Formal Specification and Verification of Programs

4th Assignment Solutions
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Group 2

۱۸ مرداد ۱۴۰۱

توصیف سیستم مدیریت مالی یک خانواده

آ

$[id]$
$[person]$
$payModel ::= im \mid lo \mid rp$
$familyMember ::= mo \mid fa \mid ch1 \mid ch2 \mid other$
$usage ::= food \mid gift \mid salary \mid stuff \mid insurance \mid health \mid cloth \mid transport \mid equipmentAssest \mid bill \mid etc$
$message ::= OK \mid PersonNotMember \mid PaymentNotFound \mid ImprestNotBalance \mid CostNotFound \mid DateNotValid \mid DateNotMatch$
$Day == \{a : \mathbb{N}_1 \mid a < 32\}$
$Mounth == \{a : \mathbb{N}_1 \mid a < 13\}$
$Year == \{a : \mathbb{N}_1 \mid a < 1501 \land a > 1299\}$

ب

$$
\begin{array}{|l}
Family : person \leftrightarrow familyMember \\
\hline
(\exists_1\, m : familyMember, p : person \bullet p \mapsto m \in Family \land m = fa) \lor \\
\qquad \neg\, (\exists\, m : familyMember, p : person \bullet p \mapsto m \in Family \land m = fa) \\
(\exists_1\, m : familyMember, p : person \bullet p \mapsto m \in Family \land m = mo) \lor \\
\qquad \neg\, (\exists\, m : familyMember, p : person \bullet p \mapsto m \in Family \land m = mo) \\
(\exists_1\, m : familyMember, p : person \bullet p \mapsto m \in Family \land m = ch1) \lor \\
\qquad \neg\, (\exists\, m : familyMember, p : person \bullet p \mapsto m \in Family \land m = ch1) \\
(\exists_1\, m : familyMember, p : person \bullet p \mapsto m \in Family \land m = ch2) \lor \\
\qquad \neg\, (\exists\, m : familyMember, p : person \bullet p \mapsto m \in Family \land m = ch2) \\
\forall\, p_1, p_2 : person \mid p_1 \in dom(Family) \land p_2 \in dom(Family) \bullet \exists\, m_1, m_2 : familyMember \mid \\
\qquad m_1 \neq m_2 \bullet p_1 \mapsto m_1 \in Family \land p_2 \mapsto m_2 \in Family \land p_1 \neq p_2
\end{array}
$$

$$
\begin{array}{|l}
Payments : id \mathrel{⤔} Payment
\end{array}
$$

$$Costs : person \rightarrow \mathbb{P}\ Cost$$

$$CostsId : id \rightarrowtail\!\!\!\!\twoheadrightarrow Cost$$

$$ImprestRemind : person \rightarrow (Payment \leftrightarrow \mathbb{N})$$

$$\begin{array}{|l}
\multicolumn{1}{l}{\textit{Date}} \\
\hline
year : Year \\
mounth : Mounth \\
day : Day \\
\hline
mounth \leq 6 \Rightarrow day \leq 31 \\
mounth \geq 7 \wedge mounth \neq 12 \Rightarrow day \leq 30 \\
mounth = 12 \wedge MOD(year - 1303, 4) = 0 \Rightarrow day \leq 30 \\
mounth = 12 \wedge MOD(year - 1303, 4) \neq 0 \Rightarrow day \leq 29 \\
\hline
\end{array}$$

$$\begin{array}{|l}
\multicolumn{1}{l}{\textit{Payment}} \\
\hline
paymentValue : \mathbb{N} \\
payer : person \\
payee : person \\
payMode : payModel \\
paymentDate : \mathbb{N} \times \mathbb{N} \times \mathbb{N} \\
\hline
paymentValue > 0 \\
payer \neq payee \\
\hline
\end{array}$$

$$\begin{array}{|l}
\multicolumn{1}{l}{\textit{Cost}} \\
\hline
costValue : \mathbb{N} \\
costDate : Date \\
costUsage : usage \\
imprestId : id \\
\hline
costValue > 0 \\
\hline
\end{array}$$

ج

$$\begin{array}{|l}
\multicolumn{1}{l}{\textit{FamilyPayment}} \\
\hline
family : Family \\
costs : Costs \\
payments : Payments \\
costId : CostsId \\
imprestRemind : ImprestRemind \\
\hline
\forall p : payment \mid p \in dom(range(imprestRemind)) \bullet p \in range(payments) \wedge p.payMode = imprest \\
\forall p : person \mid p \in dom(imprestRemind) \bullet p \in dom(family) \\
\forall p : person \mid p \in dom(costs) \bullet p \in dom(family) \\
\forall p : Payment \mid p \in range(payments) \bullet p.payer \in dom(family) \wedge p.payee \in dom(family) \\
\forall c : Cost \mid c \in range(costId) \bullet \exists p : Person \bullet c \in costs(p) \\
\hline
\end{array}$$

د

$$\begin{array}{|l}
\multicolumn{1}{l}{\mathit{FamilyPaymentInit}} \\
\mathit{FamilyPayment}' \\
\mathit{family}? : \mathit{Family} \\
\hline
\mathit{family}' = \mathit{family}? \\
\mathit{costs}' = \varnothing \\
\mathit{payments}' = \varnothing \\
\mathit{imprestRemind}' = \varnothing \\
\mathit{costId}' = \varnothing \\
\hline
\end{array}$$

$$\exists\, \mathit{State}' \bullet \mathit{StateInit}$$

$$\begin{array}{l}
\exists\, \mathit{FamilyPaymen}' \bullet \mathit{FamilyPaymenInit} \\
\quad \Leftrightarrow \exists\, \mathit{FamilyPaymen}' \bullet \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad [\mathit{definitionofFamilyPaymenInint}] \\
\qquad\quad [\mathit{FamilyPaymen}'; \mathit{family}? : \mathit{Family} \mid \\
\qquad\qquad\quad \mathit{family}' = \mathit{family}? \land \\
\qquad\qquad\quad \mathit{costs}' = \varnothing \land \\
\qquad\qquad\quad \mathit{payments}' = \varnothing \land \\
\qquad\qquad\quad \mathit{imprestRemind}' = \varnothing \\
\qquad\qquad\quad \mathit{costId}' = \varnothing] \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad [\mathit{Schemaquantification}] \\
\quad \Leftrightarrow [\mathit{family}? : \mathit{Family} \mid \\
\qquad\quad \exists\, \mathit{FamilyPaymen}' \bullet \\
\qquad\qquad\quad \mathit{family}' = \mathit{family}? \land \\
\qquad\qquad\quad \mathit{costs}' = \varnothing \land \\
\qquad\qquad\quad \mathit{payments}' = \varnothing \land \\
\qquad\qquad\quad \mathit{imprestRemind}' = \varnothing \\
\qquad\qquad\quad \mathit{costId}' = \varnothing] \\
\quad \Leftrightarrow [\mathit{family}? : \mathit{Family} \mid \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad [\mathit{definitionofFamilyPayment}'] \\
\qquad\quad \exists\, \mathit{family}' : \mathit{Family} \bullet \\
\qquad\quad \exists\, \mathit{costs}' : \mathit{Costs} \bullet \\
\qquad\quad \exists\, \mathit{payments}' : \mathit{Payments} \bullet \\
\qquad\quad \exists\, \mathit{imprestRemind}' : \mathit{ImprestRemind} \bullet \\
\qquad\quad \exists\, \mathit{costId}' : \mathit{CostId} \bullet \\
\qquad\qquad\quad \forall\, p : \mathit{payment} \mid p \in \mathit{dom}(\mathit{range}(\mathit{imprestRemind}')) \bullet \\
\qquad\qquad\qquad\quad p \in \mathit{range}(\mathit{payments}') \land p.\mathit{payMode} = \mathit{imprest} \land \\
\qquad\qquad\quad \forall\, p : \mathit{person} \mid p \in \mathit{dom}(\mathit{imprestRemind}') \bullet p \in \mathit{dom}(\mathit{family}') \land \\
\qquad\qquad\quad \forall\, p : \mathit{person} \mid p \in \mathit{dom}(\mathit{costs}') \bullet p \in \mathit{dom}(\mathit{family}') \land \\
\qquad\qquad\quad \forall\, p : \mathit{Payment} \mid p \in \mathit{range}(\mathit{payments}') \bullet \\
\qquad\qquad\qquad\quad p.\mathit{payer} \in \mathit{dom}(\mathit{famil}'y) \land p.\mathit{payee} \in \mathit{dom}(\mathit{family}') \\
\qquad\qquad\quad \forall\, c : \mathit{Cost} \mid c \in \mathit{dom}(\mathit{costId}') \bullet \exists\, p : \mathit{Person} \bullet c \in \mathit{costs}'(p)] \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad [\mathit{one} - \mathit{point\,rule},\ 5\ \mathit{times}] \\
\quad \Leftrightarrow [\mathit{family}? : \mathit{Family}]
\end{array}$$

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$$
\begin{array}{l}
AddCost_0 \\
\hline
\Delta FamilyPayment \\
cost? : Cost \\
pe : Person \\
py : Payment \\
v : \mathbb{N} \\
i : id \\
i0 : id \\
\hline
i = cost?.imprestId \\
payments(i).payMode = imprest \\
i \in dom(imprestRemind) \\
py = payments\ (i) \\
v = ((imprestRemind\ pe)\ py) \\
cost?.costValue \leq v \\
pe = py.payee \\
family' = family \\
costs' = costs \oplus \{\ pe \mapsto costs\ pe \cup \{cost?\}\ \} \\
((imprestRemind'\ pe)\ py) = v - cost?.costValue \\
(py.paymentDate.1 < cost?.costDate.year) \lor (py.paymentDate.1 = cost?.costDate.year \land \\
\quad\quad py.paymentDate.2 < cost?.costDate.mounth) \lor \\
\quad\quad (py.paymentDate.1 = cost?.costDate.year \land \\
\quad\quad\quad\quad py.paymentDate.2 = cost?.costDate.mounth \land \\
\quad\quad\quad\quad\quad\quad py.paymentDate.3 \leq cost?.costDate.day) \\
\exists\, x : usage \bullet x = cost?.costUsage \\
\exists\, t : id \mid t \notin dom(costsId) \bullet i0 = t \\
costsId' = CostsId \cup \{i0 \mapsto cost?\} \\
\hline
\end{array}
$$

$$
\begin{array}{l}
AddPayment_0 \\
\hline
\Delta FamilyPayment \\
payment? : Payment \\
p1 : Person \\
p2 : Person \\
i : id \\
v : \mathbb{N} \\
\hline
family' = family \\
costs' = costs \\
v = payment?.paymentvalue \\
p1 = payment?.payer \\
p2 = payment?.payee \\
p1 \in dom(family) \\
p2 \in dom(family) \\
p1 \neq p2 \\
\exists\, t : id \mid t \notin dom(payments) \bullet i = t \\
payments' = payments \cup \{i \mapsto payment?\} \\
payment?.payMode = imprest \Rightarrow imprestRemind' = imprestRemind \oplus \\
\quad\quad \{p2 \mapsto imprestRemind\ p2 \cup \{(payment? \mapsto v)\}\} \\
payment?.payMode \neq imprest \Rightarrow imprestRemind' = imprestRemind \\
\exists\, x : payModel \bullet x = payment?.payMode \\
\exists\, x : Date \bullet x.year = payment?.paymentDate.1 \land x.mounth = payment?.paymentDate.2 \land \\
\quad\quad\quad\quad x.day = payment?.paymentDate.3 \\
\hline
\end{array}
$$

$$
\begin{array}{|l}
\multicolumn{1}{l}{\mathit{Success}} \\ \hline
\Xi \mathit{FamilyPayment} \\
o! : \mathit{message} \\ \hline
o! = \mathit{OK} \\ \hline
\end{array}
$$

$$
\begin{array}{|l}
\multicolumn{1}{l}{\mathit{PersonNotMember}} \\ \hline
\Xi \mathit{FamilyPayment} \\
\mathit{payment}? : \mathit{Payment} \\
o! : \mathit{message} \\ \hline
p1 = \mathit{payment}?.\mathit{payer} \\
p2 = \mathit{payment}?.\mathit{payee} \\
p1 \notin \mathit{dom}(\mathit{family}) \lor p2 \notin \mathit{dom}(\mathit{family}) \\
o! = \mathit{PersonNotMember} \\ \hline
\end{array}
$$

$$
\begin{array}{|l}
\multicolumn{1}{l}{\mathit{PaymentNotFound}} \\ \hline
\Xi \mathit{FamilyPayment} \\
\mathit{cost}? : \mathit{Cost} \\
o! : \mathit{message} \\
i : \mathit{id} \\ \hline
i = \mathit{cost}?.\mathit{imprestId} \\
i \notin \mathit{dom}(\mathit{payments}) \\
o! = \mathit{PaymentNotFound} \\ \hline
\end{array}
$$

$$
\begin{array}{|l}
\multicolumn{1}{l}{\mathit{ImprestNotBalance}} \\ \hline
\Xi \mathit{FamilyPayment} \\
\mathit{cost}? : \mathit{Cost} \\
o! : \mathit{message} \\
v : \mathbb{N} \\ \hline
v = \mathit{payments}(\mathit{cost}?.\mathit{imprestId}).\mathit{paymentValue} \\
v < \mathit{cost}?.\mathit{costValue} \\
o! = Imprest_{N}ot_{B}alance \\ \hline
\end{array}
$$

$$
\begin{array}{|l}
\multicolumn{1}{l}{\mathit{CostNotFound}} \\ \hline
\Xi \mathit{FamilyPayment} \\
\mathit{cost}? : \mathit{Cost} \\
o! : \mathit{message} \\ \hline
\lnot\ (\exists\, i : \mathit{id} \bullet \mathit{costId}\ i = \mathit{cost}?) \\
o! = Cost_{N}ot_{F}ound \\ \hline
\end{array}
$$

$$
\begin{array}{|l}
\multicolumn{1}{l}{\mathit{DateNotValid}} \\ \hline
\Xi \mathit{FamilyPayment} \\
\mathit{payment}? : \mathit{Payment} \\
o! : \mathit{message} \\ \hline
\lnot\ (\exists\, x : \mathit{Date} \bullet x.\mathit{year} = \mathit{payment}?.\mathit{paymentDate}.1 \land x.\mathit{mounth} = \mathit{payment}?.\mathit{paymentDate}.2 \land \\
\qquad\qquad x.\mathit{day} = \mathit{payment}?.\mathit{paymentDate}.3) \\
o! = \mathit{DateNotValid} \\ \hline
\end{array}
$$

$$
\begin{array}{|l}
\multicolumn{1}{l}{DateNotMatch} \\ \hline
\Xi FamilyPayment \\
cost? : Cost \\
py : Payment \\
o! : message \\ \hline
py = payments\ (cost?.imprestId) \\
\neg\ ((py.paymentDate.1 < cost?.costDate.year) \lor (py.paymentDate.1 = cost?.costDate.year \land \\
\qquad py.paymentDate.2 < cost?.costDate.mounth) \lor \\
\qquad (py.paymentDate.1 = cost?.costDate.year \land \\
\qquad\qquad py.paymentDate.2 = cost?.costDate.mounth \land \\
\qquad\qquad\qquad py.paymentDate.3 \leq cost?.costDate.day)) \\
o! = DateNotMatch \\ \hline
\end{array}
$$

$$
\begin{array}{l}
AddCost == (AddCost_0 \land Success) \lor PaymentNotFound \lor \\
\qquad\qquad ImprestNotBalance \lor CostNotFound \lor DateNotMatch
\end{array}
$$

$$
AddPayment == (AddPayment_0 \land Success) \lor PersonNotMember \lor DateNotValid
$$

$$
\begin{array}{|l}
\multicolumn{1}{l}{ListOfCost} \\ \hline
\Xi FamilyPayment \\
d1? : Date \\
d2? : Date \\
o! : person \rightarrow \mathbb{P}\ \ Cost \\ \hline
\forall p : Person \mid p \in dom(costs) \land p \in dom(o!) \bullet \\
\qquad \forall c : Cost \mid c \in costs(p) \land c \in o!(p) \bullet (c.costDate.year > d1?.year \lor \\
\qquad (c.costDate.year = d1?.year \land c.costDate.mounth > d1?.mounth) \lor \\
\qquad (c.costDate.year = d1?.year \land c.costDate.mounth = d1?.mounth \land c.costDate.day \geq d1?.day)) \land \\
\qquad (c.costDate.year < d2?.year \lor (c.costDate.year = d2?.year \land c.costDate.mounth < d2?.mounth) \lor \\
\qquad (c.costDate.year = d2?.year \land c.costDate.mounth = d2?.mounth \land c.costDate.day \leq d2?.day)) \\ \hline
\end{array}
$$

$$
\begin{array}{|l}
\multicolumn{1}{l}{ListOfPayment} \\ \hline
\Xi FamilyPayment \\
d1? : Date \\
d2? : Date \\
o! : person \rightarrow \mathbb{P}\ \ Payment \\ \hline
\forall p : Person \mid p \in dom(costs) \bullet \forall y : Peyment \mid y \in range(payments) \land y \in o!(p) \land y.payer = p \bullet \\
\qquad (y.paymentDate.1 > d1.year \lor (y.paymentDate.1 = d1.year \land y.paymentDate.2 > d1.mounth) \lor \\
\qquad (y.paymentDate.1 = d1.year \land y.paymentDate.2 = d1.mounth \land y.paymentDate.3 \geq d1.day)) \land \\
\qquad (y.paymentDate.1 < d2.year \lor (y.paymentDate.1 = d2.year \land y.paymentDate.2 < d2.mounth) \lor \\
\qquad (y.paymentDate.1 = d2.year \land y.paymentDate.2 = d2.mounth \land y.paymentDate.3 \leq d2.day)) \\ \hline
\end{array}
$$

سوال ۳

$$
\begin{array}{|l}
\hline
\mathit{CostL} \\
\mathit{costValue} : \mathbb{N} \\
\mathit{costDate} : \mathit{Date} \\
\mathit{costUsage} : \mathit{usage} \\
\mathit{imprestId} : \mathit{id} \\
\hline
\mathit{costValue} > 0 \\
\hline
\end{array}
$$

$$
\begin{array}{|l}
\hline
\mathit{FamilyPaymenG} \\
\mathit{family} : \mathit{Family} \\
\mathit{costs} : \mathit{person} \rightarrow \mathbb{P}\ \mathit{CostL} \\
\mathit{payments} : \mathit{id} ⤖ \mathit{Payment} \\
\mathit{costId} : \mathit{id} ⤖ \mathit{CostL} \\
\mathit{imprestRemind} : \mathit{person} \rightarrow (\mathit{Payment} \leftrightarrow \mathbb{N}) \\
\hline
\forall\, p : \mathit{payment} \mid p \in \mathit{dom}(\mathit{range}(\mathit{imprestRemind})) \bullet p \in \mathit{range}(\mathit{payments}) \land p.\mathit{payMode} = \mathit{imprest} \\
\forall\, p : \mathit{person} \mid p \in \mathit{dom}(\mathit{imprestRemind}) \bullet p \in \mathit{dom}(\mathit{family}) \\
\forall\, p : \mathit{person} \mid p \in \mathit{dom}(\mathit{costs}) \bullet p \in \mathit{dom}(\mathit{family}) \\
\forall\, p : \mathit{Payment} \mid p \in \mathit{range}(\mathit{payments}) \bullet p.\mathit{payer} \in \mathit{dom}(\mathit{family}) \land p.\mathit{payee} \in \mathit{dom}(\mathit{family}) \\
\forall\, c : \mathit{CostL} \mid c \in \mathit{range}(\mathit{costId}) \bullet \exists\, p : \mathit{Person} \bullet c \in \mathit{costs}(p) \\
\hline
\end{array}
$$

$$
\begin{array}{|l}
\hline
\mathit{Promotion} \\
\Delta \mathit{FamilyPaymentG} \\
\Delta \mathit{CostL} \\
p? : \mathit{Person} \\
i : \mathit{id} \\
\hline
\mathit{family} = \mathit{family}' \\
p? \in \mathit{dom}(\mathit{family}) \\
i \in \mathit{dom}(\mathit{costID}) \\
p? \in \mathit{dom}(\mathit{costs}) \\
\mathit{costId}\ i = \Theta \mathit{Cost} \\
\mathit{costId}'\ i = \Theta \mathit{Cost}' \\
\Theta \mathit{Cost} \in \mathit{costs}\ p? \\
\Theta \mathit{Cost}' \in \mathit{costs}'\ p? \\
\mathit{payments} = \mathit{payments}' \\
\{p?\} ⩤ \mathit{costs} = \{p?\} ⩤ \mathit{costs}' \\
\{i\} ⩤ \mathit{costId} = \{i\} ⩤ \mathit{costId}' \\
\mathit{imprestRemind} = \mathit{imprestRemind}' \\
p? \in \mathit{dom}(\mathit{imprestRemind}) \\
\hline
\end{array}
$$

$$
\begin{array}{|l}
\hline
\mathit{addcost}_0 L \\
\Delta \mathit{Cost} \\
c? : \mathit{CostL} \\
\hline
\mathit{paymentValue}' = c?.\mathit{paymentValue} \\
\mathit{payer}' = c?.\mathit{payer} \\
\mathit{payee}' = c?.\mathit{payee} \\
\mathit{paymentDate}' = c?.\mathit{paymentDate} \\
\mathit{imprestId}' = c?.\mathit{imprestId} \\
\hline
\end{array}
$$

$$\mathit{addCost}_0 G == \exists\, \Delta \mathit{CostL} \bullet \mathit{addCost}_0 L \land \mathit{Promotion}$$

$$\begin{array}{|l}
\multicolumn{1}{l}{\textit{Success}} \\ \hline
o! : message \\ \hline
o! = OK \\ \hline
\end{array}$$

$$\begin{array}{|l}
\multicolumn{1}{l}{\textit{PaymentNotFoundG}} \\ \hline
\Xi FamilyPaymentG \\
cost? : CostL \\
o! : message \\
i : id \\ \hline
i = cost?.imprestId \\
i \notin dom(payments) \\
o! = PaymentNotFound \\ \hline
\end{array}$$

$$\begin{array}{|l}
\multicolumn{1}{l}{\textit{ImprestNotBalanceG}} \\ \hline
\Xi FamilyPaymentG \\
cost? : CostL \\
o! : message \\
v : \mathbb{N} \\ \hline
v = payments(cost?.imprestId).paymentValue \\
v < cost?.costValue \\
o! = Imprest_{N}ot_{B}alance \\ \hline
\end{array}$$

$$\begin{array}{|l}
\multicolumn{1}{l}{\textit{CostNotFoundG}} \\ \hline
\Xi FamilyPaymentG \\
cost? : CostL \\
o! : message \\ \hline
\neg\ (\exists\, i : id \bullet costId\ i = cost?) \\
o! = Cost_{N}ot_{F}ound \\ \hline
\end{array}$$

$$\begin{array}{|l}
\multicolumn{1}{l}{\textit{DateNotMatchG}} \\ \hline
\Xi FamilyPaymentG \\
cost? : CostL \\
py : Payment \\
o! : message \\ \hline
py = payments\ (cost?.imprestId) \\
\neg\ ((py.paymentDate.1 < cost?.costDate.year) \vee (py.paymentDate.1 = cost?.costDate.year \wedge \\
\qquad py.paymentDate.2 < cost?.costDate.mounth) \vee \\
\qquad (py.paymentDate.1 = cost?.costDate.year \wedge \\
\qquad\qquad py.paymentDate.2 = cost?.costDate.mounth \wedge \\
\qquad\qquad\qquad py.paymentDate.3 \leq cost?.costDate.day)) \\
o! = DateNotMatch \\ \hline
\end{array}$$

$$
\begin{array}{|l}
\multicolumn{1}{l}{\mathit{ListOfCostL}} \\
\hline
\Xi \mathit{CostL} \\
d1? : \mathit{Date} \\
d2? : \mathit{Date} \\
o! : \mathit{CostL} \\
\hline
o!.\mathit{costValue} = \mathit{costValue} \\
o!\mathit{costDate} = \mathit{costDate} \\
o!.\mathit{costUsage} = \mathit{costUsage} \\
o!.\mathit{imprestId} = \mathit{imprestId} \\
(\mathit{costDate.year} > d1?.\mathit{year} \lor (\mathit{costDate.year} = d1?.\mathit{year} \land \mathit{costDate.mounth} > d1?.\mathit{mounth}) \lor \\
\qquad (\mathit{costDate.year} = d1?.\mathit{year} \land \mathit{costDate.mounth} = d1?.\mathit{mounth} \land \mathit{costDate.day} \geq d1?.\mathit{day})) \\
(\mathit{costDate.year} < d2?.\mathit{year} \lor (\mathit{costDate.year} = d2?.\mathit{year} \land \mathit{costDate.mounth} < d2?.\mathit{mounth}) \lor \\
\qquad (\mathit{costDate.year} = d2?.\mathit{year} \land \mathit{costDate.mounth} = d2?.\mathit{mounth} \land \mathit{costDate.day} \leq d2?.\mathit{day})) \\
\hline
\end{array}
$$

$$\mathit{ListOfCostLG} == \exists \Delta \mathit{CostL} \bullet \mathit{ListOfCostL} \land \mathit{Promotion}$$

$$
\begin{array}{l}
\mathit{AddCostG} == (\mathit{AddCost}_0\, G \land \mathit{Success}) \lor \mathit{PaymentNotFoundG} \lor \\
\qquad \mathit{ImprestNotBalanceG} \lor \mathit{CostNotFoundG} \lor \mathit{DateNotMatchG}
\end{array}
$$