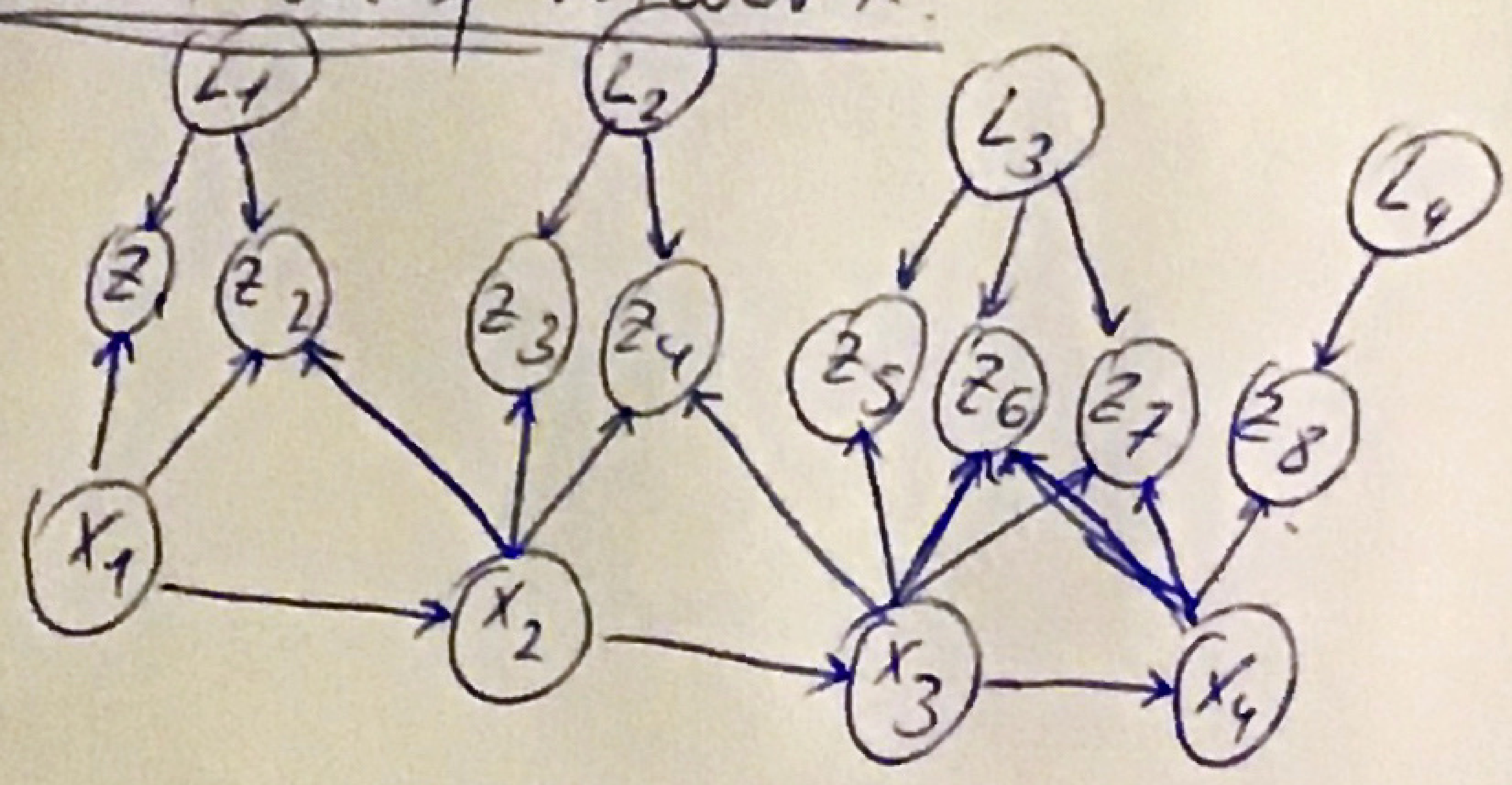
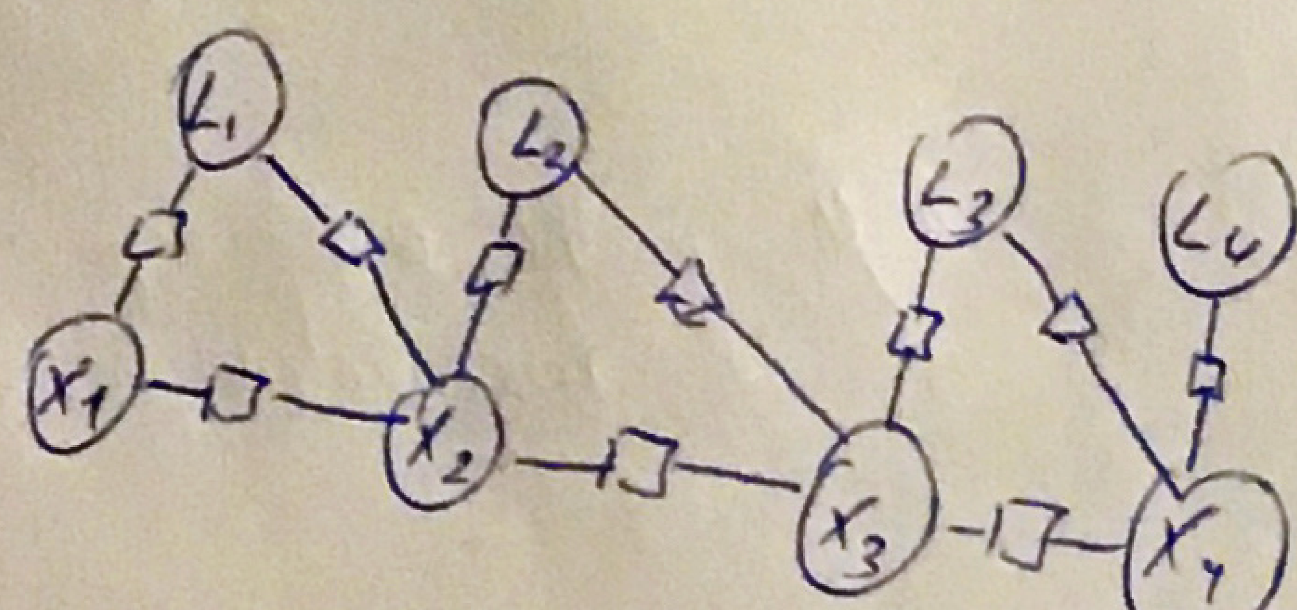
1) Graph representation of the SLAM problem.

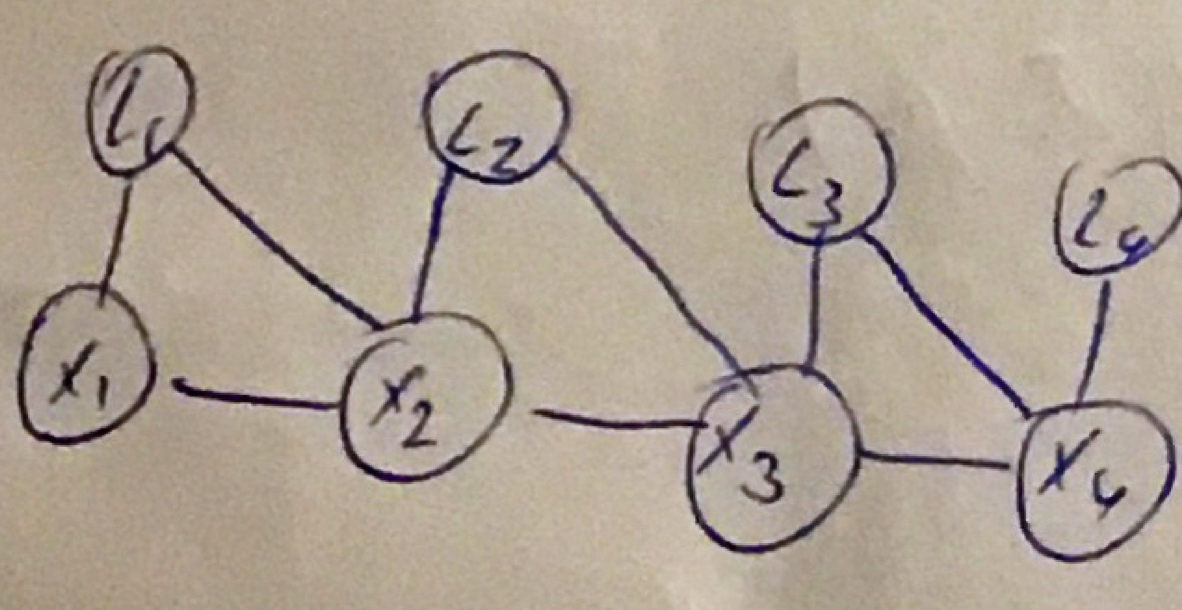
a) Bayesian belief network



b) Factor graph



c) Markov Random Field (MRF)



2) Assumptions made in:

$$p(x_{0:N}, m_{1:M}, z_{1:K}) = p(x_0) \underbrace{\prod_{i=1}^{N} p(x_i|x_{i-1},u_i)}_{\text{Motion Model}} \prod_{k=1}^{K} \overbrace{p(z_k|x_{i_k},m_{j_k})}^{\text{landmark measurement model}}$$

Assumptions:

a) Data association is solved as we know the indexes i_k and j_k corresponding to measurement z_k.

b) In addition an uniform distribution over the landmarks m is assumed.

3) What happens with the structure of Ω ~~[illegible]~~ in case of loop closure?

The matrix Ω loses its bandstructure and gets more dens which tells us that a solution in linear time is not assured anymore.

4) Linearizing the motion for one pose to pose factor.

$$\Delta\Omega_{i-1,i-1} = G^T R^{-1} G$$
$$\Delta\Omega_{i,i} = I$$
$$\Delta\Omega_{i,i-1} = -R^{-1} G$$

• $G = \Delta f \Big|_{\bar{\mu}_{i-1}}$ $\mathrm{Soff}(x_{i-1}, u_i)$

linearization: $f(x_{i-1}, u_i) \approx f(\bar{\mu}^0_{i-1}, u_i) + G(\bar{\mu}_{i-1} - x_{i-1})$

$$\Delta\xi_{i-1} = G^T R^{-1}\left(G\bar{\mu}_{i-1} - g(\bar{\mu}_{i-1}, u_i)\right)$$
$$\Delta\xi_i = -R^{-1}\left(G\bar{\mu}_{i-1} - g(\bar{\mu}_{i-1}, \mu_i)\right)$$

• $\bar{\mu}_{i-1}$ is the pose where the linearization for f takes place.

$\uparrow$

$$x_i - g(x_{i-1}, u_i) = x_i - g(\bar{\mu}_{i-1}, u_i) + G(x^0_{i-1}, \mu_i)(x_{i-1} - \bar{\mu}_{i-1})$$
$$= x_i - G x_{i-1} + G\bar{\mu}_{i-1} - g(\bar{\mu}_{i-1}, \mu_i)$$

5) Convergence condition to stop the iterations

• After each iteration we check $\|\mu_x^{i+1} - \mu_x^i\| \leq \varepsilon$ and terminate if it is true for a certain threshold ε.

• Or nr of steps.

6) Factors that may influence the number of iterations required

• We have to start relatively close to the minimum.

7) Why it may fail to converge?

the optimization procedure relies heavily on the correctnes of the graph structure (i.e. correspodences)

for example: false loop closures will lead to divergence.

1) Reject outliers

1) Graph-slam-linearize.

- 1) linearize the motion model and adjust Ω accordingly
  adjust ξ accordingly

$$\begin{pmatrix} \Delta\Omega_{i-1,i-1} \\ \Delta\Omega_{i,i} \\ \Delta\Omega_{i,i-1} \end{pmatrix} \qquad \begin{pmatrix} \Delta\xi_{i-1} \\ \Delta\xi_{i} \end{pmatrix}$$

2) linearize the measurement and adjust Ω, ξ

2) Graph-slam-reduce.

reduce $\Omega \to \hat{\Omega}$

$\xi \to \hat{\xi}$

Question 10

- Deal with outliers and spurious measurements
- Avoid oscillations and overshooting ← introduce a stepsize in the Gauss Newton
- Unknown Data Association ← as suggested in the paper of S. Thrun.
  Search through all pairs of landmarks and calculate the probability that they are the same landmark.
  If probability higher than a certain threshold, merge both landmarks.

Question 11

a) Iterative deepening : - Improves initial pose

b) Outlier Rejection based on likelihood probability

c) Data association

Dataset 1

→ converges in 8 iterations
→ using the odometry as starting point is not that bad. } ← doesn't require extra measures

Dataset 2

→ converges in 10 iterations
→ — // —
→ a little higher noise than before → change Q↑ } ← doesn't require extra measures

Dataset 3

↳ many loops → becomes problematic especially ~~when~~ false loops ~~appear~~ - influence linearization very badly
↳ solution: iterative deepening.

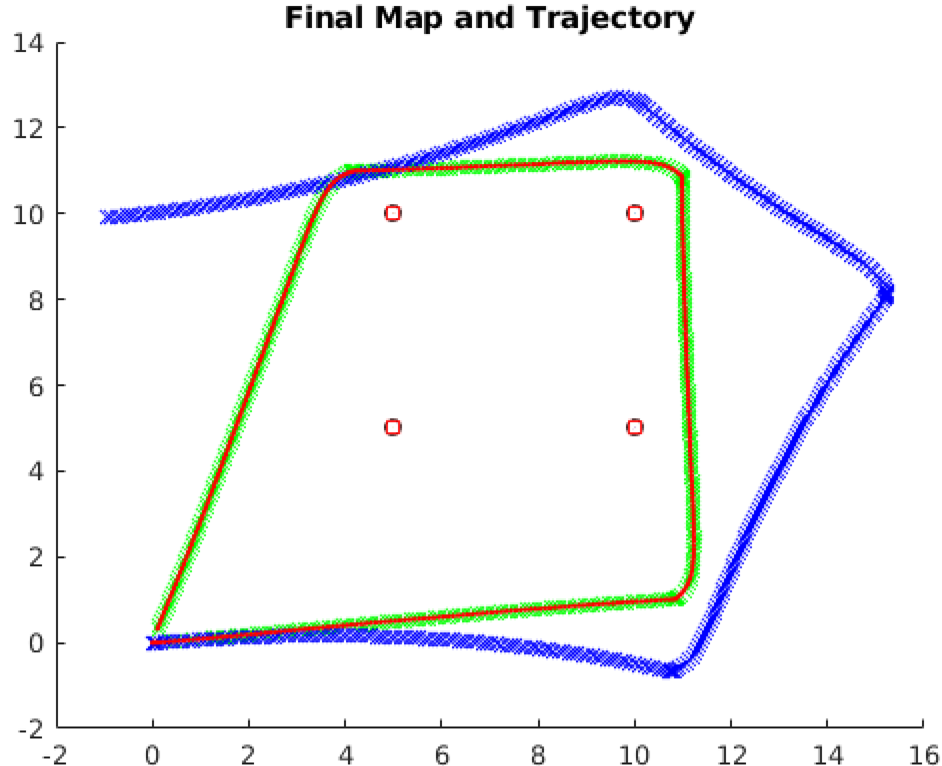
• relative ~~(a)~~ less measurements ← should be careful with outlier rejection

Dataset 4

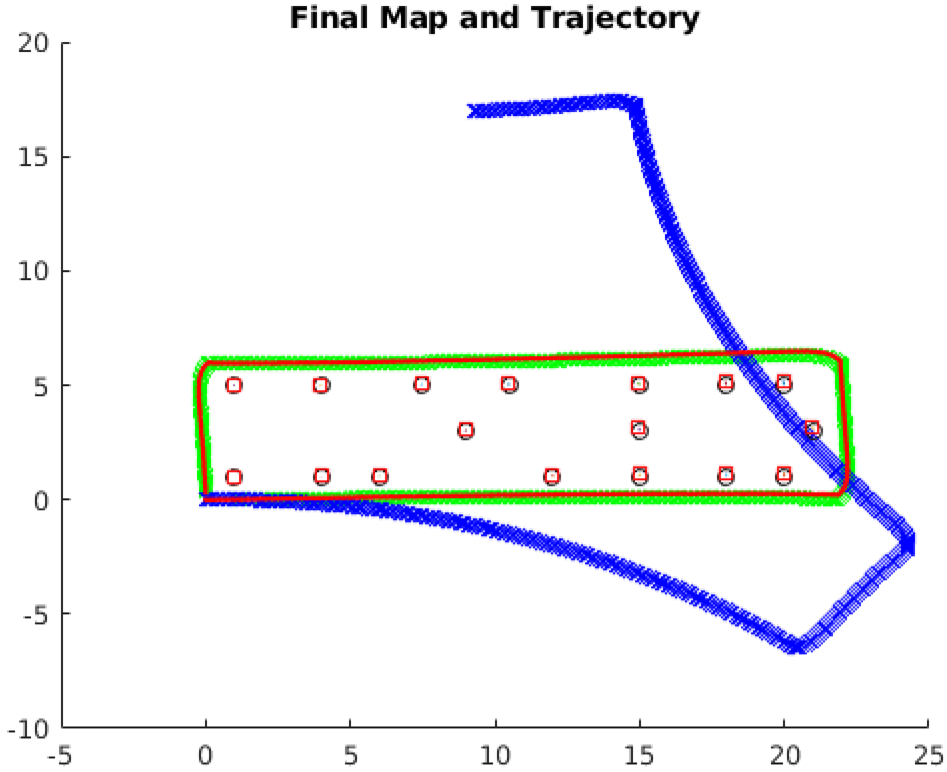
↳ measurements have many outliers ⇒ even iterative deepening doesn't help
⇒ that is why ~~we~~ introduce outlier rejection.
based on the likelihood: $p(z_k \mid x, c_k)$
~~one could go to the likelihood $p(z \mid \mu, m)$~~

Dataset 5

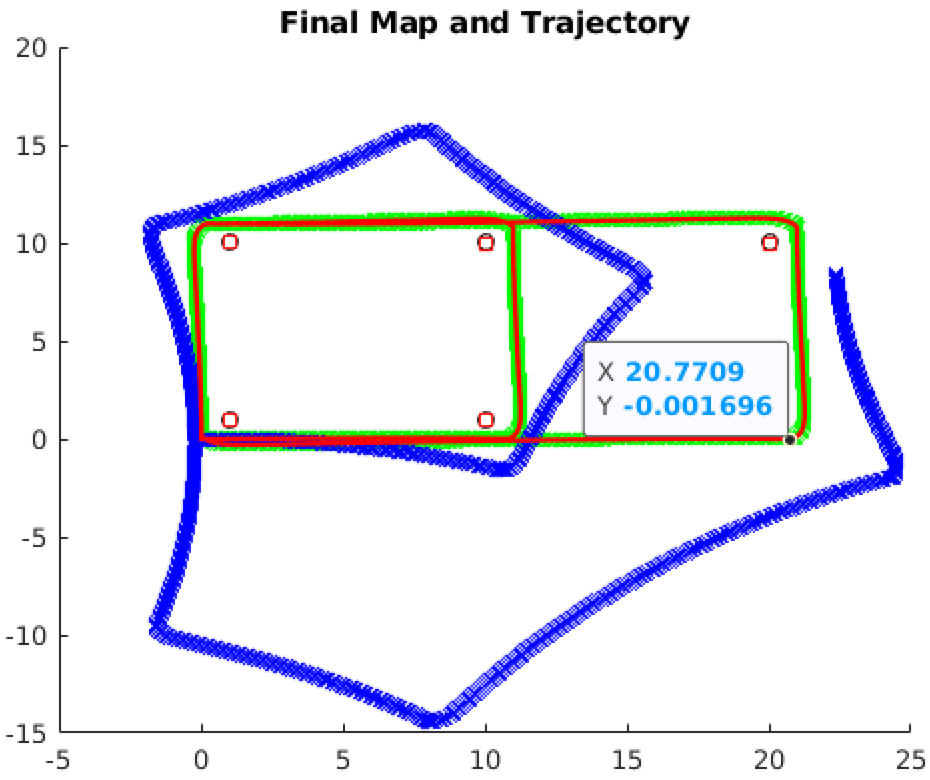
↳ no odometry → ~~a slow~~ iterative deepening with small steps and make sure outlier rejection is on since one outlier can lead on the wrong direction



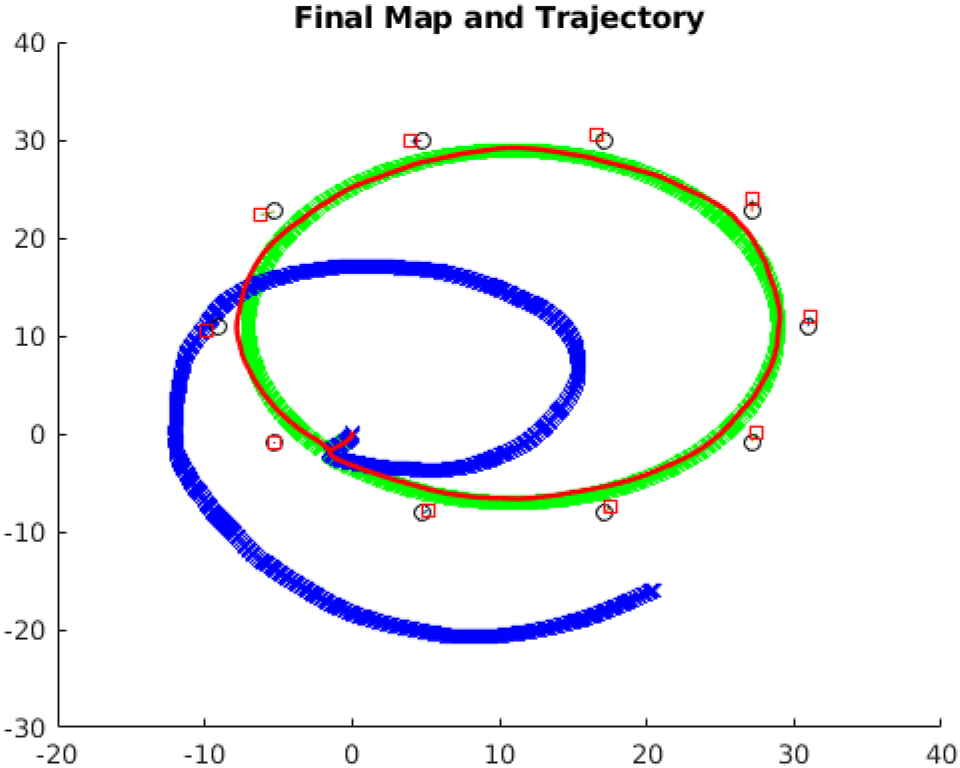
Final Map and Trajectory
14
12
10
8
6
4
2
0
-2
-2
0
2
4
6
8
10
12
14
16

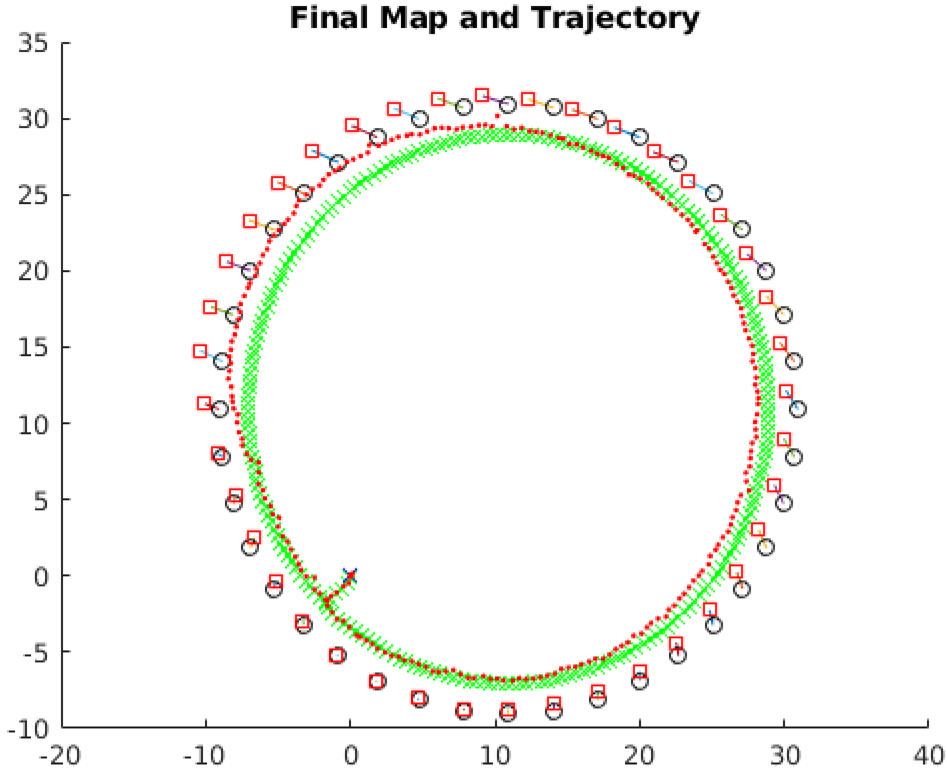


Final Map and Trajectory
20
15
10
5
0
-5
-10
-5
0
5
10
15
20
25



Final Map and Trajectory
20
15
10
5
0
-5
-10
-15
-5
0
5
10
15
20
25
X 20.7709
Y -0.001696


Final Map and Trajectory
40
30
20
10
0
-10
-20
-30
-20
-10
0
10
20
30
40


Final Map and Trajectory
35
30
25
20
15
10
5
0
-5
-10
-20
-10
0
10
20
30
40