

# CuClarabel: A conic interior point solver with GPU acceleration

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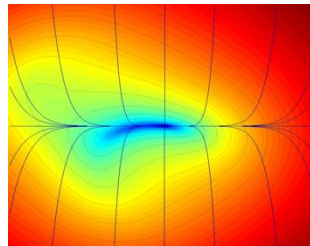
# Applications of convex optimization

## Industries

- control
- quantitative finance
- machine learning
- signal processing
- robotics
- civil
- energy
- .....



Accelerator control



Finite-element model



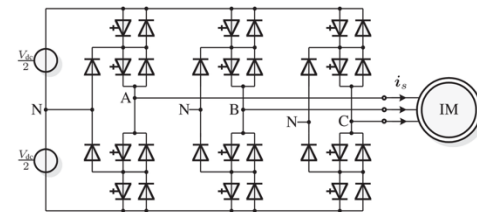
Robotics



Quantitative finance



Optimal power flow



Power electronics

# Outline

- Supported features
- Interior point method with homogeneous embedding
- GPU implementation for Clarabel

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# Problem formulation in Clarabel solver

$$\begin{aligned} &\text{minimize} && \frac{1}{2}x^\top Px + q^\top x \\ &\text{subject to} && Ax + s = b \\ &&& s \in \mathcal{K} \end{aligned}$$

$P$  is positive semidefinite

$\mathcal{K}$ : convex cone

- Zero cone
- Nonnegative cone
- Second-order cone
- Positive semidefinite cone
- Exponential cone
- Power cone

$$\mathcal{K} := \{\mathbf{0}\}^m$$

$$\mathcal{K} := \mathbb{R}_+^m$$

$$\mathcal{K}_{\text{soc}} = \{(t, x) : t \geq \|x\|, t \geq 0, x \in \mathbb{R}^{m-1}\}$$

$$\mathcal{K}_{\succeq}^n := \{\text{mat}(x) \in \mathbb{S}^n : \text{mat}(x) \succeq 0\}$$

$$\mathcal{K}_{\text{exp}} = \{(x, y, z) : y > 0, ye^{x/y} \leq z\}$$

$$\mathcal{K}_{\text{pow}(\alpha)} = \{(x, y, z) : x^\alpha y^{1-\alpha} \geq |z|, x, y \geq 0, \alpha \in (0, 1)\}$$

# Quadratic programming

Zero cone

$$\mathcal{K} := \{\mathbf{0}\}^m$$

$$\begin{array}{ll} \text{minimize} & \frac{1}{2}x^\top Px + q^\top x \\ \text{subject to} & Ax = b \end{array}$$

# Quadratic programming

Nonnegative cone

$$\mathcal{K} := \mathbb{R}_+^m$$

$$\begin{array}{ll} \text{minimize} & \frac{1}{2}x^\top Px + q^\top x \\ \text{subject to} & Ax \leq b \end{array}$$

# Conic programming

Conic form

$$\begin{array}{ll}\text{minimize} & \frac{1}{2}x^\top Px + q^\top x \\ \text{subject to} & Ax + s = b \\ & s \in \mathcal{K}\end{array}$$

- Second-order cone

$$\mathcal{K}_{\text{soc}} = \{(t, x) : t \geq \|x\|, t \geq 0, x \in \mathbb{R}^{m-1}\}$$

- Positive semidefinite cone

$$\mathcal{K}_{\succeq}^n := \{\text{mat}(x) \in \mathbb{S}^n : \text{mat}(x) \succeq 0\}$$

- Exponential cone

$$\mathcal{K}_{\text{exp}} = \{(x, y, z) : y > 0, ye^{x/y} \leq z\}$$

- Power cone

$$\mathcal{K}_{\text{pow}(\alpha)} = \{(x, y, z) : x^\alpha y^{1-\alpha} \geq |z|, x, y \geq 0, \alpha \in (0, 1)\}$$



# Second order cone programming (SOCP)

Second-order cone

$$\begin{array}{ll}\text{minimize} & \frac{1}{2}x^\top Px + q^\top x \\ \text{subject to} & Ax + s = b \\ & s \in \mathcal{K}\end{array}$$

$$\mathcal{K}_{\text{soc}} = \{(t, x) : t \geq \|x\|, t \geq 0, x \in \mathbb{R}^{m-1}\}$$

**2-norm**

# Semidefinite programming (SDP)

Positive semidefinite cone

$$\begin{array}{ll}\text{minimize} & \frac{1}{2}x^\top Px + q^\top x \\ \text{subject to} & Ax + s = b \\ & s \in \mathcal{K}\end{array}$$

$$\mathcal{K}_{\succeq}^n := \{\text{mat}(x) \in \mathbb{S}^n : \text{mat}(x) \succeq 0\}$$

$$0 \preceq X \preceq I$$

**Eigenvalue problems**

# Exponential cone programming

Exponential cone

$$\begin{array}{ll}\text{minimize} & q^\top x \\ \text{subject to} & Ax + s = b \\ & s \in \mathcal{K}\end{array}$$

$$\mathcal{K}_{\text{exp}} = \left\{ (x, y, z) : y > 0, ye^{x/y} \leq z \right\}$$

$$t \geq e^x \iff (t, 1, x) \in \mathcal{K}_{\text{exp}}$$

**Exponentials**

$$t \leq -x \log x \iff t \leq x \log(1/x) \iff (1, x, t) \in \mathcal{K}_{\text{exp}}$$

**Entropy**

# Power cone programming

Power cone

$$\begin{array}{ll}\text{minimize} & q^\top x \\ \text{subject to} & Ax + s = b \\ & s \in \mathcal{K}\end{array}$$

$$\mathcal{K}_{\text{pow}(\alpha)} = \{(x, y, z) : x^\alpha y^{1-\alpha} \geq |z|, x, y \geq 0, \alpha \in (0, 1)\}$$

$$p > 1 \quad t \geq |x|^p \iff (t, 1, x) \in \mathcal{K}_{\text{pow}}\left(\frac{1}{p}\right) \quad \text{Polynomials}$$
$$t \geq \|x\|_p \iff (r_i, t, x_i) \in \mathcal{K}_{\text{pow}}\left(\frac{1}{p}\right), \sum r_i = t \quad \text{p-norm}$$

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- Supported features
- Interior point method with homogeneous embedding
- GPU implementation for Clarabel

# Problem formulation

## Primal problem

$$\begin{array}{ll}\text{minimize} & \frac{1}{2}x^\top Px + q^\top x \\ \text{subject to} & Ax + s = b \\ & s \in \mathcal{K}\end{array}$$

## Dual problem

$$\begin{array}{ll}\text{maximize} & -\frac{1}{2}x^\top Px - b^\top z \\ \text{subject to} & Px + A^\top z + q = 0 \\ & z \in \mathcal{K}^*\end{array}$$

## Optimality: KKT condition

$$\begin{array}{ll}\text{find} & (x, s, z) \\ \text{subject to} & -Ax + b = s \\ & Px + A^\top z + q = 0 \\ & \langle s, z \rangle = 0 \\ & (s, z) \in (\mathcal{K}, \mathcal{K}^*)\end{array}$$

# Homogeneous self-dual embedding (HSDE) [1]

Primal problem

$$\begin{array}{ll}\text{minimize} & q^\top x \\ \text{subject to} & Ax + s = b \\ & s \in \mathcal{K}\end{array}$$

Dual problem

$$\begin{array}{ll}\text{maximize} & -b^\top z \\ \text{subject to} & A^\top z + q = 0 \\ & z \in \mathcal{K}^*\end{array}$$

Add scaling terms  $\tau, \kappa$

$$\begin{array}{ll}\text{find} & (x, s, z, \tau, \kappa) \\ \text{subject to} & -Ax + b\tau = s \\ & Px + A^\top z + q\tau = 0 \\ & q^\top x + b^\top z = -\kappa \\ & (s, z, \tau, \kappa) \in (\mathcal{K}, \mathcal{K}^*, \mathbb{R}_+, \mathbb{R}_+)\end{array}$$

[1] Yinyu Ye, Michael J. Todd, and Shinji Mizuno. *An  $O(\sqrt{nL})$ -iteration homogeneous and self-dual linear programming algorithm*. Mathematics of Operations Research, 19(1):53–67, 1994.

# HSDE for infeasibility detection

$$\begin{aligned} & \text{find } (x, s, z, \tau, \kappa) \\ & \text{subject to} \quad -Ax + b\tau = s \\ & \quad A^\top z + q\tau = 0 \\ & \quad q^\top x + b^\top z = -\kappa \\ & \quad (s, z, \tau, \kappa) \in (\mathcal{K}, \mathcal{K}^*, \mathbb{R}_+, \mathbb{R}_+) \end{aligned}$$

- Problem is always feasible
- The problem is homogeneous and self-dual



# HSDE for infeasibility detection

$$\begin{aligned} & \text{find } (x, s, z, \tau, \kappa) \\ & \text{subject to} \quad -Ax + b\tau = s \\ & \quad \quad \quad A^\top z + q\tau = 0 \\ & \quad \quad \quad q^\top x + b^\top z = -\kappa \\ & \quad \quad \quad (s, z, \tau, \kappa) \in (\mathcal{K}, \mathcal{K}^*, \mathbb{R}_+, \mathbb{R}_+) \end{aligned}$$

- Problem is always feasible
- The problem is homogeneous and self-dual
- $\tau^* > 0, \kappa^* = 0 \Rightarrow$  optimal solution  $(x^*/\tau^*, s^*/\tau^*, z^*/\tau^*)$
- $\tau^* = 0, \kappa^* > 0 \Rightarrow$  strongly infeasible certificate  $(x^*/\kappa^*, s^*/\kappa^*, z^*/\kappa^*)$

## Extension for quadratic cost [2]

Primal problem

$$\begin{array}{ll}\text{minimize} & \frac{1}{2}x^\top Px + q^\top x \\ \text{subject to} & Ax + s = b \\ & s \in \mathcal{K}\end{array}$$

Dual problem

$$\begin{array}{ll}\text{maximize} & -\frac{1}{2}x^\top Px - b^\top z \\ \text{subject to} & Px + A^\top z + q = 0 \\ & z \in \mathcal{K}^*\end{array}$$

Add scaling terms  $\tau, \kappa$ :

$$\begin{array}{ll}\text{find} & (x, s, z, \tau, \kappa) \\ \text{subject to} & -Ax + b\tau = s \\ & Px + A^\top z + q\tau = 0 \\ & q^\top x + b^\top z = -\kappa - \frac{1}{\tau}x^\top Px \\ & (s, z, \tau, \kappa) \in (\mathcal{K}, \mathcal{K}^*, \mathbb{R}_+, \mathbb{R}_+)\end{array}$$

[2] Erling D. Andersen and Yinyu Ye. *On a homogeneous algorithm for the monotone complementarity problem*. Mathematical Programming, 84(2):375–399, 1999.

# Homogeneous embedding for infeasibility detection

$$\begin{aligned} & \text{find } (x, s, z, \tau, \kappa) \\ & \text{subject to} \quad -Ax + b\tau = s \\ & \quad Px + A^\top z + q\tau = 0 \\ & \quad q^\top x + b^\top z = -\kappa - \frac{1}{\tau} x^\top Px \\ & \quad (s, z, \tau, \kappa) \in (\mathcal{K}, \mathcal{K}^*, \mathbb{R}_+, \mathbb{R}_+) \end{aligned}$$

- Problem is always (asymptotically) feasible
- The problem is homogeneous, **not self-dual**
- $\tau^* > 0, \kappa^* = 0 \Rightarrow$  optimal solution  $(x^*/\tau^*, s^*/\tau^*, z^*/\tau^*)$
- $\tau^* = 0, \kappa^* > 0 \Rightarrow$  strongly infeasible certificate  $(x^*/\kappa^*, s^*/\kappa^*, z^*/\kappa^*)$

# Practical benefit for quadratic costs

ECOS (quadratic cost to SOC)

$$x^\top P x = \|P^{1/2} x\|^2 \leq 2t + 1$$

$$\Leftrightarrow (t + 1, t, P^{1/2} x) \in \mathcal{K}_{\text{soc}}$$

$$\begin{bmatrix} 0 & A^T & [P^{\frac{1}{2}}]^T \\ A & -H & \\ P^{\frac{1}{2}} & & -H_{\mathcal{K}} \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta z \\ \Delta z_{\text{soc}} \end{bmatrix} = \begin{bmatrix} r_x \\ r_z \\ r_{z_{\text{soc}}} \end{bmatrix}$$

Clarabel (quadratic cost)

$$\begin{bmatrix} P & A^T \\ A & -H \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta z \end{bmatrix} = \begin{bmatrix} r_x \\ r_z \end{bmatrix}$$

- Direct LDL solve for step direction calculation (with homogeneous embedding)



# Practical benefit for quadratic costs

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$$\begin{bmatrix} 0 & A^T & [P^{\frac{1}{2}}]^T \\ A & -H & \\ P^{\frac{1}{2}} & & -H_{\mathcal{K}} \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta z \\ \Delta z_{\text{soc}} \end{bmatrix} = \begin{bmatrix} r_x \\ r_z \\ r_{z_{\text{soc}}} \end{bmatrix}$$

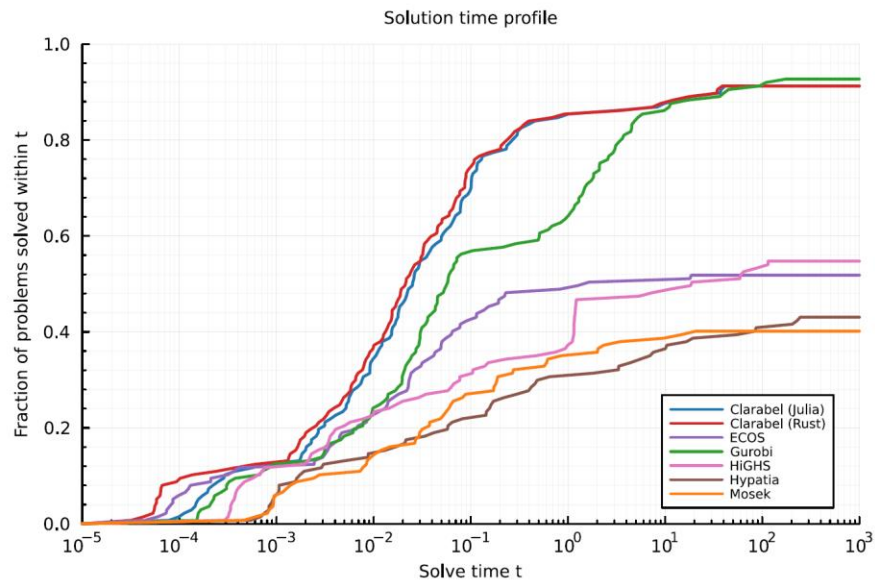
Clarabel (quadratic cost)

$$\begin{bmatrix} P & A^T \\ A & -H \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta z \end{bmatrix} = \begin{bmatrix} r_x \\ r_z \end{bmatrix}$$

- Direct LDL solve for step direction calculation (with homogeneous embedding) 
- Limited to QP, SOCP, SDP [3] 

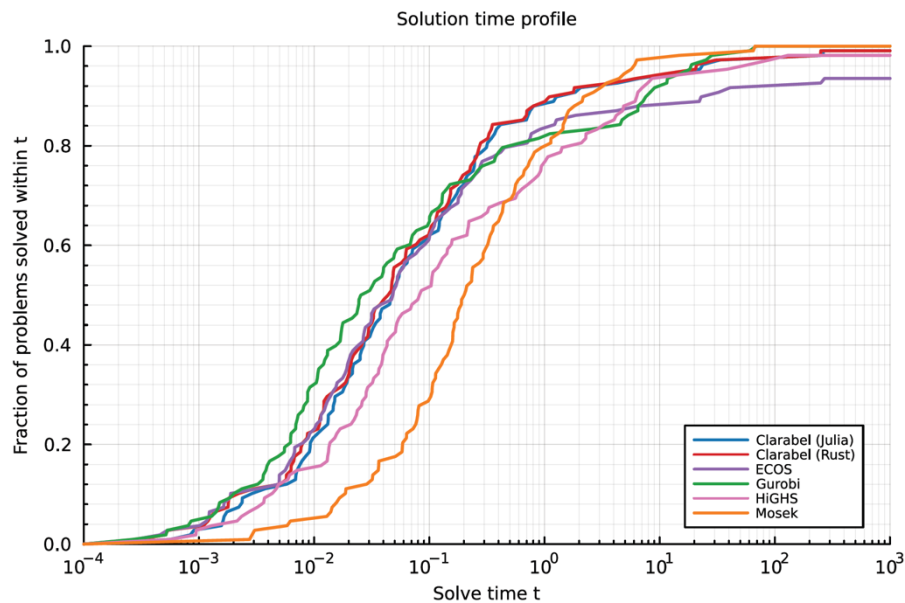
[3] Akiko Yoshise. *Interior point trajectories and a homogeneous model for nonlinear complementarity problems over symmetric cones*. SIAM Journal on Optimization, 17(4):1129–1153, 2007.

# Maros-Meszaros: (QP)



		ClarabelRs	Clarabel	ECOS	Gurobi	HiGHS	Hypatia	Mosek
Shifted GM	Full Acc.	1.0	1.02	15.49	1.64	17.92	36.61	32.67
	Low Acc.	1.0	1.1	19.8	2.9	39.99	42.99	4.07
Failure Rate (%)	Full Acc.	8.8	8.8	48.2	7.3	45.3	56.9	59.9
	Low Acc.	2.2	2.9	38.0	3.6	45.3	42.3	10.2

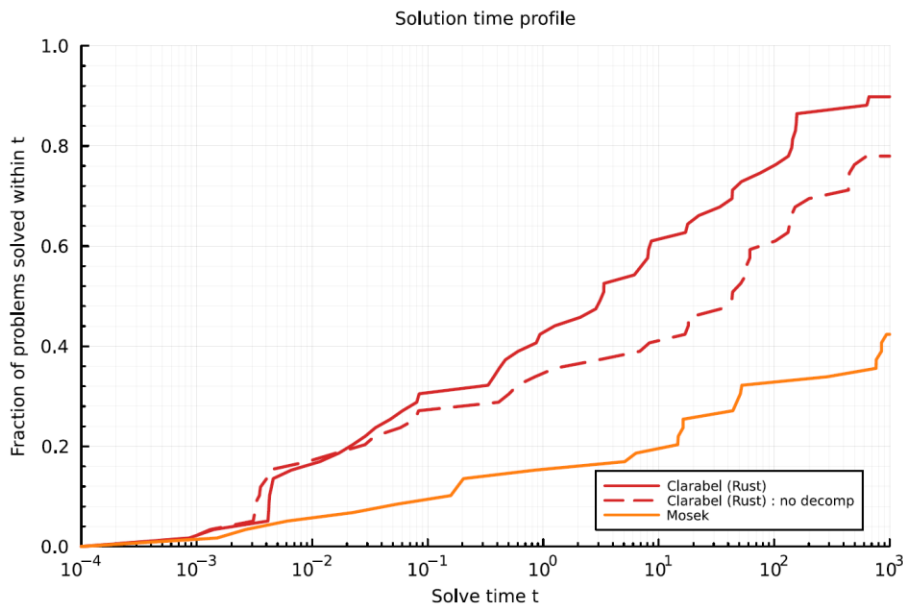
# NETLIB (LP)



		ClarabelRs	Clarabel	ECOS	Gurobi	HiGHS	Mosek
Shifted GM	Full Acc.	1.0	1.04	2.13	1.3	1.79	1.2
	Low Acc.	1.0	1.04	1.08	1.3	1.79	1.2
Failure Rate (%)	Full Acc.	0.9	0.9	6.5	0.0	1.9	0.0
	Low Acc.	0.9	0.9	0.9	0.0	1.9	0.0

# SDPLIB

- Chordal decomposition with clique merging [4]



		ClarabelRs	ClarabelRs (no decomp)	Mosek
Shifted GM	Full Acc.	1.0	2.53	9.95
	Low Acc.	1.0	2.17	1.26
Failure Rate (%)	Full Acc.	10.2	22.0	57.6
	Low Acc.	1.7	10.2	1.7

[4] Michael Garstka, Mark Cannon, and Paul Goulart. *COSMO: A conic operator splitting method for convex conic problems*. Journal of Optimization Theory and Applications, 190(3):779–810, 2021.



# Start with Clarabel

- Julia&Rust 0.11.0
- Python, C/C++, R wrappers
- Arbitrary precision (Julia)
- Default from cvxpy 1.5



The screenshot shows the Clarabel website. At the top is a cow logo and the text 'Clarabel jl/rs'. Below it is a search bar. The main navigation menu includes 'Home', 'Features', 'Credits', and 'License'. The 'Supported Languages' section is highlighted with a blue box and lists: Julia, Rust, Python, C/C++, and R. Below this is the 'Solver features' section with links to 'Callbacks', 'Chordal Decomposition', 'Problem Data Updates', and 'Linear System Solvers'. At the bottom is the 'Examples' section with a 'Version' dropdown set to 'v0.11.0'.

Home

Clarabel is an interior point numerical solver for convex optimization problems using a novel homogeneous embedding. The Clarabel package solves the following problem:

$$\begin{aligned} &\text{minimize} && \frac{1}{2}x^T Px + q^T x \\ &\text{subject to} && Ax + s = b \\ &&& s \in \mathcal{K} \end{aligned}$$

with decision variables  $x \in \mathbb{R}^n$ ,  $s \in \mathbb{R}^m$  and data matrices  $P = P^T \succeq 0$ ,  $q \in \mathbb{R}^n$ ,  $A \in \mathbb{R}^{m \times n}$ , and  $b \in \mathbb{R}^m$ . The convex set  $\mathcal{K}$  is a composition of convex cones.

Clarabel is available in either a native [Julia](#) or a native [Rust](#) implementation. Additional language interfaces (Python, C/C++ and R) are available for the Rust version.

### Features

- Versatile: Clarabel solves linear programs (LPs), quadratic programs (QPs), second-order cone programs (SOCPs) and semidefinite programs (SDPs). It also solves problems with exponential, power cone and generalized power cone constraints.
- Quadratic objectives: Unlike interior point solvers based on the standard homogeneous self-dual embedding (HSDE) model, Clarabel handles quadratic objective without requiring any epigraphical reformulation of its objective function. It can therefore be significantly faster than other HSDE-based solvers for problems with quadratic objective functions.
- Infeasibility detection: Infeasible problems are detected using using a homogeneous embedding technique.
- Arbitrary precision types: You can solve problems with any floating point precision, e.g. Float32 or Julia's BigFloat type in Julia and f32 or f64 types in Rust.
- Open Source: Our code is available on GitHub and distributed under the Apache 2.0 License. The Julia implementation is [here](#). The Rust implementation and Python interface is [here](#).

### Credits

- Industrial use:  
control, finance, energy, civil.....

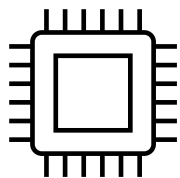
<https://oxfordcontrol.github.io/ClarabelDocs/stable/>



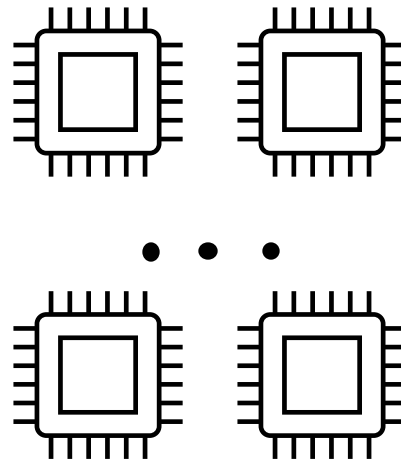
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# When the dimensionality scales up...



Serial



Parallel

# Classes of operations

## Basic parallel operations

- Add, multiplication...
- Sum, min/max...

## Cone-wise kernel operations

- Update scaling matrices
- Step size computation
- Compute r.h.s. of the KKT system
- ...

$$\mathcal{K} = \mathcal{K}_1 \times \cdots \times \mathcal{K}_p$$

## Operations related to linear systems (cuDSS [5,6])

- KKT matrix factorization
- Linear system solves

[5] <https://docs.nvidia.com/cuda/cudss/index.html>

[6] <https://github.com/exanauts/CUDSS.jl>

# Data structure

## Basic parallel operations

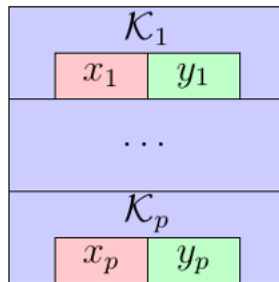
- Add, multiplication...
- Sum, min/max...

## Cone-wise kernel operations

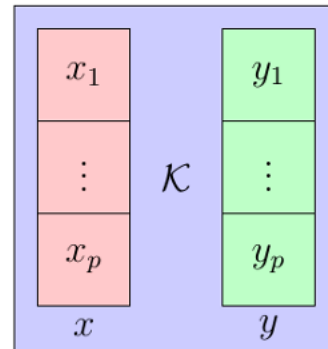
- Update Hessian
- Step size computation
- Compute r.h.s. of the KKT system
- ...

## Operations related to linear systems (cuDSS)

- KKT matrix factorization
- Linear system solves



AoS



SoA

# Stream-wise parallelism

---

**Algorithm 1** Algorithmic sketch for the scaling matrix  $H^k$  update

---

**Require:** Current iterate  $v^k$ , streams  $st_{soc}$ ,  $st_{exp}$ ,  $st_{pow}$ ,  $st_{sdp}$ .

```

1: function update_H( $v^k$ )
2:   Update  $H_{zero}^k$  // Zero cone ( $t = 1$ )
3:   Update  $H_{lin}^k$  // Nonnegative cone ( $t = 2$ )
4:
5:   // Second-order cones
6:    $H_{soc}^k = \text{kernel\_soc\_update\_H}(st_{soc})$  //  $t = 3$  to  $i$ 
7:
8:   // Exponential cones
9:    $H_{exp}^k = \text{kernel\_exp\_update\_H}(st_{exp})$  //  $t = i + 1$  to  $j$ 
10:
11:  // Power cones
12:   $H_{pow}^k = \text{kernel\_pow\_update\_H}(st_{pow})$  //  $t = j + 1$  to  $l$ 
13:
14:  // Positive semidefinite cones
15:   $H_{sdp}^k = \text{kernel\_sdp\_update\_H}(st_{sdp})$  //  $t = l + 1$  to  $p$ 
16:
17:  // Synchronize scaling update
18:  CuDeviceSynchronize()
19:
20:  return  $H^k$  // Output the scaling matrix  $H^k$ 
21: end

```

---

$$\begin{bmatrix} P & A^T \\ A & -H \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta z \end{bmatrix} = \begin{bmatrix} b_x \\ b_z \end{bmatrix}.$$

$$H = \begin{bmatrix} H_1 & & \\ & \ddots & \\ & & H_p \end{bmatrix}$$

# Dynamic parallelism for SOCP

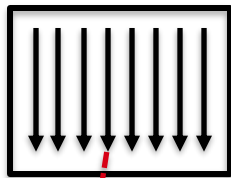
Barrier:  $F(t_i, x_i) = -\ln(t_i^2 - \|x_i\|_2^2), \quad (t_i, x_i) \in \mathbb{R}^{n+1}$

Uncertainty in dimension  $n$

# Dynamic parallelism for SOCP

Barrier:  $F(t_i, x_i) = -\ln(t_i^2 - \|x_i\|_2^2), \quad (t_i, x_i) \in \mathbb{R}^{n+1}$

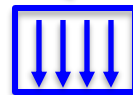
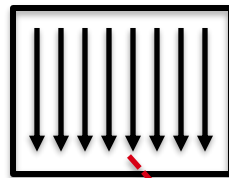
Simple parallelism (1-layer)



Thread i:



Dynamic parallelism (2-layers)



$\mathcal{K}_i$

Reduction for  $\|x_i\|^2$



# Batched SDP

## Cone related Matrix factorizations

- Cholesky
- SVD

## Supported batched operations

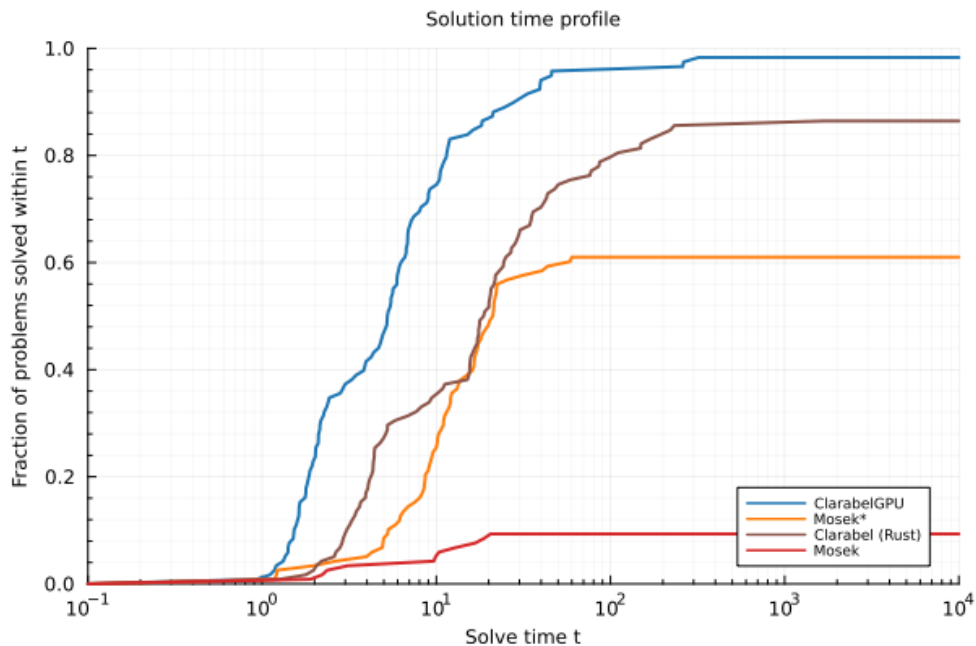
- cuSolver (limited to the same dimensionality  $\leq 32$ )

# Hardware

- CPU
  - Intel (R) Xeon (R) w9-3475X
  - 72 cores, 4.8GHz, 256GB DDR5
- GPU
  - NVIDIA GeForce RTX 4090
  - 1.29 TFLOPS (FP64), 24GB GDDR6X

# SOCP relaxation for OPF

- Pglib-opf



# Multi-stages portfolio optimization

Horizon	<u>iterations</u>			<u>total time (s)</u>		
	ClarabelGPU	Mosek*	Gurobi	ClarabelGPU	Mosek*	Gurobi
5	15	<b>12</b>	21	<b>0.803</b>	1.33	1.73
10	16	<b>9</b>	21	<b>1.78</b>	3.16	6.43
15	16	<b>11</b>	23	<b>2.86</b>	6.75	15.8
20	17	<b>12</b>	22	<b>4.49</b>	11.3	26.2
25	17	<b>9</b>	22	<b>5.73</b>	14.9	37.4
30	17	<b>9</b>	22	<b>7.18</b>	19.6	73.8

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	ClarabelGPU	Mosek*	Gurobi	ClarabelGPU	Mosek*	Gurobi
5	15	12	21	<b>0.803</b>	1.33	1.73
10	16	9	21	<b>1.78</b>	3.16	6.43
15	16	11	23	<b>2.86</b>	6.75	15.8
20	17	12	22	<b>4.49</b>	11.3	26.2
25	17	9	22	<b>5.73</b>	14.9	37.4
30	17	9	22	<b>7.18</b>	19.6	73.8

T = 30:  $\text{nnz}(P)=225000$ ,  $\text{nnz}(A) = 5398830$

Setup time: 4.70s,  $\approx 65\%$

Gain more for parametric programming!

# Multi-stages portfolio optimization (SOCP)

Horizon	<u>iterations</u>				<u>total time (s)</u>			
	ClarabelGPU	Mosek*	Clarabel	Mosek	ClarabelGPU	Mosek*	Clarabel	Mosek
5	21	15	21	14	<b>0.882</b>	1.24	12.1	1.65
10	22	<b>15</b>	22	<b>15</b>	<b>1.79</b>	3.87	53.7	4.63
15	22	14	22	<b>12</b>	<b>2.8</b>	7.49	99.5	7.42
20	22	15	22	<b>11</b>	<b>3.91</b>	12.5	167	13.2
25	24	18	24	<b>14</b>	<b>9.82</b>	20.2	245	19.8
30	23	<b>12</b>	23	13	<b>11.3</b>	22.1	312	28

# Exponential programming (entropy optimization)

Cone	<u>iterations</u>				<u>total time (s)</u>			
	ClarabelGPU	Mosek*	Clarabel	Mosek	ClarabelGPU	Mosek*	Clarabel	Mosek
2000	20	15	20	<b>10</b>	<b>0.476</b>	0.794	2.84	0.609
4000	20	16	20	<b>9</b>	<b>1.4</b>	3.02	14	2.28
6000	21	16	21	<b>9</b>	<b>3.16</b>	8.85	90.5	5.89
8000	21	16	21	<b>10</b>	<b>5.98</b>	17.7	104	12.1
10000	21	16	23	<b>10</b>	<b>19.2</b>	31.6	239	36.5

# SDP-constrained FEM

SDP cones  $\in \mathbb{S}_+^3$

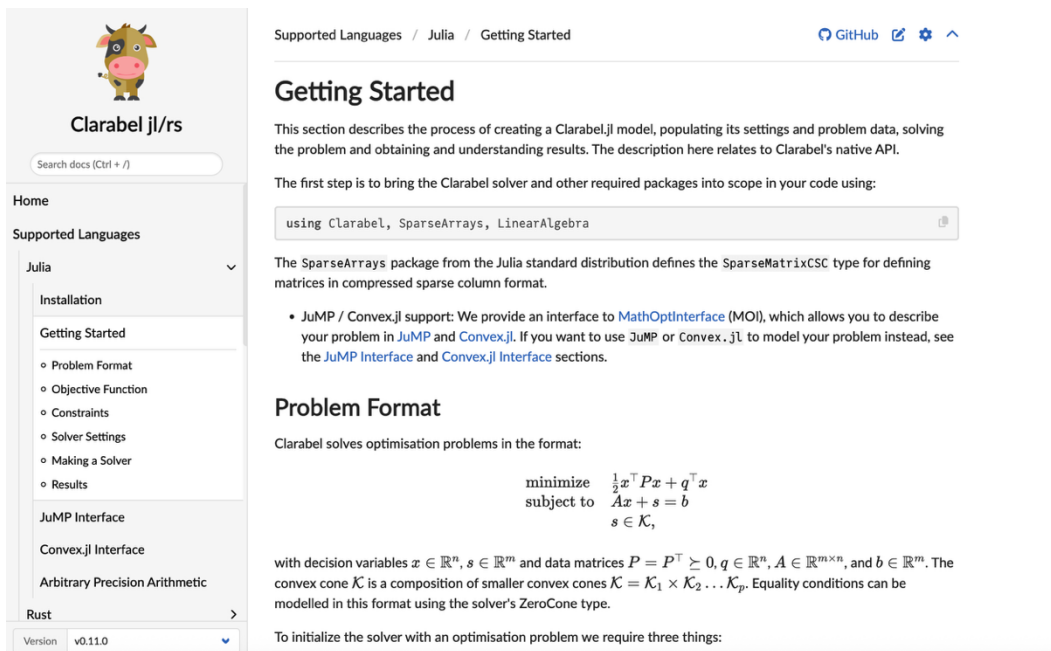
Problem	<u>iterations</u>			<u>total time (s)</u>		
	ClarabelGPU	Mosek*	Mosek	ClarabelGPU	Mosek*	Mosek
GEO3D_FELA_SDP_1048	23	24	<b>22</b>	<b>2.18</b>	6.38	2.66
GEO3D_OBEFM_SDP_1048	19	<b>13</b>	15	<b>2.47</b>	5.73	3.59
GEO3D_FELA_SDP_4444	<b>30</b>	31	38	<b>10.6</b>	38.8	18.6
GEO3D_OBEFM_SDP_4444	20	<b>13</b>	-	<b>10.8</b>	33.2	-
GEO3D_FELA_SDP_9263	-	<b>33</b>	34	-	55.6	<b>40.2</b>
GEO3D_OBEFM_SDP_9263	21	<b>14</b>	-	<b>24.4</b>	34.5	-

SDP\_9263 classes: 74008 sdp cones



# How to use it in Julia

- Julia
  - Default input
  - JuMP



Clarabel.jl/rs

Supported Languages / Julia / Getting Started

## Getting Started

This section describes the process of creating a Clarabel.jl model, populating its settings and problem data, solving the problem and obtaining and understanding results. The description here relates to Clarabel's native API.

The first step is to bring the Clarabel solver and other required packages into scope in your code using:

```
using Clarabel, SparseArrays, LinearAlgebra
```

The SparseArrays package from the Julia standard distribution defines the SparseMatrixCSC type for defining matrices in compressed sparse column format.

- JuMP / Convex.jl support: We provide an interface to MathOptInterface (MOI), which allows you to describe your problem in JuMP and Convex.jl. If you want to use JuMP or Convex.jl to model your problem instead, see the JuMP Interface and Convex.jl Interface sections.

## Problem Format

Clarabel solves optimisation problems in the format:

$$\begin{aligned} & \text{minimize} && \frac{1}{2} x^T P x + q^T x \\ & \text{subject to} && A x + s = b \\ & && s \in \mathcal{K}, \end{aligned}$$

with decision variables  $x \in \mathbb{R}^n$ ,  $s \in \mathbb{R}^m$  and data matrices  $P = P^T \succeq 0$ ,  $q \in \mathbb{R}^n$ ,  $A \in \mathbb{R}^{m \times n}$ , and  $b \in \mathbb{R}^m$ . The convex cone  $\mathcal{K}$  is a composition of smaller convex cones  $\mathcal{K} = \mathcal{K}_1 \times \mathcal{K}_2 \dots \mathcal{K}_p$ . Equality conditions can be modelled in this format using the solver's ZeroCone type.

To initialize the solver with an optimisation problem we require three things:

`set_optimizer_attribute(model, "direct_solve_method", :cudss)`

# Python use

- Python
  - juliacall

```
from juliacall import Main as jl
import numpy as np
import cupy as cp
from cupyx.scipy.sparse import csr_matrix
# Load Clarabel in Julia
jl.seval('using Clarabel, LinearAlgebra, SparseArrays')
jl.seval('using CUDA, CUDA.CUSPARSE')
```

```
# Update b vector
bpy = cp.array([2.0, 1.0, 1.0, 1.0], dtype=cp.float64)
bjl = jl.Clarabel.cupy_to_cuvector(jl.Float64, int(bpy.data.ptr), bpy.size)

# "_b" is the replacement of "!" in julia function
jl.Clarabel.update_b_b(jl.solver, bjl)          #Clarabel.update_b!()
```

# Python use

- Python
  - `juliacall`
  - `cvxpy` (CuClarabel)

## CuClarabel

CuClarabel is currently only available in the Julia version of Clarabel. To install CuClarabel, install [Julia](#), and then run in a Julia terminal `Pkg.add(Pkg.PackageSpec(url="https://github.com/oxfordcontrol/Clarabel.jl.git", rev="CuClarabel"))`.

Then install `cupy` and `juliacall` such that you can `import cupy` and `import juliacall` in Python.

# What's next

- Warm-starting
- Efficient data loading
- More powerful SDP support on GPU

Clarabel [6]



CuClarabel [7]



# Thank you

[6] Paul Goulart and Yuwen Chen. *Clarabel: An interior-point solver for conic programs with quadratic objectives*. arXiv, 2024

[7] Yuwen Chen and Danny Tse and Parth Nobel and Paul Goulart and Stephen Boyd. *CuClarabel: GPU Acceleration for a Conic Optimization Solver*. arXiv, 2024