Ay190 – Worksheet 1
John Pharo
Date: February 3, 2014
Fellow Planeteers: Mee Wongurailertkun, Cutter Coryell

Problem 1

Below is a table of MC calculation of π. Note that the result of the calculation converges toward the known value of π as N increases, approximately $\propto N^{1/2}$.

Calculation	Fractional Error	N
3.6	0.8726646259971648	10
3.32	0.9462628474668052	100
3.196	0.9829764247777826	1000
3.1308	1.0034472510507837	10000
3.13732	1.001361880072735	100000
3.141964	0.9998818107367853	1000000

Problem 2

Using a Monte Carlo experiment to calculate the probability of two people in a group having the same birthday (with $N = 10000$) for group sizes 2 to 50, the smallest group size to exceed a probability of 0.5 is 23 (see Figure 1). This agrees with the authority on the matter, the interwebs.

Analytically, this problem is easiest by first considering the probability that no one in the group has matching birthdays. First look at a group of only two people. The first person has a birthday on 1 day out of the 365 in a year, so there is a $\frac{364}{365}$ probability that the second person has a different birthday. Adding a third person, there are now two days to avoid, and the second person still has to avoid one, so the probability is $\frac{364}{365} \cdot \frac{363}{365}$. Generalizing to a group of n people, this probability becomes

$$P_n = \frac{365!}{(365-n)! \cdot 365^n}$$

But this is still the probability of no one having a matching birthday. Fortunately, the probability that at least two people have a matching birthday is the only other case, and the two cases are mutually exclusive, so

$$P = 1 - P_n = 1 - \frac{365!}{(365-n)! \cdot 365^n}$$

For $P \geq 0.5$, $n \geq 23$.

Knowing that 23 people is the minimum, let's check for convergence. Below is a table of calculations of the probability at 23 people for various values of N. $N = 10$ is off by a significant amount, but it converges to within a few percent for $N \geq 10000$.

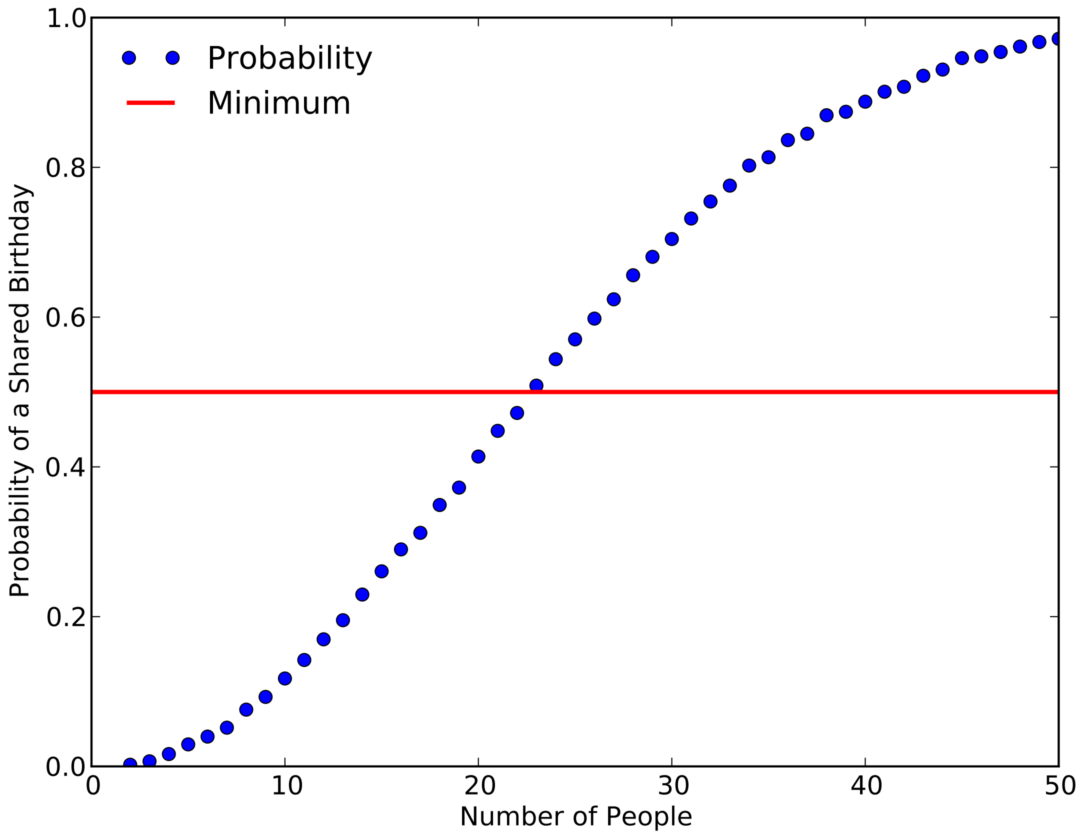

Figure 1: A plot of the probability of two people in a group having the same birthday vs group size. The first point above the red .5-probability line is 23.

Probability	Fraction	N
0.59999999999999998	0.83333333333333337	10
0.57999999999999996	0.86206896551724144	100
0.51400000000000001	0.97276264591439687	1000
0.50239999999999996	0.99522292993630579	10000
0.50643000000000005	0.98730327982149546	100000
0.50710200000000005	0.9859949280420901	1000000

Problem 3

The analytical solution is $22/3 \approx 7.333$. The simulation gets within a percent of this for $N \geq 1000$. It appears to be able to get arbitrarily close as N increases.

Value	Fraction	N
8.0	0.9166666666666663	10
7.25	1.0114942528735631	100
7.4049999999999994	0.99032185460274591	1000
7.3079999999999998	1.0034665207079001	10000
7.3167	1.0022733381624684	100000
7.3371550000000001	0.999479135078015	1000000