

Automated Theorem Proving

-- A Deep Learning Approach

Industrial Sponsor: Real Al

Gwang Hyeon CHOI, Seung Yong MOON, Zheng PAN, Justin SUN



Outline

- Introduction
- Metamath: The Foundation
- Holophrasm: An Overview
- Future Work



Outline

- Introduction
- Metamath: The Foundation
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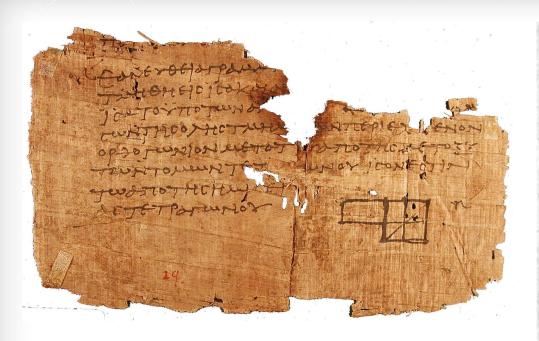
Company Background



- Research & development of safe and beneficial artificial general intelligence
- Altruistic intelligence for problem solving



Proving Math Theorems ...



PROPOSITION XIX. THEOREM 106. If two parallel lines are cut by a transversal, the corresponding angles are equal. [Converse of Prop. XIV.] Given parallel lines AB and CD and the cor. A 1 and 2. To prove 21 = 22Proof REASONS STATEMENTS. Vertical & are equal. Z1 = Z3. Alt. int. A of | lines are equal. 2 = 23.Things equal to the same thing . L1= L2 are equal to each other.

Q. E. D.

https://en.wikipedia.org/wiki/Mathematical_proof#/media/File:P._Oxy._I_29.jpg



Deep Learning

- Language translation
- Image recognition
- Alphago

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Mathematics??





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Formal Systems

Enables computers to work on proofs

Metamath, Mizar, HOL Light



Metamath

Over 19,000 proofs

- Built from 22 axioms
 - Logic + ZFC axioms



Metamath Proof Explorer Home Page

Last >

Mirrors > Home > MPE Home > Th. List > Recent

The aleph null above is the symbol for the first infinite cardinal number, discovered by Georg Cantor in 1873 (see theorem alepho).

This is the starting page for the Metamath Proof Explorer subproject (set.mm database). See the main Metamath Home Page for an overview of Metamath and download links.

Contents of this page

- Metamath Proof Explorer Overview
- How Metamath Proofs Work
- The Axioms (Propositional Calculus, Predicate Calculus, Set Theory, The Tarski-Grothendieck Axiom)
- The Theory of Classes New 13-Dec-2015
- A Theorem Sampler
- 2 + 2 = 4 Trivia
- Appendix 1: A Note on the Axioms
- Appendix 2: Traditional Textbook Axioms of Predicate Calculus
- Appendix 3: Distinct Variables (History, Notes)
 Revised 21-Dec-2016
- Appendix 4: A Note on Definitions
- Appendix 5: How to Find Out What Axioms a Proof Depends On
- Appendix 6: Notation for Function and Operation Values
- Appendix 7: Some Predicate Calculus Subsystems
- Reading Suggestions
- Bibliography
- Browsers and Fonts

Related pages

- Theorem List (Table of Contents)
- Most Recent Proofs (this mirror) (latest)
- Conventions and Style New 15-Jan-2017
- · Bibliographic Cross-Reference
- Definition List (3MB)
- Deduction Form and Natural Deduction New 7-
- Feb-2017 (Natural Deduction Rules New 9-Feb-2017)
- Weak Deduction Theorem (an older method)
- weak Deduction Theorem (an older method
- Real and Complex Numbers
- ZFC Axioms With No Distinct Variables
 ASCII Symbol Equivalents for Text-Only
- ASCII Symbol Equivalents for Text-Only
 Browsers
- <u>Ghilbert Proof Language</u> [retrieved 21-Dec-2016]

To search this site you can use <u>Google</u> [retrieved 21-Dec-2016] restricted to a mirror site. For example, to find references to infinity enter "infinity site:us.metamath.org". <u>More efficient searching</u> is possible with direct use of the <u>Metamath program</u>, once you get used to its <u>ASCII tokens</u>. See the wildcard features in "help search" and "help show statement".



Two axioms:

(A1)
$$(t = r \to (t = s \to r = s))$$

$$(A2) (t+0) = t$$

Modus Ponens:

$$(MP) \qquad ((P \land (P \to Q)) \to Q)$$



- Required 'tools'
- Assertion

th1
$$p \mid -t = t$$

Proof

tt tze tpl tt weq tt tt weq tt a2 tt tze tpl tt weq tt tze tpl tt weq tt tt weq wim tt a2 tt tze tpl tt tt a1 mp mp

```
$( Declare the constant symbols we will use $)
    c 0 + = -> () term wff | - $.
$( Declare the metavariables we will use $)
    $v t r s P O $.
$( Specify properties of the metavariables $)
    tt Sf term t S.
    tr $f term r $.
    ts $f term s $.
    wp $f wff P $.
    wa $f wff 0 $.
$( Define "term" (part 1) $)
    tze $a term 0 $.
$( Define "term" (part 2) $)
    tpl  $a term ( t + r ) $.
$( Define "wff" (part 1) $)
    weg a wff t = r .
$( Define "wff" (part 2) $)
   wim $a wff ( P -> Q ) $.
$( State axiom a1 $)
    al $a |- (t = r -> (t = s -> r = s)) $.
$( State axiom a2 $)
    a2 \$ a | - (t + 0) = t \$.
       min $e |- P $.
       mai $e |- ( P -> Q ) $.
$( Define the modus ponens inference rule $)
       mp $a |- 0 $.
  Prove a theorem $)
   th1 Sp - t = t S=
  $( Here is its proof: $)
      tt tze tpl tt weg tt tt weg tt a2 tt tze tpl
      tt weg tt tze tpl tt weg tt tt weg wim tt a2
      tt tze tpl tt tt al mp mp
```



Reverse Polish Notation (RPN)

Usual notation	RPN
2+3	23+
4+6-8	46+8-

Hypotheses: operands

Theorems: operators



Proof tt tze tpl tt weq ... a1 mp mp

Stack



Proof tt tze tpl tt weq ... a1 mp mp

tt \$f term t \$.

A hypothesis

Stack



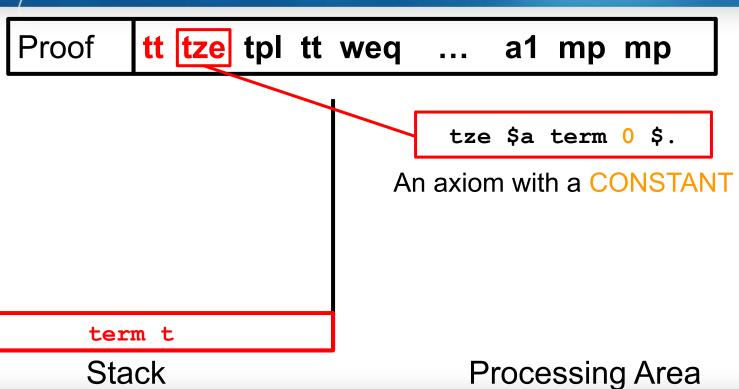
Proof tt tze tpl tt weq ... a1 mp mp

term t

Stack



Stack



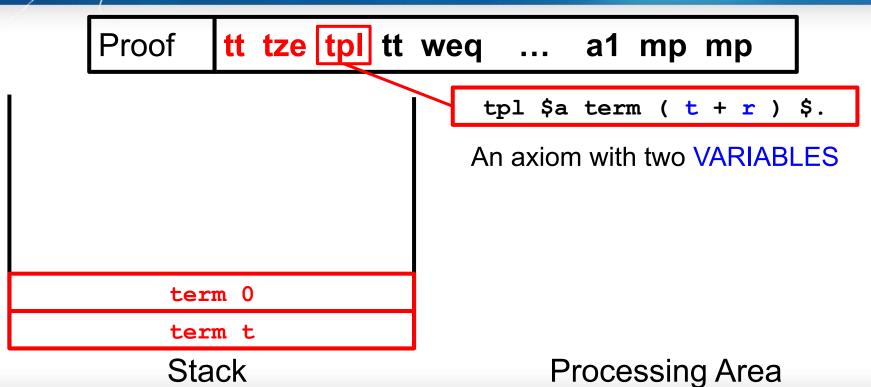


Proof tt tze tpl tt weq ... a1 mp mp

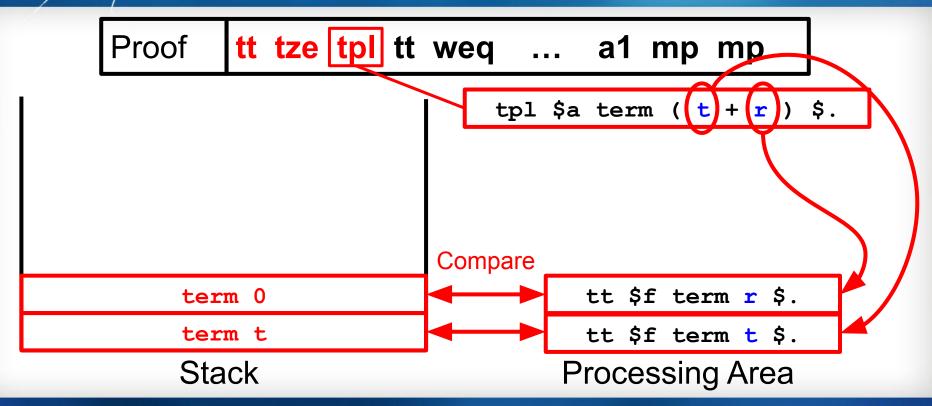
term 0
term t

Stack

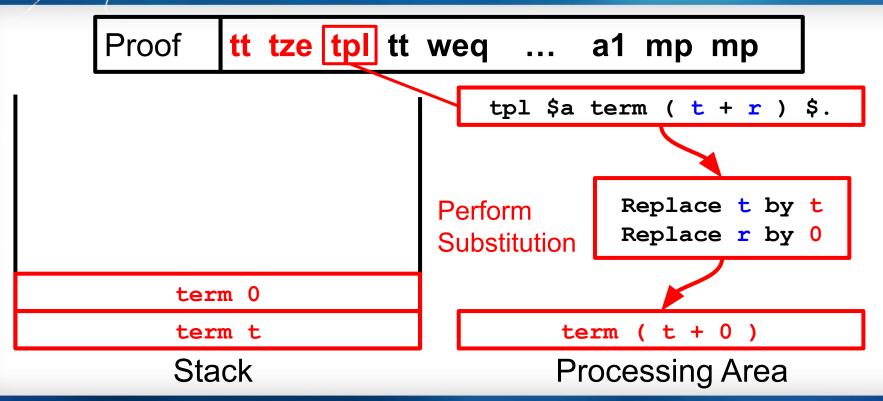














Proof tt tze tpl tt weq ... a1 mp mp

Remove the objects which were used to perform the substitution

Stack

term (t+0)



Proof tt tze tpl tt weq ... a1 mp mp

```
term (t+0)
```

Stack



```
Proof
       tt tze tpl tt weq ... a1 mp mp
                   term t
               term (t + 0)
                  Stack
```



```
Proof tt tze tpl tt weq ... a1 mp mp
```

```
wff ( t + 0 ) = t
```



Proof tt tze tpl tt weq ... a1 mp mp



```
Proof tt tze tpl tt weq ... a1 mp mp
```

```
|- ( ( t + 0 ) = t -> t = t ) )
|- ( t + 0 ) = t
```



Proof tt tze tpl tt weq ... a1 mp mp

```
| - t = t
```



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Holophrasm

Daniel P.Z. Whalen (2016), Holophrasm: a neural Automated Theorem Prover for higher-order logic. : arxiv:1608.02644

Github: https://github.com/dwhalen/holophrasm

Automated theorem prover based on metamath, python, and RNN.

Found proofs for 388 of 2720, or 14.3% of the test propositions



```
$( Declare the constant symbols we will use $)
    c 0 + = -> () term wff | - $.
$( Declare the metavariables we will use $)
    $v t r s P Q $.
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    wim $a wff ( P -> 0 ) $.
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$( State axiom a2 $)
    a2 \$ a | - (t + 0) = t \$.
       min $e |- P $.
      mai $e |- ( P -> 0 ) $.
$( Define the modus ponens inference rule $)
      mp $a |- Q $.
    $1
$( Prove a theorem $)
    th1 $p |- t = t $=
  $( Here is its proof: $)
       tt tze tpl tt weg tt tt weg tt a2 tt tze tpl
       tt weg tt tze tpl tt weg tt tt weg wim tt a2
       tt tze tpl tt tt a1 mp mp
```



```
$( State axiom al $)
    al $a |- ( t = r -> ( t = s -> r = s ) ) $.
$( State axiom a2 $)
    a2 $a |- ( t + 0 ) = t $.
    ${
        min $e |- P $.
        maj $e |- ( P -> Q ) $.
$( Define the modus ponens inference rule $)
        mp $a |- Q $.
    $}
$( Prove a theorem $)
    thl $p |- t = t $=
$( Here is its proof: $)
    tt tze tpl tt weq tt tt weq wim tt a2
    tt tze tpl tt tt al mp mp
$.
```



Axiom a1

Assertion:
$$t=r \Rightarrow$$
 $(t=s \Rightarrow r=s)$



```
$( State axiom a1 $) a1 $a \mid- ( t = r -> ( t = s -> r = s ) ) $.
```



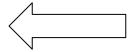
Axiom a1

Assertion : $\vdash t = r \Rightarrow$

$$(t=s \Rightarrow r=s)$$

Axiom a2

Assertion: $\vdash (t+0) = t$



(State axiom a2) a2 a - (t + 0) = t.



Axiom a1

Assertion:
$$t=r \Rightarrow$$
 $(t=s \Rightarrow r=s)$

Axiom a2

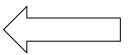
Assertion :
$$\vdash (t+0) = t$$

Modus Ponens(mp)

Hypotheses :
$$\vdash P$$

$$\vdash P \Longrightarrow Q$$

Assertion :
$$\vdash Q$$



```
${
    min $e |- P $.
    maj $e |- ( P -> Q ) $.
$( Define the modus ponens inference rule $)
    mp $a |- Q $.
$}
```



Axiom a1

Assertion:
$$t=r \Rightarrow$$
 $(t=s \Rightarrow r=s)$

Axiom a2

Assertion :
$$\vdash (t+0) = t$$

Modus Ponens(mp)

Hypotheses :
$$\vdash P$$

$$\vdash P \Longrightarrow Q$$

Assertion :
$$\vdash Q$$

Theorem th1

Assertion :
$$\vdash t = t$$



\$(Prove a theorem \$)
 th1 \$p |- t = t \$:



Axiom a1

Assertion:
$$t=r \Rightarrow (t=s \Rightarrow r=s)$$

Axiom a2

Assertion :
$$\vdash (t+0) = t$$

Modus Ponens(mp)

Hypotheses :
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Assertion :
$$\vdash Q$$

Theorem th1

Assertion : $\vdash t = t$



Axiom a1

Assertion:
$$t=r \Rightarrow$$
 $(t=s \Rightarrow r=s)$

Axiom a2

Assertion :
$$\vdash (t+0) = t$$

Modus Ponens(mp)

Hypotheses :
$$\vdash P$$

$$\vdash P \Longrightarrow Q$$

Assertion : $\vdash Q$

Theorem th1

Assertion : $\vdash \alpha = \alpha$

 $\alpha = \alpha$



Axiom a1

Assertion:
$$t=r \Rightarrow (t=s \Rightarrow r=s)$$

Axiom a2

Assertion:
$$\vdash (t+0) = t$$

Modus Ponens(mp)

Hypotheses :
$$\vdash P$$
 $\vdash P \Rightarrow Q$

Assertion : $\vdash Q$

Theorem th1

```
Assertion : \vdash \alpha = \alpha
```

$\alpha = \alpha$

```
$( Prove a theorem $)
    th1 $p |- t = t $=
    $( Here is its proof: $)
        tt tze tpl tt weq tt tt weq tt a2 tt tze tpl
        tt weq tt tze tpl tt weq tt tt weq wim tt a2
        tt tze tpl tt tt a1 mp mp
    $.
```



Axiom a1

Assertion:
$$t=r \Rightarrow$$
 $(t=s \Rightarrow r=s)$

Axiom a2

Assertion:
$$\vdash (t+0) = t$$

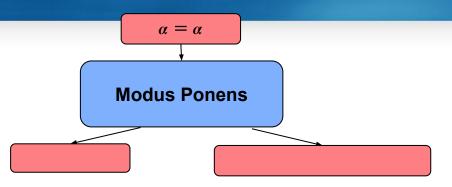
Modus Ponens(mp)

Hypotheses :
$$\vdash P$$

$$\vdash P \Longrightarrow Q$$

Assertion : $\vdash Q$

Theorem th1





Axiom a1

Assertion:
$$t=r \Rightarrow$$
 $(t=s \Rightarrow r=s)$

Axiom a2

Assertion:
$$\vdash (t+0) = t$$

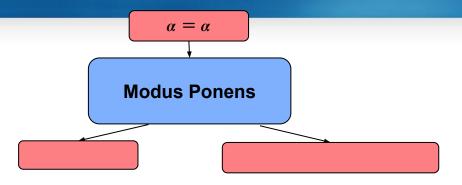
Modus Ponens(mp)

Hypotheses :
$$\vdash P$$

$$\vdash P \Longrightarrow Q$$

Assertion : $\vdash Q$

Theorem th1



```
$( Here is its proof: $)
    tt tze tpl tt weq tt tt weq tt a2 tt tze tpl
    tt weq tt tze tpl tt weq tt tt weq wim tt a2
    tt tze tpl tt tt a1 mp mp
```



Axiom a1

Assertion:
$$t=r \Rightarrow$$
 $(t=s \Rightarrow r=s)$

Axiom a2

Assertion:
$$\vdash (t+0) = t$$

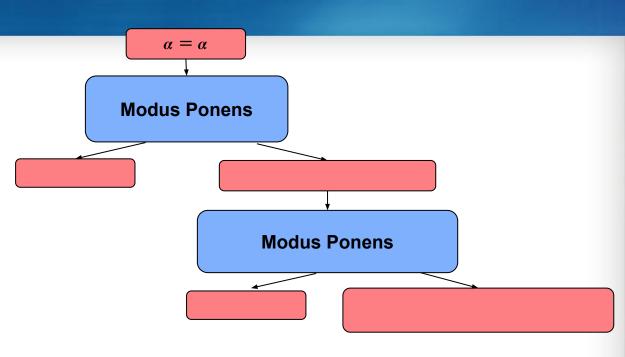
Modus Ponens(mp)

Hypotheses : $\vdash P$

$$\vdash P \Longrightarrow Q$$

Assertion : $\vdash Q$

Theorem th1





Axiom a1

Assertion:
$$t=r \Rightarrow (t=s \Rightarrow r=s)$$

Axiom a2

Assertion: $\vdash (t+0) = t$

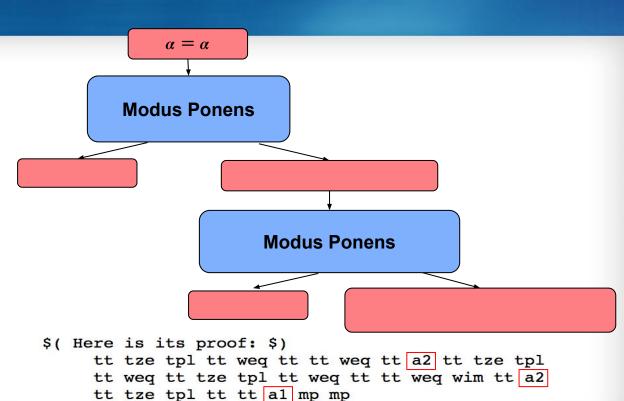
Modus Ponens(mp)

Hypotheses : ⊢ *P*

$$\vdash P \Longrightarrow Q$$

Assertion : $\vdash Q$

Theorem th1





Axiom a1

Assertion:
$$t=r \Rightarrow$$
 $(t=s \Rightarrow r=s)$

Axiom a2

Assertion:
$$\vdash (t+0) = t$$

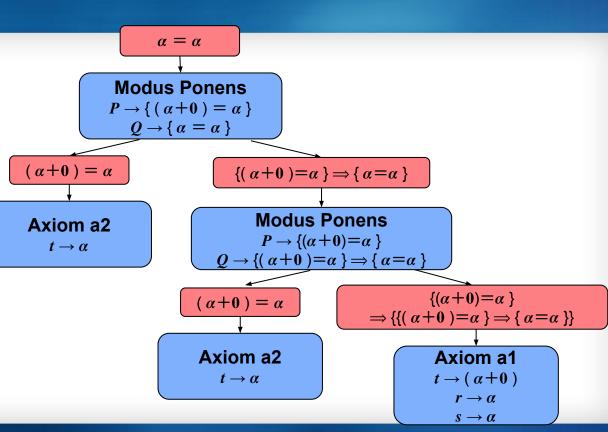
Modus Ponens(mp)

Hypotheses : $\vdash P$

$$\vdash P \Longrightarrow Q$$

Assertion : $\vdash Q$

Theorem th1





Axiom a1

Assertion:
$$t=r \Rightarrow$$
 $(t=s \Rightarrow r=s)$

Axiom a2

Assertion: $\vdash (t+0) = t$

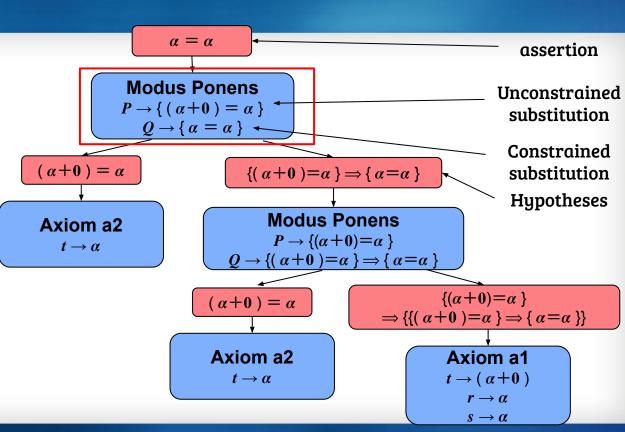
Modus Ponens(mp)

Hypotheses : $\vdash P$

$$\vdash P \Longrightarrow Q$$

Assertion : $\vdash Q$

Theorem th1





So this theorem consists of

- 1. Assertion : $\alpha = \alpha$
- 2. Hypotheses(& axioms): a1, a2, mp
- 3. Free variables : α , t, P, ...
- 4. Pairs of disjoint variables : (t, s), (P, Q), ... (restricting substitutions)



Q. How to find this proof tree?

A. Of course, Deep Learning!



- Given a theorem, find a proof tree using deep learning.
- 3 neural networks
 - Relevance
 - Generative
 - Payoff



Theorem mp2b

Hypotheses

Assertion

$$\vdash \varphi$$

$$\vdash \varphi \to \psi$$

$$\vdash \psi \to \chi$$



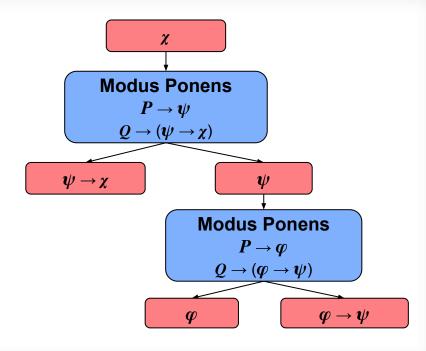
 $\vdash Q$

Axiom Modus Ponens

Hypotheses

Assertion

$$\vdash P$$
$$\vdash P \to Q$$





Theorem mp2b

Hypotheses Assertion

$$\vdash \varphi$$

$$\vdash \varphi \to \psi$$

$$\vdash \psi \to \chi$$

Axiom Modus Ponens

Hypotheses Assertion $\vdash P$

$$\vdash P \\ \vdash P \to Q$$

Assertion



Hypotheses

$$\vdash \varphi$$

$$\vdash \varphi \to \psi$$

$$\vdash \psi \to \chi$$



Theorem mp2b

Hypotheses

Assertion

$$\vdash \varphi$$

$$\vdash \varphi \to \psi$$

$$\vdash \psi \to \chi$$

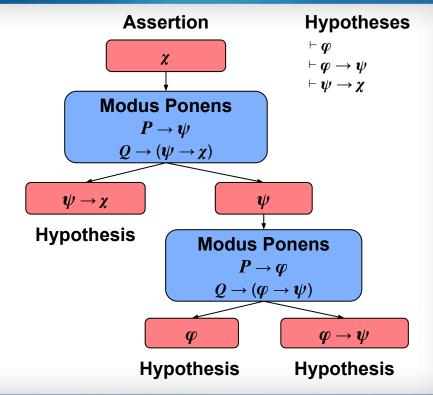
Axiom Modus Ponens

Hypotheses

Assertion

$$\vdash P \\ \vdash P \to Q$$

$$\vdash Q$$





Relevance network

 Given an assertion, find the next proposition that will be used to prove the assertion.

Assertion

χ

Hypotheses

$$\vdash \varphi$$

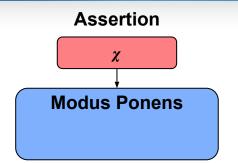
$$\vdash \varphi \to \psi$$

$$\vdash \psi \to \chi$$



Relevance network

 Given an assertion, find the next proposition that will be used to prove the assertion.



Hypotheses

$$\vdash \varphi$$
$$\vdash \varphi \to \psi$$
$$\vdash \psi \to \chi$$



Relevance network

- Inputs
 - an assertion and a set of hypotheses of theorem we want to prove

```
Theorem mp2b

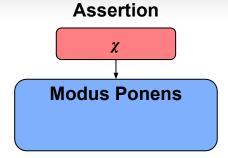
Hypotheses Assertion
\vdash \varphi \qquad \qquad \vdash \chi
\vdash \varphi \rightarrow \psi
\vdash \psi \rightarrow \chi
```

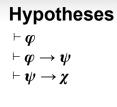
- Uses RNN
 - GRU



Generative network

 Given an assertion, and the proposition, find the substitutions for the unconstrained variables of the proposition.





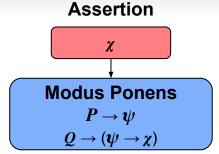
Axiom Modus Ponens

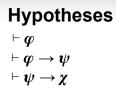
$$\begin{array}{ll} \text{Hypotheses} & \quad \text{Assertion} \\ \vdash P & \quad \vdash Q \\ \vdash P \rightarrow Q & \end{array}$$



Generative network

 Given an assertion, and the proposition, find the substitutions for the unconstrained variables of the proposition.





Axiom Modus Ponens

$$\begin{array}{ll} \text{Hypotheses} & \text{Assertion} \\ \vdash P & \vdash Q \\ \vdash P \rightarrow Q \end{array}$$



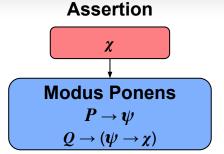
Generative network

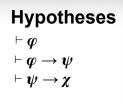
- Inputs
 - a set of hypotheses of the theorem we want to prove
 - a set of hypotheses of the proposition

- Uses sequence to sequence model
 - GRU



 repeat the previous procedures on the remaining assertions to find a proof tree.



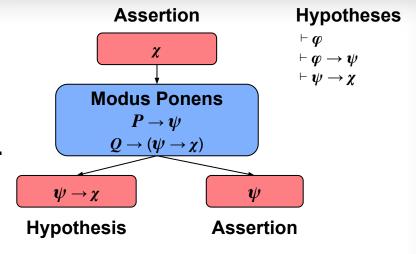


Axiom Modus Ponens

Hypotheses Assertion
$$\vdash P$$
 $\vdash P \rightarrow O$



 repeat the previous procedures on the remaining assertions to find a proof tree.

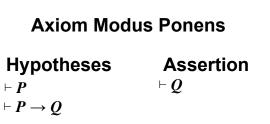


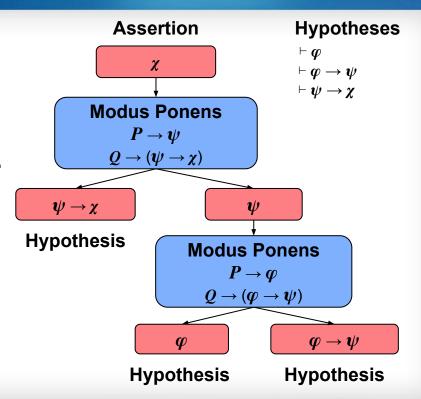
Axiom Modus Ponens

Hypotheses Assertion $\vdash P$ $\vdash Q$



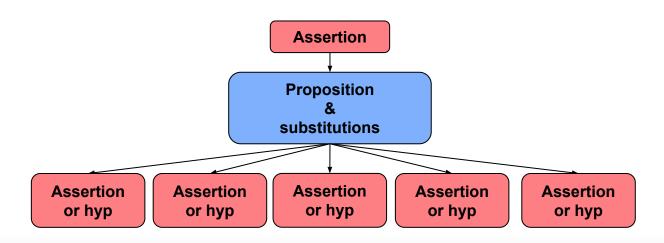
 repeat the previous procedures on the remaining assertions to find a proof tree.





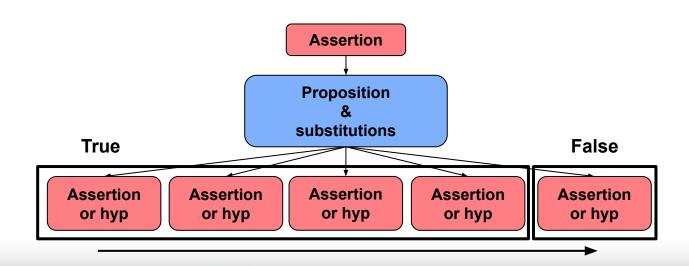


What if there are too many assertions?



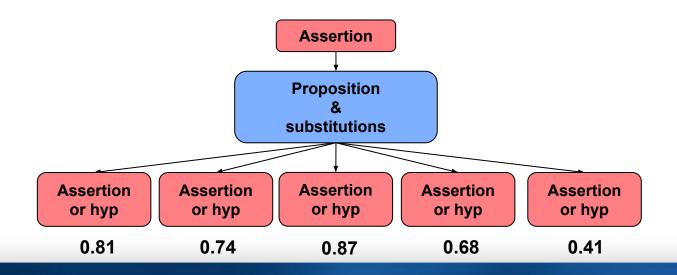


What if there are too many assertions?



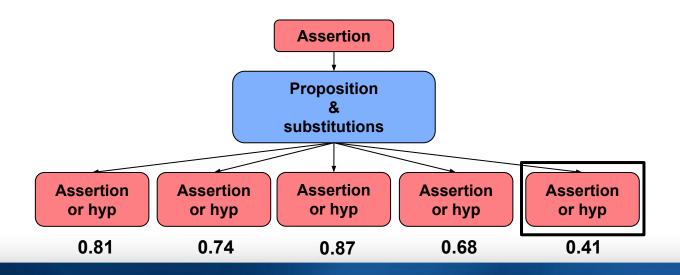


 Given a set of assertions, find the least promising assertion (least probable to be proved)





 Given a set of assertions, find the least promising assertion (least probable to be proved)





- Input: an assertion and a set of hypotheses of theorem
- Uses RNN
 - GRU
 - Bidirectional network



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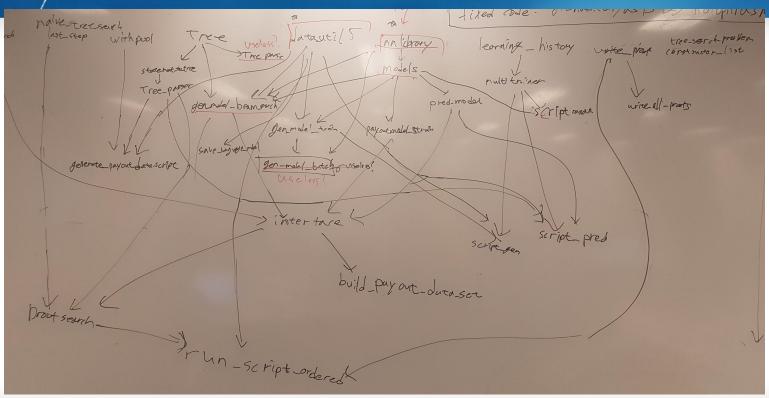


Future Work

Reconstruct Holophrasm and make descriptions



Future Work





Future Work

Accelerating Deep Learning Research with the Tensor2Tensor Library

Monday, June 19, 2017

Posted by Łukasz Kaiser, Senior Research Scientist, Google Brain Team

Deep Learning (DL) has enabled the rapid advancement of many useful technologies, such as machine translation, speech recognition and object detection. In the research community, one can find code open-sourced by the authors to help in replicating their results and further advancing deep learning. However, most of these DL systems use unique setups that require significant engineering effort and may only work for a specific problem or architecture, making it hard to run new experiments and compare the results.

Today, we are happy to release Tensor2Tensor (T2T), an open-source system for training deep learning models in TensorFlow. T2T facilitates the creation of state-of-the art models for a wide variety of ML applications, such as translation, parsing, image captioning and more, enabling the exploration of various ideas much faster than previously possible. This release also includes a library of datasets and models, including the best models from a few recent papers (Attention Is All You Need, Depthwise Separable Convolutions for Neural Machine Translation and One Model to Learn Them All) to help kick-start your own DL research.

Translation Model	Training time	BLEU (difference from baseline)
Transformer (T2T)	3 days on 8 GPU	28.4 (+7.8)
SliceNet (T2T)	6 days on 32 GPUs	26.1 (+5.5)
GNMT + Mixture of Experts	1 day on 64 GPUs	26.0 (+5.4)
ConvS2S	18 days on 1 GPU	25.1 (+4.5)
GNMT	1 day on 96 GPUs	24.6 (+4.0)
ByteNet	8 days on 32 GPUs	23.8 (+3.2)
MOSES (phrase-based baseline)	N/A	20.6 (+0.0)

BLEU scores (higher is better) on the standard WMT English-German translation task.





TensorFlow 1.2 has arrived!

We're excited to announce the release of TensorFlow 1.2! Check out the release notes for all the latest.

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Thank you for listening!



Thank you for listening! Any questions?