

Foreign exchange markets: price response and spread impact

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Abstract

We carry out a detailed large-scale data analysis of price response functions in the spot foreign exchange market for different years and different time scales. Such response functions provide quantitative information on the deviation from Markovian behavior. The price response functions show an increase to a maximum followed by a slow decrease as the time lag grows, in trade time scale and in physical time scale, for all analyzed years. Furthermore, we use a price increment point (pip) bid-ask spread definition to group different foreign exchange pairs and analyze the impact of the bid-ask spread in the price response functions. We find that large pip bid-ask spreads have a stronger impact on the response. This is similar to what has been found in stock markets.

Keywords: Econophysics, Complex systems, Statistical physics, Price response function, bid-ask spread impact, Foreign exchange market

1. Introduction

A major objective of data driven research on complex systems is the identification of generic or universal statistical behavior. The tremendous success of thermodynamics and statistical mechanics serves as an inspiration when continuing this quest in complex systems beyond traditional physics. Particularly interesting are large complex systems which consist of similar, yet clearly distinguishable complex subsystems. Financial markets, for example, have well defined subsystems as foreign exchange markets, stock markets, bond markets, among others. The degree of universality found in one particular subsystem can then be assessed if this type of universality is also seen in another subsystem. If applicable, useful information on the impact of specific system features on this universality may then be inferred.

In spite of the considerable interest, a thorough statistical analysis of the microstructure in foreign exchange markets was hampered by limited access to data. This changed, and nowadays such data analyses are possible down to the level of ticks and over long time scales.

Here, we carry out such a study for finance, because a tremendous amount of data is available in general

[1, 2, 3, 4, 5], and in particular in foreign exchange markets [6, 7, 8], to cite some examples. Markets may be viewed as macroscopic complex systems with an internal microscopic structure that is to a large extent accessible by big data analysis [9]. Stock markets and foreign exchange markets are clearly distinct, but share many common features. In previous analyses, we studied response functions in stock markets to shed light on non-Markovian behavior. Here, we extend that to the spot foreign exchange markets. To our surprise, we did not find such an investigation in the literature. Hence, we believe that this study is a rewarding effort. It helps to examine the behavior of the functions applied to the foreign exchange market and it is suitable to compare the similarities and differences to other markets.

The foreign exchange market has attracted a lot of attention in the last 20 years. Electronic trading has changed an opaque market to a fairly transparent one with transaction costs that are a fraction of their former level. The large amount of data that is now available to the public makes possible different kinds of data analysis. Intense research is currently carried out in different directions [10, 11, 12, 13, 14, 7, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24].

McGroarty et al. [25] found that smaller volumes cause larger bid-ask spreads for technical reasons related to the measurement, whereas Hau et al. [19, 26] claim that larger bid-ask spreads caused smaller volumes due to the traders' behavior.

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Burnside et al. [17] found the bid-ask spreads to be between two and four times larger for emerging market currencies than for developed country currencies. According to Huang and Masulis [20], bid-ask spreads increase when the foreign exchange market volatility increases, and decrease when the competition between the dealers increases. Ding and Hiltrop [15] showed that the Electronic Broking Services (EBS) reduces bid-ask spreads significantly, but dealers with information advantage tend to quote relatively wider bid-ask spreads. King [27] analyzed the foreign exchange futures market and observed that the number of transactions is negatively related with bid-ask spread, whereas volatility in general is positively related. Serbinenko and Rachev [12] focus on the three major market characteristics, namely efficiency, liquidity and volatility, and found that the market is efficient in a weak form. Menkhoff and Schmeling [23] used orders from the Russian interbank for Russian rouble/US dollar rate. They analyzed the price impact in different regions of Russia, and found that regions that are centers of political and financial decision making have high permanent price impact.

Price response functions are a powerful tool to obtain dynamical information because they measure price changes implied by execution of market orders. Specifically, they measure how a buy or sell order at time t influences on average the price at a later time $t + \tau$. It was shown in different works [28, 29, 30, 31, 32, 33, 34, 35, 36] that the price response functions increase to a maximum and then slowly decrease as the time lag grows.

Little is known about price response functions or related quantities in the foreign exchange markets [10, 22, 37]. Melvin and Melvin [22] simulate their proposed model for different foreign exchange markets region to analyze the impact of a one-standard-deviation shock using impulse response functions. The general pattern of response was a fairly steep drop over the first couple of days followed by a few days of gradual decline until the response is not statistically different from zero. Mancini et al. [10] model the price impact and return reversal to analyze liquidity. Their model predicts that more liquid assets should exhibit narrower bid-ask spreads and lower price impact.

To the best of our knowledge, no large-scale data analysis of response functions for the spot foreign exchange market has been carried out. Response functions are important observables as they give information on non-Markovian behavior. It is the purpose of the present study to close this gap. Based on a series of detailed empirical results obtained on trade by trade data, we show that the price response functions in the

foreign exchange markets behave qualitatively similar as the ones in correlated stocks markets. We consider different time scales, years and currency pairs to compute the price response functions. Finally, we shed light on the bid-ask spread impact in the response functions for foreign exchange pairs. We use a pip bid-ask spread definition to group different foreign exchange pairs and show that large pip bid-ask spreads have a stronger impact on the response. To facilitate the reproduction of our results, the source code for the data analysis is available in Ref. [38].

The paper is organized as follows: in Sect. 2 we introduce the foreign exchange market. In Sect. 3 we present our data set of spot foreign exchange pairs and briefly describe the physical and trade time scale. We define the time scale to be used in Sect. 4, and compute the price response functions for foreign exchange pairs in Sect. 5. In Sect. 6 we show how the bid-ask spread impact the values of the response functions. Our conclusions follow in Sect. 7.

2. Foreign exchange market overview

In Sect. 2.1 we describe the basic characteristics of the foreign exchange market. In Sect. 2.2 we establish the fundamental quantities used in the price response definitions.

2.1. Foreign exchange market

The foreign exchange market is a 24-hour global decentralized or over-the-counter (OTC) market for the trading of currencies closing only on the weekends. The foreign exchange market is the most volatile, liquid and largest of all financial markets [10, 11, 12, 13, 14, 39, 40, 41, 42], and it has a paramount importance for the world economy. It affects employment, inflation, international capital flows, among others [13]. The major participants trading in this market include governments, central banks, global funds, retail clients and corporations [41, 42]. Trading of currency in the foreign exchange market involves the purchase and sale of two currencies at the same time [40, 41, 42]. The value of one of the currencies in that pair is relative to the value of the other. The price one currency can be exchanged with another currency is the foreign exchange rate. The foreign exchange market is a closed system. As one value increases another value has to decrease. All foreign exchange rates cannot appreciate, in contrast to the stock market [40, 42].

Depending on the country, the currencies can be “free float” or “fixed float”. Free-floating currencies relative

value is determined by free-market forces. Some example of free-floating currencies include the U.S. dollar, Japanese yen and Colombian peso. On the other hand, a fixed float is where a government through the central bank set the currency's relative value to other currencies, usually by pegging it to some standard. Examples of fixed floating currencies include the Chinese Yuan and the Indian Rupee [40]. In our case, we only use free float currencies.

In the foreign exchange market, the trading day begins in Australia and Asia. Then the markets in Europe open and finally the markets in America [13, 39, 41, 42]. As the market close time in New York overlaps the market open time in Australia and Asia, the markets do not formally close during the week. Thus, using the New York time as reference, the market opens on Sunday at 19h00 and closes on Friday at 17h00. London, New York and Tokyo are the largest trading centers of foreign exchange trading [43]

Currency markets are divided into spot market, forward market, future market, currency swaps market and currency options [41, 42, 43]. The spot and forward exchange markets are OTC markets [40]. In our work we particularly focus on the spot market, where as his name suggest, the trades are settled on the spot [40, 42]. In a spot market, as the currency transactions are carried in the OTC markets, information concerning open interest and volume is unavailable. The transactions in this market represent up to the 40% of the total market transactions in the foreign exchange market. This estimations are made by the Bank for International Settlements (BIS) based on a central bank survey of foreign exchange and derivatives market activities in major financial centers [44]. The most traded currencies in the spot market are the U.S. dollar, euro, Japanese yen, British pound and Swiss franc [40].

In general, three categories of currency pairs are defined: majors, crosses, and exotics. The “major” foreign exchange currency pairs are the most frequently traded currencies that are paired with the U.S. dollar. The “crosses” are those majors pairs paired between them and that exclude the U.S. dollar. Finally, the “exotic” pairs usually consist of a major currency alongside a thinly traded currency or an emerging market economy currency. The majors are the most liquid pairs, in contrast with the exotics, who can be much more volatile. In this work, we will refer as the “major currency pairs” to the pairs of most traded currencies paired with the U.S. dollar, including the so called commodity currencies: Canadian dollar, Australian dollar and New Zealand dollar. The pairs and their corresponding symbol can be seen in Table 1.

Table 1: Analyzed currency pairs.

Currency pair	Symbol
euro/U.S dollar	EUR/USD
British pound/U.S. dollar	GBP/USD
Japanese yen/U.S. dollar	JPY/USD
Australian dollar/U.S. dollar	AUD/USD
U.S. dollar/Swiss franc	USD/CHF
U.S. dollar/Canadian dollar	USD/CAD
New Zealand dollar/U.S. dollar	NZD/USD

The term pip (Price Increment Point) is commonly used in the foreign exchange market instead of tick. The precise definition of a pip is a matter of convention. Usually, it refers to the incremental value in the fifth non-zero digit position from the left. It is not related to the position of the decimal point. For example, one pip in the exchange rate USD/JPY of 124.21 would be 0.01, while one pip for EUR/USD of 1.1021 would be 0.0001 [13, 25, 39, 42, 45].

Compared with other markets like the stock market, there are some key characteristics that differentiate the foreign spot exchange market. There are fewer rules, there are no clearing houses and central bodies that oversee the market. Some investors do not have to pay fees or commissions as on other markets. It is possible to trade at any time of day and regarding the risk and reward, **it is possible to get in, and out whenever the investor wants. In the foreign exchange market, the providers use a bid-ask spread as a measure for the transaction cost** [41].

2.2. Key concepts

In spot foreign exchange markets, orders are executed at the best available buy or sell price. Orders often fail to result in an immediate transaction, and are stored in a queue called the limit order book [13, 39, 46, 47, 48, 49]. The order book is visible for all traders and its main purpose is to ensure that all traders have the same information on what is offered on the market. For a detailed description of the operation of the markets, we suggest to see Ref. [34].

At any given time there is a best (lowest) offer to sell with price $a(t)$, and a best (highest) bid to buy with price $b(t)$ [31, 40, 49, 50, 51, 52]. The price gap between them is called the bid-ask spread $s(t) = a(t) - b(t)$ [14, 31, 32, 40, 50, 52, 53, 54, 55]. Bid-ask spreads are significantly positively related to price. Currencies with more liquidity tend to have lower bid-ask spreads [28, 50, 56, 57]. In spot foreign exchange markets, the existing bid-ask spread in any currency will vary

depending on the currency trader, the currency being traded and the conditions in the market. Although the foreign exchange market is often cited as the world's largest financial market, this description fails to consider the considerable differences in trading volume and liquidity across different currency pairs [7, 41]. These differences can be directly seen in the bid-ask spread. The bid-ask spread will tend to increase for currencies that do not generate a large volume of trading [41]. Furthermore, the bid-ask spread is directly related with the transaction costs to the dealer [14, 27, 41].

As we have the quotes prices in the data, we need to infer the trade price. We consider a basic definition of the price given by [10, 18, 21]. The average of the best ask and the best bid is the midpoint price, which is defined as [14, 31, 32, 34, 49, 52, 53, 55]

$$m(t) = \frac{a(t) + b(t)}{2}. \quad (1)$$

Price changes are typically characterized as returns. Using the midpoint price $m(t)$ of a currency pair at time t , the return $r(t, \tau)$, at time t and time lag τ is simply the relative variation of the price from t to $t + \tau$ [31, 58, 59, 60, 61, 62],

$$r(t, \tau) = \frac{m(t + \tau) - m(t)}{m(t)}. \quad (2)$$

The distribution of returns is strongly non-Gaussian and its shape continuously depends on the return period τ . Small τ values have fat tails return distributions [31]. The trade signs are defined for general cases as

$$\varepsilon(t) = \text{sign}(S(t) - m(t - \delta)), \quad (3)$$

where δ is a positive time increment and $S(t)$ is the price. Hence we have

$$\varepsilon(t) = \begin{cases} +1, & \text{If } S(t) \text{ is higher than the last } m(t) \\ -1, & \text{If } S(t) \text{ is lower than the last } m(t) \end{cases}. \quad (4)$$

Here, $\varepsilon(t) = +1$ indicates that the trade was triggered by a market order to buy and a trade triggered by a market order to sell yields $\varepsilon(t) = -1$ [31, 32, 63, 64, 65]. In our implementation of the price response functions we need a more specific definition of trade signs. We give a deeper explanation of trade signs depending on the time scale in Sect. 4.

Generally, price response functions measure price changes implied by execution of market orders. Here, different time scales have to be distinguished. The superscript scale refers to the time scale used, whether physical time scale (scale = p) or trade time scale

(scale = t). The price response functions are defined as follows

$$R_i^{(\text{scale})}(\tau) = \left\langle r_i^{(\text{scale})}(t - 1, \tau) \varepsilon_i^{(\text{scale})}(t) \right\rangle_{\text{average}}, \quad (5)$$

where the index i corresponds to currency pairs in the market, $r_i^{(\text{scale})}$ is the return of the pair i in a time lag τ in the corresponding scale and $\varepsilon_i^{(\text{scale})}$ is the trade sign of the pair i in the corresponding scale. Finally, the subscript average refers to how the price response function is averaged, either to the physical time scale (average = P) or relative to the trade time scale (average = T). The main objective of this work is to analyze the price response functions for the spot foreign exchange markets.

For correlated financial markets, the price response function increases to a maximum and then slowly decreases with increasing τ . This result is observed empirically in trade time scale and in physical time scale [34, 35].

3. Data set

The spot foreign exchange financial data was obtained from [HistData.com](https://www.histdata.com/). We use a tick-by-tick database in generic ASCII format for different years and currency pairs. This tick-by-tick data is sampled for each transaction. The data comprises the date time stamp (YYYYMMDD HHMMSSNNN), the best bid and best ask quotes prices in the Eastern Standard Time (EST) time zone. With both best bid and best ask quotes it is easy to compute the pip bid-ask spread of the data. No information about the size of each transaction is provided. Also, the identity of the participants is not given. Furthermore, trading volumes in spot foreign exchange market are not aggregated and the only volumes that are possible to find are the Broker Specific Volumes. Therefore, the data provider decided to remove the volume information from the delivered data.

Our aim is to compute the price response functions in two different time scales: trade time scale and physical time scale. We give a complete description on how the scales are defined in Sect. 4. Regarding the data for this definitions, for the trade time scale we use the data as it is, considering that it is sampled for each transaction. On the physical time scale, for each exchange rate, we process the irregularly spaced raw data to construct second-by-second price series, each containing 86,400 observations per day. For every second, the midpoint of best bid and ask quotes are used to construct one-second log-returns.

Another goal in this paper is to compare the price response functions in different calendar years to see the

differences and similarities along time. To analyze the price response functions in Sect. 5, we select the seven major currency pairs (see Table 1) in three different years: 2008, 2014 and 2019.

Additionally, we analyze the pip bid-ask spread impact in price response functions (Sect. 6). We select 46 currency pairs in three different years (2011, 2015 and 2019). The selected pairs are listed in Appendix A.

The selection of the calendar years to be analyzed was made considering the availability of the data, the completeness of the time series and the option to have a constant gap between the years.

In order to avoid overnight effects and any artifact due to the opening and closing of the foreign exchange market, we systematically discard the first ten and the last ten minutes of trading in a given week [32, 34, 36, 53, 63]. Therefore, we only consider trades of the same week from Sunday 19:10:00 to Friday 16:50:00 New York local time. We will refer to this interval of time as the “market time”.

4. Time scale

In Sect. 4.1 we describe the physical time scale and the trade time scale. In Sect. 4.2 and Sect. 4.3 we define the trade and the physical time scales, respectively.

4.1. Time definition

Due to the nature of the data, there are several options to define time for analyzing data. In general, the time series are labeled in calendar time (hours, minutes, seconds, milliseconds). Moreover, tick-by-tick data available on financial markets all over the world are time stamped up to the nanosecond [66], but the order of magnitude of the guaranteed precision is much larger, usually one second or a few hundreds of milliseconds [54, 58]. In several papers are used different time definitions (calendar time, physical time, event time, trade time, tick time) [58, 67, 68]. The spot foreign exchange market data used in the analysis only has the quotes. In consequence, we have to infer the trades during the market time. As we have tick-by-tick resolution, we can use either trade time scale or physical time scale.

The trade time scale increases by one unit each time a transaction happens, which in our case is every time the quotes change. The advantage of this count is that limit orders far away in the order book do not increase the time by one unit. The main outcome of trade time scale is its “smoothing” of data and the aggregational normality [58].

The physical time scale is increased by one unit each time a second passes. This means that computing the responses in this scale involves sampling [36, 67], which has to be done carefully when dealing for example with several stocks with different liquidity. This sampling is made in the trade signs and in the midpoint prices.

We use these two definitions of time scale to compute the price response function. Between these two scales, there is not a judgment which is better or worse. Their use directly depends on the application. Thus, our aim is to present how the price response function behaves under these two different time scales.

4.2. Trade time scale

As a first approximation, we use the trade sign classification in trade time scale proposed in Ref. [36] and used in Refs. [34, 35, 69, 70] that reads

$$\varepsilon^{(t)}(t, n) = \begin{cases} \text{sgn}(m(t, n) - m(t, n-1)), & \text{if} \\ m(t, n) \neq m(t, n-1) & \\ \varepsilon^{(t)}(t, n-1), & \text{otherwise} \end{cases} \quad (6)$$

Here, t distinguish each second where one or more trades can happen. $\varepsilon^{(t)}(t, n) = +1$ implies a trade triggered by a market order to buy, and a value $\varepsilon^{(t)}(t, n) = -1$ indicates a trade triggered by a market order to sell.

In the second case of Eq. (6), if two consecutive trades with the same trading direction do not exhaust all the available volume at the best quote, the trades would have the same price, and they will thus have the same trade sign.

With this classification we obtain trade signs for every single trade in the data set. According to Ref. [36], the average accuracy of the classification is 85% for the trade time scale.

4.3. Physical time scale

We use the trade sign definition in physical time scale proposed in Ref. [36] and used in Refs. [35, 69], that depends on the classification in Eq. (6) and reads

$$\varepsilon^{(p)}(t) = \begin{cases} \text{sgn}\left(\sum_{n=1}^{N(t)} \varepsilon^{(t)}(t, n)\right), & \text{If } N(t) > 0 \\ 0, & \text{If } N(t) = 0 \end{cases} \quad (7)$$

where $N(t)$ is the number of trades in an interval of one second following t . Here, $\varepsilon^{(p)}(t) = +1$ implies that the imbalance of trades in the second t are triggered by a market order to buy, and a value $\varepsilon^{(p)}(t) = -1$ indicates an imbalance of sell market orders. In this definition, there are two ways to obtain $\varepsilon^{(p)}(t) = 0$. First, there are no trades in a particular second and thus no trade sign. Second, the sum of the trade signs in a given second

amounts to zero, indicating an exact balance of buy and sell market orders.

Market orders show opposite trade directions as compared to limit orders executed simultaneously. An executed sell limit order corresponds to a buyer-initiated market order. An executed buy limit order corresponds to a seller-initiated market order. In this case we do not compare every single trade sign in a second, but the net trade sign obtained for every second with the definition, see Eq. (7). According to Ref. [36], this definition has an average accuracy up to 82% in the physical time scale.

5. Price response functions

In Sect. 5.1 we analyze the responses functions in trade time scale and in Sect. 5.2 we analyze the responses functions in physical time scale.

5.1. Response functions on trade time scale

The price response function in trade time scale is defined as [34]

$$R_i^{(t)}(\tau) = \left\langle r_i^{(t)}(t-1, \tau) \varepsilon_i^{(t)}(t, n) \right\rangle_T. \quad (8)$$

To compute the response functions on trade time scale, we use both, the trade signs and the returns from the tick-by-tick original data during a week in market time. Then, the response is averaged by the number of trades.

The results of Fig. 1 show the price response functions of the seven foreign exchange major pairs used in the analysis (see Table 1) for three different years. The results found for all the years are entirely in line with price responses seen in other financial markets, particularly with correlated financial markets. The response functions have an initial increasing trend to a maximum, that flattens out and saturates at some level, and eventually slowly decrease. This shape is explained by an initial increase caused by autocorrelated transaction flow. The flattening out is due to the market liquidity adapting to this flow and assuring diffusive prices [71]. For our selected pairs, a time lag of $\tau = 10^3$ trades is enough to see an increase to a maximum followed by a decrease. Thus, the trend in the price response functions is eventually reversed. The response signal is much more noisier in the year 2008 for the first seconds in the time lag. This behavior is because of the smaller amount of data of the corresponding year. In general, more data was recorded in recent years than in past years. In the three years analyzed, the more liquid currency pairs have a smaller response in comparison with the non-liquid pairs. The strength of the

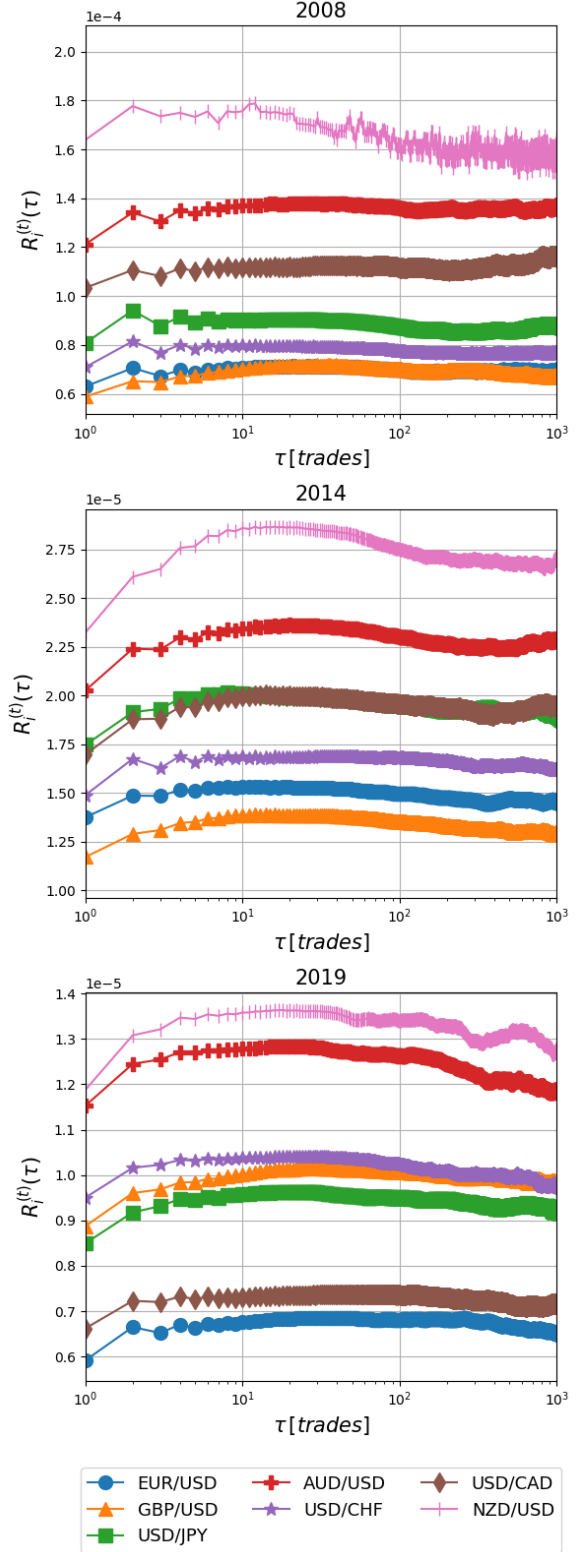


Figure 1: Price response functions $R_i^{(t)}(\tau)$ versus time lag τ on a logarithmic scale in trade time scale for the years 2008 (top), 2014 (middle) and 2019 (bottom).

response function varies from one year to the other. In 2008 the strength of the signal was one order of magnitude stronger than the response in 2014, but the signals in 2014 have approximately twice the strength of the signals of 2019. This behavior can be explained by the fact that in recent times algorithm trading has been used intensively. Thus, many more trades were carried out in the last years, which means, the impact of each trade is reduced, and then the response functions tend to decrease compared with previous years.

5.2. Response functions on physical time scale

One important detail to compute the price response function on physical time scale is to define how the averaging of the function will be made, because the response functions highly differ when we include or exclude $\varepsilon_j^{(p)}(t) = 0$ [36]. The price responses including $\varepsilon_j^{(p)}(t) = 0$ are weaker than the excluding ones due to the omission of direct influence of the lack of trades. However, either including or excluding $\varepsilon_j^{(p)}(t) = 0$ does not change the trend of price reversion versus the time lag, but it does affect the response function strength [35]. For a deeper analysis of the influence of the term $\varepsilon_j^{(p)}(t) = 0$ in price response functions, we suggest reviewing Refs. [35, 36]. We will only take into account the price response functions excluding $\varepsilon_j^{(p)}(t) = 0$.

We define the price response functions on physical time scale, using the trade signs and the returns sampled in seconds from the original data on physical time scale. The price response function on physical time scale is defined as [34]

$$R_i^{(p)}(\tau) = \left\langle r_i^{(p)}(t-1, \tau) \varepsilon_i^{(p)}(t) \right\rangle_P \quad (9)$$

The results shown in Fig. 2 are the price response functions on physical time scale for three different years. The results show approximately the same behavior observed in currency exchange pairs in trade time scale, and in correlated financial markets, where we can see that an increase to a maximum is followed by a decrease. Thus again, the trend in the price responses is eventually reversed. An exception occurs in the year 2008, where the response at short time lags seems to decrease, to then start to slightly increase, and finally it decreases again.

The price response functions on physical time scale are smoother than the responses on trade time scale. As we reduce from trade data all the returns and trade signs in one second to one data point on physical time scale, and as this sampling gives the same weight to every data point, the curves look smoother.

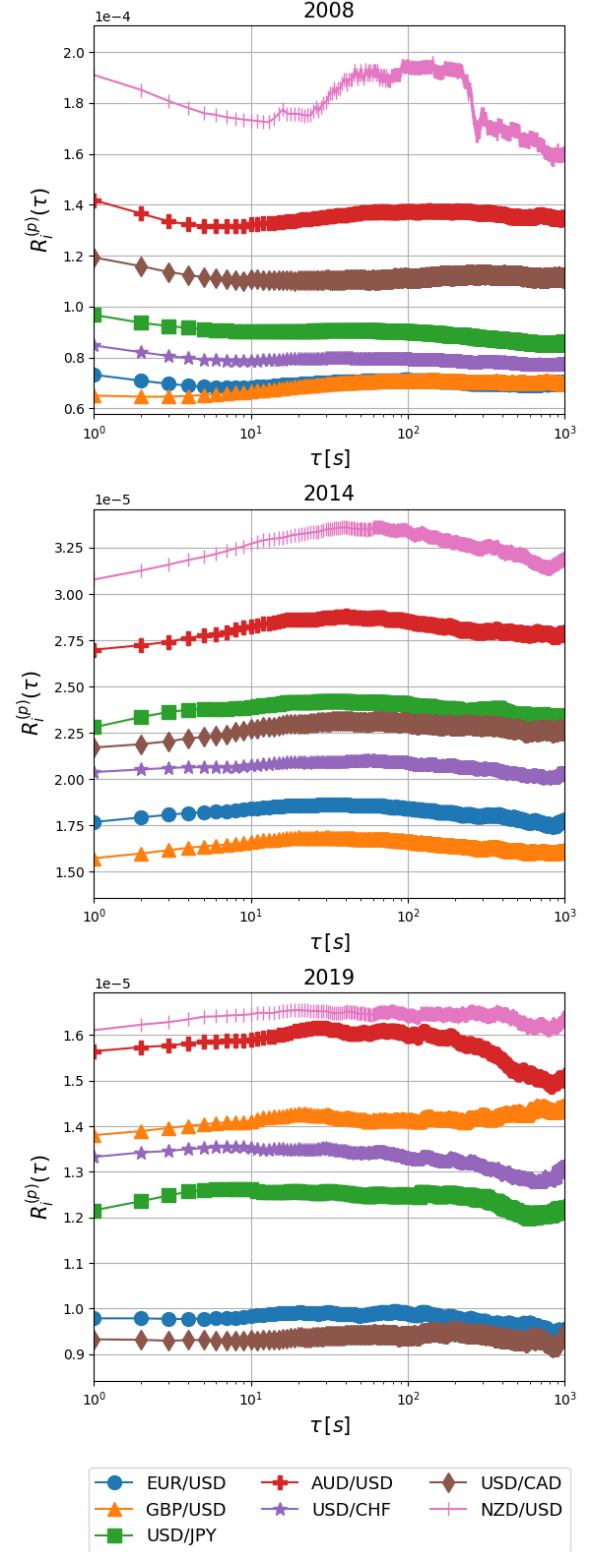


Figure 2: Price response functions $R_i^{(p)}(\tau)$ excluding $\varepsilon_i^{(p)}(t) = 0$ versus time lag τ on a logarithmic scale in physical time scale for the years 2008 (top), 2014 (middle) and 2019 (bottom).

Compared with the response functions on trade time scale, the strength of the signal of the response functions on physical time scale are similar in magnitude in the corresponding years. Thus, the strength of the signal in 2008 for trade time scale is similar to the strength of the signal in 2008 for physical time scale, and so on. This behavior is different from the one presented in correlated financial markets, where the results differ about a factor of two depending on the time scale [34].

On physical time scale, we can see that the liquid pairs have a smaller price response compared with non-liquid pairs. The liquidity of the pairs vary regarding the analyzed year. For the years 2008 and 2014, the most liquid pairs are the EUR/USD and the GBP/USD. For 2019 the most liquid pairs are EUR/USD and USD/CAD. Therefore, the price response of a foreign exchange pair with large activity is smaller to the small impact of each trade. Also, the former year responses have stronger signals. We consider the same argument of algorithm trading to explain why the signals in recent years are weaker than in older years.

6. Bid-ask spread impact in price response functions

To analyze the bid-ask spread impact in price response functions, we use 46 foreign exchange pairs from three different years (Appendix A). As we showed in Sect. 2, due to the difference in the position of the decimal points in the price between foreign exchange pairs, to compare them we need to introduce a “scaling factor” with the purpose of bringing the pip to the left of the decimal point. For example, the scaling factor for the USD/JPY is 100 and the one for the EUR/USD is 10000.

The pip bid-ask spread is defined as [25]

$$s_{\text{pip}} = (a(t) - b(t)) \cdot \text{scaling factor}. \quad (10)$$

With the s_{pip} we can group the foreign exchange pairs and check how the average strength of the price response functions on trade time scale and physical time scale behave. For each pair we compute the pip bid-ask spread of the order book at each time a trade accrued. Then we average the bid-ask spread during the trade weeks in the different years. With this value we group the foreign exchange pairs in several groups considering different interval values. We start to reduce the number of groups and find out that for the years 2011 and 2015 two clusters and for 2019 three clusters are clearly distinguishable.

Depending on the year, we identify different numbers of groups according to the pip bid-ask spread s_{pip} . For

the years 2011 and 2015, we use two intervals to select the foreign exchange pairs groups ($s_{\text{pip}} < 10$ and $10 \leq s_{\text{pip}}$). In the year 2019 we use three intervals to select the foreign exchange pairs groups ($s_{\text{pip}} < 4$, $4 \leq s_{\text{pip}} < 10$ and $10 \leq s_{\text{pip}}$). The detailed information of the foreign exchange pairs, bid-ask spread and the groups can be seen in Appendix A. With the groups of currency pairs defined, we average the price response functions of each group.

In Fig. 3 we show the average response functions for the corresponding groups in three different years. From year to year the groups can vary depending on the pip bid-ask spread. The average price response function for the pairs with smaller pip bid-ask spreads (more liquid) have on average the weakest signal in the figure for all the years and both time scales. On the other hand, the average price responses for the pairs with larger pip bid-ask spread (less liquid) have on average the strongest signal for all the years in both time scales.

From Sect. 5 we expect the increase-maximum-decrease behavior. This behavior can be seen in the figures of the year 2015 for both time scales and in the figure of the year 2019 in trade time scale. In these figures the average price response functions follow an increase, reach a maximum and then start to slowly decrease. For the other figures, on average, the response functions start to decrease from the beginning.

The response in trade time scale seems to be noisier, with large changes in the first time steps. This aggregate noise can be related with the crosses and exotics pairs, who tend to fluctuate more.

For the years 2015 and 2019 in physical time scale, the increase-maximum-decrease behavior is not that well defined as in Sect. 5 or as in the average response in trade time scale. However, some groups tend to behave in the expected way. The groups that do not follow the trend, seem to have an instantaneous high response that slowly decreases with time. This behavior is mostly noticeable in the year 2019.

For the year 2011 in physical time scale, the increase-maximum-decrease shape is not present. For both groups the response decreased almost immediately.

In the three plots, the foreign exchange market seems to have a global influence over all the pairs in the corresponding years. A similar behavior can be seen in correlated financial markets [34].

7. Conclusion

Price response functions provide quantitative information on the deviation from Markovian behavior. They

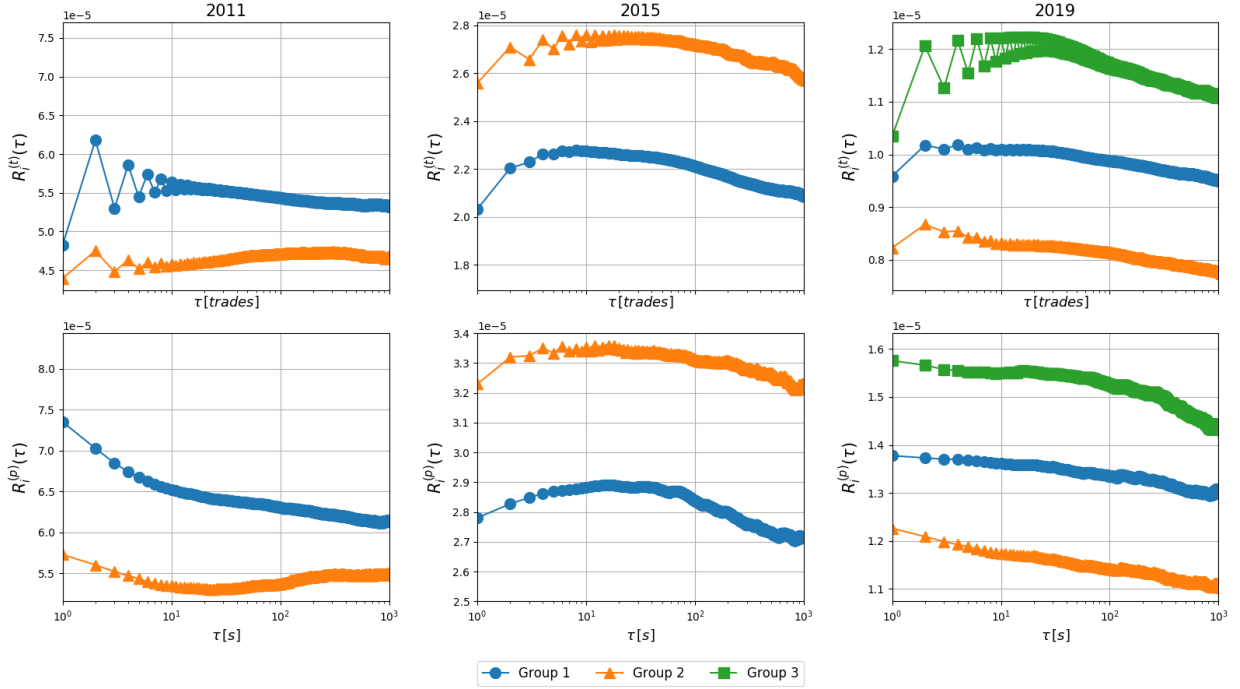


Figure 3: Average price response functions $R_i^{(l)}(\tau)$ versus time lag τ on a logarithmic scale in trade time scale (Top) and $R_{ii}^{(p)}(\tau)$ excluding $\epsilon_i^{(p)}(t) = 0$ versus time lag τ on a logarithmic scale in physical time scale (Bottom) for 46 foreign exchange pairs divided in representative groups in three different years (2011, 2015 and 2019).

measure price changes resulting from execution of market orders. We used these functions in big data analysis for spot foreign exchange markets. Such a study was, to the best of our knowledge, never done before.

We analyzed price response functions in spot foreign exchange markets for different years and different time scales. We used trade time scale and physical time scale to compute the price response functions for the seven major foreign exchange pairs for three different years. These major pairs are highly relevant in the dynamics of the market. The use of different time scales and calendar years in the work had the intention to display the different behaviors the price response function could take when the time parameters differ.

The price response functions were analyzed according to the time scales. On trade time scale, the signals were noisier. For both time scales we observe that the signal for all the pairs increases to a maximum and then starts to slowly decrease. However, for the year 2008 the shape of the signals is not as well defined as in the other years. The increase-decrease behavior observed in the spot foreign exchange market was also reported in correlated financial markets [34, 35]. These results show that the price response functions conserve their behavior

in different years and in different markets. The shape of the price response functions is qualitatively explained considering an initial increase caused by the autocorrelated transaction flow. To assure diffusive prices, price response flattens due to market liquidity adapting to the flow in the initial increase.

On both scales, the more liquid pairs have a smaller price response function compared with the non-liquid pairs. As the liquid pairs have more trades during the market time, the impact of each trade is reduced. Comparing years and scales, the price response signal is stronger in past than in recent years. As algorithmic trading has gained great relevance, the quantity of trades has grown in recent years, and in consequence, the impact in the response has decreased.

Finally, we checked the pip bid-ask spread impact in price response functions for three different years. We used 46 foreign exchange pairs and grouped them depending on the conditions of the corresponding year analyzed. We employ the year average pip bid-ask spread of every pair for each year. For all the year and time scales, the price response function signals were stronger for the groups of pairs with larger pip bid-ask spreads and weaker for the group of pairs with smaller bid-ask

spreads. For the average of the price response functions, it was only possible to see the increase-maximum-decrease behavior in the year 2015 in both scales, and in the year 2019 on trade time scale. Hence, the noise in the cross and exotic pairs due to the lack of trading compared with the majors seems stronger. A general average price response behavior for each year and time scale was spotted for the groups, suggesting a market effect on the foreign exchange pairs in each year.

Comparing the response functions in stock and spot currency exchange markets from a more general viewpoint, we find a remarkable similarity. It triggers the conclusion that the order book mechanism generates in a rather robust fashion the observed universal features in these two similar, yet different subsystems within the financial system.

8. Author contribution statement

TG proposed the research. JCHL developed the method of analysis. The idea to analyze the bid-ask spread impact was due to JCHL. JCHL carried out the analysis. Both authors contributed equally to analyzing the results and writing the paper.

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Appendix A. Foreign exchange pairs used to analyze the bid-ask spread impact

We analyze the bid-ask spread impact in the price response functions for 46 foreign exchange pairs in the foreign exchange market for the years 2011, 2015 and 2019. In Table A.2, we list the pairs in their corresponding pip bid-ask spread groups.

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Table A.2: Foreign exchange pairs used in Sect. 6.

Symbol	Pair	Category	Scaling factor	2011	2015	2019
AUD/CAD	Australian dollar/Canadian dollar	Cross	10000	G1	G1	G1
AUD/CHF	Australian dollar/Swiss franc	Cross	10000	G1	G1	G1
AUD/JPY	Australian dollar/Japanese yen	Cross	100	G1	G1	G1
AUD/NZD	Australian dollar/New Zealand dollar	Cross	10000	G2	G1	G1
AUD/USD	Australian dollar/U. S. dollar	Major	10000	G1	G1	G1
CAD/CHF	Canadian dollar/Swiss franc	Cross	10000	G1	G1	G1
CAD/JPY	Canadian dollar/Japanese yen	Cross	100	G1	G1	G1
CHF/JPY	Swiss franc/Japanese yen	Cross	100	G1	G1	G1
EUR/AUD	euro/Australian dollar	Cross	10000	G2	G1	G1
EUR/CAD	euro/Canadian dollar	Cross	10000	G1	G1	G1
EUR/CHF	euro/Swiss franc	Cross	10000	G1	G1	G1
EUR/CZK	euro/Czech koruna	Exotic	10000	G2	G2	G3
EUR/GBP	euro/British pound	Cross	10000	G1	G1	G1
EUR/HUF	euro/Hungarian forint	Exotic	100	G2	G2	G3
EUR/JPY	euro/Japanese yen	Cross	100	G1	G1	G1
EUR/NOK	euro/Norwegian krone	Exotic	10000	G2	G2	G3
EUR/NZD	euro/New Zealand dollar	Cross	10000	G2	G1	G2
EUR/PLN	euro/Polish zloty	Exotic	10000	G2	G2	G3
EUR/SEK	euro/Swedish krona	Exotic	10000	G2	G2	G3
EUR/TRY	euro/Turkish lira	Exotic	10000	G2	G2	G3
EUR/USD	euro/U. S. dollar	Major	10000	G1	G1	G1
GBP/AUD	British pound/Australian dollar	Cross	10000	G1	G1	G2
GBP/CAD	British pound/Canadian dollar	Cross	10000	G2	G1	G2
GBP/CHF	British pound/Swiss franc	Cross	10000	G2	G1	G1
GBP/JPY	British pound/Japanese yen	Cross	100	G1	G1	G1
GBP/NZD	British pound/New Zealand dollar	Cross	10000	G2	G1	G2
GBP/USD	British pound/U. S. dollar	Major	10000	G1	G1	G1
NZD/CAD	New Zealand dollar/Canadian dollar	Cross	10000	G1	G1	G2
NZD/CHF	New Zealand dollar/Swiss franc	Cross	10000	G2	G1	G1
NZD/JPY	New Zealand dollar/Japanese yen	Cross	100	G1	G1	G1
NZD/USD	New Zealand dollar/U. S. dollar	Major	10000	G1	G1	G1
SGD/JPY	Singapore dollar/Japanese yen	Exotic	100	G1	G1	G1
USD/CAD	U. S. dollar/Canadian dollar	Major	10000	G1	G1	G1
USD/CHF	U. S. dollar/Swiss franc	Major	10000	G1	G1	G1
USD/CZK	U. S. dollar/Czech koruna	Exotic	10000	G2	G2	G3
USD/DKK	U. S. dollar/Danish krone	Exotic	10000	G1	G1	G2
USD/HKD	U. S. dollar/Hong Kong dollar	Exotic	10000	G1	G1	G2
USD/HUF	U. S. dollar/Hungarian forint	Exotic	100	G2	G2	G3
USD/JPY	U. S. dollar/Japanese yen	Major	100	G1	G1	G1
USD/MXN	U. S. dollar/Mexican peso	Exotic	10000	G2	G2	G3
USD/NOK	U. S. dollar/Norwegian krone	Exotic	10000	G2	G2	G3
USD/PLN	U. S. dollar/Polish zloty	Exotic	10000	G2	G2	G3
USD/SEK	U. S. dollar/Swedish krona	Exotic	10000	G2	G2	G3
USD/SGD	U. S. dollar/Singapore dollar	Exotic	10000	G1	G1	G1
USD/TRY	U. S. dollar/Turkish lira	Exotic	10000	G2	G1	G3
USD/ZAR	U. S. dollar/South African rand	Exotic	10000	G2	G2	G3

* G1 = group 1, G2 = group 2 and G3 = group 3.

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