A Dynamic Framework for Identification and Estimation of Nonseparable Production Functions

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Introduction

- This paper identifies and estimates a nonseparable production function with unobserved heterogeneity
- Allows for interactions in the unobservable (productivity) with inputs, in a sense, allowing for non-Hicks neutral productivity shocks
- The conditional output distribution is allowed to vary with productivity
- Inputs can contain unobservable demand shocks which would violate identification arguments of OP, LP, ACF
- The identification argument does not rely on the control function approach so there are no identification issues between using value-added and gross-output production functions
- A more flexible model for productivity and non-linear persistence
- Productivity can be allowed to depend on the magnitude and sign of current and future innovation shocks

The Production Function

- Consider a nonseparable model for a firm's gross-output production function (in logs)

$$y_{it} = f_t(k_{it}, l_{it}, m_{it}, \omega_{it}, \eta_{it}) \tag{1}$$

- Allows for non-linear interactions between inputs and unobserved productivity
- Assume the following

Assumption 1 (Production Function)

1. *The unanticipated production shocks η_{it} are iid over firms and time.*
2. *The unanticipated production shock η_{it} follows a standard uniform distribution independent of $(k_{it}, l_{it}, m_{it}, \omega_{it},)$.*
3. $\tau \to Q_t^y(k_{it}, l_{it}, m_{it}, \omega_{it}, \tau)$ *is strictly increasing on* $(0, 1)$.

Productivity

- Productivity evolves according to an exogenous first-order Markov process given by

$$\omega_{it} = Q_t^{\omega}(\omega_{it-1}, \xi_{it}), \quad \xi_{it} \sim Uniform(0, 1), \tag{2}$$

  where $\xi_{i1}, \ldots, \xi_{iT}$ are independent uniform random variables which represent innovation shocks to productivity
- The function Q^{ω} is a nonlinear function that allows the persistence in productivity in firms to be nonlinear across different quantiles

Assumption 2 (Productivity)

1. *The productivity innovation shocks ξ_{it} are iid across firms and time.*
2. *ξ_{it} follows a standard uniform distribution independent of previous period productivity ω_{it-1}.*
3. *$\tau \to Q_t^{\omega}(\omega_{it-1}, \tau)$ is strictly increasing on $(0, 1)$.*

Flexible Inputs

- Labor and Material inputs are chosen to maximize current period profits
- Therefore they are a function of current period state variables

$$l_{it} = Q_t^{\ell}(k_{it}, \omega_{it}, \epsilon_{\ell,it}), \quad \epsilon_{\ell,it} \sim Uniform(0,1), \tag{3}$$

$$m_{it} = Q_t^{m}(k_{it}, l_{it}, \omega_{it}, \epsilon_{m,it}), \quad \epsilon_{m,it} \sim Uniform(0,1), \tag{4}$$

- $\epsilon_{l,it}$ and $\epsilon_{m,it}$ are iid unobservable input demand shocks that are assumed to be independent of current period state variables
- In the control function approach, with material inputs as a proxy, this function could not be inverted as an expression of productivity only
- This framework is similar to that of Hu, Huang, and Sasaki, 2020
- This can also be extended to the case where labor has adjustment frictions

Flexible Inputs

Assumption 3 (Flexible Inputs)

1. *The unobserved input demand shocks* $\epsilon_{l,it}$ *and* $\epsilon_{m,it}$ *are iid across firms and time.*
2. $\epsilon_{l,it}$ *and* $\epsilon_{m,it}$ *follow a standard uniform distribution independent of* (k_{it}, ω_{it}).
3. $\tau \rightarrow Q_t^{\ell}(k_{it}, \omega_{it}, \tau)$ *and* $\tau \rightarrow Q_t^{m}(k_{it}, \omega_{it}, \tau)$ *are strictly increasing on* $(0, 1)$.

Capital and Investment

- Capital accumulates to the following generalized law of motion

$$K_{it} = \kappa_t(K_{it-1}, I_{it-1}, \upsilon_{it-1}) \tag{5}$$

where I_{it-1} denotes firm investment in the prior period

- Eliminates the deterministic relationship of capital with respect to previous period state and choice variables
- Assume this error term is independent of the arguments in the capital accumulation law
- In each period, a firm chooses investment to maximize its discounted future profits:

$$I_{it}^* = \iota_t(K_{it}, \omega_{it}) = \underset{I_t \geq 0}{\operatorname{argmax}} \Big[\Pi_t(K_{it}, \omega_{it}) - c(I_{it}) + \beta \mathbb{E}\big[V_{t+1}(K_{it+1}, \omega_{it+1}) | \mathcal{I}_t \big] \Big], \tag{6}$$

Capital and Investment

- $\pi_t(\cdot)$ is current period profits as a function of the state variables $c(\cdot)$ is the cost function
- Empirical investment rule is

$$i^*_{it} = Q^i_t(k_{it}, \omega_{it}, \zeta_{it}), \quad \zeta_{it} \sim Uniform(0,1). \tag{7}$$

Assumption 4 (Capital Accumulation and Investment)

1. *The unobserved investment demand shocks ζ_{it} is iid across firms and time.*
2. *ζ_{it} follows a standard uniform distribution independent of (k_{it}, ω_{it}).*
3. *The production shock η_{it} and ζ_{it} are independent conditional on $(k_{it}, l_{it}, m_{it}, \omega_{it})$. In addition, υ_{it} is independent of η_{it} conditional on $(k_{it}, l_{it}, m_{it}, \omega_{it})$*
4. *$\tau \to Q^i_t(k_{it}, \omega_{it}, \tau)$ is strictly increasing on $(0,1)$*

Nonparametric Identification

- I show that the conditional densities corresponding to production, inputs, and productivity are nonparametrically identified
- Identification is similar to Hu, Huang, and Sasaki, 2020
- Let $Z_t = (l_t, k_t, m_t, k_{t+1})$ denote conditioning variables

Assumption 5 (Conditional Independence)

1. $f(y_t|y_{t+1}, I_t, \omega_t, Z_t) = f(y_t|\omega_t, Z_t)$
2. $f(I_t|y_{t+1}, \omega_t, Z_t) = f(I_t|\omega_t, Z_t)$

- First equality states that conditional on ω_t and Z_t, y_{t+1} and I_t do not provide any additional information about y_t
- Second equality states that conditional on ω_t and Z_t, y_{t+1} does not provide any additional information about I_t
- Satisfied by mutual independence assumptions on η_t and ζ_t conditional on $(\omega_t, k_t, l_t, m_t)$

Nonparametric Identification

- Begin by relating a conditional density as a function of observable to densities containing unobserved productivity
- Using the conditional independence assumption, I can write

$$f_{y_t, I_t | y_{t+1}, Z_t} = \int f_{y_t | Z_t, \omega_t} f_{I_t | Z_t, \omega_t} f_{\omega_t | y_{t+1}, Z_t} d\omega_t \tag{8}$$

- The identification strategy follows HS by using a eigenvalue-eigenfunction decomposition of integral operators of (8)

Definition 1

(Integral Operator) *Let a and b denote random variables with supports $\mathcal{A}$ and $\mathcal{B}$. Given two corresponding spaces $\mathcal{G}(\mathcal{A})$ and $\mathcal{G}(\mathcal{B})$ of functions with domains $\mathcal{A}$ and $\mathcal{B}$, let $L_{b|a}$ denote the operator mapping $g \in \mathcal{G}(\mathcal{A})$ to $L_{b|a} g \in L_{b|a}\mathcal{G}(\mathcal{B})$ defined by*

$$[L_{b|a} g](b) \equiv \int_{\mathcal{A}} f_{b|a}(b|a) g(a) da,$$

where $f_{b|a}$ denotes the conditional density of b given a.

Nonparametric Identification

- The observed density in (8) can be written in operator notation

$$L_{y_t, I_t | y_{t+1}, Z_t} = L_{y_t | Z_t, \omega_t} \Delta_{I_t | Z_t, \omega_t} L_{\omega_t | y_{t+1}, Z_t} \tag{9}$$

- Will show that under a set of assumptions, the conditional density is identified from an eigenvalue-eigenfunction decomposition of (9)

Assumption 6 (Injectivity)

The operators $L_{y_t|Z_t,\omega_t}$ and $L_{y_{t+1}|Z_t,\omega_t}$ are injective

- The above assumption allows us to take inverses of the operators.
- Consider the operator $L_{y_t|Z_t,\omega_t}$, injectivity of this operator can be interpreted as its corresponding density $f_{y_t|Z_t,\omega_t}(I_t|K_t,\omega_t)$ having sufficient variation in ω_t given Z_t.
- This assumption is often phrased as completeness condition in the nonparametric IV literature on the density $f_{y_t|Z_t,\omega_t}(y_t|Z_t,\omega_t)$.

Nonparametric Identification

Assumption 7 (Uniqueness)

For any $\bar{\omega}_t, \tilde{\omega}_t \in \Omega$, the set $\{f_{I|\omega,Z}(I_t|\bar{\omega}_t, Z_t) \neq f_{I|\omega,Z}(I_t|\tilde{\omega}_t, Z_t)\}$ has positive probability whenever $\bar{\omega}_t \neq \tilde{\omega}_t$

- This assumption is relatively weak
- Satisfied if there is conditional heteroskedasticity in $f_{I|\omega,Z}$
- Satisfied if any functional of its distribution is strictly increasing in ω_t
- I assume $E[I_t|\omega_t, Z_t]$ is strictly increasing in ω_t
- Similar to the invertibility condition in Olley and Pakes, 1996

Nonparametric Identification

Assumption 8 (Normalization)

There exists a functional Γ *such that* $\Gamma[f_{y|\omega,Z}(y_t|\omega_t, Z_t)] = \omega_t$

- This functional does not need to be known
- Sufficient to consider a known function of the data distribution as shown by Arellano and Bonhomme, 2016
- In my empirical application, I consider a nonseparable translog production function
- The assumption can be satisfied by the normalization $E[y_t|\omega_t, 0] = \omega_t$
- For more generalized production functions, if $E[y_t|\omega_t, Z_t]$ is strictly increasing in ω_{it}, then one could normalize $\omega_t = E[y_t|\omega_t, Z_t]$
- These restrictions are easily adaptable in estimation as it amounts to centering the coefficients in the model

Nonparametric Identification

Theorem 1 (Identification)

Under Assumptions 5, 6, 7, and 8, given the observed density $f_{y_t, I_t | y_{t+1}, Z_t}$, the equation

$$f_{y_t, I_t | y_{t+1}, Z_t} = \int f_{y_t | \omega_t, Z_t} f_{I_t | \omega_t, Z_t} f_{\omega_t | y_{t+1}, Z_t} d\omega_t \tag{10}$$

admits a unique solution for $f_{y_t | \omega_t, Z_t}$, $f_{I_t | \omega_t, Z_t}$ and $f_{\omega_t | y_{t+1}, Z_t}$

- The proof follows using Hu and Schennach, 2008
- However it does not directly identify the Markov transition function for productivity $f_{\omega_{it+1} | \omega_{it}}(\omega_{it+1} | \omega_{it})$

Nonparametric Identification

Corollary 1 (Identification of Markov Process: Stationarity Case)

Suppose that the production function is stationary, $f_{y_t|\omega_t,Z_t} = f_{y_1|\omega_1,Z_1} \forall t \in \{1, \cdots, T\}$. Then, under Assumptions 5, 6, 7, and 8, the observed density $f_{y_t,I_t|y_{t+1},Z_t}$ uniquely determines the density $f_{\omega_{t+1}|\omega_t} \forall t \in \{1, \ldots, T-1\}$

Corollary 2 (Identification of Markov Process: Non-Stationarity Case)

Under Assumptions 5, 6, 7, and 8, the observed density $f_{y_{t+1},I_{t+1}|y_{t+2},Z_{t+1}}$ uniquely determines the density $f_{\omega_{t+1}|\omega_t} \forall t \in \{1, \ldots, T-2\}$

Econometric Procedure: Production

- The production function is specified as Translog with non-Hicks neutral effects

$$
\begin{aligned}
Q_t^y(k_{it}, l_{it}, m_{it}, \omega_{it}, \tau) = \\
&\gamma_0(\tau) + (\gamma_k(\tau) + \sigma_k(\tau)\omega_{it})k_{it} + (\gamma_l(\tau) + \sigma_l(\tau)\omega_{it})l_{it} + (\gamma_m(\tau) + \sigma_m(\tau)\omega_{it})m_{it} \\
&+ (\gamma_{kl}(\tau) + \sigma_{kl}(\tau)\omega_{it})k_{it}l_{it} + (\gamma_{lm}(\tau) + \sigma_{lm}(\tau)\omega_{it})l_{it}m_{it} + (\gamma_{km}(\tau) + \sigma_{km}(\tau)\omega_{it})k_{it}m_{it} \\
&+ (\gamma_{kk}(\tau) + \sigma_{kk}(\tau)\omega_{it})k_{it}^2 + (\gamma_{ll}(\tau) + \sigma_{ll}(\tau)\omega_{it})l_{it}^2 + (\gamma_{mm}(\tau) + \sigma_{mm}(\tau)\omega_{it})m_{it}^2 + \sigma_\omega(\tau)\omega_{it}
\end{aligned}
\tag{11}
$$

- Similar model was estimated by Ackerberg and Chen (2015)
- In my approach I can simulate productivity from estimated intital conditions and Markov process to compute average derivative effects
- Provides a better picture of heterogeneity instead of reporting individual coefficients

Econometric Procedure: Productivity

- I specify productivity using 3rd order polynomial

$$Q^{\omega}(\omega_{it-1}, \tau) = \rho_0(\tau) + \rho_1(\tau)\omega_{it-1} + \rho_2(\tau)\omega_{it-1}^2 + \rho_3(\tau)\omega_{it-1}^3. \quad (12)$$

- Initial productivity

$$Q^{\omega_1}(k_{i1}, \tau) = \sum_{j=1}^{J} \rho_{\omega_1,j}(\tau)\phi_{\omega_1,j}(k_{i1}), \quad (13)$$

- I can also consider the case where productivity may evolve endogenously as Doraszelski and Jaumandreu, 2013

$$Q^{\omega}(\omega_{it-1}, r_{it-1}, \tau) = \mathbb{1}\{R_{it-1} = 0\}Q^{\omega}(\omega_{it-1}, \tau) + \mathbb{1}\{R_{it-1} > 0\}Q^{\omega,r}(\omega_{it-1}, r_{it-1}, \tau). \quad (14)$$

Econometric Procedure: Flexible Inputs

- I specify the labor input demand equation as follows:

$$Q_t^{\ell}(k_{it}, \omega_{it}, \tau) = \sum_{j=1}^{J} \alpha_{\ell,j}(\tau)\phi_{\ell,j}(k_{it}, \omega_{it}), \tag{15}$$

  where $\phi_{\ell,j}$ can be another non-linear function
- Material inputs are specified as

$$Q_t^{m}(k_{it}, l_{it}, \omega_{it}, \tau) = \sum_{j=1}^{J} \alpha_{m,j}(\tau)\phi_{m,j}(k_{it}, l_{it}, \omega_{it}), \tag{16}$$

- Again, $\phi_{m,j}$ can be a non-linear function

Econometric Procedure: Investment

- The investment demand function is specified as

$$i_{it}^* = Q_t^i(k_{it}, \omega_{it}, \tau) = \sum_{j=1}^{J} \delta_j(\tau)\phi_{i,j}(k_{it}, \omega_{it}), \tag{17}$$

  where $\phi_{\iota,j}$ is specified similarly as the labor and material input decision rule.

- In the case where investment is censored, I can write

$$Q_t^i(k_{it}, \omega_{it}, \tau) = \max\{0, \sum_{j=1}^{J} \delta_j(\tau)\phi_{i,j}(k_{it}, \omega_{it})\}, \tag{18}$$

  due to the equivariance properties of quantiles

- The censored quantile regression model avoids distributional assumptions at the cost of computational complexity
- Censored investment levels are not an issue in Compustat

Econometric Restrictions

Note that the following conditional moment restrictions hold as an implication of Assumptions 1-4

$$\mathbb{E}\left[\Psi_\tau\left(y_{it} - Q_t(y_{it}|k_{it}, l_{it}, m_{it}, \omega_{it}; \beta(\tau))\right)\middle| k_{it}, l_{it}, m_{it}\right] = 0 \tag{19}$$

$$\mathbb{E}\left[\Psi_\tau\left(l_{it} - \sum_{j=1}^{J} \alpha_{l,j}(\tau)\phi_{l,j}(k_{it}, \omega_{it})\right)\middle| k_{it}, \omega_{it}\right] = 0 \tag{20}$$

$$\mathbb{E}\left[\Psi_\tau\left(m_{it} - \sum_{j=1}^{J} \alpha_{m,j}(\tau)\phi_{m,j}(k_{it}, \omega_{it})\right)\middle| k_{it}, \omega_{it}\right] = 0 \tag{21}$$

$$\mathbb{E}\left[\Psi_\tau\left(i_{it} - \sum_{j=1}^{J} \delta_j(\tau)\phi_{\iota,j}(k_{it}, \omega_{it})\right)\middle| k_{it}, \omega_{it}\right] = 0 \tag{22}$$

For $t \geq 2$,

$$\mathbb{E}\left[\Psi_\tau\left(\omega_{it} - \rho_0(\tau) - \rho_1(\tau)\omega_{it-1} - \rho_2(\tau)\omega_{it-1}^2 - \rho_3(\tau)\omega_{it-1}^3\right)\middle| \omega_{it-1}\right] = 0, \tag{23}$$

$$\mathbb{E}\left[\Psi_\tau\left(\omega_{i1} - \sum_{j=1}^{J} \rho_{\omega_1,j}(\tau)\phi_{\omega_1,j}(k_{i1})\right)\middle| k_{i1}\right] = 0, \tag{24}$$

Econometric Restrictions

- The function $\Psi_\tau(u) = \tau - \mathbb{1}\{u < 0\}$
- Estimating the above conditional moment restrictions is infeasible due to the unobserved productivity component
- Use the unconditional moment restriction and integrate out productivity
- Let the finite and functional parameters be indexed by a finite dimensional parameter vector $\theta(\cdot)$.
- To fix ideas, consider the unconditional moment restriction corresponding to the production function

$$\mathbb{E}\left[\int_\Omega \Psi_\tau\left(y_{it} - Q_t^y(k_{it}, l_{it}, m_{it}, \omega_{it}; \beta(\tau))\right) \otimes \begin{pmatrix} k_{it} \\ l_{it} \\ m_{it} \\ \omega_{it} \end{pmatrix} g_i(\omega_i^T; \theta(\cdot)) d\omega_i^T\right] = 0, \qquad (25)$$

- The posterior density $g_i(\omega_i^T; \theta(\cdot)) = f(\omega_i^T | y_i^T, k_i^T, l_i^T, m_i^T, i_i^T, \theta(\cdot))$ involves the entire set of model parameters

Implementation

- Therefore, it is impossible to estimate the model parameters in a τ-by-τ procedure
- To eliminate the intractability of this problem, the continuous model parameters are approximated by piece-wise linear splines
- θ is a piecewise-polynomial interpolating splines on a grid $[\tau_1, \tau_2], [\tau_3, \tau_4], \ldots, [\tau_{Q-1}, \tau_Q]$, contained in the unit interval and is constant on $[0, \tau_1]$ and $[\tau_Q, 1)$
- The intercept coefficient β_0 is specified as the quantile of an exponential distribution on $(0, \tau_1]$ (indexed by λ^-) and $[\tau_{Q-1}, 1)$ (indexed by λ^+).
- The remaining functional parameters are modeled similarly.
- With piece-wise linear splines, the posterior density has a closed form expression without relying on strong distributional assumptions for estimation

Implementation

- In order to estimate the model, the integral inside the expectation of Equation (25) needs to be approximated
- This can be done using quadrature methods or Monte Carlo integration by converting the problem into a weighted quantile regression
- Due to the high-dimensionality of my application I choose to use a random-walk Metropolis Hastings algorithm to compute the integral
- This is known as a Monte Carlo Expectation Maximization (MCEM) procedure where the maximization step is performed using quantile regression
- This type of estimator is used by Arellano and Bonhomme, 2016 and Arellano, Blundell, and Bonhomme, 2017

Implementation

Given an initial parameter value $\hat{\theta}^0$. Iterate on $s = 0, 1, 2, \ldots$ in the following two-step procedure until converge to a stationary distribution:

1. *Stochastic E-Step*: Draw M values $\omega_i^{(m)} = (\omega_{i1}^{(m)}, \omega_{i2}^{(m)}, \ldots, \omega_{iT}^{(m)})$ from

$$g_i(\omega_i^T; \hat{\theta}^{(s)}) = f(\omega_i^T | y_i^T, k_i^T, l_i^T, m_i^T, i_i^T, ; \hat{\theta}^{(s)}) \propto$$
$$\prod_{t=1}^{T} f(y_{it} | k_{it}, l_{it}, m_{it}, \omega_{it}; \hat{\beta}^{(s)}) f(l_{it} | k_{it}, \omega_{it}; \hat{\alpha}_l{}^{(s)}) f(m_{it} | k_{it}, l_{it}, \omega_{it}; \hat{\alpha}_m^{(s)})$$
$$\times f(i_{it} | k_{it}, \omega_{it}; \hat{\delta}^{(s)}) \prod_{t=2}^{T} f(\omega_{it} | \omega_{it-1}; \hat{\rho}^{(s)}) f(\omega_{i1} | k_{i1}; \hat{\rho}_{\omega_1}^{(s)})$$

2. *Maximization Step*: For $q = 1, \ldots, Q$, solve (e.g for production function)

$$\hat{\beta}(\tau_q)^{(s+1)} = \underset{\beta(\tau_q)}{\operatorname{argmin}} \sum_{i=1}^{N} \sum_{t=1}^{T} \sum_{m=1}^{M} \psi_{\tau_q} \left(y_{it} - Q_t^y(k_{it}, l_{it}, m_{it}, \omega_{it}^{(m)}; \beta(\tau_q)) \right)$$

Empirical Implementation

- $\psi_\tau(u) = (\tau - \mathbb{1}\{u < 0\})u$ is the "check" function from quantile regression
- Repeat Step 2 for estimating the productivity process, input decision rules and investment
- Take $M = 1$ in the MCEM algorithm and the report estimates as the average of the last $\tilde{S} = S/2$ draws
- This is known as the stochastic EM algorithm (stEM) of Celeux and Diebolt, 1985
- The sequence of maximizers $\hat{\theta}^{(s)}$ is a time-homogeneous Markov chain which, if ergodic, will converge to its stationary distribution
- Nielsen, 2000 provides sufficient conditions for ergodicity and asymptotic properties of the estimator when the "M-step" is solved using maximum likelihood
- Arellano and Bonhomme, 2016 discuss the asymptotic properties of the estimator when the M-step is solved using quantile regression

Empirical Implementation

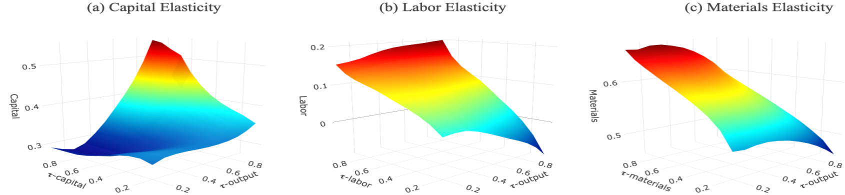
- I run the estimation procedure with 500 random walk Metropolis-Hastings steps keeping the last draw for estimation
- 200 EM steps where the average is taken over half the draws
- I take $Q = 11$ as the grid size for the interpolating spline
- Experimented with many different proposal distributions and initial values
- I use a normal distribution centered at the current draw of productivity with variance equal 0.01
- Initial values for productivity are simulated from TFP estimated from the LP model
- Replication code is available on author's Github

Application

- I apply this estimator to the same US data from Doty and Song, 2021
- I focus on more recent data past 1997
- To report my estimates, I simulate productivity data from its estimated parameters and use that to construct investment, inputs, and output using their estimated parameters
- Capital is simulated from a linear accumulation process with constant depreciation rate 0.02
- Results are not too different from reasonable specifications for the capital accumulation process
- I am interested in a variety of average and individual marginal quantile effects
- Using these estimates, I can analyze how firms react to latest shocks to production, inputs, and productivity
- How long does it take for firms to recover from bad shocks to productivity?

Production Elasticities

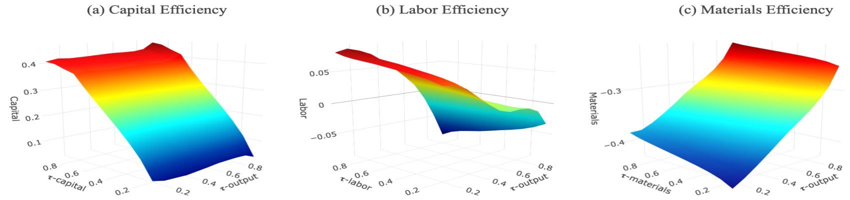
Figure 1: Output Elasticities



*Panel (a): Capital elasticity evaluated at τ_η and percentiles of capital τ_k averaged over values of (l_{it}, m_{it}) that correspond to τ_k. Panel (b): Labor elasticity evaluated at τ_η and percentiles of labor τ_l averaged over values of (k_{it}, m_{it}). Panel (c): Materials elasticity evaluated at τ_η and percentiles of materials τ_m averaged over values of (k_{it}, l_{it}).

Production Elasticities

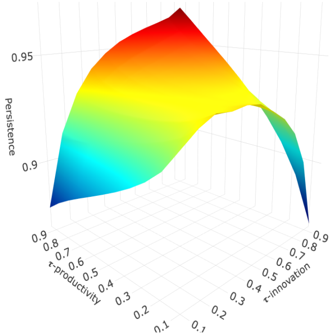
Figure 2: Non-Hicks Neutral Elasticities



*Panel (a): Capital efficiency evaluated at τ_η and percentiles of capital τ_k averaged over values of (l_{it}, m_{it}) that correspond to τ_k. Panel (b): Labor efficiency evaluated at τ_η and percentiles of labor τ_l averaged over values of (k_{it}, m_{it}). Panel (c): Materials efficiency evaluated at τ_η and percentiles of materials τ_m averaged over values of (k_{it}, l_{it}).

Productivity

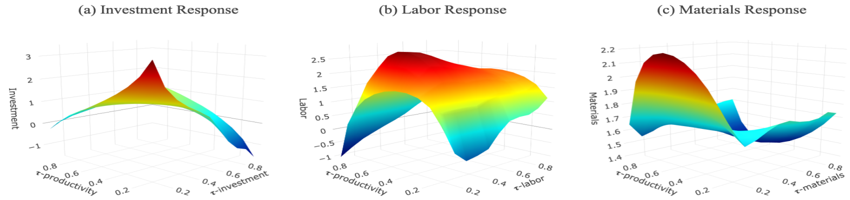
Figure 3: Productivity Persistence



*Estimates of average productivity persistence evaluated at τ_ξ and percentiles of previous productivity.

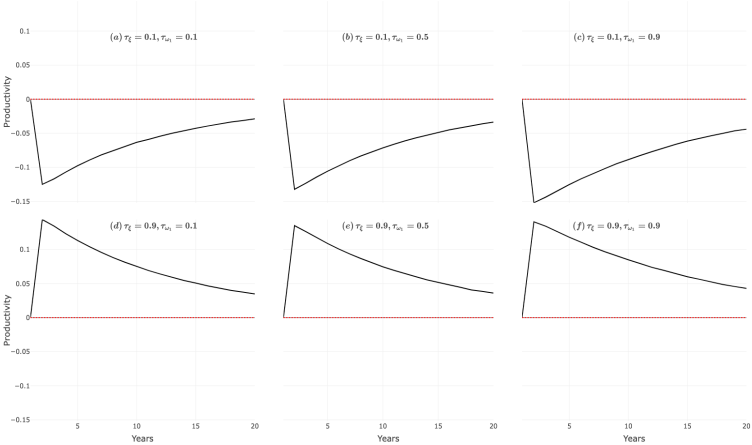
Marginal Productivities

Figure 4: Marginal Productivity of Inputs



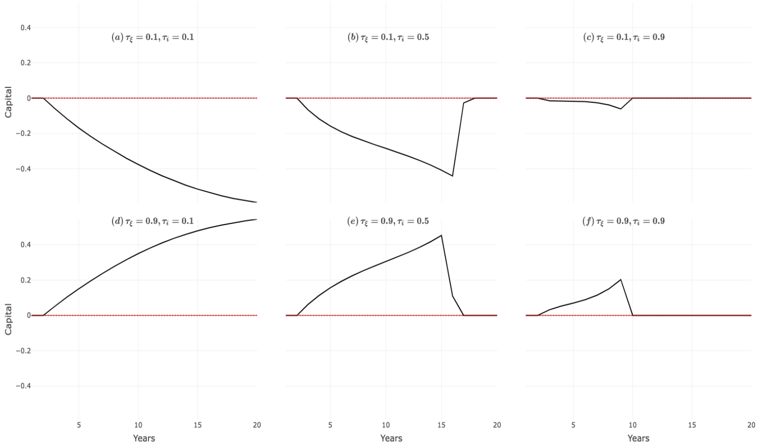
*Panel (a): Investment demand evaluated at τ_ζ and percentiles of productivity τ_ω averaged over values of k_{it}. Panel (b): Labor demand evaluated at τ_{ϵ_l} and percentiles of productivity τ_ω averaged over values of k_{it}. Panel (c): Material demand evaluated at τ_{ϵ_m} and percentiles of productivity τ_ω averaged over values of k_{it} and l_{it}

Productivity Innovation Shocks



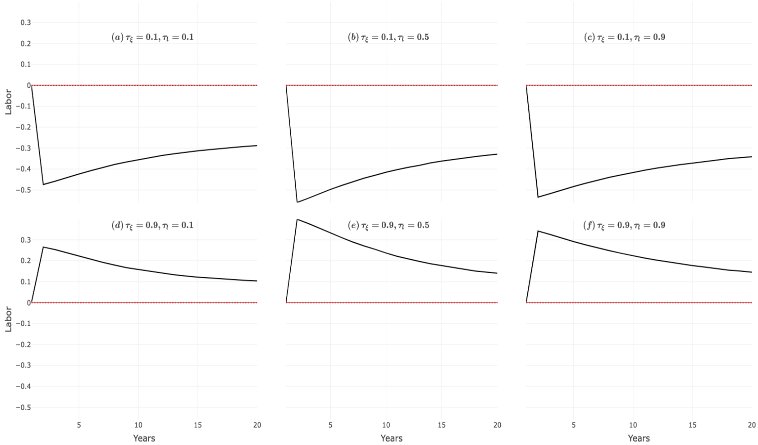
*Top row: Difference between firms hit with low productivity shock $\tau_\xi = 0.1$ and medium shock $\tau_\xi = 0.5$ at different levels of initial productivity. Bottom row: Difference between firms hit with high productivity shock $\tau_\xi = 0.9$ and medium shock $\tau_\xi = 0.5$ at different levels of initial productivity.

Productivity Innovation Shocks to Capital



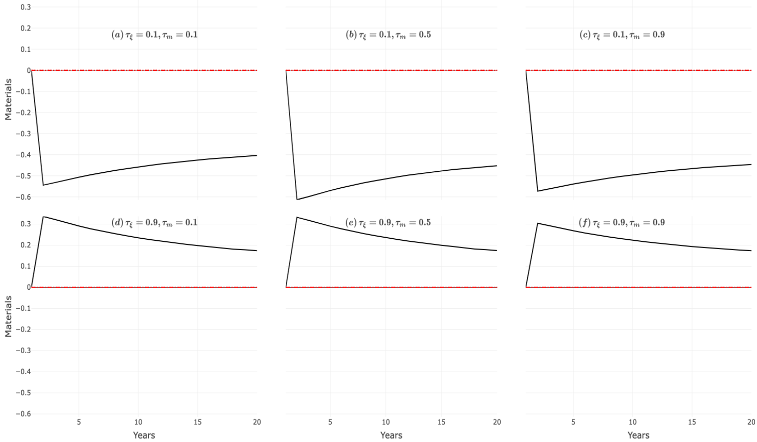
*Top row: Difference between firms hit with low productivity shock $\tau_\xi = 0.1$ and medium shock $\tau_\xi = 0.5$ at different levels of investment demand. Bottom row: Difference between firms hit with high productivity shock $\tau_\xi = 0.9$ and medium shock $\tau_\xi = 0.5$ at different levels of investment demand

Productivity Innovation Shocks to Labor



*Top row: Difference between firms hit with low productivity shock $\tau_\xi = 0.1$ and medium shock $\tau_\xi = 0.5$ at different levels of labor demand. Bottom row: Difference between firms hit with high productivity shock $\tau_\xi = 0.9$ and medium shock $\tau_\xi = 0.5$ at different levels of labor demand.

Productivity Innovation Shocks to Materials



*Top row: Difference between firms hit with low productivity shock $\tau_\xi = 0.1$ and medium shock $\tau_\xi = 0.5$ at different levels of materials demand. Bottom row: Difference between firms hit with high productivity shock $\tau_\xi = 0.9$ and medium shock $\tau_\xi = 0.5$ at different levels of materials demand.

Conclusion

- Extended the identification arguments of Hu, Huang, and Sasaki, 2020 under more general production functions and the entire distribution of productivity
- Proposed an estimation framework to document firm heterogeneity in production, input decisions, and productivity
- Asymmetric productivity persistence depends on size of innovation shock and levels of previous productivity
- Firms respond to productivity shocks by using more inputs, but this affect is asymmetric for different productivity levels and the rank of the conditional input distribution
- Asymmetric impact of innovation shocks to inputs after bad/good shocks
- Extension to multi-dimensional unobservables: fixed effects, labor-augmenting productivity
- Implications for TFP estimates? Market power?