jotix16 / **Applied-Estimation**

A repository where I try different flavors of EFK and particle filter for localization, mapping and SLAM

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EKF_Localization	Report added images and answer of questions	3 days ago
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__MACOSX	Added some function modules.	7 days ago
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Lab_1.pdf	Added some function modules.	7 days ago
Outlier_Detection.pdf	Added some function modules.	7 days ago
README.md	Final update of report	10 minutes ago

README.md

Applied Estimation

Lab 1

PART I

Linear Kalman Filter

1.**What is the difference between a "control" u_t , a "measurement" z_t and the state x_t ? Give examples of each!**

While a controll **u**_t is assumed to be exactly known we assume that a measurement **z**_t is perturbet with measurement noise. On the other hand the state **x**_t contains the values that are to be estimated using the controll and measurement. It is too assumed that the state itself is perturbed with disturbance noise. All noises are assumed to be gaussian with zero mean.

2.**Can the uncertainty in the belief increase during an update? Why (or not)?**

The uncertainty in the belief cannot increase during an update because $C_t^T X C_t$ and $\bar{\Sigma}_t$ are p.s.d which means that $K_t C_t = \bar{\Sigma}_t C_t^T X C_t$ is p.s.d and so $|I - K_t C_t| \leq |I|$. This means that during an update the uncertainty in the belief is always decreasing.

3.**During update what is it that decides the weighing between measurements and belief?**

The weighting between emasurements and beief is decided by the Kalman gain K_t which depends from the covariances of system disturbances and measurement noise, R_t and Q_t.

4.**What would be the result of using too large a covariance (Q matrix) for the measurement model?**

Using a too large covariance matrix Q_t leads to a small Kalman gain K_t. In that way the measurements would be considered very little.

5.**What would give the measurements an increased effect on the updated state estimate?**

A relatively big R_t or relatively small Q_t would give the measurements an increased effect on the updated state estimate.

6.**What happens to the belief uncertainty during prediction? How can you show that?**

During a prediction step $\Sigma_{t-1} = R_t + A_t \Sigma_{t-1} A_t^T$ the belief uncertainty normally increases because R_t is p.s.d and $A_t \Sigma_{t-1} A_t^T$ doesn't normally change the uncertainty alot compared to R_t.

One exception would be for example if A_t was a diagonal matrix with very low values.

7.**How can we say that the Kalman filter is the optimal and minimum least squared error estimator in the case of independent Gaussian noise and Gaussian priori distribution? (Just describe the reasoning not a formal proof.)** It gives the true posterior distribution for a linear Gaussian system and hast the lowest expected square error, so it is the optimal. Any other better mean is equal to the Gaussian mean.

8.**In the case of Gaussian white noise and Gaussian priori distribution, is the Kalman Filter a MLE and/or MAP estimator?** In the case of Gaussian white noise and Gaussian priori distribution the Kalman Filter is a MAP estimator since it makes use of the prior.

Extended Kalman Filter:

9.**How does the Extended Kalman filter relate to the Kalman filter?**

The Extended Kalman Filter is the usage of Kalman Filter for linearized nonlinear systems.

10.**Is the EKF guaranteed to converge to a consistent solution?**

The convergence of EKF to a consistent solution is not guaranteed. The update depends on the previous estimate and an estimate implied by linearising the measurement function, if this linearisation is far from μ_z , the measurement does not really imply μ_z and the updated state can be moved to an unreal one. Therefore, the consistency depends on the significance of the nonlinearity where the function has been linearised.

11.**If our filter seems to diverge often, can we change any parameter to try and reduce this?**

We can increase the relative size of the measurement covariance, as divergence occurs most likely on update phase. If instead it is due to a poor data association, the matching threshold can be changed. Another possibility is increasing the sampling rate.

Localization:

12.**If a robot is completely unsure of its location and measures the range r to a know landmark with Gaussian noise, what does the posterior belief of its location p(x, y, θ|r) look like? A formula is not needed but describe it at least.**

The robot will know that it is approximately at a distance r from the landmark but will not know in which direction. Since Gaussian distribution is not multimodal the posterior distribution would include all possible possitions which would correspond to a Gaussian distribution with mean at the landmark and variance big enough to include all possible location within r distance from landmark. The posterior of the angle would stay uniform.

If the estimation was done for the polar coordinates the posterior belief would have the form of a donat which translates in a Gaussian distribution for the distance r and normal distribution for θ.

13.**If the above measurement also included a bearing, how would the posterior look?**

If in addition to the distance the measurement included a bearing the robots location would be described by a Gaussian distribution with mean on the real location.

14.**If the robot moves with relatively good motion estimation (prediction error is small) but a large initial uncertainty in heading θ, how will the posterior look after traveling a long distance without seeing any features?**

The posterior would have the form of a circle since the distance travelled is known very good but the heading not. If the uncertainty of the heading θ is not totally uncertain we would have a banana form of the posterior built by Gaussian ellipses.

15.**If the above robot then sees a point feature and measures range and bear-ing to it, how might the EKF update go wrong?** The EKF update may go wrong since the linearization on the update step can be done repsectively to location which could be far away from the real position of the robot and which would make the update diverge.

PART II

Warm up

16.**What are the dimensions of ϵ_k and δ_k ? What parameters do you need to define in order to uniquely characterize a white Gaussian?**

The dimensions are [2x1] and [1x1] respectively. In order to uniquely define white Gaussian we need the covariance matrix and the mean which is normally 0 for white noise.

17.**Make a table showing the roles/usages of the variables (x, xhat, P, G, D, Q, R, wStdP, wStdV, vStd, u, PP). To do this one must go beyond simply reading the comments in the code. Hint: Some of these variables are part of our estimation model and some are for sim-ulating the car motion.**

Variable	Role/usage
x	The state to be estimated (position and velocity)
xhat	The estimated state
P	Belief uncertainty of estimated state (Covariance matrix)
G	Matrix which gives the linear combination how the noise influences the predict step
D	Matrix which gives the linear combination how the noise influences the measurements
Q	Covariance matrix of model noise in the measurement
R	Covariance matrix of model noise
wStdP	Variance amplitude of the real normally distributed position noise
wStdV	Variancve amplitude of the real normally distributed velocity nose
vStd	Max amplitude of the real normally distributed measurement noise
u	Control signal used in the prediction step(no uncertainties)
PP	List of belief uncertainty matrixes

18.**Please answer this question with one paragraph of text that summarizes broadly what you learn/deduce from changing the parametersin the code as described below. Choose two illustrative sets of plots to include as demonstration. What do you expect if you increase/decrease the covariance matrix of the modeled (not the actually simulated) process noise/measurement noise by a factor of 100 (only one change per run)? Characterize your expectations. Confirm your expectations using the code (save the corresponding figures so you can analyze them in your report). Do the same analysis for the case of increasing/decreasing both parameters by the same factor at the same time. Hint: It is the mean and covariance behavior over time that we are asking about.**

Relativel increasing R corresponds to an increase in K and so a bigger consideration of the measurements. We can see this as agressivity property of the estimation. If R would be on the other hand small relatively to Q (or Q relatively big) the covergence(decreasing of P) and the mean would change much slower. In general we have a big covariance if both Q and R are big since the estimating is done under a high uncartainty.

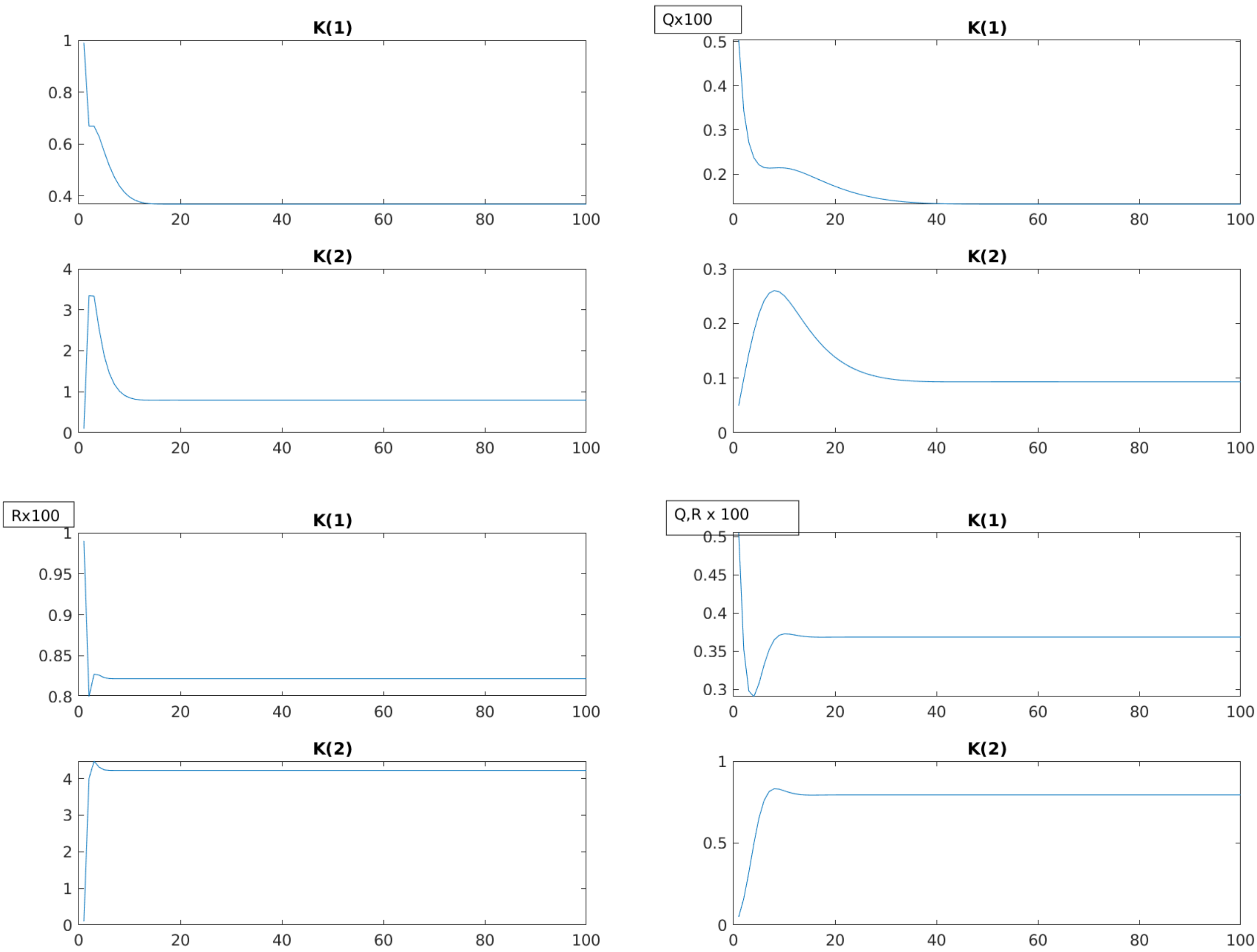

Fig 1. Kalman Gains for different modelled noise covariances Q and R

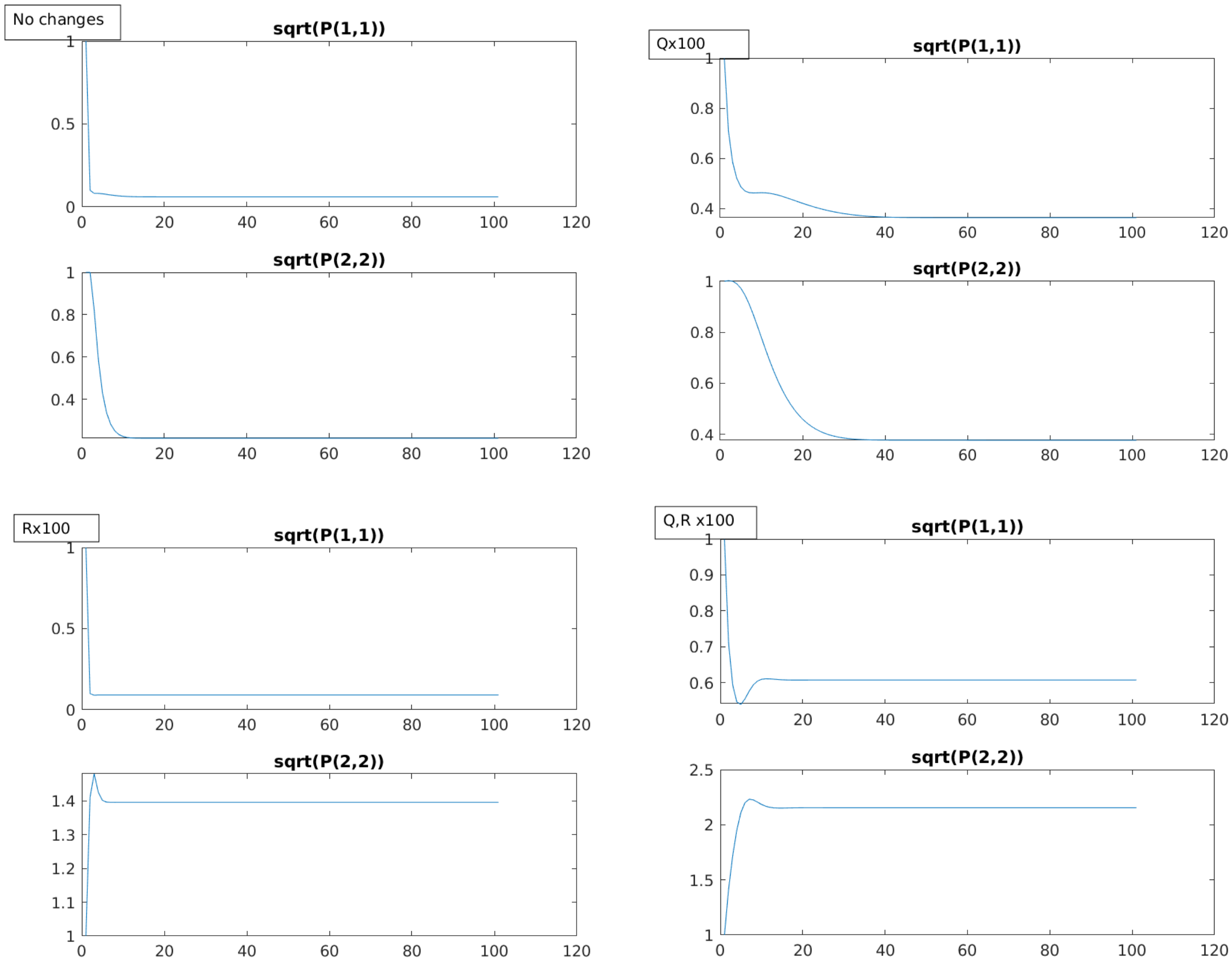

Fig 2. Uncertainty of the belief for different modelled noise covariances Q and R

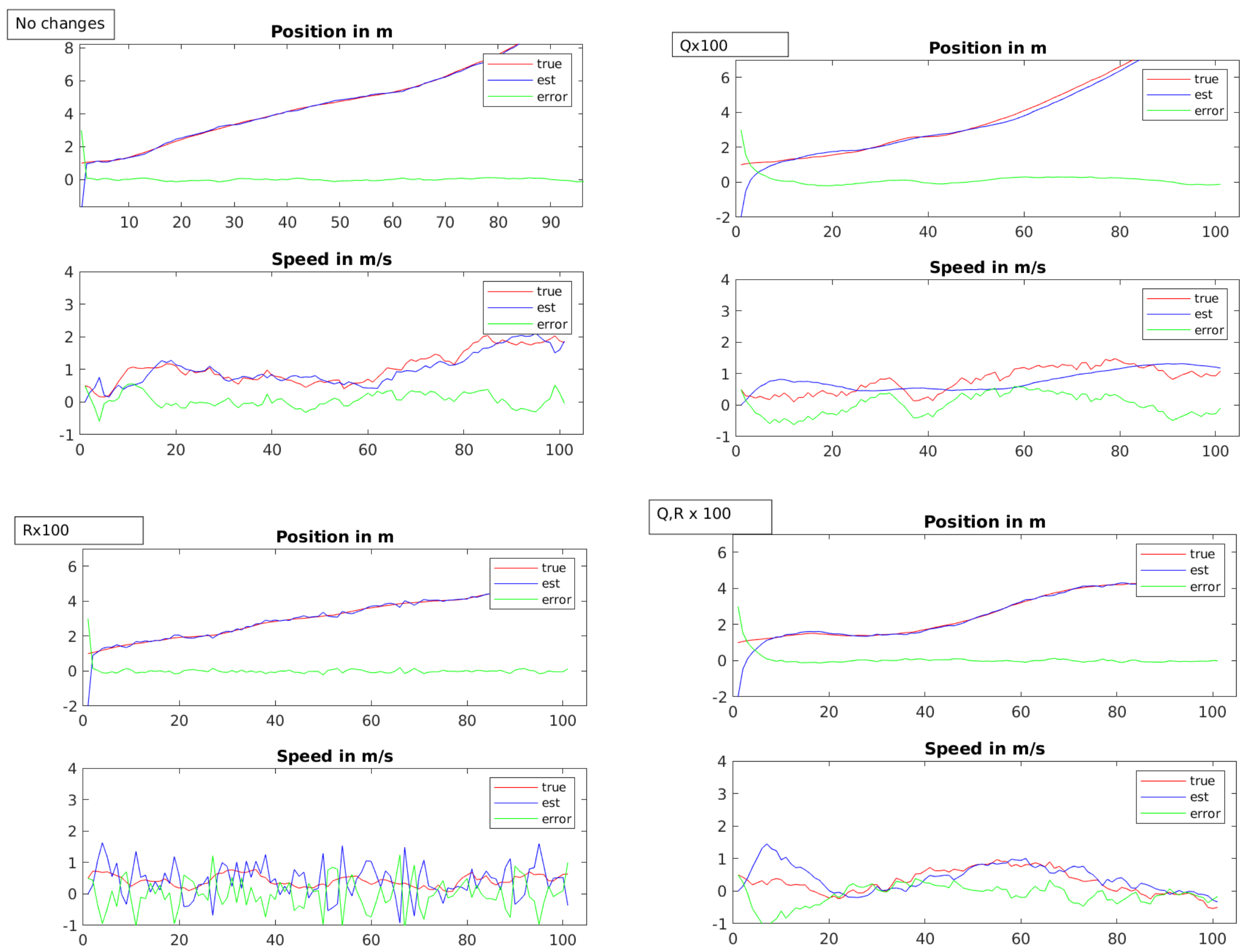

Fig 3. State estimation and corresponding error for different modelled noise covariances Q and R

19.**How do the initial values for P and $\hat{x}$ affect the rate of convergence and the error of the estimates (try both much bigger and much smaller)?**

An increase on P leads to a much bigger error in the beginning and a much faster convergence rate than P/100. $\hat{x}$ on the other hand doesn't change the convergence rate. It shows the initial position, so the close te the real initial position it is the faster it converges and so the smaller the error of the estimator. Otherwise, the further from the real initial position the bigger the error initially. In general this is much of a bigger problem in nonlinear systems, but this is fortunately not the case in our example.

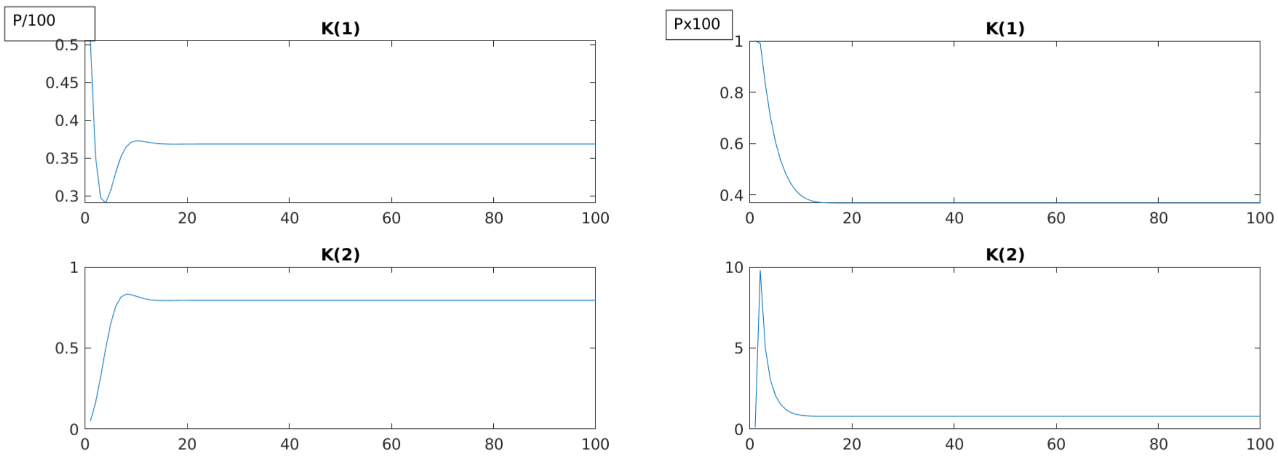

Fig 4. Kalman Gains for different initial uncertainty(covariance matrixes P)

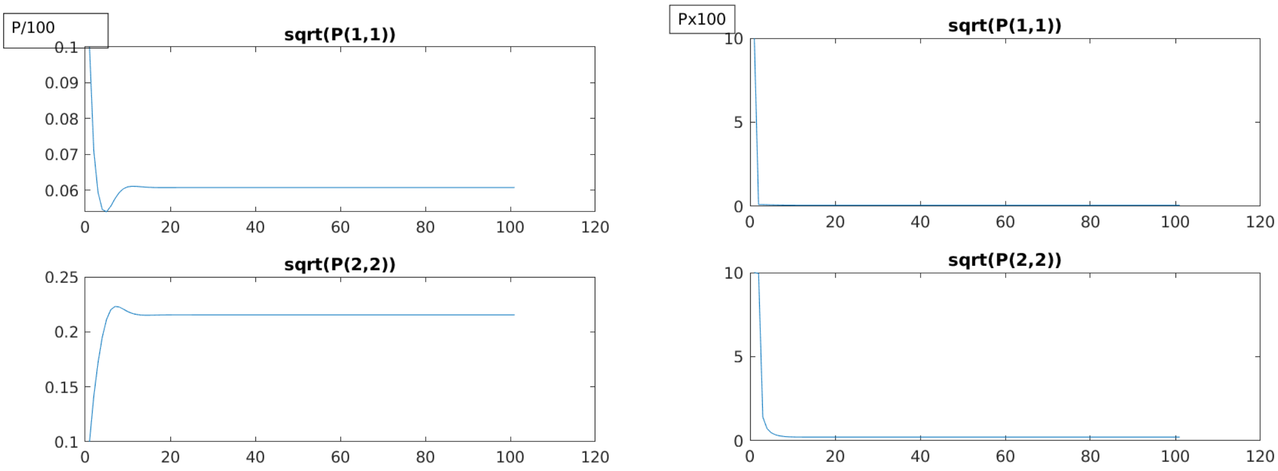

Fig 5. Uncertainty of the belief for different initial uncertainty(covariance matrixes P)

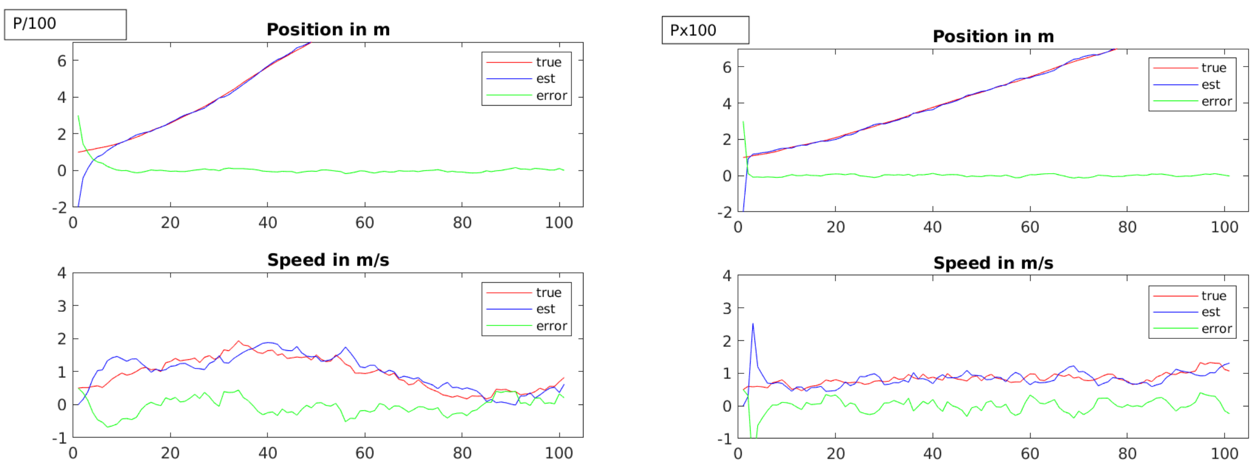

Fig 6. State estimation and corresponding error for different initial uncertainty(covariance matrixes P

EKF Localization

20.**Which parts of (2) and (3) are responsible for prediction and update steps?**

For the prediction step: the second equation in (3): $\bar{bel}(\mathbf{x}_t)$ and for the update step: the first equation in (3): $bel(\mathbf{x}_t)$.

21.**In the Maximum Likelihood data association, we assumed that the measurements are independent of each other. Is this a valid assumption? Explain why.**

It is usually invalid assumption since for example a beam of a range scan are often not independent.

22.**What are the bounds for δ_M in (8)? How does the choice of δ_M affect the outlier rejection process? What value do you suggest for λ_M when we have reliable measurements all arising from features in our map, that is all our measurements come from features on our map? What about a scenario with unreliable measurements with many arising from so called clutter or spurious measurements?**

δ_M can take values between 0 and 1 as it is a probability. A high value would give a threshold for the Mahalanobis distance defining regions so far that the probability of being in that region is very small, 1-δ_M, If there are many expected outliers, our scenario includes unreliable measurements, δ_M should be smaller so the outliers are rejected, if we expect no outliers because we have reliable measurements from features on our map δ_M would approach 1. That means λ_M big when we expect few outliers or none, and smaller when there are many unreliable measurements.

23.**Can you think of some down-sides of the sequential update approach (Algorithm 3)? Hint: How does the first (noisy) measurement affect the intermediate results?**

With sequential update the risk of diverging is higher since this individual measurements are noisy. After a first noisy measurement or an outlier we keep using the wrong estimation of the pose which is more likely to diverge. Whereas the batch update averages out the noise.

24.**How can you modify Algorithm 4 to avoid redundant re- computations?**

Exploit the symmetry of covariance matrixes.

25.**What are the dimensions of $\bar{v}_t$ and $\bar{H}_t$ in Algorithm 4? What were the corresponding dimensions in the sequential update algorithm? What does this tell you?**

$\bar{v}_t$ has dimensions [MN x 1] and $\bar{H}_t$ has dimensions [MN x P], where N is the number of observations, M is the length of each measurement and P is the length of state vector. In the sequential update the dimensions were [P x 1] and [MxP] respectively. This tells us that the complexity for the batch update is much bigger expecially for inversing the [MN x MN] matrix.

Simulations

Dataset 1

For the first datasets we see that average errors are relative small which can very good be explained by the high accuracy of the measurements and encoders. Measurement and process noise should be modelled with low covariances.

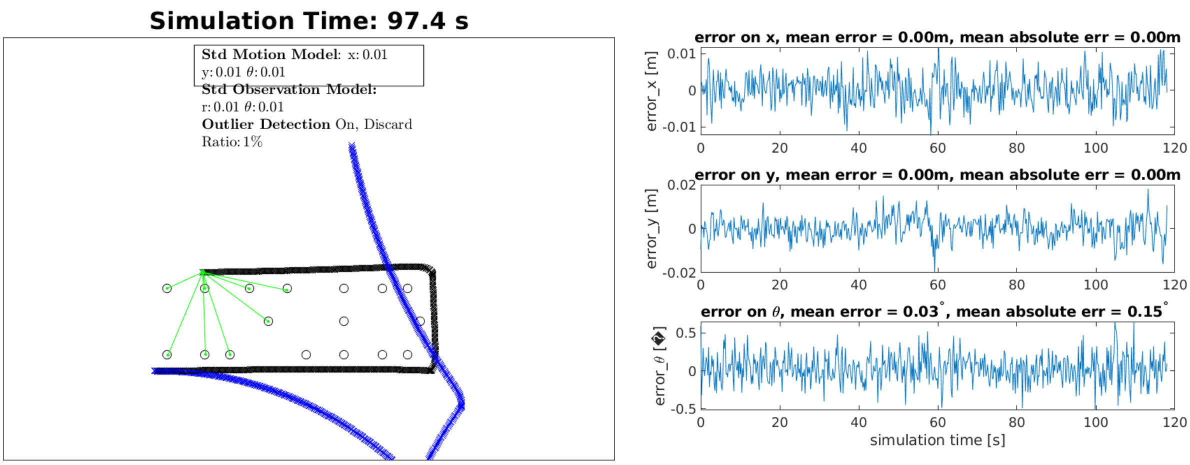

Fig 7. Visalization and error statistics of dataset 1

Dataset 2

The third dataset includes a high number of outlier measurements which have to be detected and avoided. That is why the threshold has to be properly be selected and batch update performed. Logically we see a bigger mean error in this case,, especially for the theta angle.

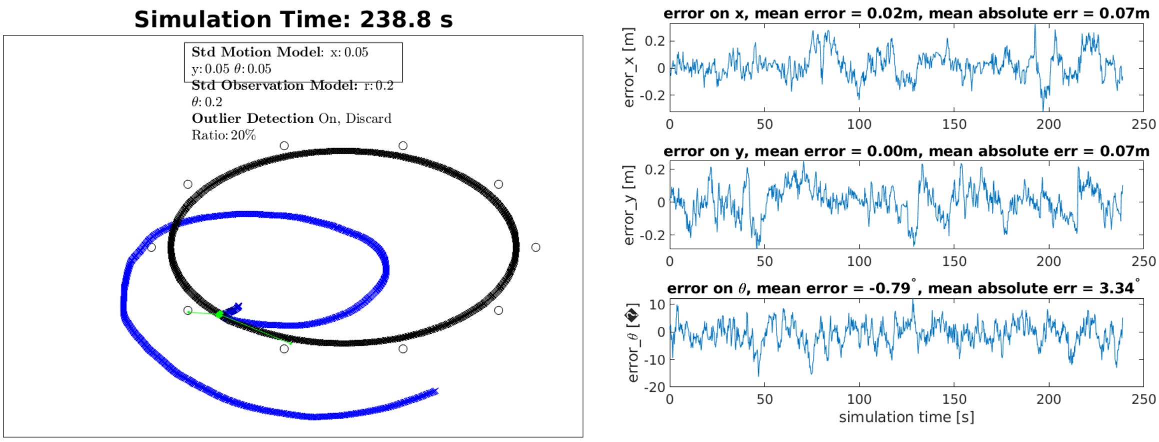

Fig 8. Visalization and error statistics of dataset 2

Dataset 3

The third dataset distinguishes of the two others because of the different simulated noises. While association does not play a big role if turned off, once the batch update is turned off, the estimation deteriorates. Averaging off noise and outliers with batch update is vital in this case.

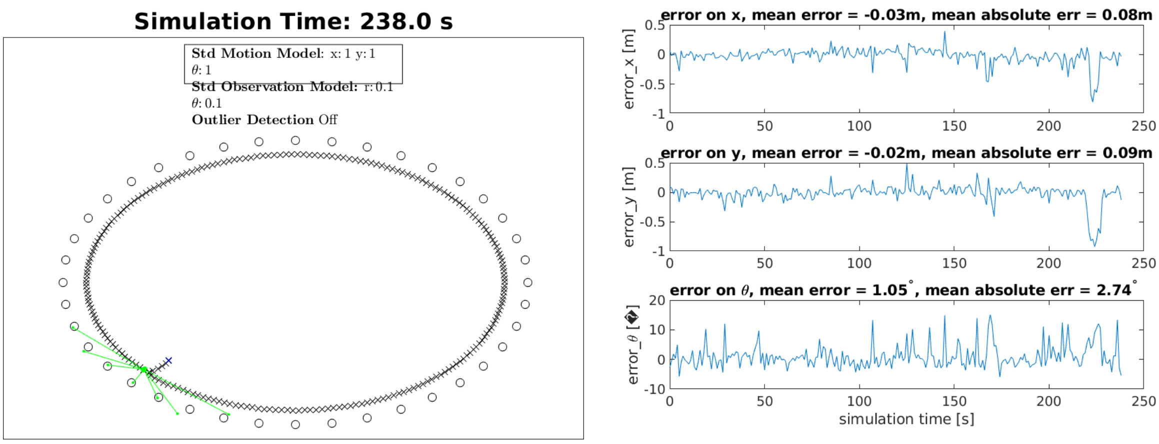

Fig 9. Visalization and error statistics of dataset 3