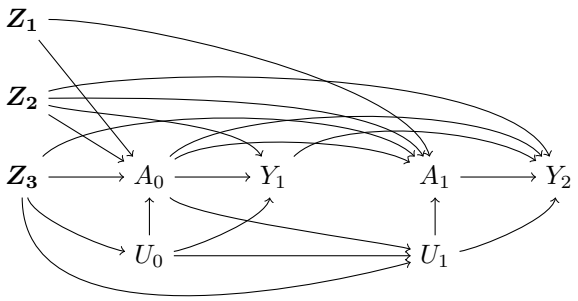
The Future of Mendelian Randomization Studies

MR Data Challenge

Part 3: Long-term exposure to "A" in women

For simplicity, only 2 time points are depicted in the following DAG. The data-generating model will include a total of 6 time points: ages 20 to 30; 30 to 40; 40 to 50; 50 to 60; 60 to 70; 70 to 80.



Note: $\boldsymbol{Z_1}$ represents the set of SNPs located on chromosome 3 and 4; $\boldsymbol{Z_2}$ represents the set of SNPs located on chromosome 9; and $\boldsymbol{Z_3}$ represents the set of SNPs located on chromosome 17. Let $\boldsymbol{Z} = (\boldsymbol{Z_1}, \boldsymbol{Z_2}, \boldsymbol{Z_3})$.

The data-generating model is as follows (note that data for $k = 0$ and $k = 1$ are generated in part 2):

$$
\begin{aligned}
&k = 2, \ldots, K \text{ where } K = 5 \\
U_{k,i} &\sim \mathcal{N}\left(\boldsymbol{\beta}_{\boldsymbol{ZU}}^{'}\boldsymbol{Z_i} + \beta_{UU}U_{k-1,i} + \beta_{AU}\frac{A_{k-1,i} - 50}{10},\ 1\right) \\
A_{k,i} &\sim \mathcal{N}\left(25 + (\boldsymbol{\beta}_{\boldsymbol{ZA_{base}}} + \boldsymbol{\beta}_{\boldsymbol{ZA_\Delta}}\boldsymbol{k})^{'}\boldsymbol{Z_i} + \beta_{UA}U_{k,i} + \beta_{AA}^{'}A_{k-1,i},\ 5\right) \\
Y_{k+1,i} &\sim Bin\left(1,\ \text{expit}\left(\log\frac{\lambda}{1-\lambda} + \boldsymbol{\beta}_{\boldsymbol{ZY}}^{'}\boldsymbol{Z_i} + \beta_{UY}U_{k,i} + \beta_{AY_k}\frac{A_{k,i} - 50}{10} + \beta_{AY_{k-1}}\beta_{AY_\Delta}\frac{A_{k-1,i} - 50}{10}\right)\right)
\end{aligned}
$$

where

$$
\begin{aligned}
\boldsymbol{\beta}_{\boldsymbol{ZU}} &= (0, 0, 0, 0, 0, 0, 0, 0, 0, 0.25, 0.10, 0.30) \\
\beta_{UU} &= 0.50 \\
\beta_{AU} &= 0.35 \\
\boldsymbol{\beta}_{\boldsymbol{ZA_{base}}} &= (\beta_{ZA_{base,1}}, \beta_{ZA_{base,2}}, \ldots, \beta_{ZA_{base,11}}) \text{ where } \beta_{ZA_{base,j}} \sim \mathcal{U}_{[1,5]} \text{ for } 1 \leq j \leq 11 \\
\boldsymbol{\beta}_{\boldsymbol{ZA_\Delta}} &= (\beta_{ZA_{\Delta,1}}, \beta_{ZA_{\Delta,2}}, \ldots, \beta_{ZA_{\Delta,11}}) \text{ where } \beta_{ZA_{\Delta,p}} \sim \mathcal{U}_{[-\frac{1}{k}\beta_{ZA_{base,p}},\ \frac{1}{k}\beta_{ZA_{base,p}}]} \\
\beta_{UA} &= 5 \\
\beta_{AA} &= 0.5 \\
\boldsymbol{\beta}_{\boldsymbol{ZY}} &= (0, 0, 0, 0.1, 0.05, 0.3, 0.2, 0.01, 0, 0, 0) \\
\beta_{UY} &= 0.3 \\
\beta_{AY_k} &= \begin{cases} 0.2 & \text{if } k = 0 \\ 0.3 & \text{if } k = 1 \\ 0.7 & \text{if } k = 2 \\ 0.3 & \text{if } k = 3 \\ 0.2 & \text{if } k = 4 \\ 0.1 & \text{if } k = 5 \end{cases} \\
\beta_{AY_\Delta} &= 0.5
\end{aligned}
$$