**3D Multiscale MATLAB Program**

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This document explains the 3D Multiscale program written in MATLAB. The scope of the document is the convention and construct of the program and the inputs and outputs at various steps. This document should be read in conjunction with the depth-resolved x-ray diffraction data reduction program’s documentation and the 3D Moving Least Squares Finite Element (MLS-FE) solver’s documentation. Note that we deal only with elastic strains.

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1. **Background**

The goal of the program is to build various components necessary for the lattice strain measurement data and the MLS-FE solver to work together to solve the multiscale problem. To ensure that the solution method that spans two size scales (crystal scale and aggregate scale), conventions used in the two size scales must to consistent. It is also important to ensure that a proper set of inputs and outputs are generated in each step of the process.

1. **Convention**

*Strain* in the matrix form is the following.

Similarly, *stress* in matrix form is the following.

For this work, we use the modified Voigt-Mandel (VM) notation[[1]](#footnote-1) to vectorize the 2nd order symmetric tensors. The following is the notation.

*Anisotropic elasticity* is defined as the following.

Note that in Hosford, the same relationship is presented as the following.

The shear strain, ij, is related to the shear strain, ij, by the following.



Thus, in terms of stiffness, the components of that relates the shears and the components of that relates the shears are related by the following.

Similarly, in terms of compliance, the relationship is the following.

To use single crystal elastic coefficients from Hosford while maintaining the VM notation that is employed in this work, the coefficients that correspond to the shear terms need to be modified accordingly.

1. **Strain Projection Operator**

The projection of a second order tensor is performed as the following.

In this equation, **qq** is the strain projected along unit vector, **q**. Using vectorized strain tensor, the projection operator, {**p**}, can be defined as the following.

1. **Coordinate Transformation**

Transformation of stress (or strain) matrix is performed as the following.

In this equation, the stress tensor, [****], is transformed from a right-handed coordinate system to the stress in a prime coordinate system, [****'], by a rotation matrix, [**R**]. The rotation matrix transforms a vector, {**v**}, defined in the original right handed coordinate system to the vector in the prime coordinate system, {**v**’}.

Explicitly writing out the stress transformation operation, the following is obtained.

Using the modified stress vectorization employed in this work, the stress transformation is rewritten as the following.

Writing the above as a matrix operation, the following is obtained.

Similarly, the strain tensor can be transformed by the same operation as the strain and the stress tensors are vectorized exactly the same way.

The matrix that transforms the vectrorized stress (strain) is from here on referred to as [**T**][[2]](#footnote-2).

The 4th order tensor such as the stiffness and the compliance matrices can be transformed using [**T**] as the following.

Example:

*Need to put in an example demonstrating the transformation*

1. **Formulation of the Strain Pole Figure Inversion Problem**

Let us assume that there exists an orientation space contains all possible transformations that relate the crystal coordinate system to the sample coordinate system bounded by the crystal symmetry. Further, let us assume that the orientation space is represented using finite elements. The number of independent nodes in the FE representation of orientation is 76[[3]](#footnote-3). Furthermore, let us assume that there are enough crystals in the diffraction volume (DV) such that orientation dependent quantites can be obtained.

* 1. **Lattice strain distribution function (LSDF) and stress orientation distribution function (SODF)**

The LSDF, ****(**r**), is arranged as the following.

In this equation, {****1} is the strain vector at node 1 of the orientation space finite element mesh, {****2} is the strain vector at node 2 of the orientation space finite element mesh, and {****76} is the strain vector at node 76 of the orientation space finite element mesh. The size of ****(**r**) is (1 x (76x6)).

Correspondingly, the SODF, ****(**r**), is defined as the following.

Note that the SODF and LSDF are related by the single crystal Hooke’s law. If we are working with SODF and LSDF in sample coordinate system, the following can be written.

In this equation [**S**(**r**)] is the orientation dependent single crystal compliance. In essence, [**S**(**r**)] is simply the transformation of [**S**] in the crystal frame using [**R**(**r**)] or corresponding [**T**(**r**)].

* 1. **LSDF Projection Operator**

For a given scattering vector, **q**, the measured lattice strain is the weighted projection of strain tensors along the fiber in orientation space as defined in the projection operator for vectorized strain. If we define a projection operator [**A**] that projects the lattice strain distribution function (LSDF) or the lattice strain tensor field over orientation space on to a particular **q**, it is of the following form.

In this equation, [**AOP**] is the projection operator that takes a scalar field over orientation space and projects it along a particular **q** and {hkl}, and [**P**] is a strain projection matrix that consists of {**p**} as the following.

In this equation, fn is the value of the orientation distribution function (ODF) at a particular nodal point in orientation space. The size is [**P**] is (76 x (76 x 6)). The size of [**A**OP] is (1 x 76). Thus, [**A**] is a (1 x (76 x 6)) matrix if only 1 scattering vector and {hkl} is considered. In essence, we are projecting the strains along a particular q with weights and then taking the fiber average[[4]](#footnote-4). If we consider 1800 scattering vectors, the size of [**A**] is (1800 x (76 x 6)), the size of ****(**r**) is (1 x (76x6)) and the size of the lattice strain data, {**qq**}, or strain pole figure (SPF) data is (1800 x 1).

Note that the projection operator [**A**] contains information about the orientation distribution function (ODF) such that preferred orientation is taken in to account in the projection as the weights. In other words, the strains from points in fiber with higher values ODF are weighed more than the strain from points in fiber with lower values of ODF.

Thus, the simplest inversion formulation is the following.

Knowing [**A**] and {**qq**}, the {****(**r**)} can be found.

1. **Linking SPF to MLS-FE Program**

In the previous section, the SPF inversion method was formulated. The role of MLS-FE Program is to enforce and link the macroscopic boundary conditions and equilibrium of the body to the microscopic stresses.

Thus, the aforementioned inversion method needs to be reformulated in terms of stresses.

Using the single crystal Hooke’s law, the inversion method is reformulated.

In this equation, [**S**(**r**)] is the orientation dependent crystal compliance matrix which has the following form.

The size of [S(r)] is ((6x76) x (6 x 76)).

* 1. **Spherical harmonics representation of the SODF**

The SODF can be represented in terms of spherical harmonic functions over the orientation space and corresponding spherical harmonic weight. For a particular component of ****(**r**) denoted as ****ij(**r**) the following can be written.

In this equation, **h**m(**r**) is the mth spherical harmonic function, wm is the corresponding weight. The number of spherical harmonic functions used here is 23 (or 10 depending on the suitability). The number of spherical harmonic functions used can depend on many factors and is beyond the scope of this documentation.

The spherical harmonic functions used in this work is generated from a large mesh over orientation space (For cubic symmetry, a 20x refinement over orientation space was used. For hexagonal symmetry, a 11x refinement over orientation space was used.). These spherical harmonic functions are then projected down to the smaller refinement mesh over orientation. This process is used to ensure that the set of spherical harmonic functions used for the residual stress calculation remains consistent over the refinement of the orientation space mesh and platform.

If ****(**r**) is arranged as then ****(**r**) can be written as the following.

In this equation, [**h**(**r**)] is the matrix of spherical harmonic functions of size (76 x 23) and {**w**}s are the corresponding weights for each stress component. Note that the arrangement of stress here are different from the stress arranged in the previous section. This is bookkeeping and will change the overall arrangement of [**h**(**r**)]. It is presented here in this arrangement for clarity. We will use [**H**] for the collection of [**h**(**r**)] and {**w**} for {{**w**11} {**w**22} {**w**33} {**w**12} {**w**13} {**w**23}}T.

Substituting this SODF representation into the aforementioned equation the following is obtained.

This is for a single DV. However, it can be expanded to many DV where inversion of SPF data at many DVs is performed simultaneously. This is simply done by arranging [**A**], [**S**(**r**)], and [**H**] in a large diagonal matrices and using a large {**w**} and {**qq**} vectors. It is warned here that the operation involves large matrices and vectors and a lot of bookkeeping.

* 1. **Linking the Two Scales**

The macroscopic FE formulation enforces equilibrium and the boundary condition on the body. The information it needs is the diffraction volume stress or the sum of crystal stresses over orientation space.

In the previous equation, the SODF is represented by the following.

The SODF represented by {**w**} and [**H**] can be summed over orientation space by the following to obtain the crystal average stress at a particular diffraction volume, {****}(**x**).

Note that {****}(**x**) is a 6 x 1 vector and division by in the shear terms is to be consistent with the stress notation at the continuum level. Furthermore, {f} needs to be normalized properly such that it is a probability density function.

1. **Ordering of DV data**

The macroscopic FE mesh and the DV data must be consistently arranged. For the interference fit sample, the FE mesh is organized such that for a given angular and radial position, the elements are arranged from smallest thickness to the largest thickness. Then for the same radial position but for a different angular position the same arrangement is made. This is repeated up to maximum angular positions. Then for the next radial position, the arrangement restarts.

It is also important to note that the macroscopic code vectorizes the stress (or strain) in the following order – {11, 22, 33, 23, 13, 12}. Thus, the crystal average stress needs to be rearranged to match the macroscopic stress vector.

1. **Implementation**

The framework describe up to this point is implemented in MATLAB. It is recommended that a modern computer is used to run the MATLAB scripts.

The scripts are organized in steps. Information generated or processed at a particular step is used at a later step. Thus, the idea is to run the scripts in series. Note that the scripts are organized with the interference fit sample in mind but can be modified such that a general sample geometry can be accommodated.

1. step0\_Generate\_Input.m – generates input.mat files that define various variables that are used throughout the bi-scale optimization.
2. step1\_Generate\_BaseA.m – generates the operator that projects the SODF to a SPF with particular coverage. It is also important to note that the projection operator generated here is a common projection operator that can be used for all diffraction volumes associated with the bi-scale optimization. In the next step, appropriate rows of the projection operator generated here are removed if the SPF coverage is different for a particular diffraction volume.
3. step2\_Build\_matx\_A.m – builds diffraction volume-specific projection operators based on the projection operator built in step 1. The reason for this is because generating the projection matrix is expensive and time consuming.
4. step3\_Assemble\_matx\_A.m – concatenates the diffraction volume-specific projection operators into a large projection operator that deals with the entire set of diffraction volumes simultaneously. Furthermore, summation operator for the SODF is also built here.
5. step4\_Generate\_DVGrid.m – diffraction volume grid are specified here. Note that the ordering of diffraction volume grid needs to match the ordering used to build the projection operator and summation operator built in step 3. However, the ordering the diffraction volume grid does not have to resemble the ordering or finite element nodes for the macroscopic field. The only requirement is that the finite element nodes encompass the diffraction grid such that the macroscopic field can be computed.
6. step5\_Generate\_FEMesh.m – generates the finite element mesh necessary for macroscopic stress field calculation.
7. step6\_Generate\_matx\_K.m – generates macroscopic finite element matrices based on the finite element mesh generated in step 5 and the diffraction volume grid generated in step 4.
8. step7\_Multiscale\_Solver.m – solves the bi-scale optimization problem posed by the matrices generated in steps 3 and 7.
9. step8 – various plotting routines.
10. **The relationship between a field in orientation space and a field on a pole figure surface**
    1. **Spherical harmonic functions for cubic symmetry**

The set of spherical harmonic functions used in this work for cubic symmetry are shown in the following pages. Here, only the first 23 spherical harmonic functions are shown.

* 1. **Projection of the spherical harmonic functions on to the pole figure surface**

The projection operator that takes a scalar field over orientation space and projects it along a particular **q** and {hkl} was defined as [**AOP**]. Using appropriate [**AOP**], each spherical harmonic function is projected on the pole figure surface. The results are shown in the following pages. It is interesting to note that for each spherical harmonic function, projected pole figures are similar across the four crystallographic planes considered here. Indeed, when we take the dot product between two pole figures obtained when a particular spherical harmonic function is projected on to the pole figure surface, they are parallel. The magnitudes on the pole figure is different across the pole figures obtained when a particular spherical harmonic function is projected on to the pole figure surface.

This is the case for all spherical harmonic functions considered here. It is also important to note that depending on the experimental pole figure coverage, certain coefficients associated with the spherical harmonic functions cannot be determined[[5]](#footnote-5).

There are some spherical harmonic modes whose pole figures are not similar (spherical harmonic functions 13 for example). For the spherical harmonic functions considered here, it is probably the refinement of the orientation space mesh and the pole figure surface that cause the disparity. In fact, projections of a spherical harmonic function to pole figures should be similar up to spherical harmonic function number 80[[6]](#footnote-6).

* 1. **Projection of the synthetic lattice strain distribution functions on to the strain pole figure surface**

The projection of a spherical harmonic function onto the pole figure surface is mimicking the projection of a scalar field over orientation space onto the pole figure surface. It was found that the pole figures for several {hkl}s obtained by projecting a particular spherical harmonic functions on to the pole figure surface are similar.

This is also the case when an orientation dependent strain is considered because for each scattering vector, the normal strain field over orientation space is the projection of the orientation dependent strain tensor on to the scattering vector and the normal strain field is then projected on to the scattering vector. Figures in the following pages illustrate this idea.

These figures are generated as following. First, a synthetic state of elastic strain is generated as the following.

For each component of strain, the kth spherical harmonic function is used to generate the synthetic state of strain. This synthetic state of elastic strain is projected on to the strain pole figure surface as the following.

Because this operation is linear, it is expected that the strain pole figures for different {hkl}s will have the same patterns for a particular spherical harmonic function occupying a particular component of strain.

These projections of scalar and tensor field over orientation space imply that

1. If the pole figure coverage is insufficient, the coefficients associated with spherical harmonic functions may not be resolved.
2. If the uncertainty associated with measured strains is not small enough, the coefficients associated with spherical harmonic functions may not be resolved (or the coefficients may have some uncertainty propagated from the lattice strain uncertainty).
3. While we mainly deal with the SODFs, using the LSDF (assuming items 1 and 2 are resolved) and associated lattice strain measurements and the macroscopic stress, the anisotropic elastic moduli can be resolved.

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| **Spherical harmonic functions** | **{111}** | **{200}** | **{220}** | **{311}** |
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Projection of individual spherical harmonic functions onto [111], {200}, {220}, and {311} pole figure surfaces.

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| **Spherical harmonic functions** | **{111}** | **{200}** | **{220}** | **{311}** |
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Projection of individual spherical harmonic functions onto [111], {200}, {220}, and {311} pole figure surfaces.

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Projection of individual spherical harmonic functions onto [111], {200}, {220}, and {311} pole figure surfaces.

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Projection of individual spherical harmonic functions onto [111], {200}, {220}, and {311} pole figure surfaces.

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Projection of individual spherical harmonic functions onto [111], {200}, {220}, and {311} pole figure surfaces.

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Projection of individual spherical harmonic functions onto [111], {200}, {220}, and {311} pole figure surfaces.

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| **Spherical harmonic functions** | **{111}** | **{200}** | **{220}** | **{311}** |
| C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m1.png  **Mode 1** |  | Z:\Cu200.sig1.shm1.png | Z:\Cu220.sig1.shm1.png | Z:\Cu311.sig1.shm1.png |
| C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m2.png  **Mode 2** |  | Z:\Cu200.sig1.shm2.png | Z:\Cu220.sig1.shm2.png | Z:\Cu311.sig1.shm2.png |
| C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m3.png  **Mode 3** |  | Z:\Cu200.sig1.shm3.png | Z:\Cu220.sig1.shm3.png | Z:\Cu311.sig1.shm3.png |
| C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m4.png  **Mode 4** |  | Z:\Cu200.sig1.shm4.png | Z:\Cu220.sig1.shm4.png | Z:\Cu311.sig1.shm4.png |

Projection of synthetic orientation dependent strain onto [111], {200}, {220}, and {311} strain pole figure surfaces when a particular spherical harmonic function is occupying 11.

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| **Spherical harmonic functions** | **{111}** | **{200}** | **{220}** | **{311}** |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m5.png**  **Mode 5** |  | Z:\Cu200.sig1.shm5.png | Z:\Cu220.sig1.shm5.png | Z:\Cu311.sig1.shm5.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m6.png**  **Mode 6** |  | Z:\Cu200.sig1.shm6.png | Z:\Cu220.sig1.shm6.png | Z:\Cu311.sig1.shm6.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m7.png**  **Mode 7** |  | Z:\Cu200.sig1.shm7.png | Z:\Cu220.sig1.shm7.png | Z:\Cu311.sig1.shm7.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m8.png**  **Mode 8** |  | Z:\Cu200.sig1.shm8.png | Z:\Cu220.sig1.shm8.png | Z:\Cu311.sig1.shm8.png |

Projection of synthetic orientation dependent strain onto [111], {200}, {220}, and {311} strain pole figure surfaces when a particular spherical harmonic function is occupying 11.

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| **Spherical harmonic functions** | **{111}** | **{200}** | **{220}** | **{311}** |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m9.png**  **Mode 9** |  | Z:\Cu200.sig1.shm9.png | Z:\Cu220.sig1.shm9.png | Z:\Cu311.sig1.shm9.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m10.png**  **Mode 10** |  | Z:\Cu200.sig1.shm10.png | Z:\Cu220.sig1.shm10.png | Z:\Cu311.sig1.shm10.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m11.png**  **Mode 11** |  | Z:\Cu200.sig1.shm11.png | Z:\Cu220.sig1.shm11.png | Z:\Cu311.sig1.shm11.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m12.png**  **Mode 12** |  | Z:\Cu200.sig1.shm12.png | Z:\Cu220.sig1.shm12.png | Z:\Cu311.sig1.shm12.png |

Projection of synthetic orientation dependent strain onto [111], {200}, {220}, and {311} strain pole figure surfaces when a particular spherical harmonic function is occupying 11.

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| --- | --- | --- | --- | --- |
| **Spherical harmonic functions** | **{111}** | **{200}** | **{220}** | **{311}** |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m13.png**  **Mode 13** |  | Z:\Cu200.sig1.shm13.png | Z:\Cu220.sig1.shm13.png | Z:\Cu311.sig1.shm13.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m14.png**  **Mode 14** |  | Z:\Cu200.sig1.shm14.png | Z:\Cu220.sig1.shm14.png | Z:\Cu311.sig1.shm14.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m15.png**  **Mode 15** |  | Z:\Cu200.sig1.shm15.png | Z:\Cu220.sig1.shm15.png | Z:\Cu311.sig1.shm15.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m16.png**  **Mode 16** |  | Z:\Cu200.sig1.shm16.png | Z:\Cu220.sig1.shm16.png | Z:\Cu311.sig1.shm16.png |

Projection of synthetic orientation dependent strain onto [111], {200}, {220}, and {311} strain pole figure surfaces when a particular spherical harmonic function is occupying 11.

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| --- | --- | --- | --- | --- |
| **Spherical harmonic functions** | **{111}** | **{200}** | **{220}** | **{311}** |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m17.png**  **Mode 17** |  | Z:\Cu200.sig1.shm17.png | Z:\Cu220.sig1.shm17.png | Z:\Cu311.sig1.shm17.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m18.png**  **Mode 18** |  | Z:\Cu200.sig1.shm18.png | Z:\Cu220.sig1.shm18.png | Z:\Cu311.sig1.shm18.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m19.png**  **Mode 19** |  | Z:\Cu200.sig1.shm19.png | Z:\Cu220.sig1.shm19.png | Z:\Cu311.sig1.shm19.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m20.png**  **Mode 20** |  | Z:\Cu200.sig1.shm20.png | Z:\Cu220.sig1.shm20.png | Z:\Cu311.sig1.shm20.png |

Projection of synthetic orientation dependent strain onto [111], {200}, {220}, and {311} strain pole figure surfaces when a particular spherical harmonic function is occupying 11.

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| --- | --- | --- | --- | --- |
| **Spherical harmonic functions** | **{111}** | **{200}** | **{220}** | **{311}** |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m21.png**  **Mode 21** |  | Z:\Cu200.sig1.shm21.png | Z:\Cu220.sig1.shm21.png | Z:\Cu311.sig1.shm21.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m22.png**  **Mode 22** |  | Z:\Cu200.sig1.shm22.png | Z:\Cu220.sig1.shm22.png | Z:\Cu311.sig1.shm22.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m23.png**  **Mode 23** |  | Z:\Cu200.sig1.shm23.png | Z:\Cu220.sig1.shm23.png | Z:\Cu311.sig1.shm23.png |

Projection of synthetic orientation dependent strain onto [111], {200}, {220}, and {311} strain pole figure surfaces when a particular spherical harmonic function is occupying 11.

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| --- | --- | --- | --- | --- |
| **Spherical harmonic functions** | **{111}** | **{200}** | **{220}** | **{311}** |
| C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m1.png  **Mode 1** |  | Z:\Cu200.sig2.shm1.png | Z:\Cu220.sig2.shm1.png | Z:\Cu311.sig2.shm1.png |
| C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m2.png  **Mode 2** |  | Z:\Cu200.sig2.shm2.png | Z:\Cu220.sig2.shm2.png | Z:\Cu311.sig2.shm2.png |
| C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m3.png  **Mode 3** |  | Z:\Cu200.sig2.shm3.png | Z:\Cu220.sig2.shm3.png | Z:\Cu311.sig2.shm3.png |
| C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m4.png  **Mode 4** |  | Z:\Cu200.sig2.shm4.png | Z:\Cu220.sig2.shm4.png | Z:\Cu311.sig2.shm4.png |

Projection of synthetic orientation dependent strain onto [111], {200}, {220}, and {311} strain pole figure surfaces when a particular spherical harmonic function is occupying 22.

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| --- | --- | --- | --- | --- |
| **Spherical harmonic functions** | **{111}** | **{200}** | **{220}** | **{311}** |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m5.png**  **Mode 5** |  | Z:\Cu200.sig2.shm5.png | Z:\Cu220.sig2.shm5.png | Z:\Cu311.sig2.shm5.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m6.png**  **Mode 6** |  | Z:\Cu200.sig2.shm6.png | Z:\Cu220.sig2.shm6.png | Z:\Cu311.sig2.shm6.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m7.png**  **Mode 7** |  | Z:\Cu200.sig2.shm7.png | Z:\Cu220.sig2.shm7.png | Z:\Cu311.sig2.shm7.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m8.png**  **Mode 8** |  | Z:\Cu200.sig2.shm8.png | Z:\Cu220.sig2.shm8.png | Z:\Cu311.sig2.shm8.png |

Projection of synthetic orientation dependent strain onto [111], {200}, {220}, and {311} strain pole figure surfaces when a particular spherical harmonic function is occupying 22.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Spherical harmonic functions** | **{111}** | **{200}** | **{220}** | **{311}** |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m9.png**  **Mode 9** |  | Z:\Cu200.sig2.shm9.png | Z:\Cu220.sig2.shm9.png | Z:\Cu311.sig2.shm9.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m10.png**  **Mode 10** |  | Z:\Cu200.sig2.shm10.png | Z:\Cu220.sig2.shm10.png | Z:\Cu311.sig2.shm10.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m11.png**  **Mode 11** |  | Z:\Cu200.sig2.shm11.png | Z:\Cu220.sig2.shm11.png | Z:\Cu311.sig2.shm11.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m12.png**  **Mode 12** |  | Z:\Cu200.sig2.shm12.png | Z:\Cu220.sig2.shm12.png | Z:\Cu311.sig2.shm12.png |

Projection of synthetic orientation dependent strain onto [111], {200}, {220}, and {311} strain pole figure surfaces when a particular spherical harmonic function is occupying 22.

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| --- | --- | --- | --- | --- |
| **Spherical harmonic functions** | **{111}** | **{200}** | **{220}** | **{311}** |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m13.png**  **Mode 13** |  | Z:\Cu200.sig2.shm13.png | Z:\Cu220.sig2.shm13.png | Z:\Cu311.sig2.shm13.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m14.png**  **Mode 14** |  | Z:\Cu200.sig2.shm14.png | Z:\Cu220.sig2.shm14.png | Z:\Cu311.sig2.shm14.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m15.png**  **Mode 15** |  | Z:\Cu200.sig2.shm15.png | Z:\Cu220.sig2.shm15.png | Z:\Cu311.sig2.shm15.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m16.png**  **Mode 16** |  | Z:\Cu200.sig2.shm16.png | Z:\Cu220.sig2.shm16.png | Z:\Cu311.sig2.shm16.png |

Projection of synthetic orientation dependent strain onto [111], {200}, {220}, and {311} strain pole figure surfaces when a particular spherical harmonic function is occupying 22.

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| --- | --- | --- | --- | --- |
| **Spherical harmonic functions** | **{111}** | **{200}** | **{220}** | **{311}** |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m17.png**  **Mode 17** |  | Z:\Cu200.sig2.shm17.png | Z:\Cu220.sig2.shm17.png | Z:\Cu311.sig2.shm17.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m18.png**  **Mode 18** |  | Z:\Cu200.sig2.shm18.png | Z:\Cu220.sig2.shm18.png | Z:\Cu311.sig2.shm18.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m19.png**  **Mode 19** |  | Z:\Cu200.sig2.shm19.png | Z:\Cu220.sig2.shm19.png | Z:\Cu311.sig2.shm19.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m20.png**  **Mode 20** |  | Z:\Cu200.sig2.shm20.png | Z:\Cu220.sig2.shm20.png | Z:\Cu311.sig2.shm20.png |

Projection of synthetic orientation dependent strain onto [111], {200}, {220}, and {311} strain pole figure surfaces when a particular spherical harmonic function is occupying 22.

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| --- | --- | --- | --- | --- |
| **Spherical harmonic functions** | **{111}** | **{200}** | **{220}** | **{311}** |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m21.png**  **Mode 21** |  | Z:\Cu200.sig2.shm21.png | Z:\Cu220.sig2.shm21.png | Z:\Cu311.sig2.shm21.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m22.png**  **Mode 22** |  | Z:\Cu200.sig2.shm22.png | Z:\Cu220.sig2.shm22.png | Z:\Cu311.sig2.shm22.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m23.png**  **Mode 23** |  | Z:\Cu200.sig2.shm23.png | Z:\Cu220.sig2.shm23.png | Z:\Cu311.sig2.shm23.png |

Projection of synthetic orientation dependent strain onto [111], {200}, {220}, and {311} strain pole figure surfaces when a particular spherical harmonic function is occupying 22.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Spherical harmonic functions** | **{111}** | **{200}** | **{220}** | **{311}** |
| C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m1.png  **Mode 1** |  | Z:\Cu200.sig3.shm1.png | Z:\Cu220.sig3.shm1.png | Z:\Cu311.sig3.shm1.png |
| C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m2.png  **Mode 2** |  | Z:\Cu200.sig3.shm2.png | Z:\Cu220.sig3.shm2.png | Z:\Cu311.sig3.shm2.png |
| C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m3.png  **Mode 3** |  | Z:\Cu200.sig3.shm3.png | Z:\Cu220.sig3.shm3.png | Z:\Cu311.sig3.shm3.png |
| C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m4.png  **Mode 4** |  | Z:\Cu200.sig3.shm4.png | Z:\Cu220.sig3.shm4.png | Z:\Cu311.sig3.shm4.png |

Projection of synthetic orientation dependent strain onto [111], {200}, {220}, and {311} strain pole figure surfaces when a particular spherical harmonic function is occupying 33.

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| --- | --- | --- | --- | --- |
| **Spherical harmonic functions** | **{111}** | **{200}** | **{220}** | **{311}** |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m5.png**  **Mode 5** |  | Z:\Cu200.sig3.shm5.png | Z:\Cu220.sig3.shm5.png | Z:\Cu311.sig3.shm5.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m6.png**  **Mode 6** |  | Z:\Cu200.sig3.shm6.png | Z:\Cu220.sig3.shm6.png | Z:\Cu311.sig3.shm6.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m7.png**  **Mode 7** |  | Z:\Cu200.sig3.shm7.png | Z:\Cu220.sig3.shm7.png | Z:\Cu311.sig3.shm7.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m8.png**  **Mode 8** |  | Z:\Cu200.sig3.shm8.png | Z:\Cu220.sig3.shm8.png | Z:\Cu311.sig3.shm8.png |

Projection of synthetic orientation dependent strain onto [111], {200}, {220}, and {311} strain pole figure surfaces when a particular spherical harmonic function is occupying 33.

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| --- | --- | --- | --- | --- |
| **Spherical harmonic functions** | **{111}** | **{200}** | **{220}** | **{311}** |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m9.png**  **Mode 9** |  | Z:\Cu200.sig3.shm9.png | Z:\Cu220.sig3.shm9.png | Z:\Cu311.sig3.shm9.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m10.png**  **Mode 10** |  | Z:\Cu200.sig3.shm10.png | Z:\Cu220.sig3.shm10.png | Z:\Cu311.sig3.shm10.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m11.png**  **Mode 11** |  | Z:\Cu200.sig3.shm11.png | Z:\Cu220.sig3.shm11.png | Z:\Cu311.sig3.shm11.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m12.png**  **Mode 12** |  | Z:\Cu200.sig3.shm12.png | Z:\Cu220.sig3.shm12.png | Z:\Cu311.sig3.shm12.png |

Projection of synthetic orientation dependent strain onto [111], {200}, {220}, and {311} strain pole figure surfaces when a particular spherical harmonic function is occupying 33.

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| --- | --- | --- | --- | --- |
| **Spherical harmonic functions** | **{111}** | **{200}** | **{220}** | **{311}** |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m13.png**  **Mode 13** |  | Z:\Cu200.sig3.shm13.png | Z:\Cu220.sig3.shm13.png | Z:\Cu311.sig3.shm13.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m14.png**  **Mode 14** |  | Z:\Cu200.sig3.shm14.png | Z:\Cu220.sig3.shm14.png | Z:\Cu311.sig3.shm14.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m15.png**  **Mode 15** |  | Z:\Cu200.sig3.shm15.png | Z:\Cu220.sig3.shm15.png | Z:\Cu311.sig3.shm15.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m16.png**  **Mode 16** |  | Z:\Cu200.sig3.shm16.png | Z:\Cu220.sig3.shm16.png | Z:\Cu311.sig3.shm16.png |

Projection of synthetic orientation dependent strain onto [111], {200}, {220}, and {311} strain pole figure surfaces when a particular spherical harmonic function is occupying 33.

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| --- | --- | --- | --- | --- |
| **Spherical harmonic functions** | **{111}** | **{200}** | **{220}** | **{311}** |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m17.png**  **Mode 17** |  | Z:\Cu200.sig3.shm17.png | Z:\Cu220.sig3.shm17.png | Z:\Cu311.sig3.shm17.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m18.png**  **Mode 18** |  | Z:\Cu200.sig3.shm18.png | Z:\Cu220.sig3.shm18.png | Z:\Cu311.sig3.shm18.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m19.png**  **Mode 19** |  | Z:\Cu200.sig3.shm19.png | Z:\Cu220.sig3.shm19.png | Z:\Cu311.sig3.shm19.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m20.png**  **Mode 20** |  | Z:\Cu200.sig3.shm20.png | Z:\Cu220.sig3.shm20.png | Z:\Cu311.sig3.shm20.png |

Projection of synthetic orientation dependent strain onto [111], {200}, {220}, and {311} strain pole figure surfaces when a particular spherical harmonic function is occupying 33.

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| --- | --- | --- | --- | --- |
| **Spherical harmonic functions** | **{111}** | **{200}** | **{220}** | **{311}** |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m21.png**  **Mode 21** |  | Z:\Cu200.sig3.shm21.png | Z:\Cu220.sig3.shm21.png | Z:\Cu311.sig3.shm21.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m22.png**  **Mode 22** |  | Z:\Cu200.sig3.shm22.png | Z:\Cu220.sig3.shm22.png | Z:\Cu311.sig3.shm22.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m23.png**  **Mode 23** |  | Z:\Cu200.sig3.shm23.png | Z:\Cu220.sig3.shm23.png | Z:\Cu311.sig3.shm23.png |

Projection of synthetic orientation dependent strain onto [111], {200}, {220}, and {311} strain pole figure surfaces when a particular spherical harmonic function is occupying 33.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Spherical harmonic functions** | **{111}** | **{200}** | **{220}** | **{311}** |
| C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m1.png  **Mode 1** |  | Z:\Cu200.sig4.shm1.png | Z:\Cu220.sig4.shm1.png | Z:\Cu311.sig4.shm1.png |
| C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m2.png  **Mode 2** |  | Z:\Cu200.sig4.shm2.png | Z:\Cu220.sig4.shm2.png | Z:\Cu311.sig4.shm2.png |
| C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m3.png  **Mode 3** |  | Z:\Cu200.sig4.shm3.png | Z:\Cu220.sig4.shm3.png | Z:\Cu311.sig4.shm3.png |
| C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m4.png  **Mode 4** |  | Z:\Cu200.sig4.shm4.png | Z:\Cu220.sig4.shm4.png | Z:\Cu311.sig4.shm4.png |

Projection of synthetic orientation dependent strain onto [111], {200}, {220}, and {311} strain pole figure surfaces when a particular spherical harmonic function is occupying 12.

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| **Spherical harmonic functions** | **{111}** | **{200}** | **{220}** | **{311}** |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m5.png**  **Mode 5** |  | Z:\Cu200.sig4.shm5.png | Z:\Cu220.sig4.shm5.png | Z:\Cu311.sig4.shm5.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m6.png**  **Mode 6** |  | Z:\Cu200.sig4.shm6.png | Z:\Cu220.sig4.shm6.png | Z:\Cu311.sig4.shm6.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m7.png**  **Mode 7** |  | Z:\Cu200.sig4.shm7.png | Z:\Cu220.sig4.shm7.png | Z:\Cu311.sig4.shm7.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m8.png**  **Mode 8** |  | Z:\Cu200.sig4.shm8.png | Z:\Cu220.sig4.shm8.png | Z:\Cu311.sig4.shm8.png |

Projection of synthetic orientation dependent strain onto [111], {200}, {220}, and {311} strain pole figure surfaces when a particular spherical harmonic function is occupying 12.

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| --- | --- | --- | --- | --- |
| **Spherical harmonic functions** | **{111}** | **{200}** | **{220}** | **{311}** |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m9.png**  **Mode 9** |  | Z:\Cu200.sig4.shm9.png | Z:\Cu220.sig4.shm9.png | Z:\Cu311.sig4.shm9.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m10.png**  **Mode 10** |  | Z:\Cu200.sig4.shm10.png | Z:\Cu220.sig4.shm10.png | Z:\Cu311.sig4.shm10.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m11.png**  **Mode 11** |  | Z:\Cu200.sig4.shm11.png | Z:\Cu220.sig4.shm11.png | Z:\Cu311.sig4.shm11.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m12.png**  **Mode 12** |  | Z:\Cu200.sig4.shm12.png | Z:\Cu220.sig4.shm12.png | Z:\Cu311.sig4.shm12.png |

Projection of synthetic orientation dependent strain onto [111], {200}, {220}, and {311} strain pole figure surfaces when a particular spherical harmonic function is occupying 12.

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| **Spherical harmonic functions** | **{111}** | **{200}** | **{220}** | **{311}** |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m13.png**  **Mode 13** |  | Z:\Cu200.sig4.shm13.png | Z:\Cu220.sig4.shm13.png | Z:\Cu311.sig4.shm13.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m14.png**  **Mode 14** |  | Z:\Cu200.sig4.shm14.png | Z:\Cu220.sig4.shm14.png | Z:\Cu311.sig4.shm14.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m15.png**  **Mode 15** |  | Z:\Cu200.sig4.shm15.png | Z:\Cu220.sig4.shm15.png | Z:\Cu311.sig4.shm15.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m16.png**  **Mode 16** |  | Z:\Cu200.sig4.shm16.png | Z:\Cu220.sig4.shm16.png | Z:\Cu311.sig4.shm16.png |

Projection of synthetic orientation dependent strain onto [111], {200}, {220}, and {311} strain pole figure surfaces when a particular spherical harmonic function is occupying 12.

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| --- | --- | --- | --- | --- |
| **Spherical harmonic functions** | **{111}** | **{200}** | **{220}** | **{311}** |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m17.png**  **Mode 17** |  | Z:\Cu200.sig4.shm17.png | Z:\Cu220.sig4.shm17.png | Z:\Cu311.sig4.shm17.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m18.png**  **Mode 18** |  | Z:\Cu200.sig4.shm18.png | Z:\Cu220.sig4.shm18.png | Z:\Cu311.sig4.shm18.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m19.png**  **Mode 19** |  | Z:\Cu200.sig4.shm19.png | Z:\Cu220.sig4.shm19.png | Z:\Cu311.sig4.shm19.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m20.png**  **Mode 20** |  | Z:\Cu200.sig4.shm20.png | Z:\Cu220.sig4.shm20.png | Z:\Cu311.sig4.shm20.png |

Projection of synthetic orientation dependent strain onto [111], {200}, {220}, and {311} strain pole figure surfaces when a particular spherical harmonic function is occupying 12.

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| **Spherical harmonic functions** | **{111}** | **{200}** | **{220}** | **{311}** |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m21.png**  **Mode 21** |  | Z:\Cu200.sig4.shm21.png | Z:\Cu220.sig4.shm21.png | Z:\Cu311.sig4.shm21.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m22.png**  **Mode 22** |  | Z:\Cu200.sig4.shm22.png | Z:\Cu220.sig4.shm22.png | Z:\Cu311.sig4.shm22.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m23.png**  **Mode 23** |  | Z:\Cu200.sig4.shm23.png | Z:\Cu220.sig4.shm23.png | Z:\Cu311.sig4.shm23.png |

Projection of synthetic orientation dependent strain onto [111], {200}, {220}, and {311} strain pole figure surfaces when a particular spherical harmonic function is occupying 12.

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| --- | --- | --- | --- | --- |
| **Spherical harmonic functions** | **{111}** | **{200}** | **{220}** | **{311}** |
| C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m1.png  **Mode 1** |  | Z:\Cu200.sig5.shm1.png | Z:\Cu220.sig5.shm1.png | Z:\Cu311.sig5.shm1.png |
| C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m2.png  **Mode 2** |  | Z:\Cu200.sig5.shm2.png | Z:\Cu220.sig5.shm2.png | Z:\Cu311.sig5.shm2.png |
| C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m3.png  **Mode 3** |  | Z:\Cu200.sig5.shm3.png | Z:\Cu220.sig5.shm3.png | Z:\Cu311.sig5.shm3.png |
| C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m4.png  **Mode 4** |  | Z:\Cu200.sig5.shm4.png | Z:\Cu220.sig5.shm4.png | Z:\Cu311.sig5.shm4.png |

Projection of synthetic orientation dependent strain onto [111], {200}, {220}, and {311} strain pole figure surfaces when a particular spherical harmonic function is occupying 13.

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| **Spherical harmonic functions** | **{111}** | **{200}** | **{220}** | **{311}** |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m5.png**  **Mode 5** |  | Z:\Cu200.sig5.shm5.png | Z:\Cu220.sig5.shm5.png | Z:\Cu311.sig5.shm5.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m6.png**  **Mode 6** |  | Z:\Cu200.sig5.shm6.png | Z:\Cu220.sig5.shm6.png | Z:\Cu311.sig5.shm6.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m7.png**  **Mode 7** |  | Z:\Cu200.sig5.shm7.png | Z:\Cu220.sig5.shm7.png | Z:\Cu311.sig5.shm7.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m8.png**  **Mode 8** |  | Z:\Cu200.sig5.shm8.png | Z:\Cu220.sig5.shm8.png | Z:\Cu311.sig5.shm8.png |

Projection of synthetic orientation dependent strain onto [111], {200}, {220}, and {311} strain pole figure surfaces when a particular spherical harmonic function is occupying 13.

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| --- | --- | --- | --- | --- |
| **Spherical harmonic functions** | **{111}** | **{200}** | **{220}** | **{311}** |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m9.png**  **Mode 9** |  | Z:\Cu200.sig5.shm9.png | Z:\Cu220.sig5.shm9.png | Z:\Cu311.sig5.shm9.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m10.png**  **Mode 10** |  | Z:\Cu200.sig5.shm10.png | Z:\Cu220.sig5.shm10.png | Z:\Cu311.sig5.shm10.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m11.png**  **Mode 11** |  | Z:\Cu200.sig5.shm11.png | Z:\Cu220.sig5.shm11.png | Z:\Cu311.sig5.shm11.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m12.png**  **Mode 12** |  | Z:\Cu200.sig5.shm12.png | Z:\Cu220.sig5.shm12.png | Z:\Cu311.sig5.shm12.png |

Projection of synthetic orientation dependent strain onto [111], {200}, {220}, and {311} strain pole figure surfaces when a particular spherical harmonic function is occupying 13.

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| --- | --- | --- | --- | --- |
| **Spherical harmonic functions** | **{111}** | **{200}** | **{220}** | **{311}** |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m13.png**  **Mode 13** |  | Z:\Cu200.sig5.shm13.png | Z:\Cu220.sig5.shm13.png | Z:\Cu311.sig5.shm13.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m14.png**  **Mode 14** |  | Z:\Cu200.sig5.shm14.png | Z:\Cu220.sig5.shm14.png | Z:\Cu311.sig5.shm14.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m15.png**  **Mode 15** |  | Z:\Cu200.sig5.shm15.png | Z:\Cu220.sig5.shm15.png | Z:\Cu311.sig5.shm15.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m16.png**  **Mode 16** |  | Z:\Cu200.sig5.shm16.png | Z:\Cu220.sig5.shm16.png | Z:\Cu311.sig5.shm16.png |

Projection of synthetic orientation dependent strain onto [111], {200}, {220}, and {311} strain pole figure surfaces when a particular spherical harmonic function is occupying 13.

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| --- | --- | --- | --- | --- |
| **Spherical harmonic functions** | **{111}** | **{200}** | **{220}** | **{311}** |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m17.png**  **Mode 17** |  | Z:\Cu200.sig5.shm17.png | Z:\Cu220.sig5.shm17.png | Z:\Cu311.sig5.shm17.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m18.png**  **Mode 18** |  | Z:\Cu200.sig5.shm18.png | Z:\Cu220.sig5.shm18.png | Z:\Cu311.sig5.shm18.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m19.png**  **Mode 19** |  | Z:\Cu200.sig5.shm19.png | Z:\Cu220.sig5.shm19.png | Z:\Cu311.sig5.shm19.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m20.png**  **Mode 20** |  | Z:\Cu200.sig5.shm20.png | Z:\Cu220.sig5.shm20.png | Z:\Cu311.sig5.shm20.png |

Projection of synthetic orientation dependent strain onto [111], {200}, {220}, and {311} strain pole figure surfaces when a particular spherical harmonic function is occupying 13.

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| **Spherical harmonic functions** | **{111}** | **{200}** | **{220}** | **{311}** |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m21.png**  **Mode 21** |  | Z:\Cu200.sig5.shm21.png | Z:\Cu220.sig5.shm21.png | Z:\Cu311.sig5.shm21.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m22.png**  **Mode 22** |  | Z:\Cu200.sig5.shm22.png | Z:\Cu220.sig5.shm22.png | Z:\Cu311.sig5.shm22.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m23.png**  **Mode 23** |  | Z:\Cu200.sig5.shm23.png | Z:\Cu220.sig5.shm23.png | Z:\Cu311.sig5.shm23.png |

Projection of synthetic orientation dependent strain onto [111], {200}, {220}, and {311} strain pole figure surfaces when a particular spherical harmonic function is occupying 13.

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| --- | --- | --- | --- | --- |
| **Spherical harmonic functions** | **{111}** | **{200}** | **{220}** | **{311}** |
| C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m1.png  **Mode 1** |  | Z:\Cu200.sig6.shm1.png | Z:\Cu220.sig6.shm1.png | Z:\Cu311.sig6.shm1.png |
| C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m2.png  **Mode 2** |  | Z:\Cu200.sig6.shm2.png | Z:\Cu220.sig6.shm2.png | Z:\Cu311.sig6.shm2.png |
| C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m3.png  **Mode 3** |  | Z:\Cu200.sig6.shm3.png | Z:\Cu220.sig6.shm3.png | Z:\Cu311.sig6.shm3.png |
| C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m4.png  **Mode 4** |  | Z:\Cu200.sig6.shm4.png | Z:\Cu220.sig6.shm4.png | Z:\Cu311.sig6.shm4.png |

Projection of synthetic orientation dependent strain onto [111], {200}, {220}, and {311} strain pole figure surfaces when a particular spherical harmonic function is occupying 23.

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| **Spherical harmonic functions** | **{111}** | **{200}** | **{220}** | **{311}** |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m5.png**  **Mode 5** |  | Z:\Cu200.sig6.shm5.png | Z:\Cu220.sig6.shm5.png | Z:\Cu311.sig6.shm5.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m6.png**  **Mode 6** |  | Z:\Cu200.sig6.shm6.png | Z:\Cu220.sig6.shm6.png | Z:\Cu311.sig6.shm6.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m7.png**  **Mode 7** |  | Z:\Cu200.sig6.shm7.png | Z:\Cu220.sig6.shm7.png | Z:\Cu311.sig6.shm7.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m8.png**  **Mode 8** |  | Z:\Cu200.sig6.shm8.png | Z:\Cu220.sig6.shm8.png | Z:\Cu311.sig6.shm8.png |

Projection of synthetic orientation dependent strain onto [111], {200}, {220}, and {311} strain pole figure surfaces when a particular spherical harmonic function is occupying 23.

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| **Spherical harmonic functions** | **{111}** | **{200}** | **{220}** | **{311}** |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m9.png**  **Mode 9** |  | Z:\Cu200.sig6.shm9.png | Z:\Cu220.sig6.shm9.png | Z:\Cu311.sig6.shm9.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m10.png**  **Mode 10** |  | Z:\Cu200.sig6.shm10.png | Z:\Cu220.sig6.shm10.png | Z:\Cu311.sig6.shm10.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m11.png**  **Mode 11** |  | Z:\Cu200.sig6.shm11.png | Z:\Cu220.sig6.shm11.png | Z:\Cu311.sig6.shm11.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m12.png**  **Mode 12** |  | Z:\Cu200.sig6.shm12.png | Z:\Cu220.sig6.shm12.png | Z:\Cu311.sig6.shm12.png |

Projection of synthetic orientation dependent strain onto [111], {200}, {220}, and {311} strain pole figure surfaces when a particular spherical harmonic function is occupying 23.

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| **Spherical harmonic functions** | **{111}** | **{200}** | **{220}** | **{311}** |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m13.png**  **Mode 13** |  | Z:\Cu200.sig6.shm13.png | Z:\Cu220.sig6.shm13.png | Z:\Cu311.sig6.shm13.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m14.png**  **Mode 14** |  | Z:\Cu200.sig6.shm14.png | Z:\Cu220.sig6.shm14.png | Z:\Cu311.sig6.shm14.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m15.png**  **Mode 15** |  | Z:\Cu200.sig6.shm15.png | Z:\Cu220.sig6.shm15.png | Z:\Cu311.sig6.shm15.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m16.png**  **Mode 16** |  | Z:\Cu200.sig6.shm16.png | Z:\Cu220.sig6.shm16.png | Z:\Cu311.sig6.shm16.png |

Projection of synthetic orientation dependent strain onto [111], {200}, {220}, and {311} strain pole figure surfaces when a particular spherical harmonic function is occupying 23.

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| **Spherical harmonic functions** | **{111}** | **{200}** | **{220}** | **{311}** |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m17.png**  **Mode 17** |  | Z:\Cu200.sig6.shm17.png | Z:\Cu220.sig6.shm17.png | Z:\Cu311.sig6.shm17.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m18.png**  **Mode 18** |  | Z:\Cu200.sig6.shm18.png | Z:\Cu220.sig6.shm18.png | Z:\Cu311.sig6.shm18.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m19.png**  **Mode 19** |  | Z:\Cu200.sig6.shm19.png | Z:\Cu220.sig6.shm19.png | Z:\Cu311.sig6.shm19.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m20.png**  **Mode 20** |  | Z:\Cu200.sig6.shm20.png | Z:\Cu220.sig6.shm20.png | Z:\Cu311.sig6.shm20.png |

Projection of synthetic orientation dependent strain onto [111], {200}, {220}, and {311} strain pole figure surfaces when a particular spherical harmonic function is occupying 23.

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| **Spherical harmonic functions** | **{111}** | **{200}** | **{220}** | **{311}** |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m21.png**  **Mode 21** |  | Z:\Cu200.sig6.shm21.png | Z:\Cu220.sig6.shm21.png | Z:\Cu311.sig6.shm21.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m22.png**  **Mode 22** |  | Z:\Cu200.sig6.shm22.png | Z:\Cu220.sig6.shm22.png | Z:\Cu311.sig6.shm22.png |
| **C:\Users\jspark\Desktop\prjInversion\SH.Modes\cubic\m23.png**  **Mode 23** |  | Z:\Cu200.sig6.shm23.png | Z:\Cu220.sig6.shm23.png | Z:\Cu311.sig6.shm23.png |

Projection of synthetic orientation dependent strain onto [111], {200}, {220}, and {311} strain pole figure surfaces when a particular spherical harmonic function is occupying 23.

1. It is modified VM notation because typical VM notation will vectorize by listing 11, 22, 33, 23, 13, and 12. Here, we vectorize by 11, 22, 33, 12, 13, and 23. [↑](#footnote-ref-1)
2. Note that matrix or vector transpose is a plain T and the transformation matrix is a **bold T** written as [**T**]. [↑](#footnote-ref-2)
3. The number of independent nodes is set as 76 to demonstrate the strain pole figure inversion. It can be increased to the desired refinement of the fundamental region. [↑](#footnote-ref-3)
4. It is also possible to take the fiber average of the strain tensors then perform the strain projection of the fiber averaged strain tensor. [↑](#footnote-ref-4)
5. This is explained further in *Preferred Orientation in Deformed Metals and Rocks: An Introduction to Modern Texture Analysis* (edited by H.R. Wenk) and *Texture and Anisotropy* (edited by U.F. Kocks et al). [↑](#footnote-ref-5)
6. This is explained further in *Preferred Orientation in Deformed Metals and Rocks: An Introduction to Modern Texture Analysis* (edited by H.R. Wenk). [↑](#footnote-ref-6)