Exercise 1

Show: $\mathcal{L}(x,\theta,\phi) = \mathbb{E}_{q_\phi(z|x)}\left[\log \frac{p_\theta(x,z)}{q_\phi(z|x)}\right] \leq \log p(x)$

(1) $\mathcal{L}(x,\theta,\phi) = \mathbb{E}_{q_\phi(z|x)}\left[\log \frac{p_\theta(x,z)}{q_\phi(z|x)}\right] \leq \log \mathbb{E}_{q_\phi(z|x)}\left[\frac{p_\theta(x,z)}{q_\phi(z,x)}\right]$

Jensen's inequality & concativity of log.

(2)

(3) $= \log \int q_\phi(z|x) \frac{p_\theta(x,z)}{q_\phi(z,x)} dz = \log \int p_\theta(x,z) dz$

$= \log p_\theta(x)$ ☒

Exercise 2

If $\hat{p}_N(x_{1:T})$ is an unbiased estimator of $p(x)$.

Prove: Bound $\mathcal{L}_N(x,p) \leq \log p(x)$.

$\mathcal{L}_N(x,p) = \mathbb{E}[\log \hat{p}_N(x)] \overset{\text{log concavity and Jensen's inequality}}{\leq} \log \mathbb{E}[\hat{p}_N(x)] \underset{\text{unbiasedness}}{=} \log p(x)$