



Automated Theorem Proving:

A Deep Learning Approach

Industrial Sponsor: Real AI

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Outline

- Introduction
- Holophrasm: The Program
 - Metamath Database
 - Proof Tree
 - The Neural Networks
- Improving Holophrasm: TFLearn
- Conclusion



Outline

- **Introduction**
- Holophrasm: The Program
 - Metamath Database
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Company Background



- Aims to develop safe and beneficial Artificial General Intelligence (AGI)
- Monitors deep learning development
- Conducts research, like Automated Theorem Proving



Proving Math Theorems ...

$A = \pi r^2$
 $C = 2\pi r$

$V = \frac{1}{3} \pi r^2 h$

$V = \pi r^2 h$

	30°	45°	60°
sin	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
cos	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
tan	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

$\int \sin x dx = -\cos x + C$
 $\int \frac{dx}{\cos^2 x} = \tan x + C$
 $\int \tan x dx = -\ln|\cos x| + C$
 $\int \frac{dx}{\sin x} = \ln\left|\frac{x}{2}\right| + C$
 $\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a}$
 $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln\left|\frac{x-a}{x+a}\right|$

$\tan(\theta)$

θ/rad

$ax^3 + bx + c = 0$
 $a(x^3 + \frac{b}{a}x + \frac{c}{a}) = 0$
 $x^3 + 2\frac{b}{2a}x + (\frac{b}{2a})^2 - (\frac{b}{2a})^2 +$
 $(x + \frac{b}{2a})^2 - \frac{b^2 - 4ac}{4a^2} = 0$



Inspiration: AlphaGo



Monte Carlo Tree Search + Artificial Neural Networks

Image Source: <https://www.theguardian.com/technology/2017/may/23/alphago-google-ai-beats-ke-jie-china-go>



Holophrasm

- Written in Python by Daniel P. Z. Whalen, in 2016
- Tree search
- Three neural networks
- Finds proofs for 14.3% of the test propositions





Objectives

- Study the codes and understand how it works
- Try to improve the program



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Metamath Database

- Over 19,000 propositions
- Built from 22 axioms



Metamath Proof Explorer Home Page

[First >](#)
[Last >](#)

[Mirrors](#) > [Home](#) > [MPE Home](#) > [Th. List](#) > [Recent](#)

The aleph null above is the symbol for the first infinite cardinal number, discovered by Georg Cantor in 1873 (see theorem [aleph0](#)).

This is the starting page for the Metamath Proof Explorer subproject (set.mm database). See the main [Metamath Home Page](#) for an overview of Metamath and download links.

Contents of this page

- [Metamath Proof Explorer Overview](#)
- [How Metamath Proofs Work](#)
- [The Axioms](#) (Propositional Calculus, Predicate Calculus, Set Theory, The Tarski-Grothendieck Axiom)
- [The Theory of Classes](#) New 13-Dec-2015
- [A Theorem Sampler](#)
- [2 + 2 = 4 Trivia](#)
- [Appendix 1: A Note on the Axioms](#)
- [Appendix 2: Traditional Textbook Axioms of Predicate Calculus](#)
- [Appendix 3: Distinct Variables](#) (History, Notes)
- Revised 21-Dec-2016
- [Appendix 4: A Note on Definitions](#)
- [Appendix 5: How to Find Out What Axioms a Proof Depends On](#)
- [Appendix 6: Notation for Function and Operation Values](#)
- [Appendix 7: Some Predicate Calculus Subsystems](#)
- [Reading Suggestions](#)
- [Bibliography](#)
- [Browsers and Fonts](#)

Related pages

- [Theorem List \(Table of Contents\)](#)
- [Most Recent Proofs \(this mirror\)](#) (latest)
- [Conventions and Style](#) New 15-Jan-2017
- [Bibliographic Cross-Reference](#)
- [Definition List \(3MB\)](#)
- [Deduction Form and Natural Deduction](#) New 7-Feb-2017 ([Natural Deduction Rules](#) New 9-Feb-2017)
- [Weak Deduction Theorem](#) (an older method)
- [Real and Complex Numbers](#)
- [ZFC Axioms With No Distinct Variables](#)
- [ASCII Symbol Equivalents for Text-Only Browsers](#)
- [Ghilbert Proof Language](#) [retrieved 21-Dec-2016]

To search this site you can use [Google](#) [retrieved 21-Dec-2016] restricted to a mirror site. For example, to find references to infinity enter "infinity site:us.metamath.org". **More efficient searching** is possible with direct use of the [Metamath program](#), once you get used to its [ASCII tokens](#). See the wildcard features in "help search" and "help show statement".



Example Proposition

- Proposition (context)

Hypotheses

Assertion

Proof

Theorem **mp2b** ¹¹

Hypotheses

Ref	Expression
mp2b.1	$\vdash \varphi$
mp2b.2	$\vdash (\varphi \rightarrow \psi)$
mp2b.3	$\vdash (\psi \rightarrow \chi)$

Assertion

Ref	Expression
mp2b	$\vdash \chi$

Proof of Theorem **mp2b**

Step	Hyp	Ref	Expression
1		mp2b.1	$\dots \vdash \varphi$
2		mp2b.2	$\dots \vdash (\varphi \rightarrow \psi)$
3	<u>1, 2</u>	<u>ax-mp</u> ¹⁰	$\dots \vdash \psi$
4		mp2b.3	$\dots \vdash (\psi \rightarrow \chi)$
5	<u>3, 4</u>	<u>ax-mp</u> ¹⁰	$\vdash \chi$



Tautological Proposition

- Proposition (context)

Hypotheses

Assertion

Proof

Theorem **2p2e4** 9811

Assertion

Ref	Expression
2p2e4	$\vdash (2 + 2) = 4$

Proof of Theorem **2p2e4**

Step	Hyp	Ref	Expression
1		df-2 <small>9773</small>	$\dots 3 \vdash 2 = (1 + 1)$
2	1	oveq2i <small>5804</small>	$\dots 2 \vdash (2 + 2) = (2 + (1 + 1))$
3		df-4 <small>9775</small>	$\dots 3 \vdash 4 = (3 + 1)$
4		df-3 <small>9774</small>	$\dots 4 \vdash 3 = (2 + 1)$
5	4	oveq1i <small>5803</small>	$\dots 3 \vdash (3 + 1) = ((2 + 1) + 1)$
6		2cn <small>9785</small>	$\dots 4 \vdash 2 \in \mathbb{C}$
7		ax-1cn <small>8764</small>	$\dots 4 \vdash 1 \in \mathbb{C}$
8	6, 7, 7	addassi <small>8814</small>	$\dots 3 \vdash ((2 + 1) + 1) = (2 + (1 + 1))$
9	3, 5, 8	3eqtri <small>2282</small>	$\dots 2 \vdash 4 = (2 + (1 + 1))$
10	2, 9	eqtr4i <small>2281</small>	$\vdash (2 + 2) = 4$



Axiomatic Proposition

- Proposition (context)

Hypotheses

Assertion

Proof

Axiom **ax-mp** 10

Hypotheses

Ref	Expression
min	$\vdash \varphi$
maj	$\vdash (\varphi \rightarrow \psi)$

Assertion

Ref	Expression
ax-mp	$\vdash \psi$



Proof Step

Type I:

A context hypothesis

- No theorem is applied

Theorem **mp2b** ¹¹

Hypotheses

Ref	Expression
mp2b.1	$\vdash \varphi$
mp2b.2	$\vdash (\varphi \rightarrow \psi)$
mp2b.3	$\vdash (\psi \rightarrow \chi)$

Assertion

Ref	Expression
mp2b	$\vdash \chi$

Proof of Theorem **mp2b**

Step	Hyp	Ref	Expression
1		mp2b.1	$\dots 3 \vdash \varphi$
2		mp2b.2	$\dots 3 \vdash (\varphi \rightarrow \psi)$
3	<u>1, 2</u>	<u>ax-mp</u> ¹⁰	$\dots 2 \vdash \psi$
4		mp2b.3	$\dots 2 \vdash (\psi \rightarrow \chi)$
5	<u>3, 4</u>	<u>ax-mp</u> ¹⁰	$\vdash \chi$



Proof Step

Type II:

An assertion a

- A theorem is applied
- Usually entailed by previous step(s)

Theorem mp2b ¹¹

Hypotheses

Ref	Expression
mp2b.1	$\vdash \varphi$
mp2b.2	$\vdash (\varphi \rightarrow \psi)$
mp2b.3	$\vdash (\psi \rightarrow \chi)$

Assertion

Ref	Expression
mp2b	$\vdash \chi$

Proof of Theorem mp2b

Step	Hyp	Ref	Expression
1		mp2b.1	$\vdash \varphi$
2		mp2b.2	$\vdash (\varphi \rightarrow \psi)$
3	1, 2	ax-mp ¹⁰	$\vdash \psi$
4		mp2b.3	$\vdash (\psi \rightarrow \chi)$
5	3, 4	ax-mp ¹⁰	$\vdash \chi$



Substitutions

Notations:

C : Context

a : A Type II step in C

T : Theorem used to derive a

a_T : Assertion of T

e_T : Hypotheses of T



Substitutions

ϕ : A set of substitutions satisfying $\phi(a_T) = a$.

Hypotheses	
Ref	Expression
mp2b.1	$\vdash \varphi$
mp2b.2	$\vdash (\varphi \rightarrow \psi)$
mp2b.3	$\vdash (\psi \rightarrow \chi)$
Assertion	
Ref	Expression
mp2b	$\vdash \chi$

Proof of Theorem mp2b

Step	Hyp	Ref	Expression
1		mp2b.1	$\vdash \varphi$
2		mp2b.2	$\vdash (\varphi \rightarrow \psi)$
3	1, 2	ax-mp 10	$\vdash \psi$
4		mp2b.3	$\vdash (\psi \rightarrow \chi)$
5	3, 4	ax-mp 10	$\vdash \chi$

Context C

Axiom ax-mp 10

Hypotheses	
Ref	Expression
min	$\vdash \varphi$
maj	$\vdash (\varphi \rightarrow \psi)$
Assertion	
Ref	Expression
ax-mp	$\vdash \psi$

a_T

Detailed syntax
breakdown of Axiom ax-
mp

Step	Hyp	Ref	Expression
1		wps 1	wff ψ

Theorem T



Substitutions

- Constrained variables (cv):
 - appear in a_T
 - Unconstrained variables (uv):
 - not appear in a_T
-
- Constrained substitutions:
 - replaces a cv
 - Unconstrained substitutions:
 - replaces a uv

Hypotheses	
Ref	Expression
mp2b.1	$\vdash \varphi$
mp2b.2	$\vdash (\varphi \rightarrow \psi)$
mp2b.3	$\vdash (\psi \rightarrow \chi)$
Assertion	
Ref	Expression
mp2b	$\vdash \chi$

Proof of Theorem mp2b			
Step	Hyp	Ref	Expression
1		mp2b.1	$\vdash \varphi$
2		mp2b.2	$\vdash (\varphi \rightarrow \psi)$
3	1, 2	ax-mp 10	$\vdash \psi$
4		mp2b.3	$\vdash (\psi \rightarrow \chi)$
5	3, 4	ax-mp 10	$\vdash \chi$

Context C

Axiom ax-mp 10

Hypotheses	
Ref	Expression
min	$\vdash \varphi$
maj	$\vdash (\varphi \rightarrow \psi)$
Assertion	
Ref	Expression
ax-mp	$\vdash \psi$

Detailed syntax
breakdown of Axiom ax-
mp

Step	Hyp	Ref	Expression
1		wps	$\vdash \text{wff } \psi$

Theorem T



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Proof Tree

Every Metamath proof has a tree structure.



Constructing a Proof Tree

Theorem **mp2b** ¹¹

Hypotheses

Ref	Expression
mp2b.1	$\vdash \varphi$
mp2b.2	$\vdash (\varphi \rightarrow \psi)$
mp2b.3	$\vdash (\psi \rightarrow \chi)$

Assertion

Ref	Expression
mp2b	$\vdash \chi$

Proof of Theorem **mp2b**

Step	Hyp	Ref	Expression
1		mp2b.1	$\dots \vdash \varphi$
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3	<u>1, 2</u>	<u>ax-mp</u> ¹⁰	$\dots \vdash \psi$
4		mp2b.3	$\dots \vdash (\psi \rightarrow \chi)$
5	<u>3, 4</u>	<u>ax-mp</u> ¹⁰	$\vdash \chi$



Constructing a Proof Tree

$\vdash \chi$

Theorem **mp2b** ¹¹

Hypotheses

Ref	Expression
mp2b.1	$\vdash \varphi$
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mp2b.3	$\vdash (\psi \rightarrow \chi)$

Assertion

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mp2b	$\vdash \chi$

Proof of Theorem **mp2b**

Step	Hyp	Ref	Expression
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4		mp2b.3	$\vdash (\psi \rightarrow \chi)$
5	<u>3, 4</u>	<u>ax-mp</u> ¹⁰	$\vdash \chi$



Constructing a Proof Tree

$\vdash \chi$

Axiom **ax-mp** ¹⁰

Hypotheses

Ref	Expression
min	$\vdash \varphi$
maj	$\vdash (\varphi \rightarrow \psi)$

Assertion

Ref	Expression
ax-mp	$\vdash \psi$

Theorem **mp2b** ¹¹

Hypotheses

Ref	Expression
mp2b.1	$\vdash \varphi$
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mp2b.3	$\vdash (\psi \rightarrow \chi)$

Assertion

Ref	Expression
mp2b	$\vdash \chi$

Proof of Theorem **mp2b**

Step	Hyp	Ref	Expression
1		mp2b.1	$\vdash \varphi$
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5	3, 4	ax-mp ¹⁰	$\vdash \chi$



Constructing a Proof Tree

$\vdash \chi$

Substitutions

Replace ψ by χ
Replace φ by ψ

Axiom **ax-mp** ¹⁰

Hypotheses

Ref	Expression
min	$\vdash \varphi$
maj	$\vdash (\varphi \rightarrow \psi)$

Assertion

Ref	Expression
ax-mp	$\vdash \psi$

Theorem **mp2b** ¹¹

Hypotheses

Ref	Expression
mp2b.1	$\vdash \varphi$
mp2b.2	$\vdash (\varphi \rightarrow \psi)$
mp2b.3	$\vdash (\psi \rightarrow \chi)$

Assertion

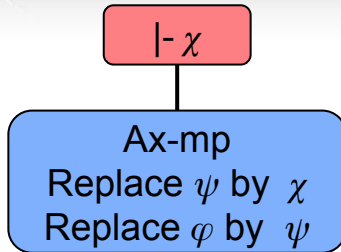
Ref	Expression
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Proof of Theorem **mp2b**

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Constructing a Proof Tree



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Assertion

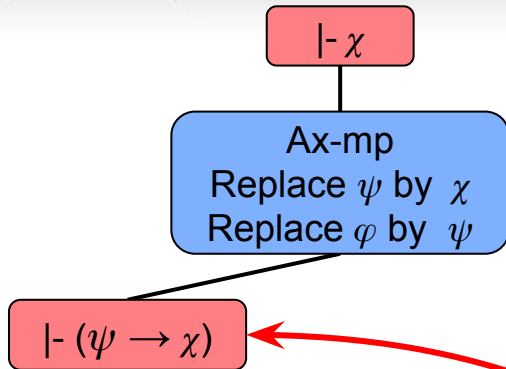
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Proof of Theorem **mp2b**

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Constructing a Proof Tree



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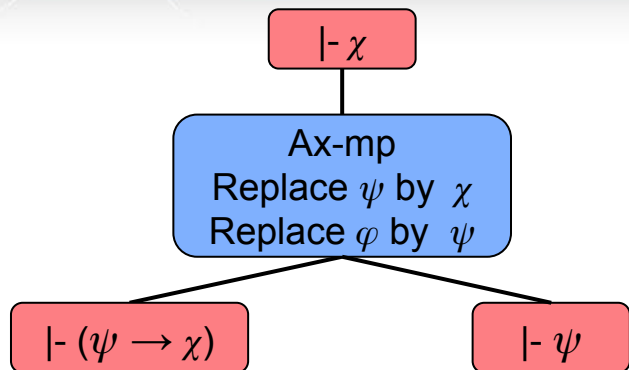
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Constructing a Proof Tree



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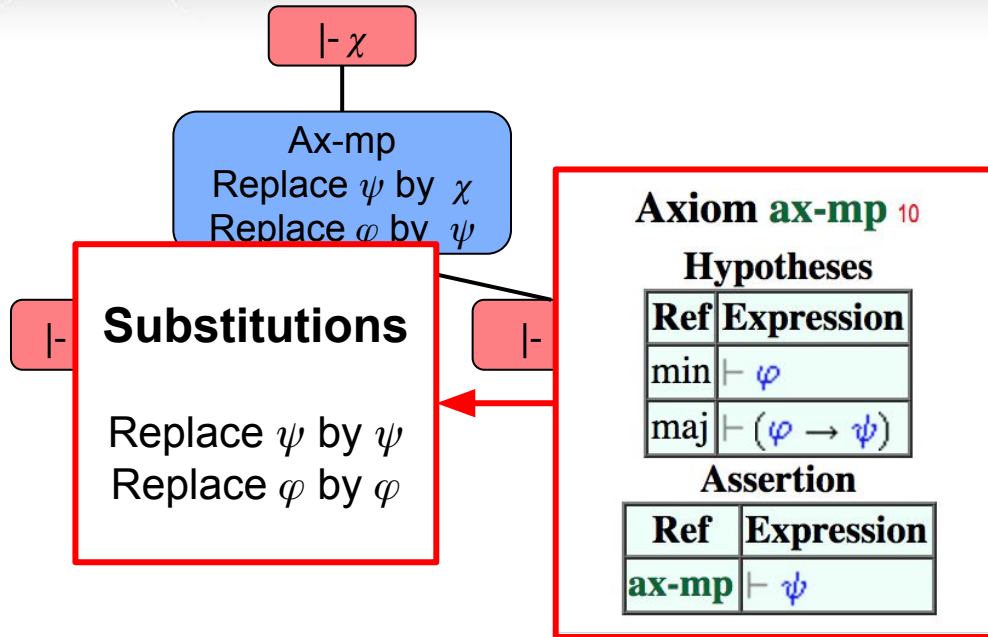
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Constructing a Proof Tree



Theorem **mp2b** 11

Hypotheses

Ref	Expression
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mp2b.3	$\vdash (\psi \rightarrow \chi)$

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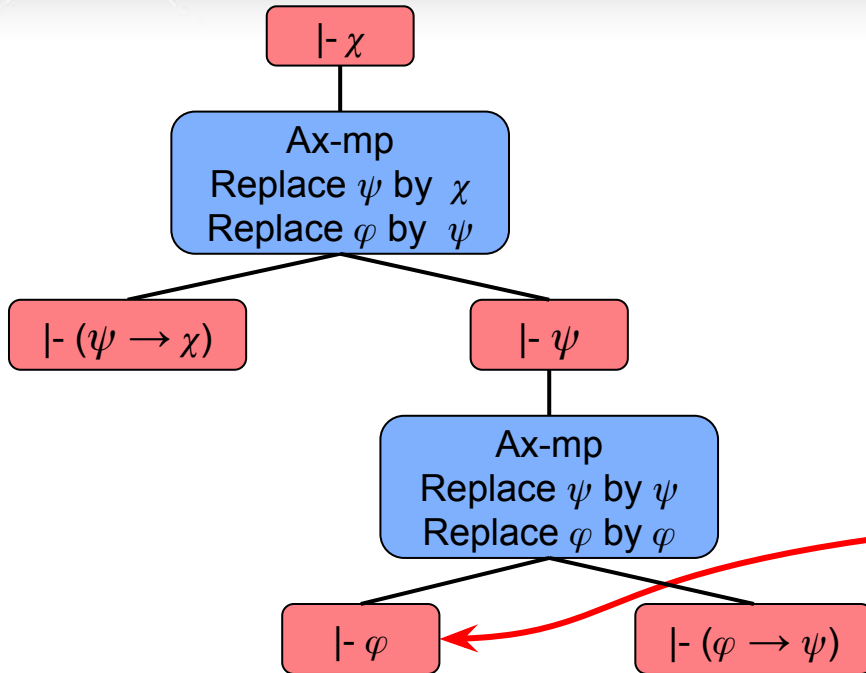
Ref	Expression
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Proof of Theorem **mp2b**

Step	Hyp	Ref	Expression
1		mp2b.1	$\vdash \varphi$
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3	1, 2	ax-mp 10	$\vdash \psi$
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5	3, 4	ax-mp 10	$\vdash \chi$



Constructing a Proof Tree



Theorem **mp2b** ¹¹

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Ref	Expression
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mp2b.3	$\vdash (\psi \rightarrow \chi)$

Assertion

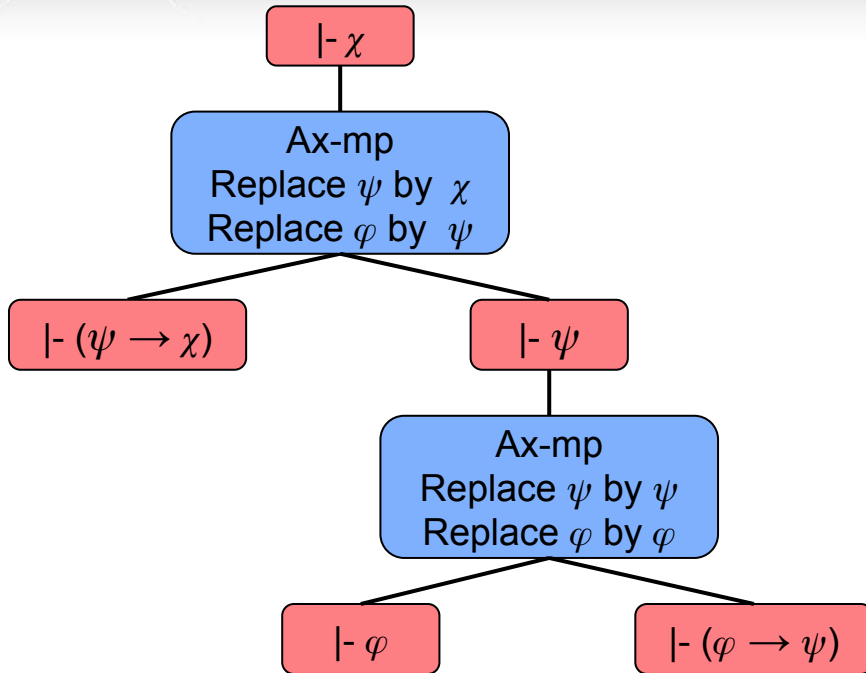
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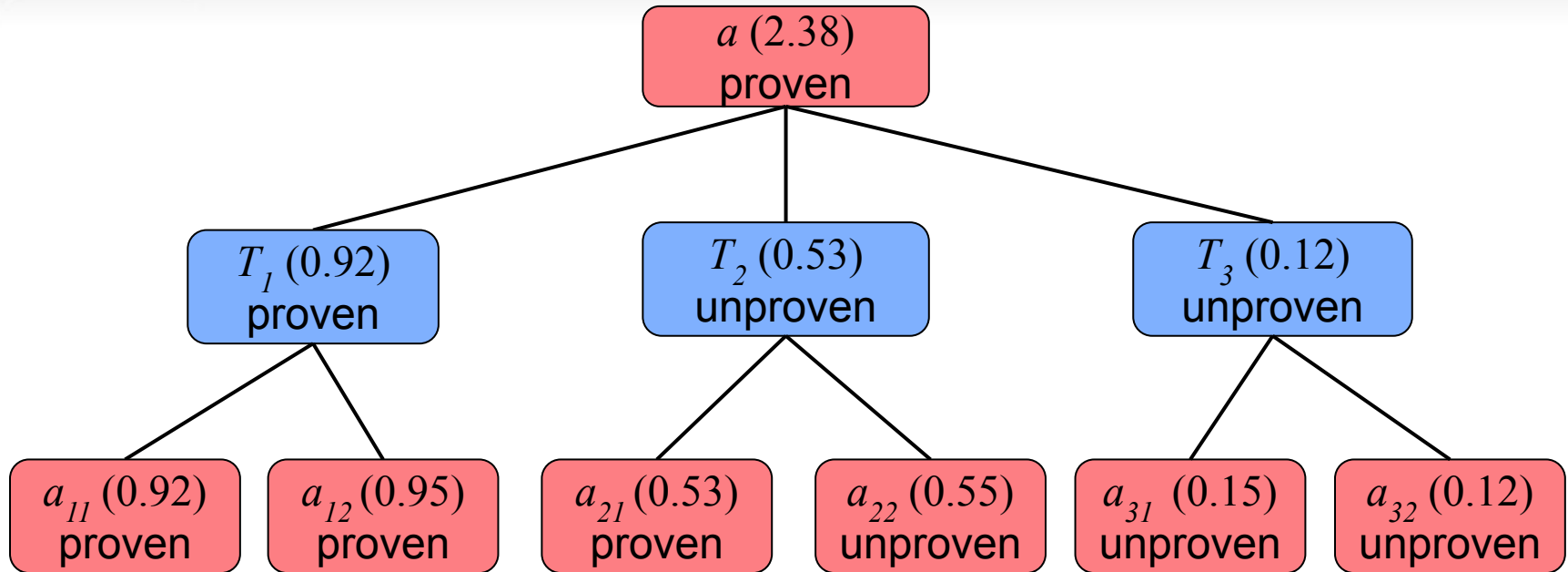
Constructing a Proof Tree



- A **red** node:
 - A statement
 - Has 0 or 1 child
- A **blue** node:
 - A theorem T
 - Parent: assertion $\phi(a_T)$
 - Children: hypotheses $\phi(e_T)$



Partial Proof Tree (PPT)





Partial Proof Tree (PPT)

- An expansion of a proof tree (if any)
- Thus, the PPT of a proven red node can be trimmed down to a standard proof tree.



Tree Search

- Suppose we want to prove a context C given:
 - its assertion a_C
 - A set of hypotheses

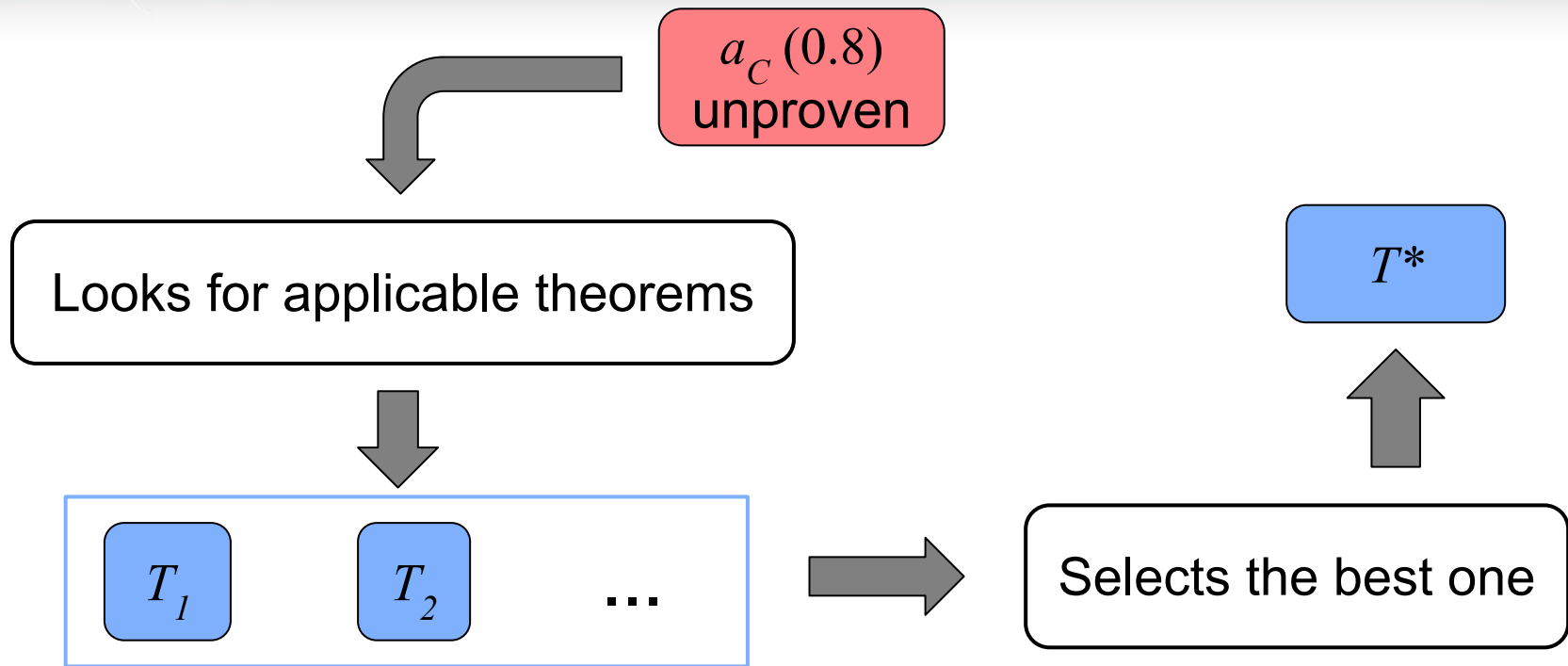


Tree Search: First Pass

$a_c(0.8)$
unproven

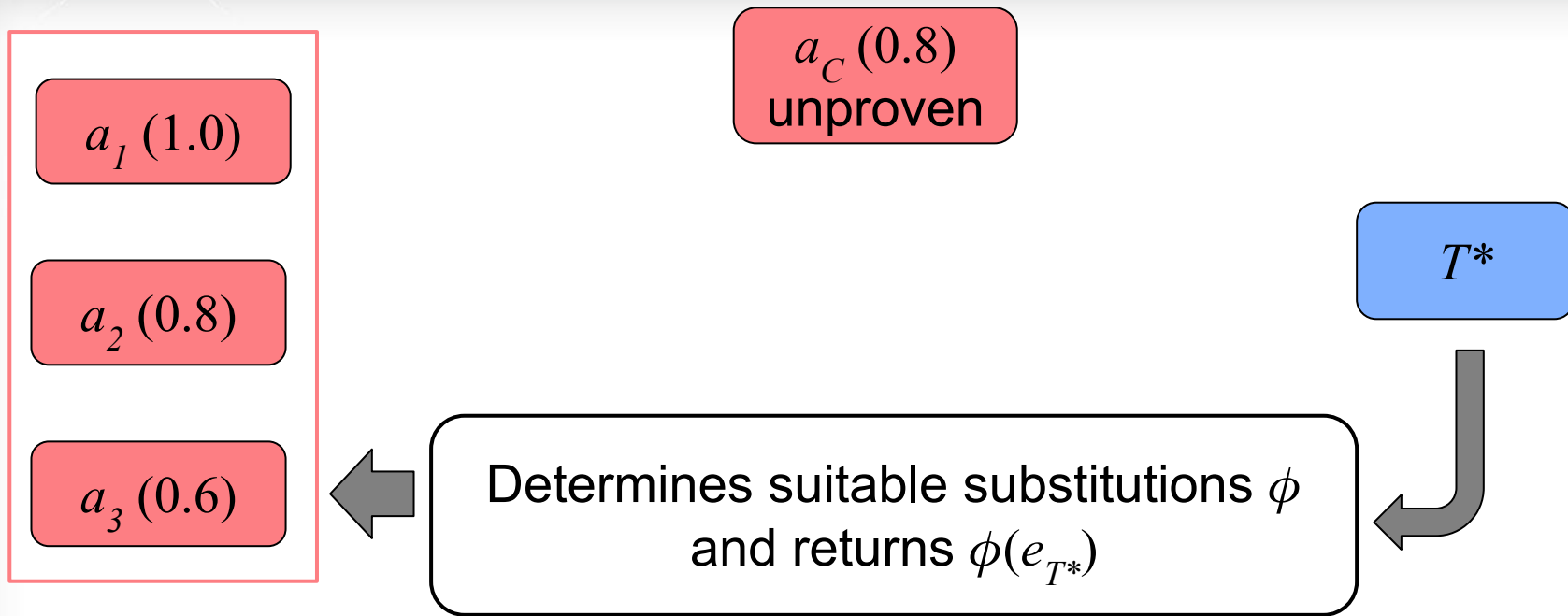


Tree Search: First Pass



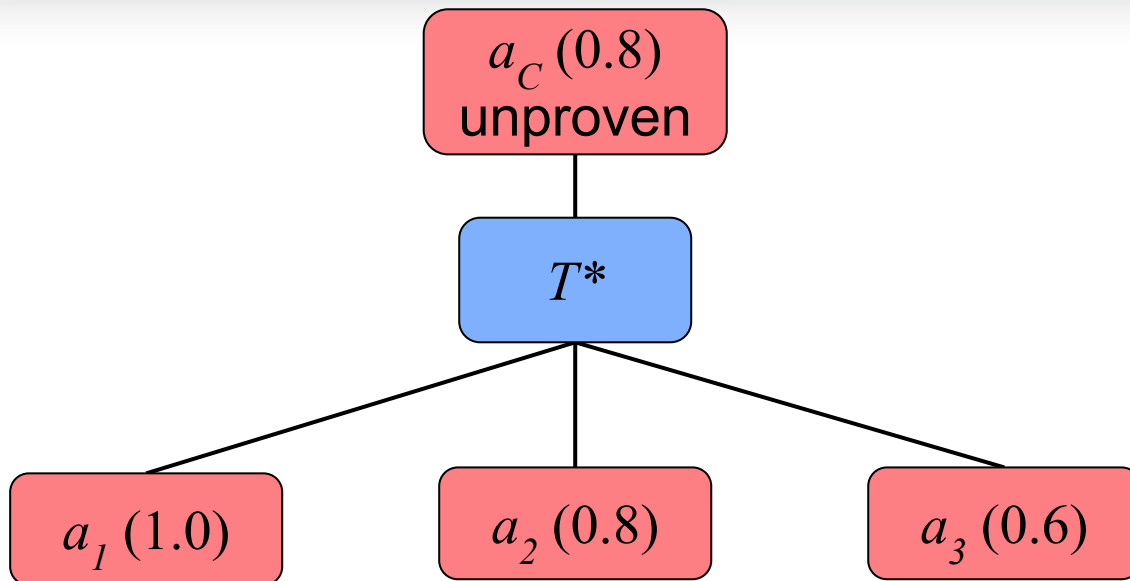


Tree Search: First Pass



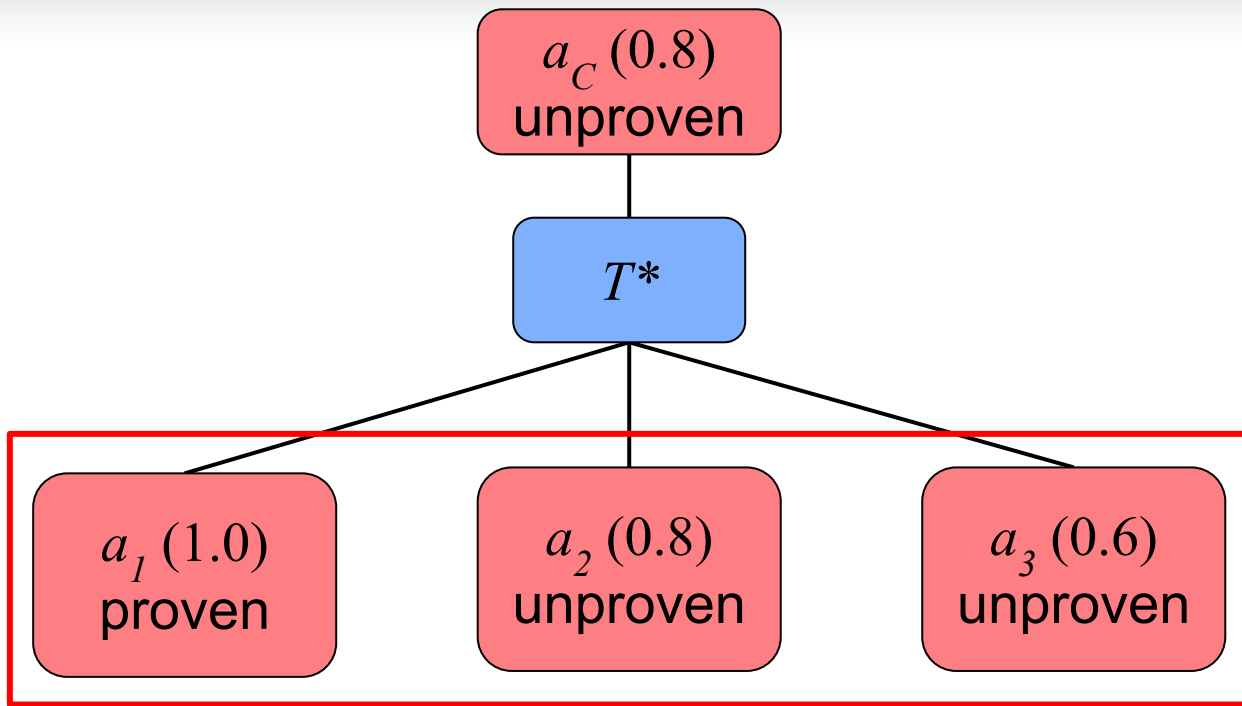


Tree Search: First Pass



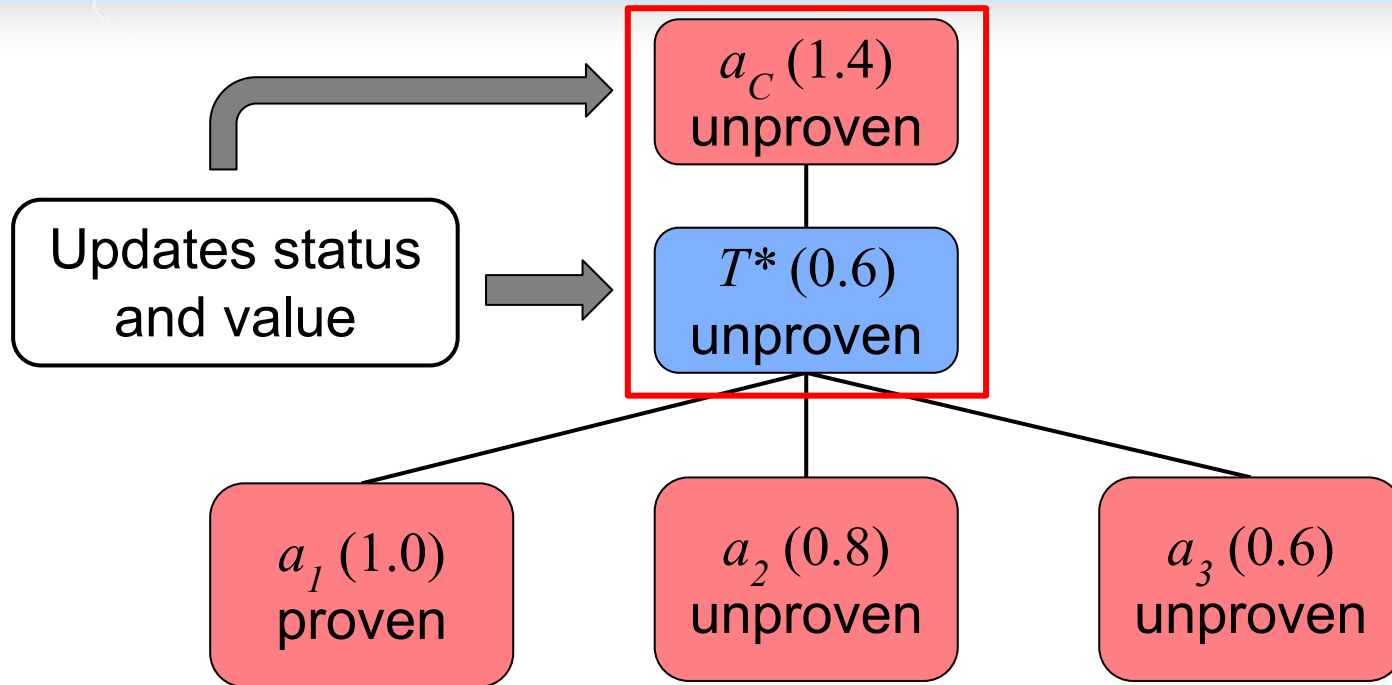


Tree Search: First Pass





Tree Search: First Pass

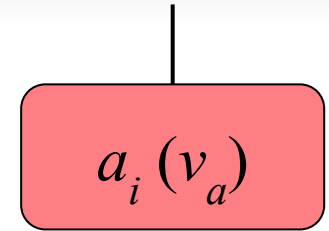




Subsequent Passes: Red Nodes

No child blue node:

- Attempts to create one
- If cannot create one,
 - marks node as dead
 - ends current pass

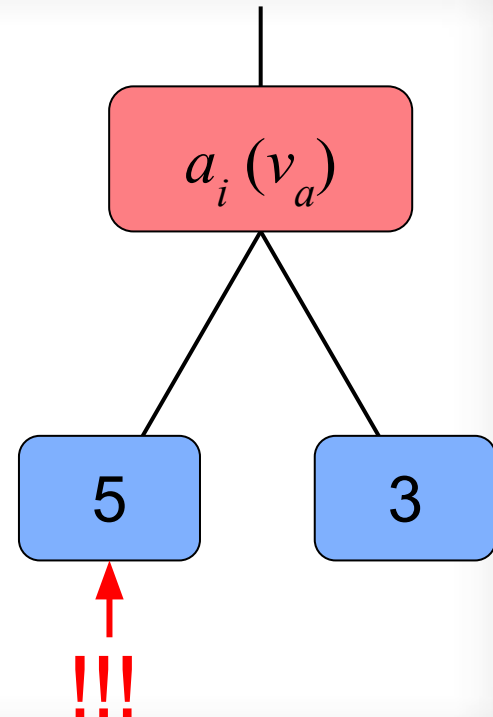




Subsequent Passes: Red Nodes

At least one child blue node:

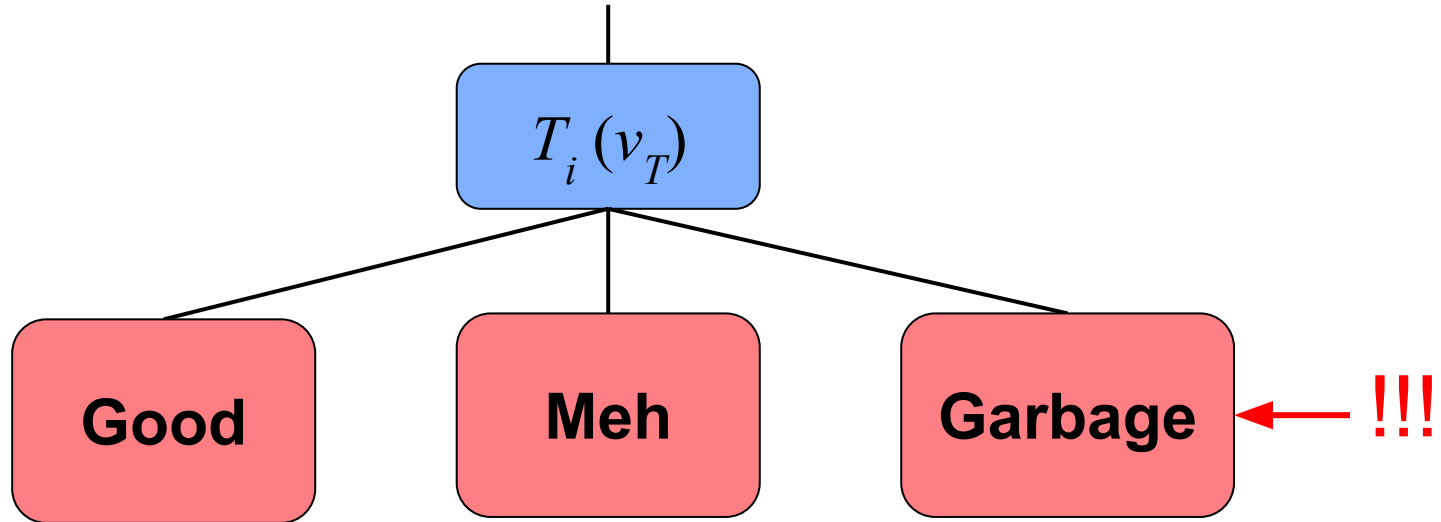
- Creates a new child blue node;
- or
- Visits the child node with the highest score





Subsequent Passes: Blue Nodes

Immediately visits its worst child red node

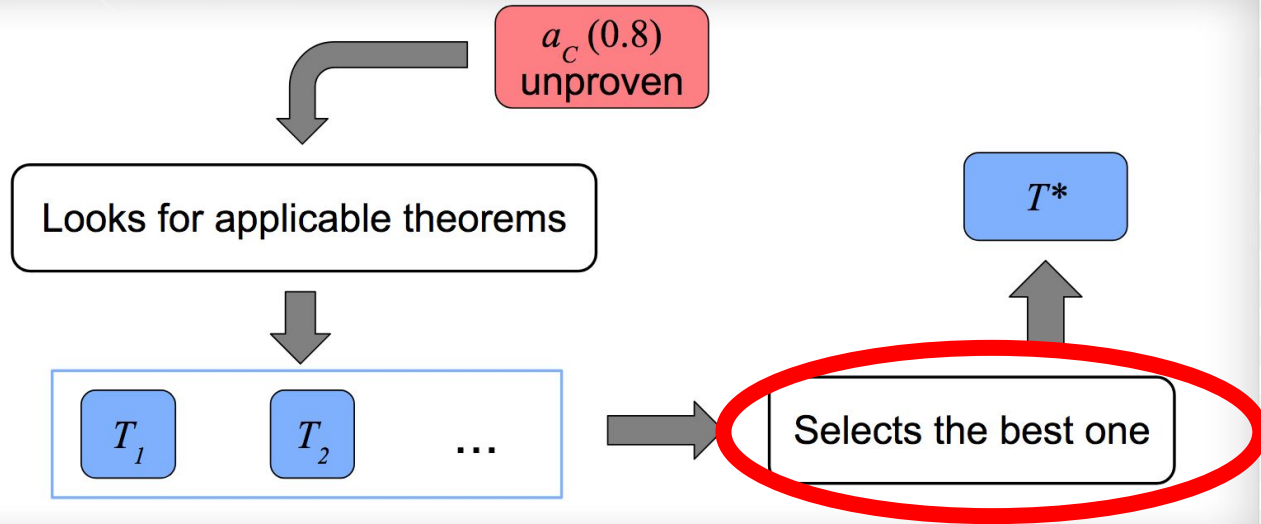




Challenges



Tree Search: First Pass



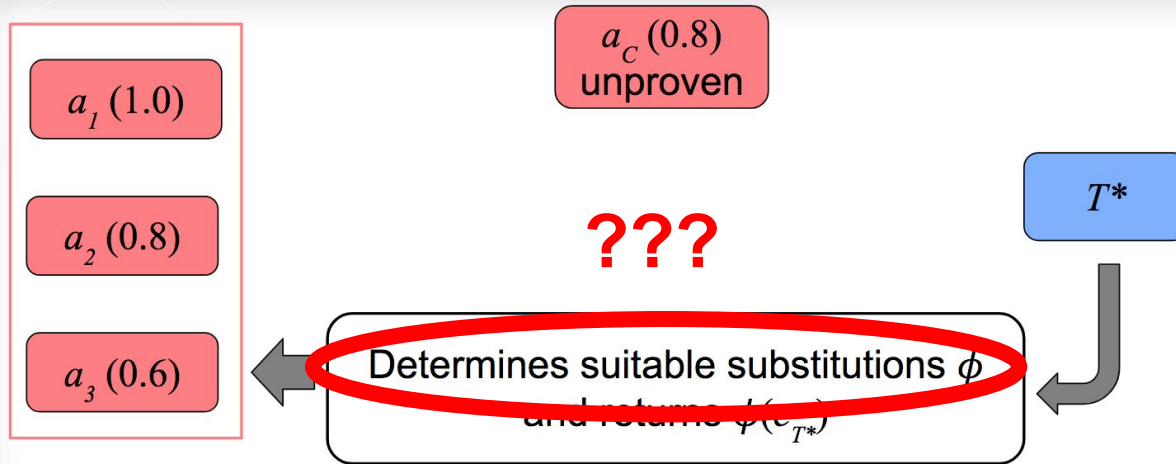
???



Challenges



Tree Search: First Pass





Challenges



Tree Search: First Pass

$a_c(0.8)$
unproven

???



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Holophrasm

- Given a theorem, find a proof tree using deep learning.
- 3 neural networks
 - Relevance
 - Generative
 - Payoff
- All networks take assertions and hypotheses as inputs
 - Need to translate into vectors



Data preprocessing

- All assertions and hypotheses have tree structure

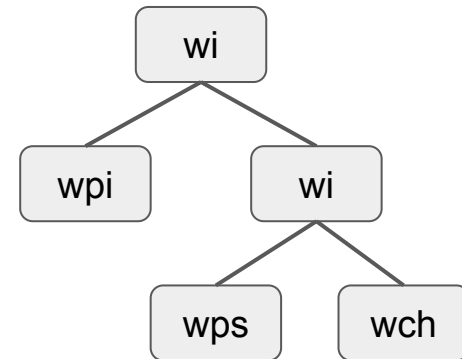
Assertion

$$\varphi \rightarrow (\psi \rightarrow \chi)$$



Tree structure

$wi(wpi(), wi(wps(), wch()))$

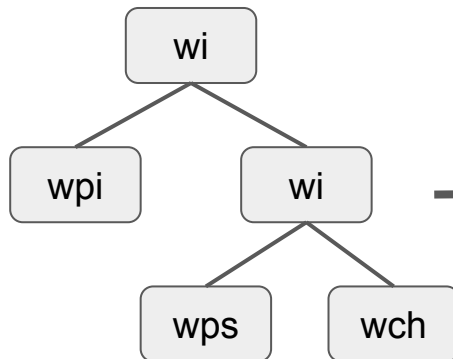




Data preprocessing

Tree structure

`wi(wpi(), wi(wps(), wch()))`



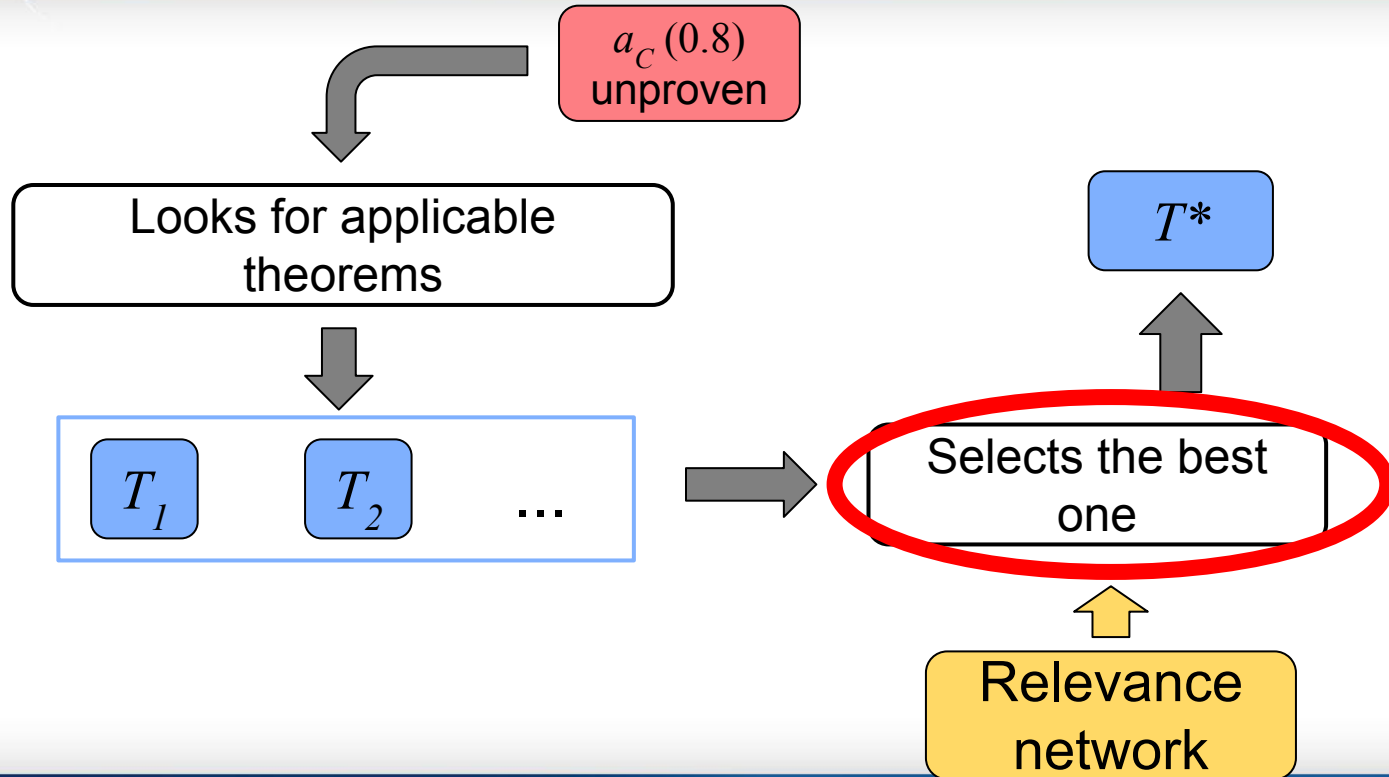
0	1	0	1	3
1	5	1	2	1
4	3	3	2	2
2	4	3	3	4
1	1	1	4	4
4	5	1	1	2
.
.
3	1	1	3	4
2	1	2	2	2
3	3	3	3	3
1	4	1	1	1
2	1	2	2	2

The content
of a node

The structure
data of a node



Relevance network



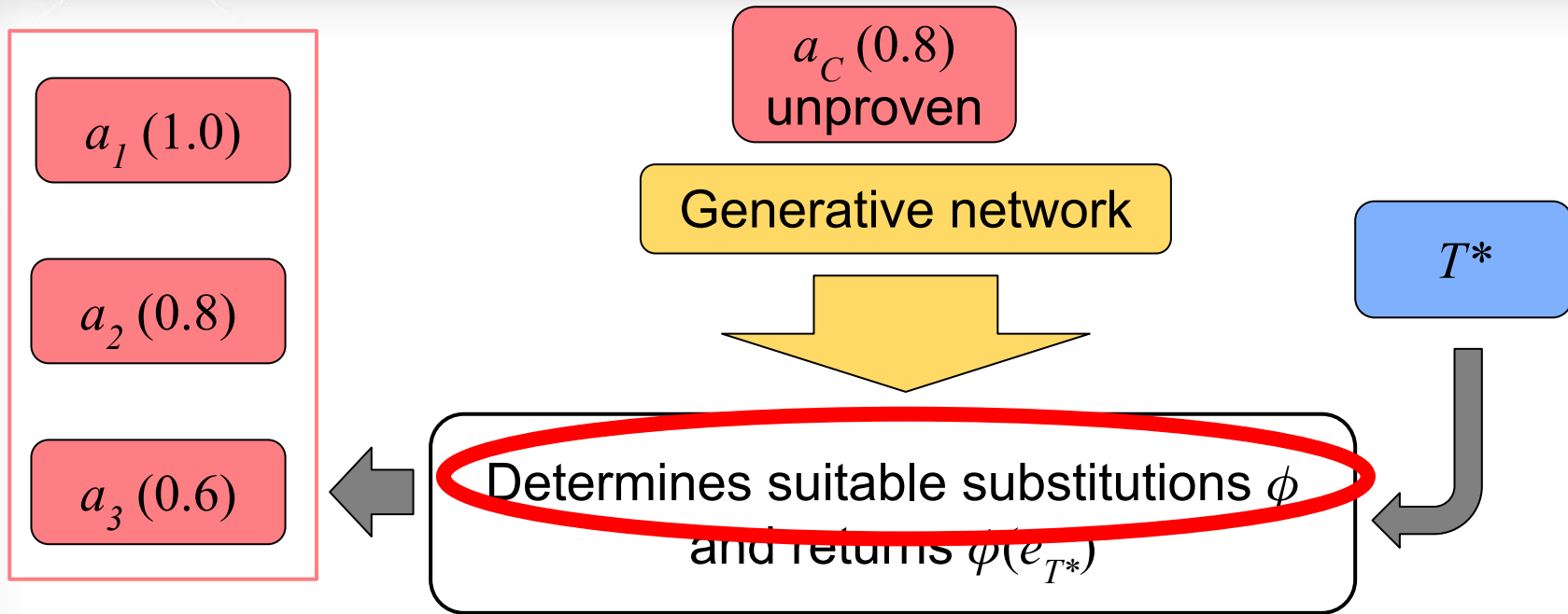


Relevance network

- Inputs
 - An assertion
 - A set of hypotheses of the context
- Output
 - Probabilities of theorems to be used in the node
- Uses RNN with GRU blocks



Generative network



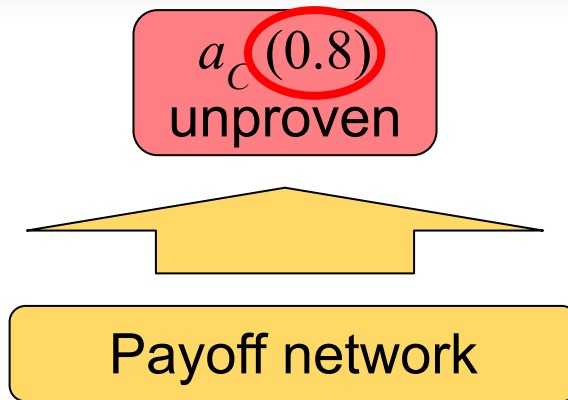


Generative network

- Inputs
 - A set of hypotheses of a proposition
 - A set of hypotheses of the context
- Outputs
 - Set of substitutions
 - Probability of substitutions to be used
- Uses Sequence to Sequence Model with GRU blocks
 - Attention



Payoff network





Payoff network

- Inputs
 - An assertion
 - A set of hypotheses of the context
- Outputs
 - Probability that assertion can be proved
- Uses RNN with GRU block
 - Bidirectional network



Outline

- Introduction
- Holophrasm: The Program
 - Metamath: A Formal Language
 - Proof Tree
 - The Neural Networks
- **Improving Holophrasm: TFLearn**
- Conclusion



TensorFlow

- He used his own library
 - Difficult to understand
- Need to replace the code with well-known library!

```
def GRUCell(hin, x, GRUparams, graph, dropout=None):
    hx = ConcatNode([hin, x], graph)
    # print hin.value.shape, x.value.shape
    # print hx.value.shape, GRUparams.Wz.value.shape
    z = DotNode(hx, GRUparams.Wz, graph)
    z = AddNode([z, GRUparams.bz], graph)
    z = SigmoidNode(z, graph)
    r = DotNode(hx, GRUparams.Wr, graph)
    r = AddNode([r, GRUparams.br], graph)
    r = SigmoidNode(r, graph)
    rh = TensorMultiplyNode(r, hin, graph)
    rhx = ConcatNode([rh, x], graph)
    htilde = DotNode(rhx, GRUparams.W, graph)
    htilde = TanhNode(htilde, graph)
    htilde = DropoutNode(htilde, dropout, graph)

    negz = MultiplyNode(-1.0, z, graph)
    negz = ScalarAddNode(1, negz, graph)

    negzh = TensorMultiplyNode(negz, hin, graph)
    zhtilde = TensorMultiplyNode(z, htilde, graph)

    hout = AddNode([negzh, zhtilde], graph)

    return hout
```


Companies using TensorFlow



Build History trend	
find	x
#259	Aug 2, 2017 2:25 AM
#258	Aug 1, 2017 2:25 AM
#257	Jul 31, 2017 12:38 PM
#256	Jul 31, 2017 2:25 AM
#255	Jul 30, 2017 2:25 AM
#254	Jul 29, 2017 2:25 AM
#253	Jul 28, 2017 2:25 AM
#252	Jul 27, 2017 2:25 AM
#251	Jul 26, 2017 2:25 AM
#250	Jul 25, 2017 2:25 AM
#249	Jul 24, 2017 2:25 AM
#248	Jul 23, 2017 2:25 AM
#247	Jul 22, 2017 2:25 AM
#246	Jul 21, 2017 2:25 AM
#245	Jul 20, 2017 2:25 AM
#244	Jul 19, 2017 2:25 AM
#243	Jul 18, 2017 2:25 AM
#242	Jul 17, 2017 2:25 AM
#241	Jul 16, 2017 4:35 AM
#240	Jul 16, 2017 12:20 AM
#239	Jul 15, 2017 2:25 AM
#238	Jul 14, 2017 2:25 AM
#237	Jul 13, 2017 2:25 AM
#236	Jul 12, 2017 2:25 AM
#235	Jul 11, 2017 2:25 AM

istent
ole n

adversarial_crypto	Comment wrong in adversarial_crypto model	a month ago
adversarial_text	Fix KL when num_classes != 2 (#1820)	a month ago
attention_ocr	Add ./ to --checkpoint_inception (#1951)	16 days ago
autoencoder	Variational Autoencoder generate() function fixed (z fed in rather th...	4 months ago
cognitive_mapping_and_planning	Update README.md	18 days ago
compression	Update username and add new line in READMEs.	2 months ago
differential_privacy	typo fix	2 months ago
domain_adaptation	Open sourcing PixelDA code	13 days ago
im2txt	Fix formatting	15 days ago
inception	Fixed error message for inception/imagenet.	2 months ago
learning_to_remember_rare_events	Convert tf.op_scope to tf.name_scope, plus a few other 1.0 upgrade ch...	5 months ago
lfads	Removing redundant print statement	6 days ago
lm_1b	Import xrange directly from six.moves for lm_1b	3 months ago
namignizer	Merge pull request #924 from h4ck3rm1k3/master	5 months ago
neural_gpu	Fix arxiv links in README of neural_gpu model	2 months ago
neural_programmer	Merge pull request #924 from h4ck3rm1k3/master	5 months ago
next_frame_prediction	Improvements to several READMEs	3 months ago
object_detection	Set training step limits on pets configs.	5 days ago
pcl_rl	code for running RL agents with various algs	28 days ago
ptn	Added training details comment.	14 days ago
real_nvp	change no-longer-existing concat_v2 to concat (#1701)	a month ago
rebar	Update README.md	9 days ago
resnet	Offset of image bytes for cifar100 should be 2	9 days ago
skip_thoughts	Fix Bazel incantations in docs	2 months ago
slim	Merge pull request #1800 from txizzle/master	6 days ago



TensorFlow

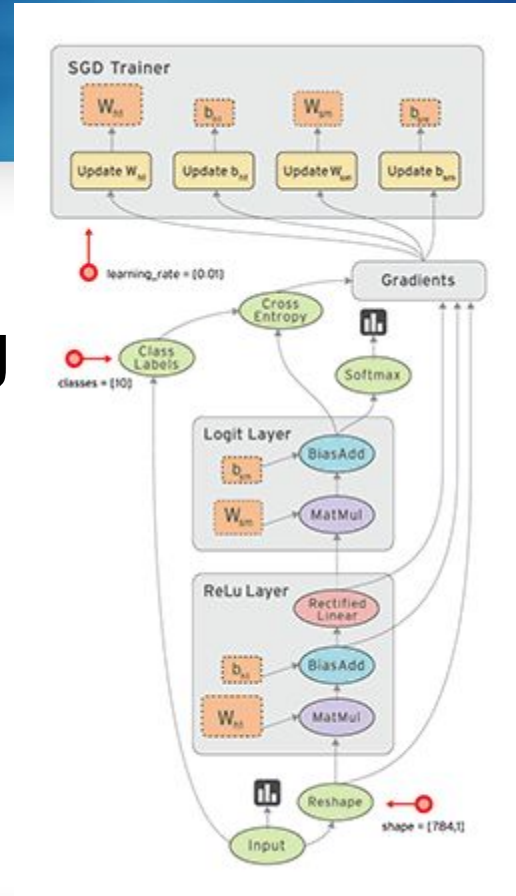
- GPU, Multithreading, Distributed training

```
train object.run many epochs/language model. plot every=10000.  
# Creates a session with log device placement set to True.  
sess = tf.Session(config=tf.ConfigProto(log_device_placement=True))  
                batch_size=100, multiprocessing=8,  
                reorder=False)  
sess = tf.Session(config=tf.ConfigProto(  
    intra_op_parallelism_threads=NUM_THREADS))  
    process before the  
    current process has finished its bootstrapping phase.  
  
    This probably means that you are not using fork to start your  
    child processes and you have forgotten to use the proper idiom  
>>> server = tf.train.Server.create_local_server()  
>>> sess = tf.Session(server.target) # Create a session on the server.  
    if __name__ == '__main__':  
        freeze_support()  
        ...  
  
    The "freeze_support()" line can be omitted if the program  
    is not going to be frozen to produce an executable.
```



TensorFlow

- TensorBoard
 - Visualization of Deep Learning
 - Graph, Loss decrease, etc.





TensorFlow

- Unfortunately, we are all beginners in developing deep learning.
- Even TensorFlow is too difficult to use!
- We don't have time!
- We need easier one.



TFLearn

- Developed by Aymeric Damien, available at tflearn.org
- Source code : [Github.com/tflearn/tflearn](https://github.com/tflearn/tflearn)
- Higher-level API of TensorFlow



TFLearn

- Very easy, short, and intuitive

```
110
111 main_net = tflearn.fully_connected(main_net, output_dim, weights_init='xavier')
112 main_net = tflearn.relu(main_net)
113 main_net = tflearn.fully_connected(main_net, output_dim, weights_init='xavier')
114 main_net = tflearn.relu(main_net)
115 main_net = tflearn.dropout(main_net, p)
116
```

Layers

Core Layers

Convolutional Layers

Recurrent Layers

Normalization Layers

Embedding Layers

Merge Layers

Estimator Layers



TFLearn

- Automatically detects GPU

```
# Creates a session with GPU acceleration set to True.  
sess = tf.Session(config=tf.ConfigProto(device_placement=True))
```

No extra code for using GPU!



TFLearn

- Easy construction of TensorBoard

```
# Create other useful summary (weights, grads, activations...)  
# according to 'tensorboard_verbose' level.  
self.create_summaries(tensorboard_verbose)
```

TFLearn library built-in code

```
model = tflearn.DNN(net, tensorboard_verbose=3)
```

What we need for constructing Tensorboard



TFLearn

- Full transparency over TensorFlow
 - If you want models not in TFLearn, you can build it with TensorFlow and TFLearn together!
 - Example :
https://github.com/tflearn/tflearn/blob/master/examples/nlp/seq2seq_example.py



TFLearn

- We implemented TFLearn into Relevance network! (but incomplete)
- 1302 lines of 4 codes -> 452 lines in 2 codes





Relevance network

- It is basically a classification problem
- But too many propositions(classes)
 - Huge computational time



→ Cat

Hypotheses

$\vdash \varphi$
 $\vdash \varphi \rightarrow \psi$
 $\vdash \psi \rightarrow \chi$

Assertion

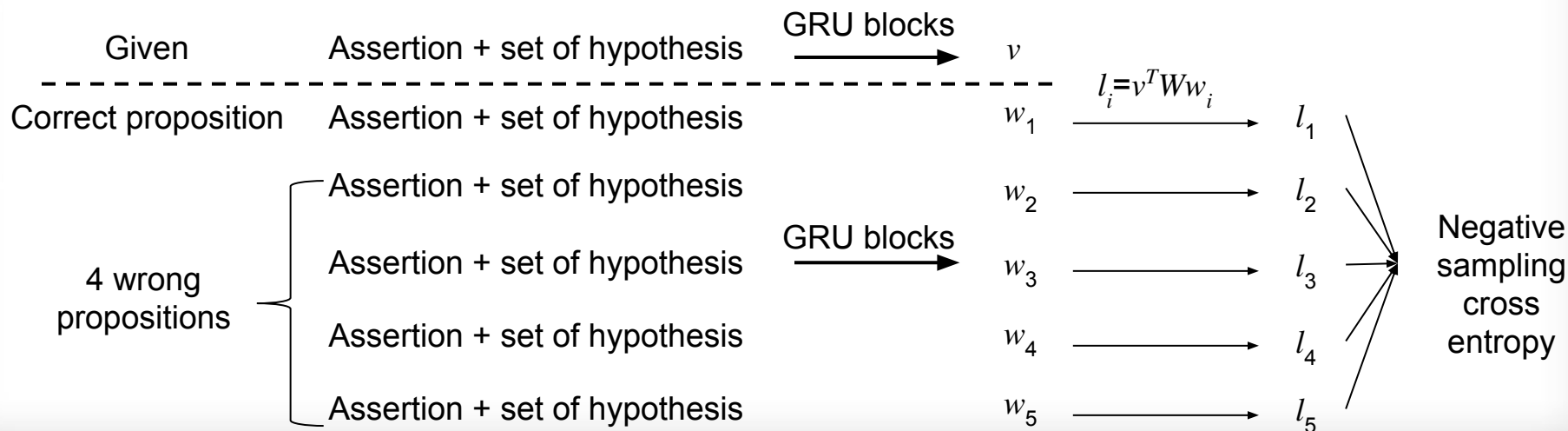
$\vdash \chi$

→ Modus ponens



Relevance network

- Negative sampling technique was used to minimize the computational cost
 - Pick 1 correct proposition and 4 wrong viable propositions

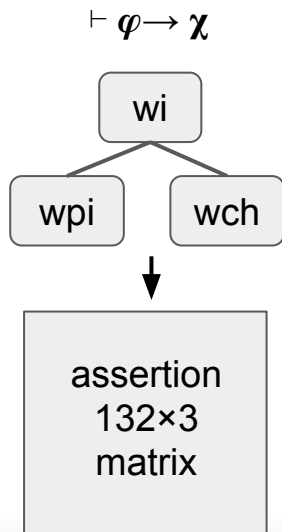




Data preprocessing

Assertion

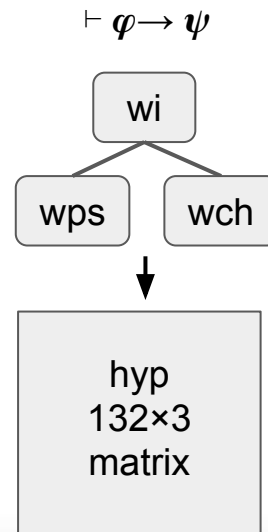
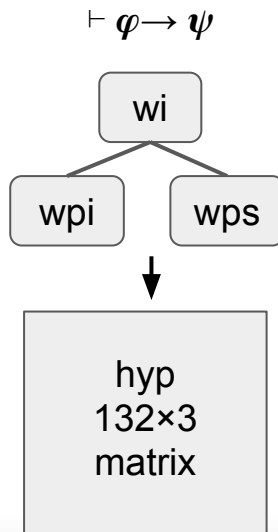
$\vdash \varphi \rightarrow \chi$



Hypotheses

$\vdash \varphi \rightarrow \psi$

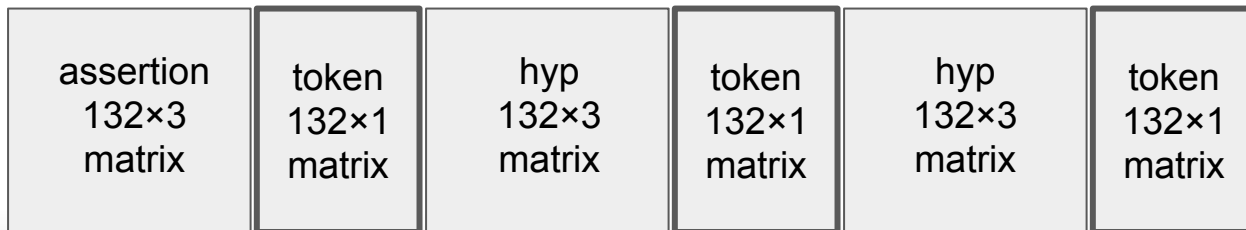
$\vdash \psi \rightarrow \chi$





Data preprocessing

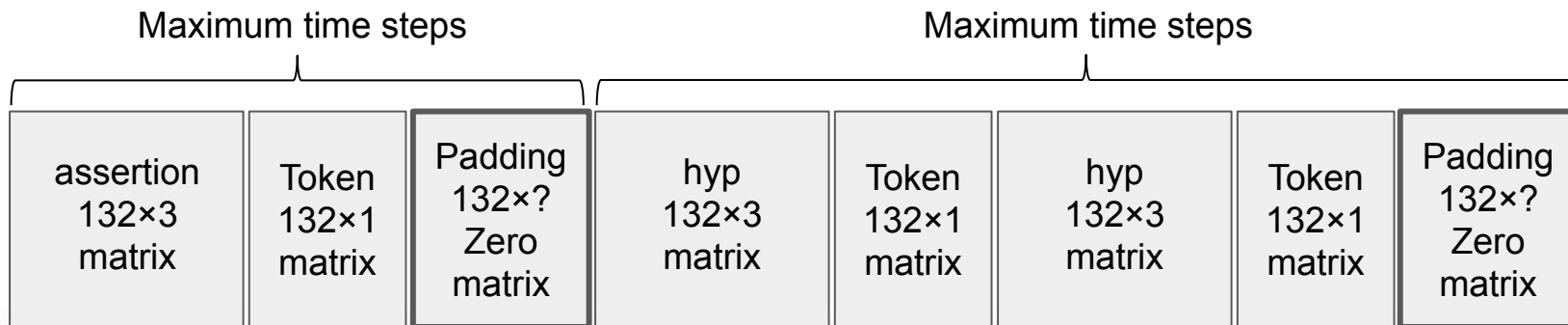
- Tokens are added between assertion and hypotheses
 - All entries are -1
- All matrices are concatenated
 - Row size is flexible





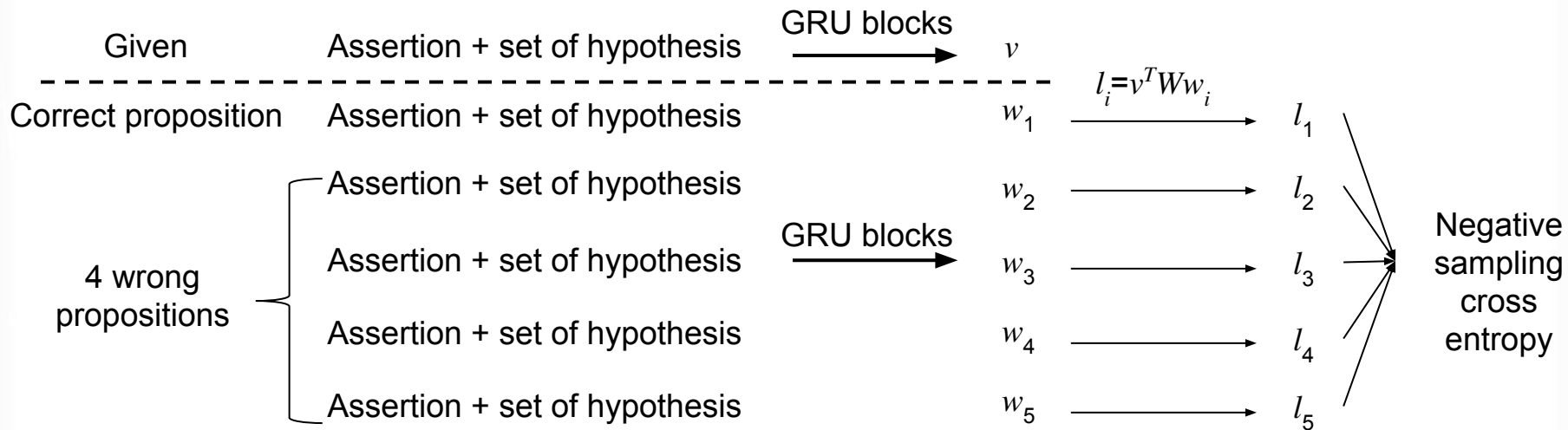
Data preprocessing

- Each column vector will be fed into one GRU Block
 - The number of GRU blocks we need varies
- To use **dynamic** RNN in TFLearn, we have to make all inputs have the same row length.
- Two zero matrices are padded



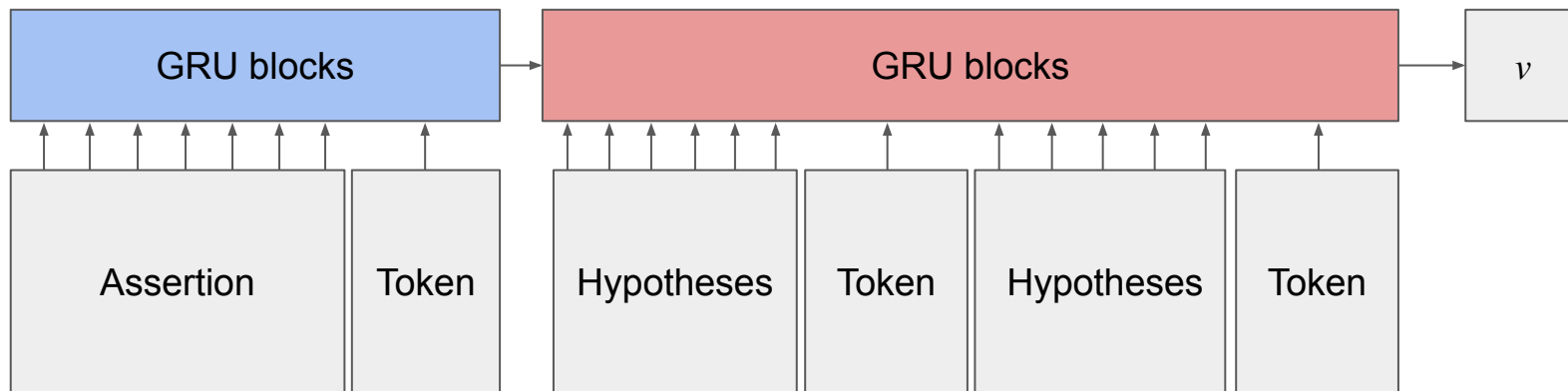
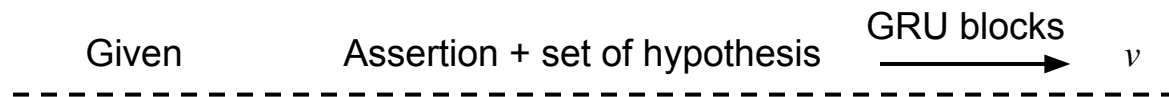


Creating the network





The first network





The second network

Correct proposition

Assertion + set of hypothesis

w_1

4 wrong
propositions

Assertion + set of hypothesis

w_2

Assertion + set of hypothesis

w_3

Assertion + set of hypothesis

w_4

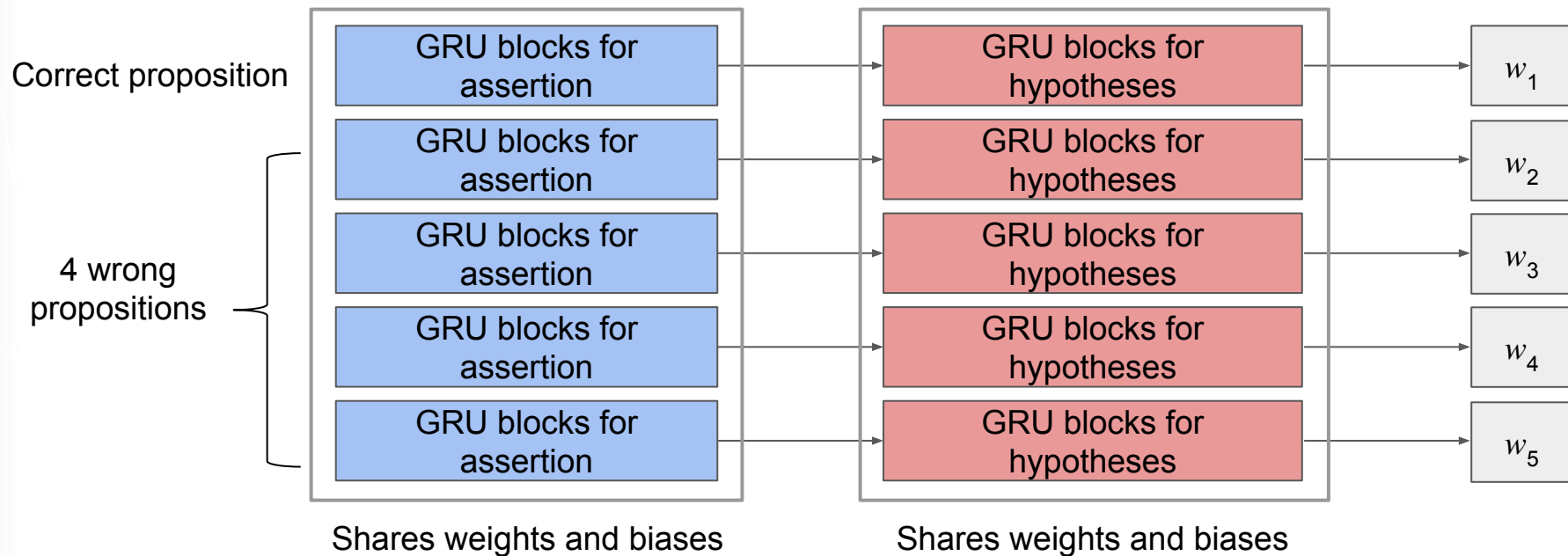
Assertion + set of hypothesis

w_5

GRU blocks
→

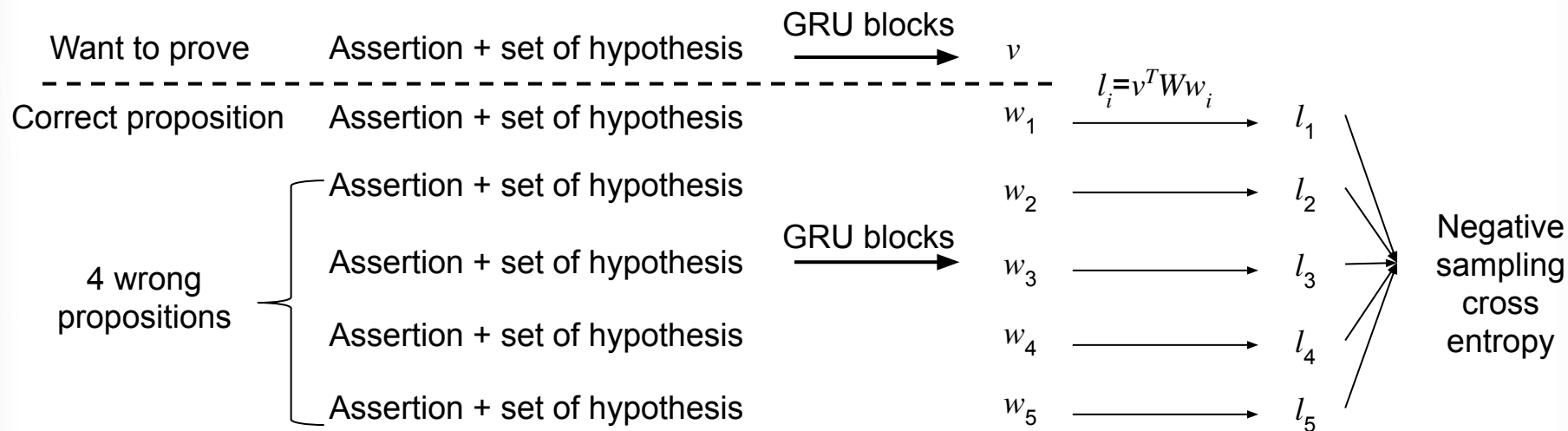


The second network





Creating the network





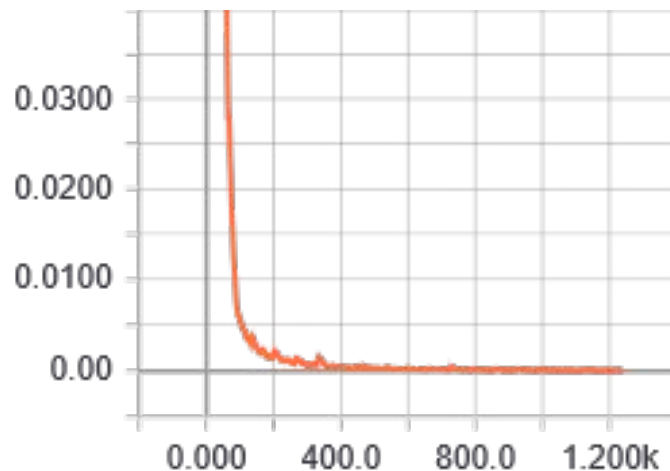
Training the network

- Uses the first 2000 propositions in Metamath database.
- 6480 training data are extracted from the propositions.
- Batch size = 100
- 20 epochs



Results

- The loss function decreases well
- We don't have enough time to test the network.
 - Need to construct a new network to test (negative sampling)





Outline

- Introduction
- Holophrasm: The Program
 - Metamath Database
 - Proof Tree
 - The Neural Networks
- Improving Holophrasm: TFLearn
- Conclusion



Conclusion

- Holophrasm: an automated theorem prover
 - Searches for a proof tree within a PPT
 - Guided by three neural networks
- Improved Holophrasm with TFLearn



Future Work

- Replace generative and payoff networks
- Test and improve



References

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Thank you for listening!



Thank you for listening!
Any questions?