LDPC endcode

Notations

N: codeword length

K: information bit length

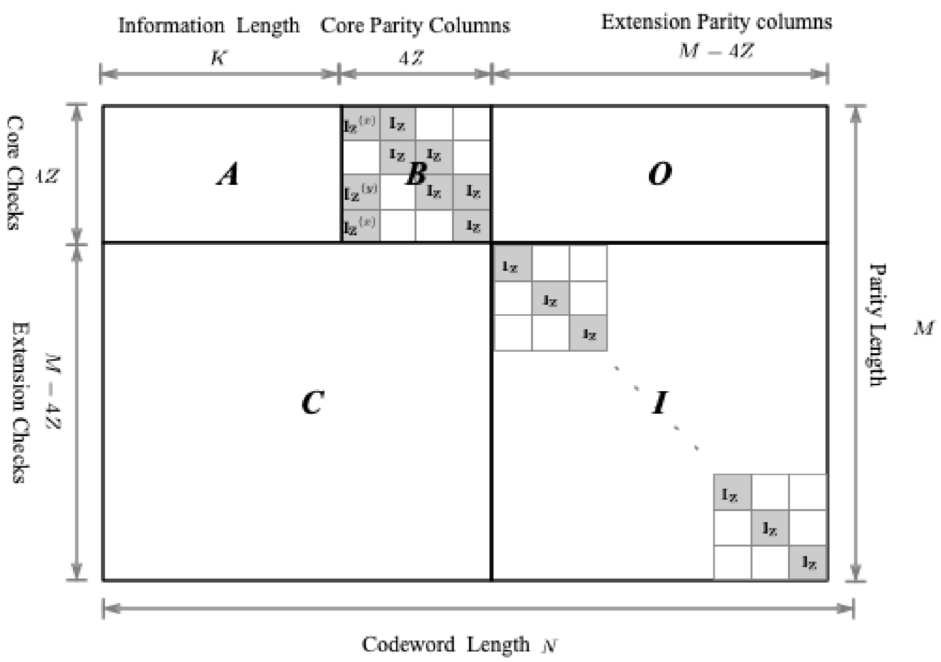
M: parity bit length

Z: lifting size

Encoding is to to compute Parity bits according the given message bits by solving the following checksum equation, where $\mathbf{c} = [\mathbf{u}^T, \mathbf{p}^T]^T$ is the encoded bit sequence.

$$\mathbf{Hc} = \mathbf{0}, \quad \mathbf{H} \in \mathbb{B}^{M \times N}, \mathbf{c} \in \mathbb{B}^{N \times 1}$$

The structure of parity check matrix



We can use the structure of parity check matrix of NR LDPC, as showing the picture below

$$\mathbf{H} = \begin{bmatrix} \mathbf{A} & \mathbf{B} & \mathbf{O} \\ \mathbf{C}_1 & \mathbf{C}_2 & \mathbf{I} \end{bmatrix} \in \mathbb{B}^{M \times N}$$
$$\mathbf{A} \in \mathbb{B}^{4Z \times K} \quad \mathbf{B} \in \mathbb{B}^{4Z \times 4Z} \quad \mathbf{O} = O^{4Z \times M-4Z}$$
$$\mathbf{C} = [\mathbf{C}_1, \mathbf{C}_2] \in \mathbb{B}^{M-4Z \times K+4Z} \mathbf{C}_1 \in \mathbb{B}^{M-4Z \times K} \quad \mathbf{C}_2 \in \mathbb{B}^{M-4Z \times 4Z} \quad \mathbf{I} = 1_{M-4Z}$$
$$\mathbf{c} = [\mathbf{u}^T, \mathbf{p}_c^T, \mathbf{p}_e^T]^T$$
$$\mathbf{u} \in \mathbb{B}^{K \times 1} \quad \mathbf{p}_a \in \mathbb{B}^{4Z \times 1} \quad \mathbf{p}_b \in \mathbb{B}^{M-4Z \times 1}$$

Solving core parity check $\mathbf{p}_c$

we have

$$\mathbf{B} = \begin{bmatrix} \mathbf{I}_Z^{(x)} & \mathbf{I}_Z & \mathbf{0}_Z & \mathbf{0}_Z \\ \mathbf{0}_Z & \mathbf{I}_Z & \mathbf{I}_Z & \mathbf{0}_Z \\ \mathbf{I}_Z^{(y)} & \mathbf{0}_Z & \mathbf{I}_Z & \mathbf{I}_Z \\ \mathbf{I}_Z^{(x)} & \mathbf{0}_Z & \mathbf{0}_Z & \mathbf{I}_Z \end{bmatrix}$$
$$(\cdot)^{(x)} \text{ right circular shift by y times}$$

let

$$\mathbf{A} = \begin{bmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \\ \mathbf{a}_3 \\ \mathbf{a}_4 \end{bmatrix} \quad \mathbf{a}_i \in \mathbb{B}^{Z \times K}, \quad \mathbf{p}_c = \begin{bmatrix} \mathbf{p}_{\mathbf{c}1} \\ \mathbf{p}_{\mathbf{c}2} \\ \mathbf{p}_{\mathbf{a}3} \\ \mathbf{p}_{\mathbf{c}4} \end{bmatrix} \quad \mathbf{p}_{\mathbf{c}i} \in \mathbb{B}^{Z \times 1},$$

we have

$$\begin{bmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \\ \mathbf{a}_3 \\ \mathbf{a}_4 \end{bmatrix} \mathbf{u} + \begin{bmatrix} \mathbf{I}_Z^{(x)} & \mathbf{I}_Z & \mathbf{0}_Z & \mathbf{0}_Z \\ \mathbf{0}_Z & \mathbf{I}_Z & \mathbf{I}_Z & \mathbf{0}_Z \\ \mathbf{I}_Z^{(y)} & \mathbf{0}_Z & \mathbf{I}_Z & \mathbf{I}_Z \\ \mathbf{I}_Z^{(x)} & \mathbf{0}_Z & \mathbf{0}_Z & \mathbf{I}_Z \end{bmatrix} \begin{bmatrix} \mathbf{p}_{\mathbf{c}1} \\ \mathbf{p}_{\mathbf{c}2} \\ \mathbf{p}_{\mathbf{c}3} \\ \mathbf{p}_{\mathbf{c}4} \end{bmatrix} = \mathbf{0}$$

adding all rows of above equations , the parity nodes $\mathbf{p}_{c_2}, \mathbf{p}_{c_3}, \mathbf{p}_{c_4}$ are eliminated, obtains

$$\mathbf{A}\mathbf{u} + \mathbf{p}_{\mathbf{a}1}^{(y)} = 0 \Rightarrow \mathbf{p}_{\mathbf{c}1} = (\mathbf{A}\mathbf{u})^{(Z-y)} \quad (\because I_Z^{(y)} \mathbf{p}_{\mathbf{c}1} = \mathbf{p}_{\mathbf{c}1}^{(y)})$$

use the first/second/third row of above simultaneous equations, we sole the rest core parith check nodes in sequence

$$
\begin{aligned}
\mathbf{p}_{a2} &= \mathbf{a}_1\mathbf{u} + \mathbf{I}_z^{(x)}\mathbf{p}_{\mathbf{c}1} \\
\mathbf{p}_{c3} &= \mathbf{a}_2\mathbf{u} + \mathbf{p}_{\mathbf{c}2} \\
\mathbf{p}_{c4} &= \mathbf{a}_3\mathbf{u} + I_z^{(y)}\mathbf{p}_{\mathbf{c}1} + \mathbf{p}_{\mathbf{c}3}
\end{aligned}
$$

Solving extensive parity check

after solving $\mathbf{p}_c,\ \mathbf{p}_e$ can be easily solved

$$\mathbf{p}_e = \mathbf{C}_1\mathbf{u} + \mathbf{C}_2\mathbf{p}_c$$

The implementation

The efficient implementation, as

$$I_Z^{(x)}[u_0, u_1, \cdots, u_{Z-1}]^T = [u_x, u_{x+1}, \cdots, u_{Z-1}, u_0, u_1, \cdots, u_{x-1}]^T$$

Therefore , in the implementation, we do not really do the matrix multiplication, rather just circularly shift the corresponding bit sequence.

Also, utilizing the sparsity property, we can ignore those $q_{i,j} = -1$. Therefore, instead work on the parity check matrix or its exponent matrix, we can directly work on the reduced shift coefficients tables (3GPP TS 38.212 Table 5.3.2-2/3) where the -1s are ignored. I.e., for BG1, there are only 316 circular shifts involved and for BG2 only 197 shifts.