Implementing efficient balanced networks with mixed-signal spike-based learning circuits

Supplementary Material

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1 Derivation of the Conditions

Our learning rule aims at minimizing the fluctuations of the membrane potential in response to a spike. This intuitively captures the idea of greedily maximizing the balance between excitatory and inhibitory input currents.
To arrive at a learning rule that is easily implemented in hardware, we reformulate the update process to a fixed update dependent on whether one of two conditions is met. We now derive those conditions.
As presented in the paper, we aim at minimizing

$$L = \frac{1}{2}(V_n^{\text{before}} - V_n^{\text{rest}} + V_n^{\text{after}} - V_n^{\text{rest}})^2$$

We impose the constraint that a recurrent connection should be changed by some minimal value ω if and only if it contributes to the reduction of the loss. Therefore, on each spike that neuron n receives, two conditions are checked:

Condition I:

$$L(\Omega_{n,k} + \omega) < L(\Omega_{n,k})$$

Condition II:

$$L(\Omega_{n,k} - \omega) < L(\Omega_{n,k})$$

We can now use the identity $V_n^{\text{after}} = V_n^{\text{before}} + \Omega_{n,k}$ (see paper for more details) and plug this into the loss:

$$L = \frac{1}{2}(V_n^{\text{before}} - V_n^{\text{rest}} + V_n^{\text{before}} + \Omega_{n,k} - V_n^{\text{rest}})^2$$

and we write down the inequality for condition I, where we replaced $\Omega_{n,k}$ with $\Omega_{n,k} + \omega$ on the RHS and with $\Omega_{n,k}$ on the LHS.

$$(2V_n^{\text{before}} + \Omega_{n,k} + \omega - 2V_n^{\text{rest}})^2 < (2V_n^{\text{before}} + \Omega_{n,k} - 2V_n^{\text{rest}})^2$$

Substituting $a = 2V_n^{\text{before}} + \Omega_{n,k} - 2V_n^{\text{rest}}$ and $b = \omega$ we can rewrite this to

$$(a+b)^2 < a^2 \iff a^2 + 2ab + b^2 < a^2 \iff^1 a < -\frac{b}{2}$$

[1]Since $b > 0$

Yielding

$$2V_n^{\text{before}} + \Omega_{n,k} - 2V_n^{\text{rest}} < -\frac{\omega}{2} \iff 2V_n^{\text{before}} + \Omega_{n,k} < -\frac{\omega}{2} + 2V_n^{\text{rest}}$$

Which is the first condition. The inequality for the second condition can be derived the same way, where one simply substitutes $\Omega_{n,k} - \omega$ on the RHS rather than $\Omega_{n,k} + \omega$.