Ay190 – Worksheet 3

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Problem 1

(a)

We will compute the integral

$$\int_0^{\pi} f(x)dx = \int_0^{\pi} \sin(x)dx = 2$$

using the Midpoint Rule, the Trapezoid Rule, and Simpson's Rule with a step size of 0.01. The results are (the ordering of the errors is (absolute, relative)):

Midpoint Rule
Value: 2.00000706508
Errors: (-7.0650798953408867e-06, -3.5325399476704433e-06)

Trapezoidal Rule
Value: 1.99994799207
Errors: (5.2007928589281605e-05, 2.6003964294640802e-05)

Simpsons Rule
Value: 1.99998737408
Errors: (1.2625922932940625e-05, 6.3129614664703126e-06)

Now let's reduce the step size by a factor of 10

Midpoint Rule
Value: 2.00000000037
Errors: (-3.6778446954599531e-10, -1.8389223477299765e-10)

Trapezoidal Rule
Value: 1.99999975037
Errors: (2.4963220757179272e-07, 1.2481610378589636e-07)

Simpsons Rule
Value: 1.99999991703
Errors: (8.2965546432944848e-08, 4.1482773216472424e-08)

Next, we'll use these methods to compute the integral

$$I = \int_0^{\pi} x\sin(x)dx = \pi$$

Midpoint Rule

Value: 3.14160174726
Errors: (-9.0936699561616763e-06, -2.8946050487387799e-06)

Trapezoidal Rule
Value: 3.14145510764
Errors: (0.00013754594899140216, 4.3782235368494702e-05)

Simpsons Rule
Value: 3.14155286739
Errors: (3.9786203026359601e-05, 1.2664341757005714e-05)

Now let's reduce the step size by a factor of 10

Midpoint Rule
Value: 3.14159252386
Errors: (1.2973284624351322e-07, 4.129524752207191e-08)

Trapezoidal Rule
Value: 3.14159213106
Errors: (5.2253375004696068e-07, 1.6632765850463738e-07)

Simpsons Rule
Value: 3.14159239292
Errors: (2.6066648084466237e-07, 8.297271784959373e-08)

Problem 2

(a)

In order to use Gauss-Laguerre Quadrature, I must modify the integral as follows:

$$\frac{8\pi(k_BT)^3}{(2\pi\hbar c)^3}\int_0^\infty \frac{x^2dx}{e^x+1} = \frac{8\pi(k_BT)^3}{(2\pi\hbar c)^3}\int_0^\infty \frac{x^2e^{-x}dx}{1+e^{-x}} = \frac{8\pi(k_BT)^3}{(2\pi\hbar c)^3}\int_0^\infty e^{-x}f(x)dx$$

Using scipy's ability to calculate the roots and weights for Gauss-Laguerre Quadrature, I was then able to calculate the number density (in m^{-3}) for $n = 10$ and $n = 100$. Larger values of n took a really long time to calculate, and the change in result was negligible.

n	10	100
n_e	$1.89822604979 * 10^{41}$	$1.8982157567 * 10^{41}$

(b)

Since $E = pc$, the integral we're looking at for this problem should look like the middle expression in Equation 1, except for an added factor of c^{-3}. Using the method described in the problem, I calculate the number density again, but with Gauss-Legendre Quadrature. This time, I did so for $n = 100$ and $n = 500$, although the difference between them appears to be lower than the float precision,as both results returned

$$n_e = 7.41900699135 * 10^{41}$$

I'm not really sure why this isn't the same as the result for part (a).