n, \underline{Y}_n)$) (approximately gain-normalized)

Output Gating

Idea: Since we have input and feedback gates, why not an ***output gate?***



Gated RNN with **Skip Connection**

Output gating allows network to be “bypassed” when not helpful

Idea: For detecting vectors in $\underline{y}$ using *learned* inner-product weights and thresholds, use an array of *“Perceptrons”* (P)

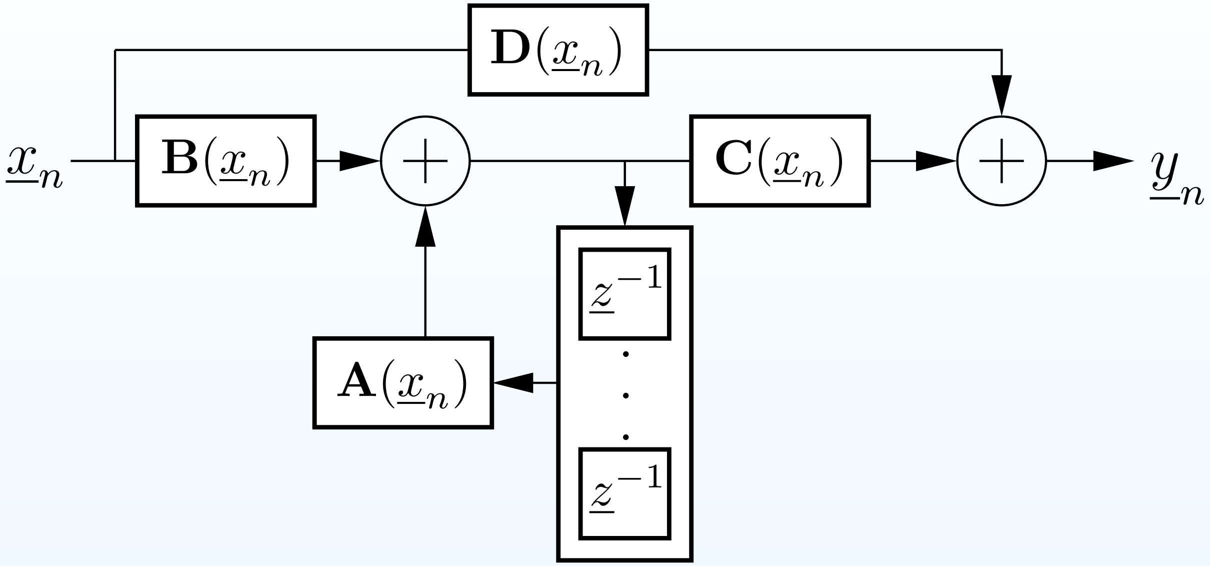
- Each P detects one or more memory vectors similar to its weight vectors
- The P outputs indicate *which weight-vectors are present* in the vector sum

Sequence Modeling

- If each vector represents a *word,* a vector sum is simply a *bag of words*
- To model a *sequence* of words, we have options:

1. Use a *positional encoding* scheme, such as
   (a) *Amplitude Decay* - Multiply the sum by a *forgetting factor* each sequence step (RNNs) - *poor choice* (conflates with angular distance on the hypersphere)
   (b) *Sinusoidal Amplitude Modulation* - Add a sinusoid with *increasing frequency* to each vector summing into the history (used in the original Transformer)
   (c) *Phase Shift* - Multiply by the sum by $e^{j\Delta}$ each sample ("RoPE") - *apparently most used today*
2. Use many vector-sum memories in parallel, positionally encoded ("State Expansion" in SSMs)

State Expansion



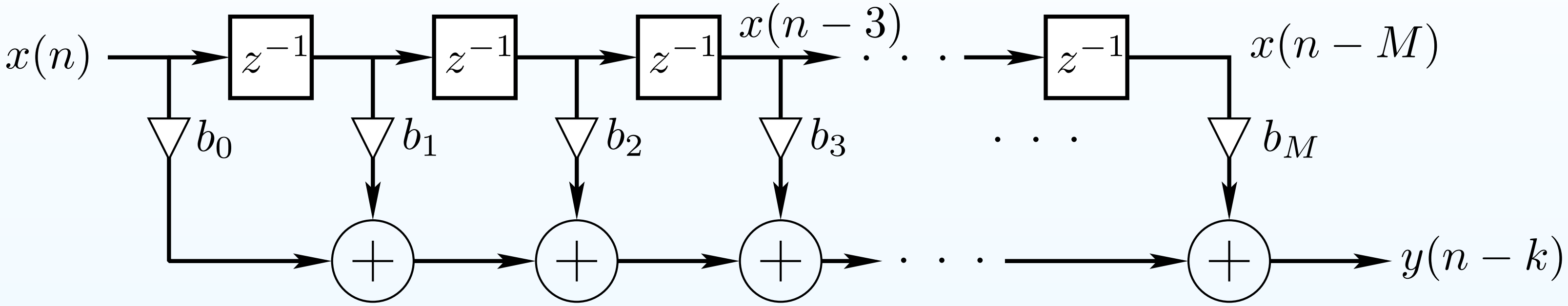
- Called a *"Structured State-space Model"* (SSM) in machine learning
- Feedback matrix $\mathbf{A}$ has been *diagonal* since "S4D" (2022)
  $\Rightarrow$ Parallel bank of vector one-poles (*gated, state-expanded RNNs*)
- Processed sequence ("context buffer") is *indefinitely long*
- Gating matrices are typically simple linear input projections, *e.g.*,

$$[\mathbf{B}(\underline{x}_n), \mathbf{C}(\underline{x}_n)] = \mathbf{L}\,\underline{x}_n$$

(See Mamba, *e.g.*)

Attention Layer

Idea: Also use *FIR Filtering*



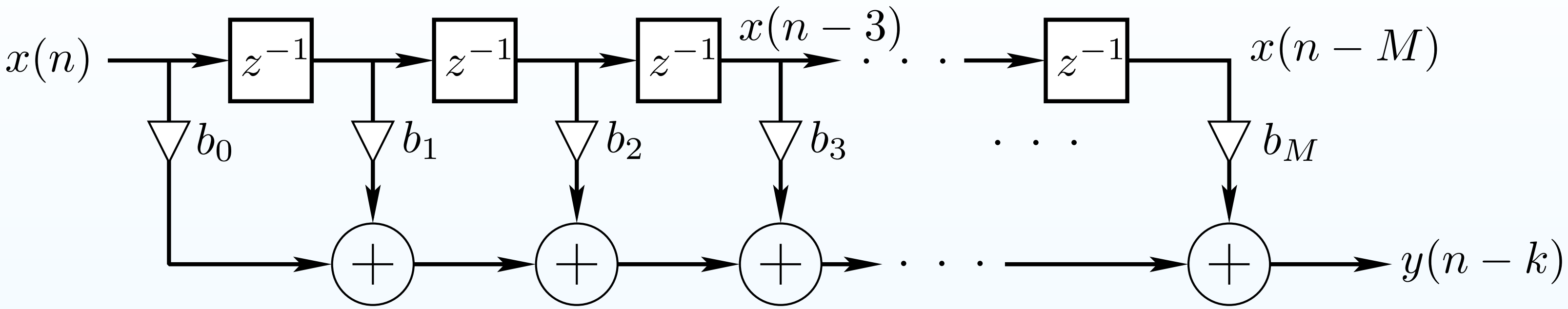
We can have *separately learned FIR coefficient matrices* $b_j[\underline{x}(n-j), k]$, which depend on the

1. input *position* j in the input sequence ("context buffer")
2. input *vector* $\underline{x}(n-j)$, $j = 0, 1, 2, \ldots, M$, (nonlinear)
3. *output-position* k being computed, $k = 0, 1, 2, \ldots, M$ ($M + 1$ outputs)

Idea: Add *relevance gating* suppressing unimportant inputs to each output ("attention")

Idea: Measure relevance using an *inner product* between the output and input positions ("dot-product attention")

Dot-Product Attention



Relevance Gating

Contribution from input $\underline{x}(n-j)$ to the nonlinear FIR sum for output $\underline{y}(n-k)$ is $(Q_k^T K_j)\underline{x}(n-j)$, where

- $Q_k[\underline{x}(n-k), k]$ is called the *query* vector for position k in the input sequence
- $K_j[\underline{x}(n-j), j]$ is called the *key* vector for position j in the input sequence

Idea: To provide more flexibility for the attention sum, replace $\underline{x}(n-j)$ in the attention sum with a learned *value vector* $V_j[\underline{x}(n-j)]$

$\Rightarrow$

Relevance of input j to output $k \propto (Q_k^T K_j)V_j$, where all three vectors are learned projections of $\underline{x}(n-j)$ for each (j,k) ("Transformer")

Multi-Head Attention

Idea: To support multiple meaning possibilities, *partition the model space* into parallel independent *attention calculations* (“multi-head attention”)

- Each *attention head* can form an independent input interpretation
- Useful for *ambiguous* sequences, especially in the lower layers
- Also introduced in the Transformer paper (2017)

Weight Tying

When the sequence model maps input to output in the same “language” (e.g., English to English), it makes sense to use the *same embedding vectors* at the input and output layers, instead of separately learning a set of weights for mapping to the final output. This is called “weight tying” (many fewer parameters, better results).

Hierarchical Blocks

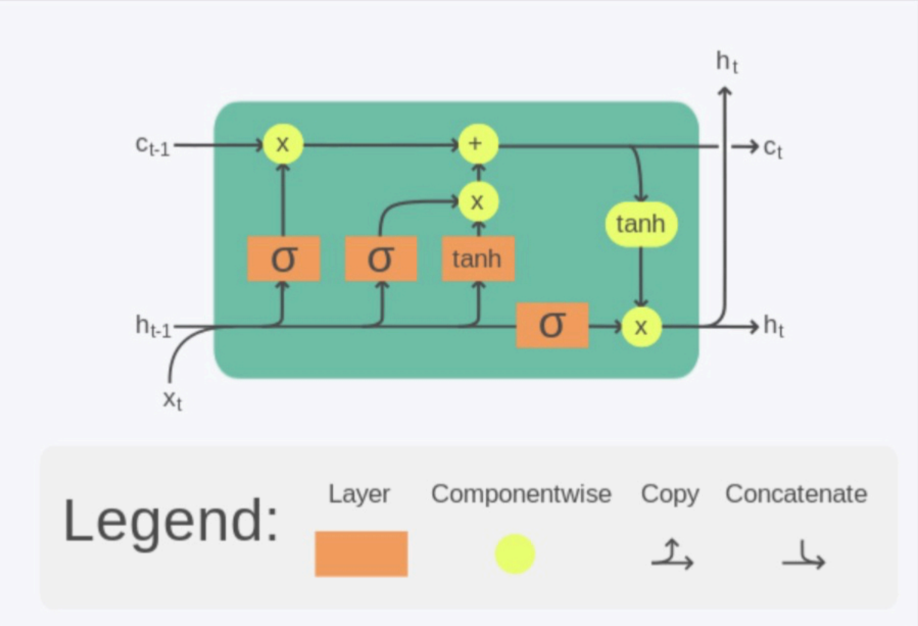
- Cascade blocks of attention + MLP and/or gated recurrence + MLP to model *hierarchical relationships* like image features or grammatical constructs
- Attention and gated RNNs are called “mixing layers” (successive inputs are combined)
- MLPs are called “point transformations” (generally mapping of any vector from one place to another)
- RMSNorm typical at the input to put it on the hypersphere — also used internally (see Hawk/Griffin e.g.)

Where Meaning Lies

Sequence Modeling Snapshots

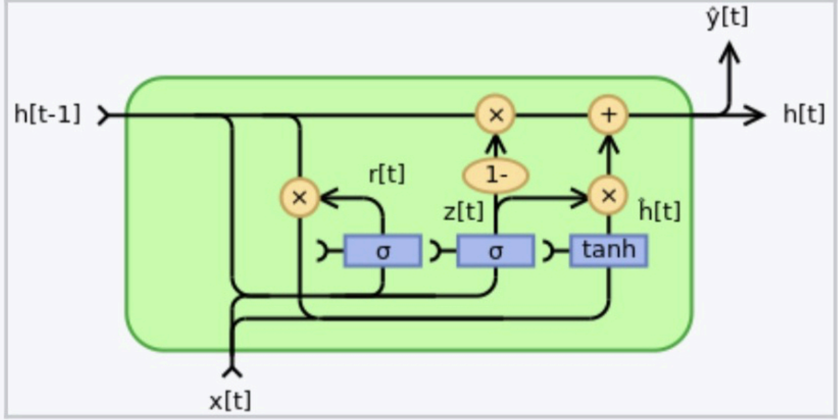
LSTM and GRU

1997: LSTM



The Long Short-Term Memory (LSTM) cell can process data sequentially and keep its hidden state through time.

2014: GRU



Gated Recurrent Unit, fully gated version

Structured State Space and Mamba

2023: Mamba (S6)

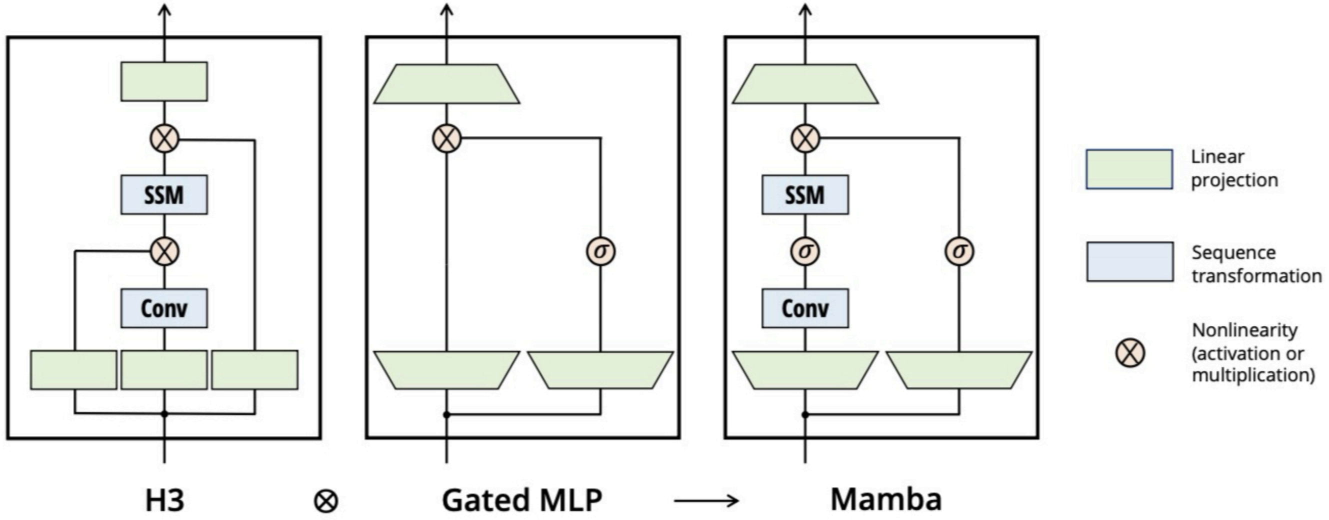


Figure 2: (**Architecture.**) Our simplified block design combines the H3 block, which is the basis of most SSM architectures, with the ubiquitous MLP block of modern neural networks. Instead of interleaving these two blocks, we simply repeat the Mamba block homogenously. Compared to the H3 block, Mamba replaces the first multiplicative gate with an activation function. Compared to the MLP block, Mamba adds an SSM to the main branch. For σ we use the SiLU / Swish activation (Hendrycks & Gimpel, 2016; Ramachandran et al., 2017).

Hawk and Griffin

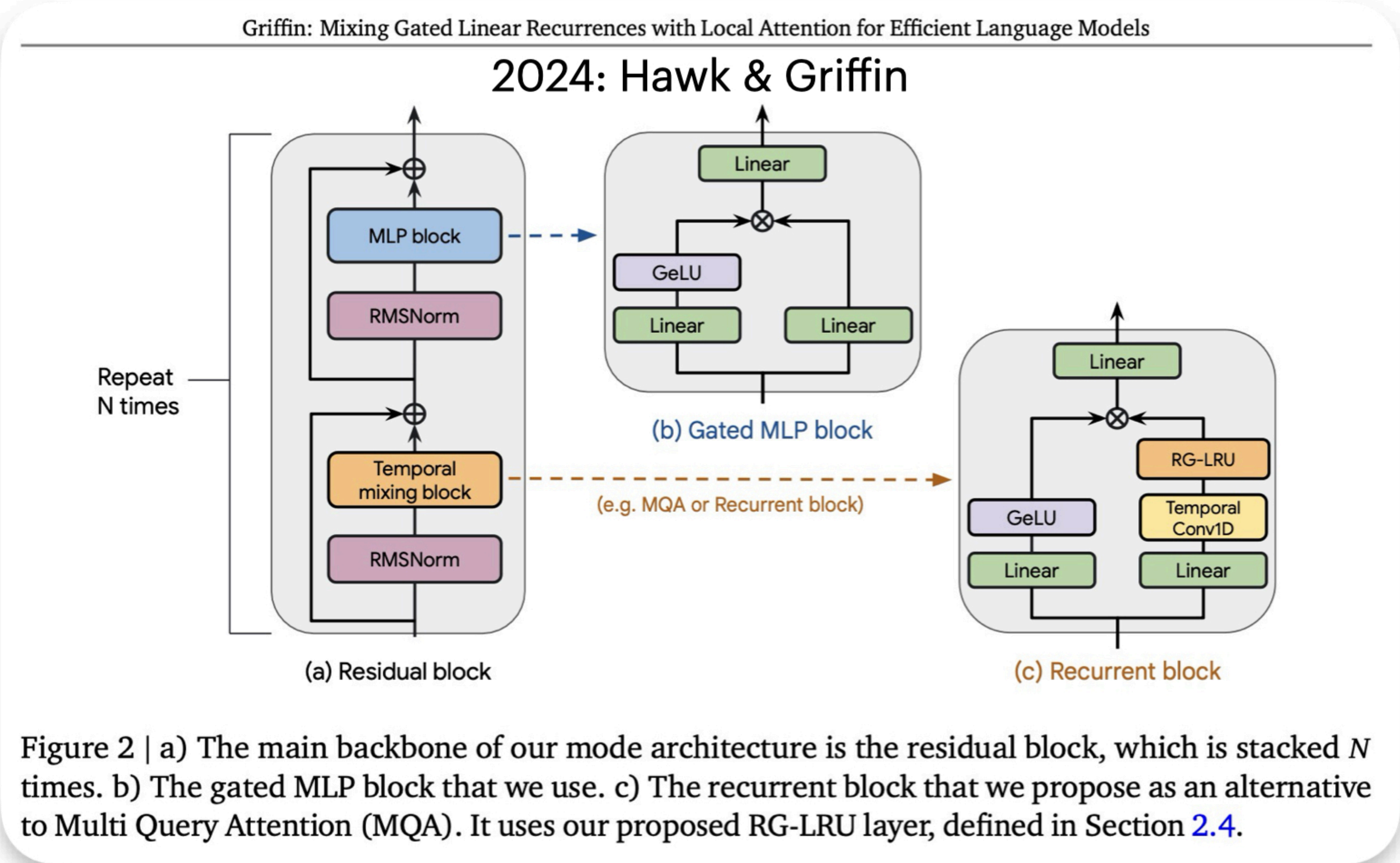
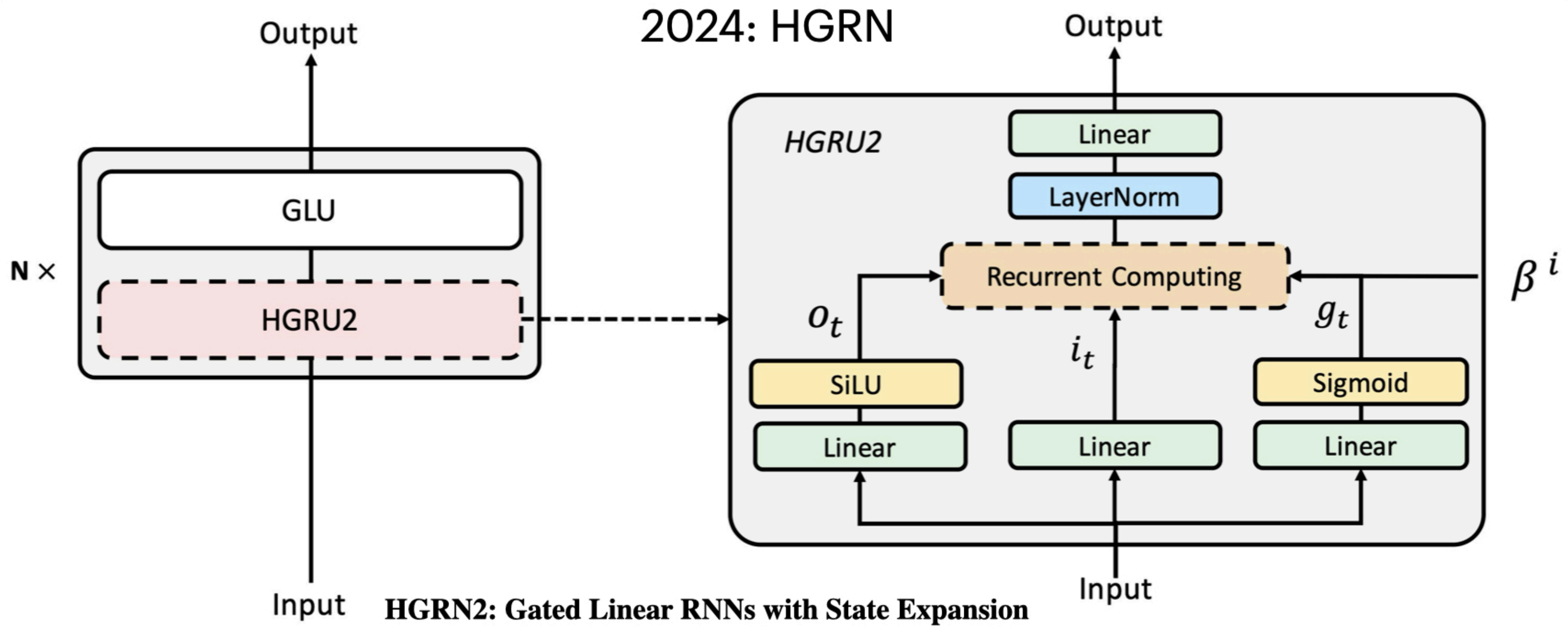


Figure 2 | a) The main backbone of our mode architecture is the residual block, which is stacked N times. b) The gated MLP block that we use. c) The recurrent block that we propose as an alternative to Multi Query Attention (MQA). It uses our proposed RG-LRU layer, defined in Section 2.4.

Gated Linear RNNs with State Expansion



HGRN2: Gated Linear RNNs with State Expansion

Figure 1: **The neural architecture of HGRN2.** Each HGRN2 layer includes a token mixer layer HGRU2 and a channel mixerlayer GLU. HGRU2 employs recurrent computation through Eq. 3, where $\mathbf{i}_t$ is the input vector, $\mathbf{g}_t$ is the forget gate (not lower bounded), β^i is the lower bound of the forget gate value, $\mathbf{o}_t$ is the output gate for layer i.

RWKV, Eagle, Finch

2024: Eagle-Finch RWKV

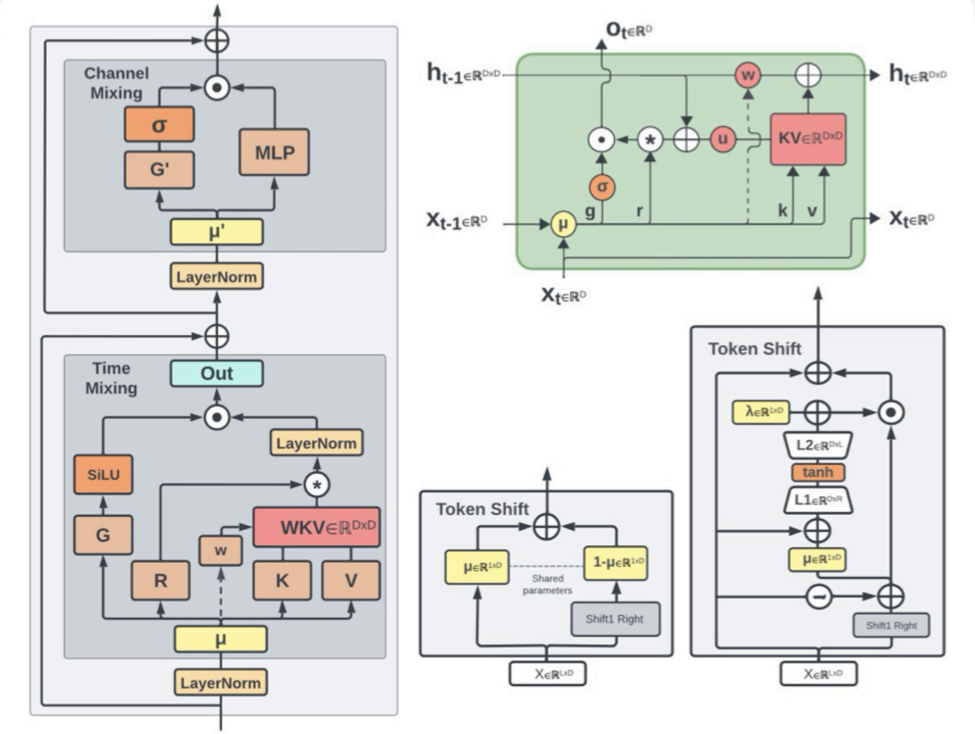


Figure 1: RWKV architecture overview. **Left:** time-mixing and channel-mixing blocks; **top-right:** RWKV time-mixing block as RNN cell; **center-bottom:** token-shift module in FeedForward module and Eagle time-mixing; **bottom-right:** token-shift module in Finch time-mixing. All shape annotations assume a single head for simplicity. Dashed arrows (left, top-right) indicate a connection in Finch, but not in Eagle.