VAE

$$\log P(z|x) = \log \mathcal{N}(z; \mu, \sigma^2 I)$$

$$\Rightarrow \quad p(z) = \sum_x p(z|x)p(x) = \sum_x \mathcal{N}(0, I)p(X) = \mathcal{N}(0, I)\sum_x P(X) = \underline{\mathcal{N}(0, I)}$$

force it

$$\Rightarrow \quad L_{\mu,\sigma^2} = L_\mu + L_{\sigma^2}$$

$$= \frac{1}{2}\sum_i (\mu_i^2) + \frac{1}{2}\sum_i (\sigma_i^2 - \log \sigma_i^2 - 1)$$

$$\log P(x) = \log \frac{P(x,z)}{P(z|x)} = \log \frac{P(x,z)}{q(z,x)} - \log \frac{P(z|x)}{q(z,x)}$$

$$\Rightarrow \int q(z|x)\log P(x) = \int q(z|x)\log \frac{P(x,z)}{q(z,x)}\,dz - \int q(z|x)\log \frac{P(x,z)}{q(z,x)}\,dz$$

$$\Rightarrow \quad \log P(x) = \underline{\text{ELBO}} + KL(q(z|x)\,\|\,P(z|x))$$

maximize ELBO $\Rightarrow$ minimize KL $\Rightarrow$ as close as possible to P

$$\text{ELBO} = \int q(z|x)\log \frac{P(z,x)}{q(z|x)}\,dz = \int q(z|x)\log P(x|z)\,dz + \int q(z|x)\log \frac{P(z)}{q(z|x)}\,dz$$

$$= \underbrace{\int q(z|x)\log P(x|z)\,dz}_{1} + \underbrace{\int q(z|x)\log \frac{P(z)}{q(z|x)}\,dz}_{2}$$

1 $\Rightarrow$ maximize $\int q(z|x)\log P(x|z)\,dz$

$= \text{maximize } E_q(\log P(x|z))$

given $q(z|x)$, maximize $p(x|z)$

$\Rightarrow$ Loss function

2 $\Rightarrow$ $-KL(q(z|x)\,\|\,p(z)) = \sum(\exp(\sigma_i) - (1+\sigma_i) + m_i^2)$

Appendix B

« Auto-Encoding Variational Bayes »

Sample $z \sim \mathcal{N}(\mu, \sigma^2)$ = sample $\varepsilon \sim \mathcal{N}(0, I)$

$$z = \mu + \varepsilon \times \sigma$$