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In[71]:= ClearAll[m, mm, c, cc, cs, ccs]
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In[362]:= (* nonempty sequence *)
m = mm /. Solve[mm == x + x mm, mm];
(* nonempty Catalan *)
c = cc /. Solve[cc == x + 2 cc x + x cc^2, cc][[1]];
(* nonempty Catalan starred *)
cs = ccs /. Solve[ccs == c ccs x + ccs x + c x + x, ccs][[1]];
(* Omega_infty(C) *)
oinfc = oinfcc /.
  Solve[oinfcc == (m + 1) x + 2 oinfcc (m + 1) x + oinfcc^2 (m + 1) x, oinfcc][[2]];
(* Omega_01(C) *)
o01c = o01cc /. Solve[o01cc == (m + 1) x + 2 oinfcc (m + 1) x^2 + 2 o01cc x +
  o01cc c x + oinfcc o01cc x + oinfcc^2 (m + 1) x^2, o01cc][[1]];
(* Omega_10(C) *)
o10c =
  o10cc /. Solve[o10cc == x (m + 1) x + 2 c x (m + 1) x + 2 o10cc (m + 1) x + c c x (m + 1) x +
  o10cc oinfcc (m + 1) x + c o10cc (m + 1) x, o10cc][[1]];
(* Omega_11(C) *)
o11c =
  o11cc /. Solve[o11cc == x (m + 1) x x + 2 o11cc x + 2 c x (m + 1) x x + 2 o10c (m + 1) x x +
  o10c oinfcc (m + 1) x x + c o10c (m + 1) x x + o11cc c x +
  c o11cc x + c c x (m + 1) x x + o10c o10c x, o11cc][[1]];
(* Omega_10(Cstar) *)
o10cs = o10ccs /.
  Solve[o10ccs == x (m + 1) x + o10c (m + 1) x + c x (m + 1) x + o10ccs (m + 1) x + c x (m + 1)
  x + o10c oinfcc (m + 1) x + c o10ccs (m + 1) x + c c x (m + 1) x, o10ccs][[1]];
(* Omega_11(Cstar) *)
o11cs = o11ccs /.
  Solve[o11ccs == (m + 1) x x + o11c x + c x (m + 1) x x + o10c (m + 1) x x + o11ccs x +
  o10cs (m + 1) x x + o10c oinfcc (m + 1) x x + c o10cs (m + 1) x x +
  o11ccs c x + c o11ccs x + o10c o01c x, o11ccs][[1]];
(* Omega_11(barCstar) *)
o11bcs = o11bccs /. Solve[o11bccs == o11cs x + o10cs (m + 1) x x, o11bccs][[1]];
(* final function enumerating Av(321|21) *)
f = ff /. Solve[ff == 1 + m + o11bcs / x, ff][[1]];
CoefficientList[Series[f, {x, 0, 25}], x]
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Out[373]= {1, 1, 2, 4, 17, 80, 382, 1813, 8547, 40 096, 187 498, 875 099,
  4 080 294, 19 019 584, 88 676 865, 413 701 822, 1 931 782 007, 9 030 586 650,
  42 269 741 911, 198 129 000 107, 930 046 910 109, 4 372 413 409 174,
  20 587 731 934 356, 97 089 049 201 228, 458 568 018 123 854, 2 169 209 354 365 436}
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