

Eötvös Experiments with Supermassive Black Holes

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ABSTRACT

By examining the locations of central black holes in two elliptical galaxies, M32 and M87, we derive constraints on the violation of the strong equivalence principle for purely gravitational objects, *i.e.* *black holes*, of less than six percent, $|\eta_N| < 0.06$.

1 INTRODUCTION

The strong equivalence principle (SEP) states that any object regardless of its composition will travel through a gravitational field in the same way. This includes objects with varying contributions of gravitational energy. In particular a black hole whose mass is entirely gravitational should travel in the same manner through the gravitational field as a star. Nordtvedt (1968a,b) argued that metric theories of gravity other than general relativity may exhibit violations of the SEP — in particular, objects whose have a large contribution of gravitational energy to their makeup may travel differently through a gravitational field than other objects.

Typically the strong equivalence principle is probed by looking for the Nordvedt effect in the Earth-Moon system or binary stars consisting of a neutron star and a white dwarf (Stairs et al. 2005). The Nordtvedt (1968c) effect results in the polarization of an orbit in the direction of the gravitational acceleration of a large body: the Sun in the case of the Earth-Moon system (Williams et al. 1976; Shapiro et al. 1976), and the Galaxy in the case binary pulsars. The recently discovered millisecond pulsar in a triple system, PSR J0337+1715, will provide further interesting constraints (Ransom et al. 2014).

Here we perform the test first proposed by Hui & Nicolis (2012). In particular we will examine the polarization of the orbits of supermassive black holes through the central regions of elliptical galaxies. We will focus on elliptical galaxies because the presence of central black holes is common in sufficiently large elliptical galaxies and the location of the centre of the stellar potential is straightforward to constrain. Interactions between galaxies are ubiquitous, so example systems are straightforward to find. In particular we will examine the small elliptical galaxies that orbit the largest neighbour of the Milky Way galaxy, the Andromeda galaxy or M31, and the massive elliptical galaxy, M87, in the Virgo cluster.

2 CALIBRATION

Let us suppose that we are studying a small stellar system in the gravitational of a large one. The stars in the small

system experience an acceleration toward the centre of the larger one

$$a_\star = \frac{GM}{d^2} \quad (1)$$

where d is the distance between the small system and the larger one and M is the mass of the larger system. These accelerations are typically ten to one hundred times larger than those exerted by large-scale structure $a \sim 600 \text{ km s}^{-1} H_0 \sim 10^{-10} \text{ cm s}^{-2}$.

Furthermore, let us assume that the a massive black hole within the smaller system experiences a different acceleration

$$a_\bullet = (1 - \Delta) a_\star \quad (2)$$

where Δ quantifies the extent of violation of the SEP. Nordtvedt (1982) presents an equivalent definition where the ratio of the inertia masses and passive gravitational mass of an object could depend on its constitution and in particular the contribution of gravitational energy,

$$\frac{m_{\text{GP}}}{m_{\text{I}}} = 1 - \eta_N \frac{U_G}{mc^2}. \quad (3)$$

In the context of the parameterized post-Newtonian treatment of deviations from general relativity (Will 2014),

$$\eta_N = 4\beta\gamma\frac{10}{3}\xi - \alpha_1 + \frac{2}{3}\alpha_2 - \frac{2}{3}\zeta_1 - \frac{1}{3}\zeta_2. \quad (4)$$

For the case of a black hole, $U_G = mc^2$ so $\Delta = \eta_N$.

The massive black hole also experiences an acceleration due to the stellar system in which it resides. Furthermore, dynamical friction with the less massive stars nearby damps any motion of the black hole relative to its equilibrium position. Near minimum of the stellar potential we can approximate it as a harmonic oscillator so

$$(1 - \Delta)\omega^2 x = a_\bullet - a_\star = -\Delta \frac{GM}{d^2} \quad (5)$$

and x is the displacement of the black hole from the centre of the small stellar system in the direction of the larger system. The value of ω depends on the structure of the small stellar system and depends on the mean density within the region surrounding the black hole. For a Hernquist (1990) model for the density distribution that well describes the stellar distribution of elliptical galaxies

$$\rho = \frac{m}{2\pi} \frac{a}{r} \frac{1}{(r+a)^3} \quad (6)$$

where R_{eff} , the half-light radius in projection is approximately $1.8153a$. We take the black hole mass to be $m_{\bullet} = 0.005m$ where m is the mass of the small system, we find that

$$\omega^2 \approx \frac{Gm}{a^3} \quad (7)$$

from n-body simulations independent of the number of particles in the simulations from 10,000 to 250,000. In these simulations we find that small perturbations of the black hole from the centre result in sinusoidal oscillations with this typical frequency, regardless of the number of particles and the softening length adopted, and the amplitude of the perturbation as long as it is smaller than the black hole's sphere of influence. For such small perturbations the black hole pulls the neighbouring stars along with it. The half-mass radius of the Hernquist model is $a/(\sqrt{2}-1)$ so the typical frequency within the half-mass radius is

$$\omega^2 = \frac{4\pi}{8} (\sqrt{2}-1)^3 \frac{Gm}{a^3} \approx 0.11 \frac{Gm}{a^3}. \quad (8)$$

The oscillation frequency of the black hole is significantly larger than the typical frequency of the system because the black hole lies near the centre of the galaxy where typical density is larger. On the other hand, if one directly used Eq. (6) to calculate the restoring force toward the centre, one would find that it is constant with displacement from the centre. This would assume that the density is actually singular at the centre but also would neglect the effect of the black hole on the neighbouring stars which would be dragged with it. We will compare this estimate with the detailed measurements of the potential near the core of the elliptical galaxy, M87, and find good agreement.

If we assume that the black hole lies within a bulge and halo approximately modelled by a Hernquist model (as in van der Marel et al. 1997b), we find that

$$\frac{x}{d} = \frac{\Delta}{1-\Delta} \frac{M}{d^3} \frac{a^3}{m}, \quad (9)$$

so the relative displacement only depends on the relative masses and sizes of the various bodies and the angular displacement does not depend on the distance to the systems from Earth. We can make this explicit by calculating the apparent displacement across the sky as

$$\Delta\theta = \frac{\Delta}{1-\Delta} \frac{M}{\delta^2} \frac{\alpha^3}{m} \cos^3 i \quad (10)$$

where i is the inclination of the line connecting the two bodies with respect to the plane of the sky, δ is the angular distance between the bodies and α is the angular size of the Hernquist radius a of the host galaxy. If smaller galaxies are distributed as an isothermal distribution about the perturbing galaxy as Koch & Grebel (2006) found for the satellites of M31, the mean value of the geometric term $|\cos^3 i|$ is $4/(3\pi) \approx 0.42$ and the median value is $\sqrt{2}/4 \approx 0.35$. Ninety percent of the time the term lies between 5×10^{-4} and 0.99, so geometric information is especially useful to give firm constraints. Of course the halo of the galaxy group must end somewhere so the lower limit on $\cos^3 i$ found here is somewhat unrealistic. On the other hand if the density of satellites is proportional to r^{-3} like the outer regions of

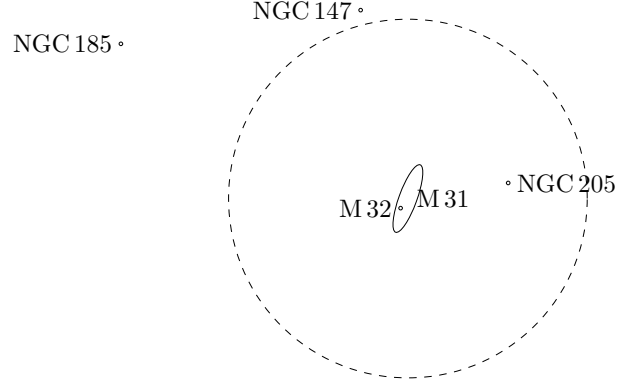


Figure 1. The geometry of the largest elliptical satellite galaxies of the Andromeda Galaxy. The Sun is to 773 kpc to the left of M32, and the radius of the dashed circle is 100 kpc.

the Navarro-Frenk-White profile (1996), the mean value of the geometric term is $3\pi/16 \approx 0.59$ and the median value is $3\sqrt{3}/8 \approx 0.65$. Ninety percent of the time the term lies between 0.03 and 0.996.

3 THE ANDROMEDA GALAXY SYSTEM

The nearest neighbour of the Milky Way has four small elliptical galaxies orbiting it: M32, NGC 205, NGC 147 and NGC 185. Fig. 1 depicts the location of the various galaxies relative to each other from the distances compiled by Koch & Grebel (2006). Of course, the distances to the various galaxies are more uncertain than their positions on the sky.

A supermassive black hole (SMBH) has already been discovered in the core of the compact elliptical galaxy M32 (van der Marel et al. 1997a) and found to be within 0.5 arcseconds of the nucleus of the galaxy (Yang et al. 2015). M32 is located about 0.4° from the centre of M31 in the plane of the sky. We can estimate the displacement of the black hole from the centre of M32 as a function of Δ in the best-case scenario of the minimal separation of M32 from M31 of about 5 kpc. In this case M32 would be well within the halo of M31, and we could use the circular velocity of M31 of 240 km/s to estimate the acceleration of M32 toward the centre of M31, a_{\star} in Eq. (1),

$$a_{\star} = \frac{v^2}{d} = 3.7 \times 10^{-8} \text{ cm s}^{-2}. \quad (11)$$

For the galaxy M32 we take $m = 8 \times 10^8 M_{\odot}$ and $R_{\text{eff}} = 100$ pc so $a = 55$ pc and

$$x = 0.55 \frac{\Delta}{1-\Delta} \text{ pc} \quad (12)$$

or

$$\Delta\theta = \frac{x}{d_{\text{M32-Sun}}} = 0.17 \frac{\Delta}{1-\Delta} \text{ arcseconds}, \quad (13)$$

so if M32 lies indeed at the close to minimum distance to M31, the existing measurement of the position of the black hole yields a constraint; either $\Delta < 0.75$ or $\Delta > 1.5$. Dierickx et al. (2014) argue that the actual position of M32 is 85 kpc

in front of the disk of M 31. Here the expected displacement is much smaller

$$x = 0.02 \frac{\Delta}{1 - \Delta} \text{pc} \quad (14)$$

and only

$$\Delta\theta = \frac{x}{d_{\text{M32-Sun}}} \frac{5 \text{ kpc}}{85 \text{ kpc}} = 0.0004 \frac{\Delta}{1 - \Delta} \text{arcseconds} \quad (15)$$

where we have taken the mass enclosed within the halo of M 31 to be $10^{12} M_{\odot}$.

We can improve upon the astrometric constraint of Yang et al. (2015) if we assume that the position of the black hole is coincident with the cusp in the surface brightness of the galaxy. This is a reasonable assumption because the black hole would entrain the neighboring stars, so the stellar nucleus should be centered on the black hole. To measure the possible deviation of the position of the black hole on the sky and the centre of the potential well of the galaxy we use images from the Wide-Field Camera 3 (WFC3) on the Hubble Space Telescope (HST) in the filter, F555W. This instrument features 0.04 arcsecond pixels and a 162×162 -arcsecond field of view. Both the Space Telescope Imaging Spectrograph (STIS) and the planetary camera on Wide-Field Planetary Camera 2 (WF/PC2) feature a finer pixel scale but at the expense of field of view. We used images from GO-11714 (PI: Bond) with total exposure time of 120 seconds. To localise the core, we used circular and elliptical apertures of 0.16 arcseconds and 0.2 arcseconds. To localise the centroid of the outer regions we used two elliptical annuli: one of inner axes 2.8 and 4.3 arcseconds and outer axes of 5.7 and 8.5 arcseconds and one half that size. The centres of the annuli are initially roughly placed within about an arcsecond of the core and updated to lie on the calculated centroid. This is iterated thirty times. The centroids of the various annuli to probe the galactic potential coincide to within 0.01 arcseconds as do the ellipses and circles centered on the stellar cusp, yielding an estimate of the precision of 0.01 arcseconds. Furthermore, the centroid of the cusp and the outer regions also agree to within 0.01 arcseconds. If we combine this constraint with the distance estimate of Koch & Grebel (2006), this yields a constant, $|\Delta| < 0.06$.

The three smaller elliptical galaxies, NGC 205, NGC 147 and NGC 185, have more favourable geometry and smaller mass densities. However, SMBHs have not yet been identified in these galaxies. Each of these galaxies has a mass of about $2 \times 10^8 M_{\odot}$ and a $R_{\text{eff}} \approx 300 \text{ pc}$, so $a \approx 165 \text{ pc}$. The two galaxies, NGC 147 and NGC 185 are each about 100 kpc from M31, so the enclosed halo mass is about $10^{12} M_{\odot}$. This yields a typical value of the displacement of

$$x = 2.3 \frac{\Delta}{1 - \Delta} \text{pc and } \Delta\theta = 0.75 \frac{\Delta}{1 - \Delta} \text{arcseconds..} \quad (16)$$

Although NGC 205 may be somewhat closer to M 31 at 50 kpc than the other galaxies, the geometry is somewhat poorer perhaps reducing the angular displacement by a factor of two counteracting the increased acceleration. This yields a similar observed displacement. If a SMBH is discovered near the centre of these galaxies and can be localised within 0.5 arcseconds, this would yield a constraint of $-2.0 < \Delta < 0.4$.

On the other hand if NGC 205 lies at the same distance from us as M 31, this minimum possible distance to M 31 is

about 7 kpc, and we obtain a much larger linear and angular displacement

$$x = 43. \frac{\Delta}{1 - \Delta} \text{pc and } \Delta\theta = 13. \frac{\Delta}{1 - \Delta} \text{arcseconds.} \quad (17)$$

In this most favourable case the constraint would be $|\Delta| < 0.04$ with 0.5-arcsecond astrometry and $|\Delta| < 8 \times 10^{-3}$ with 0.01-arcsecond astrometry.

4 VIRGO ELLIPTICAL GALAXIES

One of the largest elliptical galaxies in the Virgo Cluster, M 87, also contains a supermassive black hole and lies at the centre of the Virgo A subcluster which contains the bulk of the mass of the Virgo Cluster, about $10^{14} M_{\odot}$. The Virgo Cluster is still in the process of forming and contains at least two additional subclumps of masses of about $10^{13} M_{\odot}$. The subclump closest to M 87 is centered on M 84 and M 86 about 1.3° away. M 89 is also a similar angular distance away, so this analysis applies to it as well, but it is likely to be less massive.

For the galaxy M 87 we take $R_{\text{eff}} = 164''$ (Ferrarese et al. 2006) so $a = 90''$. We will take the mass of M 87 to be about $4 \times 10^{12} M_{\odot}$ (Wu & Tremaine 2006) and take the mass of the subclump to be $10^{13} M_{\odot}$ at a distance of 1.3° . This yields a displacement of

$$\Delta\theta = 0.08 \frac{\Delta}{1 - \Delta} \cos^3 i \text{arcseconds.} \quad (18)$$

We can examine the stellar distribution of M 87 in further detail. Ferrarese et al. (2006) find that the central region of M 87 is better fit by a “core-Sérsic” model (Sérsic 1968; Trujillo et al. 2004) that has a much less cuspy central surface brightness distribution than a Hernquist model. The break radius between the core and the outer Sérsic model is about 7 arcseconds.

Walsh et al. (2013) determine the mass of the central black hole of M 87 to be about $3.5 \times 10^9 M_{\odot}$ (approximately 10^{-3} of the mass of the galaxy) and to dominate the mass within within about 5.61 arcseconds of the centre. They also model the contribution of the stellar mass to the circular velocity in this inner region, yielding a value of about 160 km/s at $4''$ from the black hole consistent with the photometry of Ferrarese et al. (2006) and a mass-to-light ratio of 4. Assuming a smaller mass-to-light ratio would yield small constraints on Δ , roughly the expected displacement for a given value of Δ is inversely proportional to the square of the mass-to-light ratio. Within 10 arcseconds the black hole contributes about thirty percent of the mass, so we will use the circular velocity due to the stellar contribution at this radius of about 210 km/s to estimate the displacement of the black hole due to SEP violation. Using a distance to M 87 of 16.4 Mpc (Bird et al. 2010) and the same mass and angular distance to the subclump centred on M 84 as earlier yields

$$x = 4.6 \frac{\Delta}{1 - \Delta} \cos^2 i \text{pc} \quad (19)$$

and

$$\Delta\theta = 0.06 \frac{\Delta}{1 - \Delta} \cos^3 i \text{arcseconds} \quad (20)$$

where we have extrapolated the theoretical stellar mass profile of Walsh et al. (2013) out to 10 arcseconds, slightly

beyond the break radius of 7.15 arcseconds determined by Ferrarese et al. (2006) where the surface brightness profile steepens.

The position of the black hole can in principle be determined to microarcsecond precision with microwave interferometry (Broderick et al. 2011). On the other hand, the centre of the potential well is best defined in the optical using the isophotes of the galaxy. Fortunately, the supermassive black hole is also apparent in the optical. The innermost isophotes will be centered on the black hole because it will dominate the potential well, so the centres of more distant isophotes provide an estimate of the potential well that constrains the black hole.

Batcheldor et al. (2010) argued from isophotal analysis of observations with the Advanced Camera for Survey on HST that the AGN may be displaced from the centre of the galaxy by about 7 pc in a direction opposite to the observed jet (about 0.1 arcseconds). However, subsequently neither Gebhardt et al. (2011) nor Walsh et al. (2013) found evidence for displacement greater than about one parsec. Batcheldor et al. (2010) outlined several astrophysical explanations for a potential displacement such as a SMBH binary, a recent merging of black holes or a one-sided jet. The interaction with the neighbouring stars or even clusters of stars is too weak to explain such a displacement.

To measure the possible deviation of the position of the black hole on the sky and the centre of the potential well of the galaxy we use images from the Wide-Field Camera 3 (WFC3) on the Hubble Space Telescope (HST) in the ultraviolet filter, F225W. We used images from GO-12989 (PI: Renzini) with total exposure time of 5599 seconds as depicted in Fig. 2. To find the centre of the potential we calculated the light centroid within three circular annuli of inner and outer radii 24 and 36 arcseconds, 12 and 24 arcseconds, and 9 and 15 arcseconds. M 87 is an E0 galaxy so its isophotes are nearly circular (Ferrarese et al. 2006). If we take the mass of the black hole to be about $4 \times 10^9 M_\odot$, it will dominate the mass within about 6.6 arcseconds from the centre. Within 12 arcseconds it contributes less than 25% of the mass, and within 24 arcseconds it contributes about 8% of the mass; therefore, these annuli lie outside the black hole's sphere of gravitational influence. The slices of the annulus along the direction of the observed jet and potential counterjet are omitted from the calculation and so exclude light from the AGN itself. This technique contrasts with that of Batcheldor et al. (2010). They explicitly masked the noticeable jet emission as well as globular clusters. Here we remove a much larger region from the analysis both in the direction of the jet and the opposite direction. This excludes both the observed jet and a wide regions around the jets. The excluded region is symmetric, so we minimize any potential bias along the jet axis. Furthermore, the measurements that we use are in F225W with a higher angular resolution camera (WFC3 vs. ACS). In this ultraviolet filter, the emission from globular clusters is negligible. The centre of the annuli are initially roughly placed within about an arcsecond of the AGN and updated to lie on the calculated centroid. This is iterated thirty times.

To determine the location of the black hole we performed a similar centroiding on circle apertures of 0.15 and 0.2 arcseconds, also initially centred on the AGN within 0.2 arcseconds. We have repeated this process several times us-

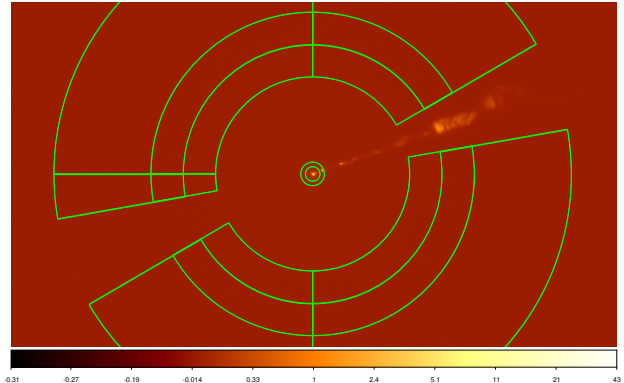


Figure 2. Central region of M 87 focussing on the AGN. Several apertures to determine the centroid of the light are depicted. The two middle annuli lie from 9 to 15 arcseconds and 12 to 24 arcseconds from the centre. The inner apertures are circular and 0.15 and 0.2 arcseconds in radius. The outermost aperture between 24 and 36 arcseconds is not depicted.

ing different starting positions, resulting in slightly different centroids all consistent within their mutual standard deviation of 0.03 arcseconds, slightly less than a pixel. This is consistent with the precision found by Walsh et al. (2013) and poorer than we found with M 32 because the surface brightness profile of M 87 is much shallower. The isophotes constrain the deviation of the black hole from the centre of the potential within 0.03 arcseconds, yielding a constraint the violation of the SEP for black holes of $-1 < \Delta < 0.3$, assuming the more conservative displacement estimate Eq. (20) and a favourable geometry of $i \approx 0^\circ$. Using additional images and an more sophisticated isophote fitting procedure could possibly yield smaller constraints.

5 CONCLUSIONS

Both the calibration and observational analysis presented here can be improved upon in several directions. From the calibration standpoint, we have consider just the force on the black hole and not the cotery of stars in its immediate gravitational influence. Moving the black hole from the centre of the stellar system will also move these stars, somewhat dilluting the Nordvedlt effect. Of course in the simulations that we did perform as the black hole oscillates about the centre of mass of the stellar system, it carries the neighbouring stars with it, but additional study is warranted where one would start the black hole in the centre of the galaxy increase the differential acceleration, say due to the effect of the small galaxy approxching the larger one, and determine where the black hole ends up.

As we have mentioned earlier, there are more sophisticated ways to determine the centre of the galaxy with isophotal fitting and by considering measurements from several observations at several wavelengths. In priciple one could find tighter constraints on the deviation of the black hole from the centre of the potential. In the case of M 32 the existing constraints are just 0.5 arcseconds because one has to link the radio coordinate system to the visual one. These could possibly be improved dramatically with further

observations. The visual core of M32 lies within 0.01 arcseconds of the centroid of the outer isophotes of the galaxy, indicating that if the black hole is coincident with the core, $|\eta_N| < 0.06$, competitive with the results from pulsar timing (Stairs et al. 2005).

Furthermore, the discovery of a supermassive black hole in one of the other elliptical satellites of M31 could provide stronger constraints from the M31 system. However, the main uncertainty in the constraints comes from the unknown geometry of the system as we saw for M32 in particular so further information on the relative distances of the various galaxies would yield better constraints on the results. Perhaps other interacting and nearly interacting galaxies in the local Universe could also provide more information and would statistically limit the violation of SEP in the motion of black holes.

The constraint on SEP from the motion of black holes probes the SEP in an essentially different limit than lunar ranging; however, the technique outlined here could become competitive with the lunar ranging results which now constrain $|\eta_N| < 4.5 \times 10^{-4}$ (Baessler et al. 1999). Improved astrometric measurements of the centre of the potential of M87 and the location of the black hole could be achieved with existing visual and UV data and the position of the black hole could be constrained further with radio interferometry provided the optical and radio astrometry could be tied together with sufficient precision. Relative astrometry with a precision $\sim 10^{-4}$ arcseconds can be achieved with Hubble photometry (e.g. Heyl et al. 2012), a factor of one hundred better than presented here, so constraints on $|\eta_N|$ of 10^{-3} could be possible with supermassive black holes. If microarcsecond astrometry can be brought to bear (e.g. Broderick et al. 2011) on M87, then constraints on η_N on the order of 10^{-4} might be possible.

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REFERENCES

- Baessler, S., Heckel, B. R., Adelberger, E. G., Gundlach, J. H., Schmidt, U., & Swanson, H. E. 1999, *Phys. Rev. Lett.*, 83, 3585
- Batcheldor, D., Robinson, A., Axon, D. J., Perlman, E. S., & Merritt, D. 2010, *Astrophys. J. Lett.*, 717, L6
- Bird, S., Harris, W. E., Blakeslee, J. P., & Flynn, C. 2010, *Astron. Astrophys.*, 524, A71
- Broderick, A. E., Loeb, A., & Reid, M. J. 2011, *Astrophys. J.*, 735, 57
- Dierickx, M., Blecha, L., & Loeb, A. 2014, *Astrophys. J. Lett.*, 788, L38
- Ferrarese, L., Côté, P., Jordán, A., Peng, E. W., Blakeslee, J. P., Piatek, S., Mei, S., Merritt, D., Milosavljević, M., Tonry, J. L., & West, M. J. 2006, *Astrophys. J. Suppl.*, 164, 334
- Gebhardt, K., Adams, J., Richstone, D., Lauer, T. R., Faber, S. M., Gültekin, K., Murphy, J., & Tremaine, S. 2011, *Astrophys. J.*, 729, 119
- Hernquist, L. 1990, *Astrophys. J.*, 356, 359
- Heyl, J. S., Richer, H., Anderson, J., Fahlman, G., Dotter, A., Hurley, J., Kalirai, J., Rich, R. M., Shara, M., Stetson, P., Woodley, K. H., & Zurek, D. 2012, *Astrophys. J.*, 761, 51 (25 pages)
- Hui, L. & Nicolis, A. 2012, *Physical Review Letters*, 109, 051304
- Koch, A. & Grebel, E. K. 2006, *Astron. J.*, 131, 1405
- Navarro, J. F., Frenk, C. S., & White, S. D. M. 1996, *Astrophys. J.*, 462, 563
- Nordtvedt, K. 1968a, *Physical Review*, 169, 1014
- . 1968b, *Physical Review*, 169, 1017
- . 1968c, *Physical Review*, 170, 1186
- Nordtvedt, Jr., K. 1982, *Reports on Progress in Physics*, 45, 631
- Ransom, S. M., Stairs, I. H., Archibald, A. M., Hessels, J. W. T., Kaplan, D. L., van Kerkwijk, M. H., Boyles, J., Deller, A. T., Chatterjee, S., Schechtman-Rook, A., Berndsen, A., Lynch, R. S., Lorimer, D. R., Karako-Argaman, C., Kaspi, V. M., Kondratiev, V. I., McLaughlin, M. A., van Leeuwen, J., Rosen, R., Roberts, M. S. E., & Stovall, K. 2014, *Nature*, 505, 520
- Sérsic, J. L. 1968, *Atlas de galaxies australes* (Cordoba, Argentina: Observatorio Astronomico)
- Shapiro, I. I., Counselman, III, C. C., & King, R. W. 1976, *Physical Review Letters*, 36, 555
- Stairs, I. H., Faulkner, A. J., Lyne, A. G., Kramer, M., Lorimer, D. R., McLaughlin, M. A., Manchester, R. N., Hobbs, G. B., Camilo, F., Possenti, A., Burgay, M., D’Amico, N., Freire, P. C., & Gregory, P. C. 2005, *Astrophys. J.*, 632, 1060
- Trujillo, I., Erwin, P., Asensio Ramos, A., & Graham, A. W. 2004, *Astron. J.*, 127, 1917
- van der Marel, R. P., de Zeeuw, P. T., Rix, H.-W., & Quinlan, G. D. 1997a, *Nature*, 385, 610
- van der Marel, R. P., Sigurdsson, S., & Hernquist, L. 1997b, *Astrophys. J.*, 487, 153
- Walsh, J. L., Barth, A. J., Ho, L. C., & Sarzi, M. 2013, *Astrophys. J.*, 770, 86
- Will, C. 2014, *Living Rev. Relativity*, 17, 4, uRL (accessed 16 May 2015): <http://www.livingreviews.org/lrr-2014-4>
- Williams, J. G., Dicke, R. H., Bender, P. L., Alley, C. O., Currie, D. G., Carter, W. E., Eckhardt, D. H., Faller, J. E., Kaula, W. M., & Mulholland, J. D. 1976, *Physical Review Letters*, 36, 551
- Wu, X. & Tremaine, S. 2006, *Astrophys. J.*, 643, 210
- Yang, Y., Li, Z., Sjouwerman, L. O., Wang, Q. D., Gu, Q., Kraft, R. P., & Yuan, F. 2015, *ArXiv e-prints*