Ay190 – Worksheet 2
John Pharo
Date: January 16, 2014
Co-Conspirators: Anthony Alvarez, Cutter Coryell

Problem 1

The following is the copied output of the Python file problem1.py.

n	x_n	Absolute Error	Relative Error
0	1.0	0.0	0.0
1	0.333333	$9.93410748107e-09$	$2.98023224432e-08$
2	0.111111	$5.29819064732e-08$	$4.76837158259e-07$
3	0.0370373	$2.16342784749e-07$	$5.84125518823e-06$
4	0.0123466	$8.71809912319e-07$	$7.06166028978e-05$
5	0.00411871	$3.48814414362e-06$	0.0008476190269
6	0.00138569	$1.39522571909e-05$	0.0101711954921
7	0.000513056	$5.58089223023e-05$	0.122054113075
8	0.000375651	0.000223235614917	1.46464886947
9	0.000943748	0.000892942551319	17.5757882376
10	0.00358871	0.00357177023583	210.909460655
11	0.0142927	0.0142870812639	2530.91358466
12	0.0571502	0.0571483257835	30370.9634027
13	0.228594	0.228593318278	364451.584977
14	0.914374	0.914373307961	4373419.18641
15	3.65749	3.6574932832	52481030.9737

Notice that, for n=15, the absolute error is almost the size of x_{15}, and consequently, the relative error is huge.

Problem 2

See Figure 1 for the plot of the absolute errors $f'(x;h_i) - f'(x)$ for the forward and central difference approximations. Note that, when changing from h_1 to h_2, the forward difference changes by one factor of the ratio of the step sizes (so $n = 1$), but the central difference changes by the ratio squared (so $n = 2$). Thus, the forward difference is first-order convergent and the central difference is second-order convergent.

Problem 3

The notes calculate the central finite difference approximation by subtracting the forward and backward Taylor expansions and solving for the first derivative. We want a second-order central approximation for the second derivative, which means the final error term must be divided by the coefficient of the second derivative and still end up $O(h^2)$. Thus we make our Taylor expansions

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2}f''(x) + \frac{h^3}{6}f'''(x) + O(h^4)$$

$$f(x-h) = f(x) - hf'(x) + \frac{h^2}{2}f''(x) - \frac{h^3}{6}f'''(x) + O(h^4)$$

Adding the two together, the first and third derivatives of f will cancel, allowing us to easily solve for the second derivative.

$$f''(x) = \frac{f(x+h) + f(x-h) - 2f(x)}{h^2} + O(h^2)$$

So we have a second-order central finite difference approximation of the second derivative.

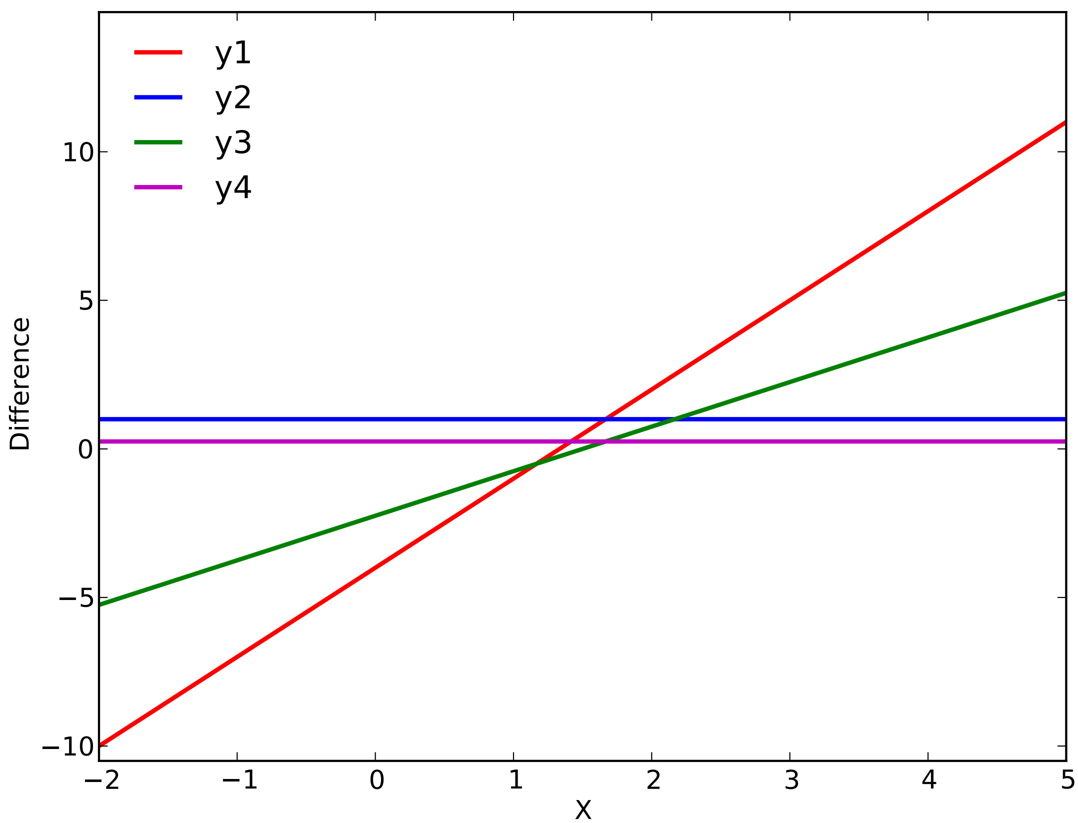

Figure 1: Plot of the absolute error of the forward and central difference approximations at h_1 and h_2. y_1 and y_3 ar ethe forward difference at h_1 and h_2, and y_2 and y_4 are the central difference approximations.

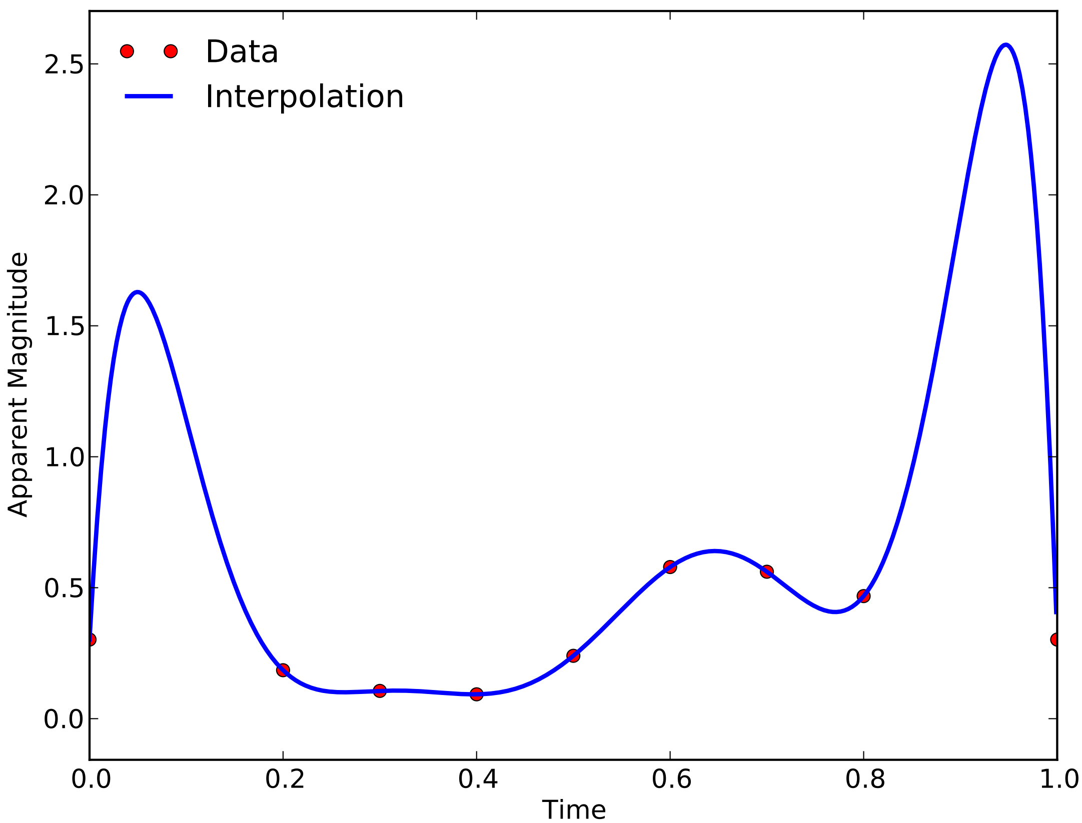

Figure 2: A Plot of the 8th-degree global Lagrange interpolation polynomial against the data points provided. Note that the interpolation matches well near the center of the interval, but it gets a bit wild near the edges.

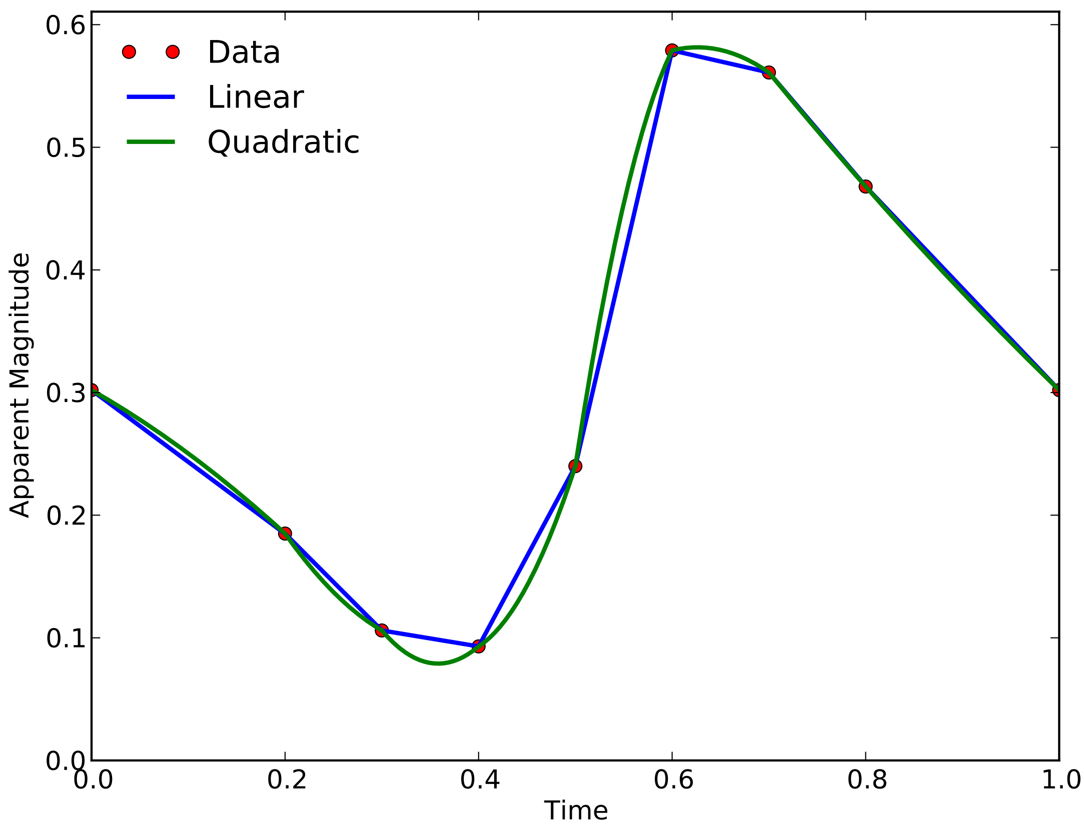

Figure 3: A plot of piecewise linear and piecewise quadratic interpolations against the data.

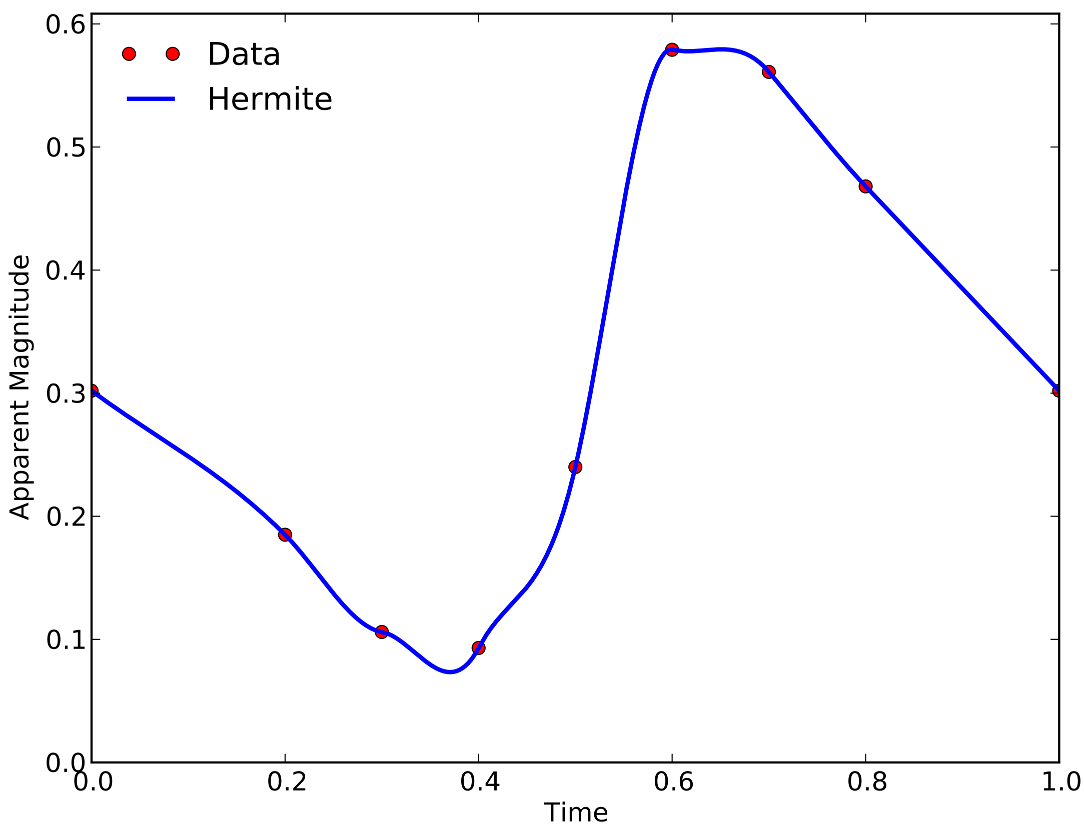

Figure 4: A plot of a piecewise cubic Hermite interpolation against the data.

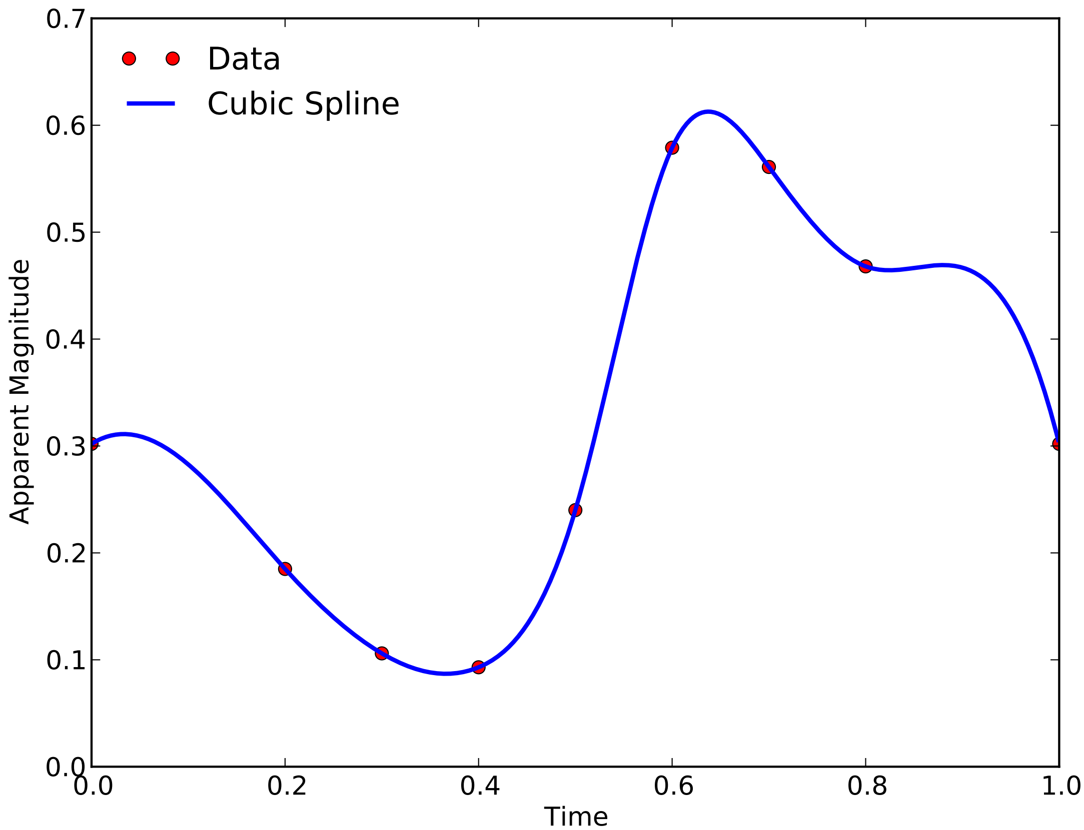

Figure 5: A plot of a natural cubic spline interpolation against the data, using scipy's interp1d function.