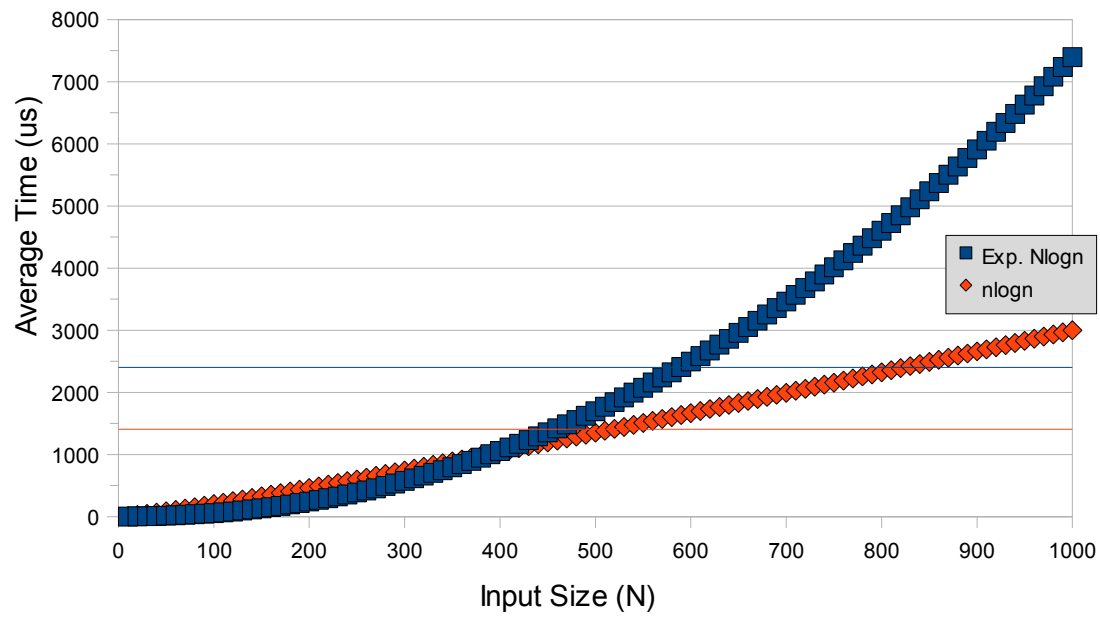
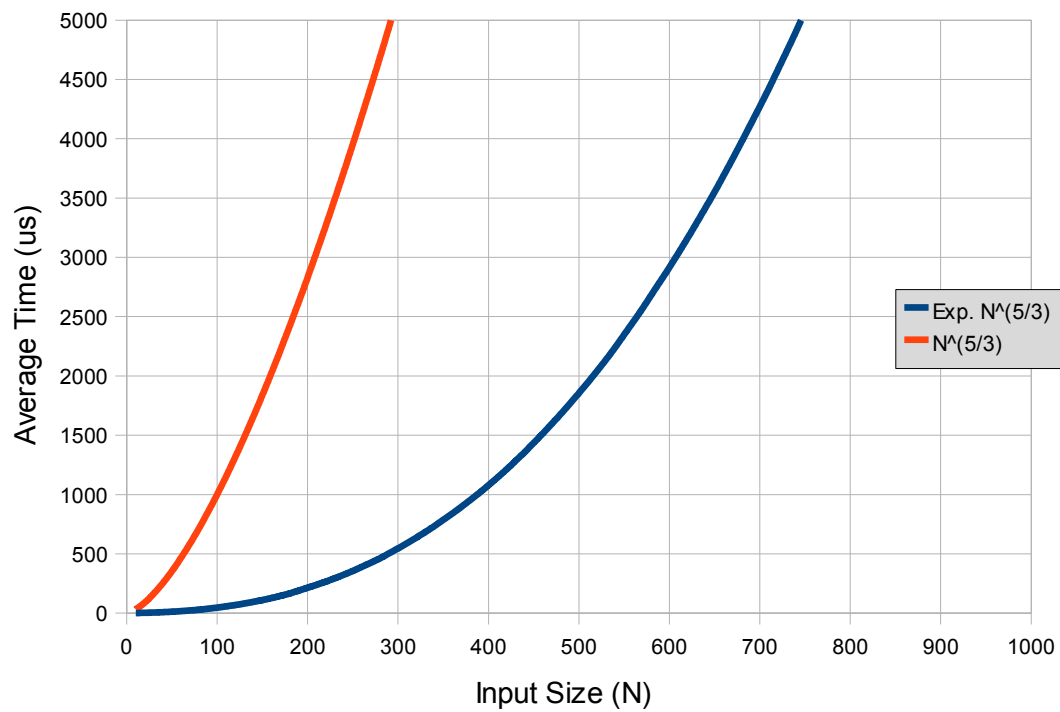


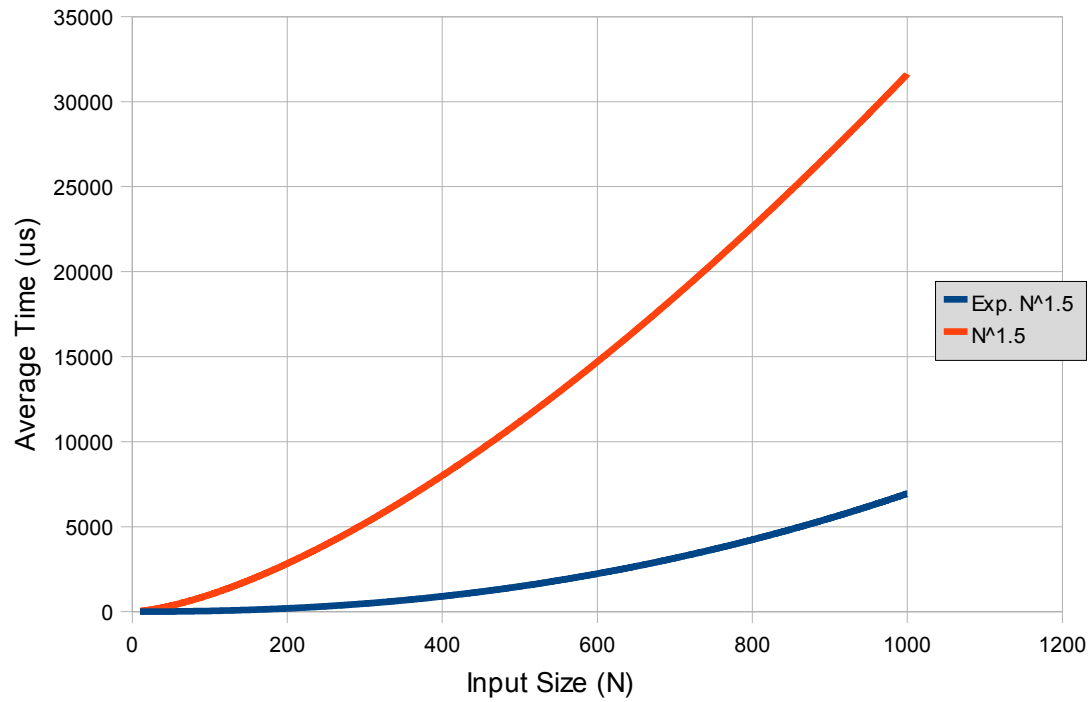
Quicksort Average Case Analysis



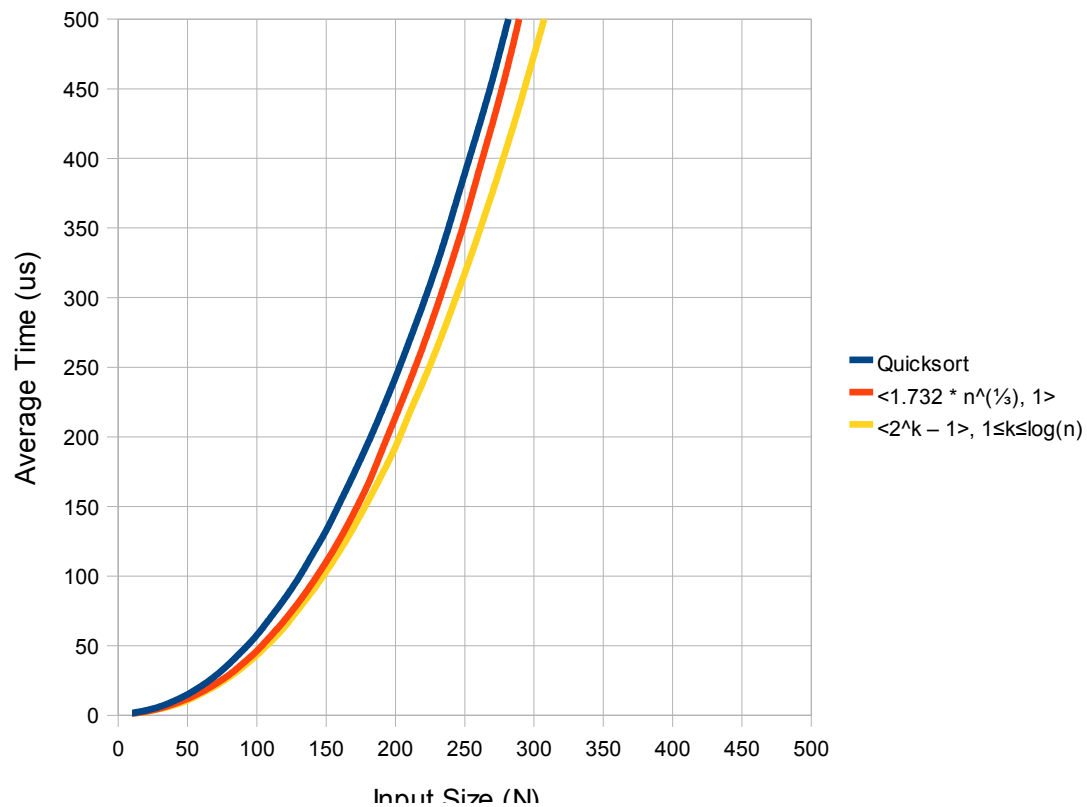
Shell Sort H-seq (1.732n^{1/3}, 1) Avg. Case Analysis



Shell Sort H-seq ($\log(n)$, $2^k - 1$) Avg. Case Analysis



Quick Sort and Shell Sort Average Case Analysis



input size (n)

Sheet2

Quick Sort Data Set, s = 500
Input Size N **Avg. Time (μs)**

| | |
|-----|---------|
| 10 | 1.65 |
| 20 | 3.48 |
| 30 | 6.25 |
| 40 | 10.16 |
| 50 | 15.01 |
| 60 | 21.09 |
| 70 | 28.24 |
| 80 | 36.98 |
| 90 | 46.82 |
| 100 | 57.6 |
| 110 | 70.37 |
| 120 | 83.73 |
| 130 | 98.46 |
| 140 | 115.28 |
| 150 | 132.75 |
| 160 | 152.8 |
| 170 | 173.51 |
| 180 | 195.09 |
| 190 | 218.17 |
| 200 | 242.63 |
| 210 | 268.53 |
| 220 | 295.53 |
| 230 | 324.04 |
| 240 | 355.91 |
| 250 | 389.15 |
| 260 | 421.97 |
| 270 | 456.55 |
| 280 | 494.23 |
| 290 | 533.1 |
| 300 | 574.24 |
| 310 | 615.74 |
| 320 | 658.32 |
| 330 | 703.39 |
| 340 | 748.72 |
| 350 | 795.22 |
| 360 | 847.84 |
| 370 | 897.72 |
| 380 | 949.13 |
| 390 | 1002.93 |
| 400 | 1057.16 |
| 410 | 1113.39 |
| 420 | 1171.98 |
| 430 | 1232.96 |
| 440 | 1293.83 |
| 450 | 1355.9 |
| 460 | 1420.06 |
| 470 | 1485.95 |

Shell Sort Data Set, s = 500

| H-seq = $1.732n^{(1/3)}$ | | H-seq = $\log n (2^k - 1)$ | |
|--|-----------------------|--|-----------------------|
| Input Size N | Avg. Time (μs) | Input Size N | Avg. Time (μs) |
| 10 | 1.25 | 10 | 1.42 |
| 20 | 2.77 | 20 | 2.68 |
| 30 | 5.04 | 30 | 4.61 |
| 40 | 8.08 | 40 | 7.27 |
| 50 | 11.94 | 50 | 10.77 |
| 60 | 16.72 | 60 | 15.64 |
| 70 | 22.45 | 70 | 21.03 |
| 80 | 29.12 | 80 | 27.4 |
| 90 | 37.26 | 90 | 34.77 |
| 100 | 46.31 | 100 | 43.25 |
| 110 | 56.95 | 110 | 52.9 |
| 120 | 68.31 | 120 | 63.52 |
| 130 | 81 | 130 | 76.23 |
| 140 | 94.91 | 140 | 89.13 |
| 150 | 110.18 | 150 | 103.21 |
| 160 | 126.73 | 160 | 118.51 |
| 170 | 145.37 | 170 | 135.25 |
| 180 | 165.56 | 180 | 153.46 |
| 190 | 189.65 | 190 | 172.49 |
| 200 | 214.17 | 200 | 192.85 |
| 210 | 238.95 | 210 | 216.66 |
| 220 | 265.37 | 220 | 239.67 |
| 230 | 293.51 | 230 | 263.93 |
| 240 | 323.65 | 240 | 290.11 |
| 250 | 355.45 | 250 | 317.44 |
| 260 | 390.26 | 260 | 345.91 |
| 270 | 425.08 | 270 | 375.8 |
| 280 | 462.07 | 280 | 406.91 |
| 290 | 503.31 | 290 | 439.62 |
| 300 | 545.88 | 300 | 474.22 |
| 310 | 588.97 | 310 | 509.8 |
| 320 | 635.14 | 320 | 547 |
| 330 | 683.13 | 330 | 585.77 |
| 340 | 732.84 | 340 | 625.59 |
| 350 | 786.04 | 350 | 669.03 |
| 360 | 839.51 | 360 | 711.73 |
| 370 | 895.67 | 370 | 756.7 |
| 380 | 954.26 | 380 | 802.41 |
| 390 | 1014.55 | 390 | 853.76 |
| 400 | 1077.76 | 400 | 902.49 |
| 410 | 1143.18 | 410 | 953.36 |
| 420 | 1212.1 | 420 | 1007.13 |
| 430 | 1283.71 | 430 | 1060.97 |
| 440 | 1357.11 | 440 | 1116.27 |
| 450 | 1435.34 | 450 | 1172.7 |
| 460 | 1513.33 | 460 | 1232.21 |

Sheet2

| | | | | | |
|-----|---------|-----|---------|-----|---------|
| 480 | 1553.5 | 470 | 1595.83 | 470 | 1292.21 |
| 490 | 1624.01 | 480 | 1679.33 | 480 | 1353.73 |
| 500 | 1695.36 | 490 | 1765.11 | 490 | 1419.57 |
| 510 | 1766.74 | 500 | 1854.7 | 500 | 1484.62 |
| 520 | 1843.94 | 510 | 1946.09 | 510 | 1551.3 |
| 530 | 1919.08 | 520 | 2040.39 | 520 | 1619.13 |
| 540 | 1996.18 | 530 | 2137.41 | 530 | 1692.62 |
| 550 | 2074.76 | 540 | 2239.86 | 540 | 1765.07 |
| 560 | 2157.07 | 550 | 2346.72 | 550 | 1840.94 |
| 570 | 2240.56 | 560 | 2455.33 | 560 | 1915.68 |
| 580 | 2323.54 | 570 | 2564.88 | 570 | 1992.29 |
| 590 | 2408.4 | 580 | 2681.7 | 580 | 2071.23 |
| 600 | 2496.03 | 590 | 2797.56 | 590 | 2151.06 |
| 610 | 2585.08 | 600 | 2914.17 | 600 | 2235.07 |
| 620 | 2675.26 | 610 | 3035.74 | 610 | 2318.41 |
| 630 | 2767.48 | 620 | 3160.62 | 620 | 2402.68 |
| 640 | 2860.41 | 630 | 3287.8 | 630 | 2488.81 |
| 650 | 2956.03 | 640 | 3418.26 | 640 | 2577.39 |
| 660 | 3053.07 | 650 | 3551.24 | 650 | 2668.91 |
| 670 | 3152.66 | 660 | 3689.5 | 660 | 2761.23 |
| 680 | 3253.08 | 670 | 3830.95 | 670 | 2854.68 |
| 690 | 3357.47 | 680 | 3975.75 | 680 | 2951.6 |
| 700 | 3461.12 | 690 | 4122.11 | 690 | 3049.72 |
| 710 | 3568.25 | 700 | 4271.19 | 700 | 3148.94 |
| 720 | 3676.75 | 710 | 4422.92 | 710 | 3249.54 |
| 730 | 3785.55 | 720 | 4580.77 | 720 | 3352.94 |
| 740 | 3898.97 | 730 | 4740.32 | 730 | 3456.93 |
| 750 | 4011.46 | 740 | 4902.81 | 740 | 3562.46 |
| 760 | 4126.21 | 750 | 5071.04 | 750 | 3669.51 |
| 770 | 4242.59 | 760 | 5240.79 | 760 | 3778.97 |
| 780 | 4359.86 | 770 | 5416.12 | 770 | 3889.78 |
| 790 | 4479.34 | 780 | 5590.6 | 780 | 4002.18 |
| 800 | 4599.9 | 790 | 5770.7 | 790 | 4116.46 |
| 810 | 4724.08 | 800 | 5955.53 | 800 | 4233.08 |
| 820 | 4850.8 | 810 | 6143.34 | 810 | 4352.02 |
| 830 | 4976.95 | 820 | 6336.54 | 820 | 4472.05 |
| 840 | 5106.17 | 830 | 6530.42 | 830 | 4593.94 |
| 850 | 5236.32 | 840 | 6729.95 | 840 | 4717.05 |
| 860 | 5367.48 | 850 | 6936.77 | 850 | 4842.02 |
| 870 | 5501.15 | 860 | 7143.19 | 860 | 4968.64 |
| 880 | 5636.7 | 870 | 7353.15 | 870 | 5097.73 |
| 890 | 5772.66 | 880 | 7569.65 | 880 | 5228.29 |
| 900 | 5909.64 | 890 | 7788.36 | 890 | 5361.16 |
| 910 | 6049.59 | 900 | 8009.4 | 900 | 5495.18 |
| 920 | 6191.73 | 910 | 8236.62 | 910 | 5631.43 |
| 930 | 6336.54 | 920 | 8467.39 | 920 | 5770.37 |
| 940 | 6482.18 | 930 | 8700.93 | 930 | 5910.73 |
| 950 | 6629.44 | 940 | 8939.76 | 940 | 6052.63 |
| 960 | 6778.28 | 950 | 9179.54 | 950 | 6196.02 |

Sheet2

| | | | | | |
|------|---------|------|----------|------|---------|
| 970 | 6927.75 | 960 | 9423.49 | 960 | 6342.63 |
| 980 | 7079.65 | 970 | 9672.48 | 970 | 6490.08 |
| 990 | 7236.53 | 980 | 9923.29 | 980 | 6640.13 |
| 1000 | 7393.28 | 990 | 10179.81 | 990 | 6792.02 |
| | | 1000 | 10440.05 | 1000 | 6945.58 |

Sheet3

Actual Quick Sort Time, $n * \log(n)$

Actual Shell Sort Time, $n^{(5/3)}$, $n^{(3/2)}$

| <u>Input Size N</u> | <u>Avg. Time</u> | <u>H-seq = $1.732n^{(1/3)}$</u> | | <u>H-seq = $\log n (2^k - 1)$</u> | |
|---------------------|------------------|--|-----------------------|--|-----------------------|
| | | <u>Input Size N</u> | <u>Avg. Time (μs)</u> | <u>Input Size N</u> | <u>Avg. Time (μs)</u> |
| 10 | 10 | 10 | 46.42 | 10 | 31.62 |
| 20 | 26.02 | 20 | 147.36 | 20 | 89.44 |
| 30 | 44.31 | 30 | 289.65 | 30 | 164.32 |
| 40 | 64.08 | 40 | 467.84 | 40 | 252.98 |
| 50 | 84.95 | 50 | 678.6 | 50 | 353.55 |
| 60 | 106.69 | 60 | 919.57 | 60 | 464.76 |
| 70 | 129.16 | 70 | 1188.95 | 70 | 585.66 |
| 80 | 152.25 | 80 | 1485.31 | 80 | 715.54 |
| 90 | 175.88 | 90 | 1807.47 | 90 | 853.81 |
| 100 | 200 | 100 | 2154.43 | 100 | 1000 |
| 110 | 224.55 | 110 | 2525.35 | 110 | 1153.69 |
| 120 | 249.5 | 120 | 2919.46 | 120 | 1314.53 |
| 130 | 274.81 | 130 | 3336.1 | 130 | 1482.23 |
| 140 | 300.46 | 140 | 3774.68 | 140 | 1656.5 |
| 150 | 326.41 | 150 | 4234.66 | 150 | 1837.12 |
| 160 | 352.66 | 160 | 4715.56 | 160 | 2023.86 |
| 170 | 379.18 | 170 | 5216.93 | 170 | 2216.53 |
| 180 | 405.95 | 180 | 5738.36 | 180 | 2414.95 |
| 190 | 432.96 | 190 | 6279.47 | 190 | 2618.97 |
| 200 | 460.21 | 200 | 6839.9 | 200 | 2828.43 |
| 210 | 487.67 | 210 | 7419.34 | 210 | 3043.19 |
| 220 | 515.33 | 220 | 8017.48 | 220 | 3263.13 |
| 230 | 543.2 | 230 | 8634.02 | 230 | 3488.12 |
| 240 | 571.25 | 240 | 9268.7 | 240 | 3718.06 |
| 250 | 599.49 | 250 | 9921.26 | 250 | 3952.85 |
| 260 | 627.89 | 260 | 10591.45 | 260 | 4192.37 |
| 270 | 656.47 | 270 | 11279.06 | 270 | 4436.55 |
| 280 | 685.2 | 280 | 11983.86 | 280 | 4685.3 |
| 290 | 714.1 | 290 | 12705.64 | 290 | 4938.52 |
| 300 | 743.14 | 300 | 13444.21 | 300 | 5196.15 |
| 310 | 772.32 | 310 | 14199.38 | 310 | 5458.11 |
| 320 | 801.65 | 320 | 14970.97 | 320 | 5724.33 |
| 330 | 831.11 | 330 | 15758.8 | 330 | 5994.75 |
| 340 | 860.7 | 340 | 16562.72 | 340 | 6269.29 |
| 350 | 890.42 | 350 | 17382.55 | 350 | 6547.9 |
| 360 | 920.27 | 360 | 18218.15 | 360 | 6830.52 |
| 370 | 950.23 | 370 | 19069.36 | 370 | 7117.09 |
| 380 | 980.32 | 380 | 19936.06 | 380 | 7407.56 |
| 390 | 1010.52 | 390 | 20818.1 | 390 | 7701.88 |
| 400 | 1040.82 | 400 | 21715.34 | 400 | 8000 |
| 410 | 1071.24 | 410 | 22627.67 | 410 | 8301.87 |
| 420 | 1101.76 | 420 | 23554.95 | 420 | 8607.44 |
| 430 | 1132.39 | 430 | 24497.07 | 430 | 8916.67 |
| 440 | 1163.12 | 440 | 25453.91 | 440 | 9229.52 |
| 450 | 1193.95 | 450 | 26425.36 | 450 | 9545.94 |
| 460 | 1224.87 | 460 | 27411.31 | 460 | 9865.9 |
| 470 | 1255.89 | | | | |

Sheet3

| | | | | | |
|-----|---------|-----|----------|-----|----------|
| 480 | 1287 | 470 | 28411.65 | 470 | 10189.36 |
| 490 | 1318.2 | 480 | 29426.28 | 480 | 10516.27 |
| 500 | 1349.49 | 490 | 30455.11 | 490 | 10846.61 |
| 510 | 1380.86 | 500 | 31498.03 | 500 | 11180.34 |
| 520 | 1412.32 | 510 | 32554.95 | 510 | 11517.42 |
| 530 | 1443.87 | 520 | 33625.77 | 520 | 11857.82 |
| 540 | 1475.49 | 530 | 34710.41 | 530 | 12201.52 |
| 550 | 1507.2 | 540 | 35808.79 | 540 | 12548.47 |
| 560 | 1538.99 | 550 | 36920.81 | 550 | 12898.64 |
| 570 | 1570.85 | 560 | 38046.38 | 560 | 13252.02 |
| 580 | 1602.79 | 570 | 39185.44 | 570 | 13608.56 |
| 590 | 1634.8 | 580 | 40337.9 | 580 | 13968.25 |
| 600 | 1666.89 | 590 | 41503.69 | 590 | 14331.05 |
| 610 | 1699.05 | 600 | 42682.72 | 600 | 14696.94 |
| 620 | 1731.28 | 610 | 43874.93 | 610 | 15065.89 |
| 630 | 1763.58 | 620 | 45080.23 | 620 | 15437.88 |
| 640 | 1795.96 | 630 | 46298.57 | 630 | 15812.87 |
| 650 | 1828.39 | 640 | 47529.87 | 640 | 16190.86 |
| 660 | 1860.9 | 650 | 48774.06 | 650 | 16571.81 |
| 670 | 1893.47 | 660 | 50031.08 | 660 | 16955.71 |
| 680 | 1926.11 | 670 | 51300.86 | 670 | 17342.52 |
| 690 | 1958.81 | 680 | 52583.34 | 680 | 17732.23 |
| 700 | 1991.57 | 690 | 53878.46 | 690 | 18124.82 |
| 710 | 2024.39 | 700 | 55186.15 | 700 | 18520.26 |
| 720 | 2057.28 | 710 | 56506.35 | 710 | 18918.54 |
| 730 | 2090.23 | 720 | 57839.01 | 720 | 19319.63 |
| 740 | 2123.23 | 730 | 59184.06 | 730 | 19723.51 |
| 750 | 2156.3 | 740 | 60541.46 | 740 | 20130.18 |
| 760 | 2189.42 | 750 | 61911.14 | 750 | 20539.6 |
| 770 | 2222.6 | 760 | 63293.04 | 760 | 20951.75 |
| 780 | 2255.83 | 770 | 64687.13 | 770 | 21366.63 |
| 790 | 2289.13 | 780 | 66093.34 | 780 | 21784.22 |
| 800 | 2322.47 | 790 | 67511.61 | 790 | 22204.48 |
| 810 | 2355.87 | 800 | 68941.91 | 800 | 22627.42 |
| 820 | 2389.33 | 810 | 70384.18 | 810 | 23053 |
| 830 | 2422.83 | 820 | 71838.36 | 820 | 23481.23 |
| 840 | 2456.39 | 830 | 73304.42 | 830 | 23912.07 |
| 850 | 2490.01 | 840 | 74782.3 | 840 | 24345.51 |
| 860 | 2523.67 | 850 | 76271.95 | 850 | 24781.55 |
| 870 | 2557.38 | 860 | 77773.34 | 860 | 25220.15 |
| 880 | 2591.14 | 870 | 79286.41 | 870 | 25661.31 |
| 890 | 2624.96 | 880 | 80811.12 | 880 | 26105.02 |
| 900 | 2658.82 | 890 | 82347.42 | 890 | 26551.25 |
| 910 | 2692.73 | 900 | 83895.28 | 900 | 27000 |
| 920 | 2726.68 | 910 | 85454.64 | 910 | 27451.25 |
| 930 | 2760.69 | 920 | 87025.47 | 920 | 27904.98 |
| 940 | 2794.74 | 930 | 88607.73 | 930 | 28361.19 |
| 950 | 2828.84 | 940 | 90201.36 | 940 | 28819.85 |
| 960 | 2862.98 | 950 | 91806.34 | 950 | 29280.97 |

Sheet3

| | | | | | |
|------|---------|------|----------|------|----------|
| 970 | 2897.17 | 960 | 93422.63 | 960 | 29744.51 |
| 980 | 2931.4 | 970 | 95050.17 | 970 | 30210.48 |
| 990 | 2965.68 | 980 | 96688.94 | 980 | 30678.85 |
| 1000 | 3000 | 990 | 98338.9 | 990 | 31149.62 |
| | | 1000 | 100000 | 1000 | 31622.78 |