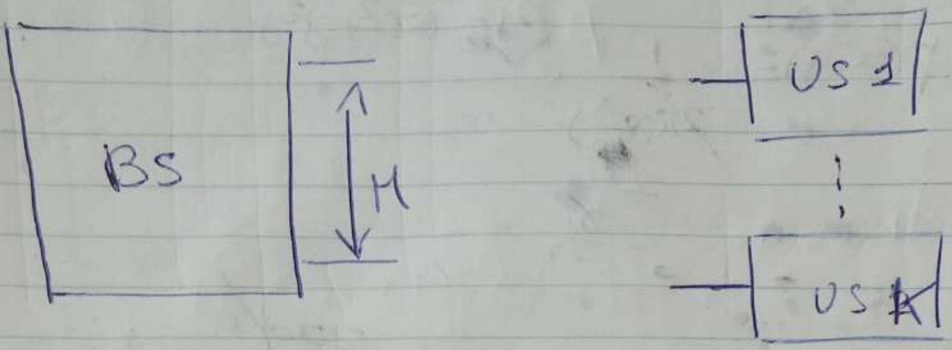
Formulation 1



$$H_{M\times K} = [h_1 \cdots h_K]$$

$$h_i = \frac{1}{\sqrt{L_p}} \sum_{j=1}^{L_p} \alpha_j \, a(\theta_j)$$

$$a(\theta_j) = \begin{bmatrix} e^{j\frac{2\pi d}{\lambda}\sin(\theta_j)\cdot 0} \\ e^{j\frac{2\pi d}{\lambda}\sin(\theta_j)\cdot 1} \\ \vdots \\ e^{j\frac{2\pi d}{\lambda}\sin(\theta_j)\cdot (M-1)} \end{bmatrix}$$

$$h_i = \frac{1}{\sqrt{L_p}} A \underline{\alpha} \qquad A = [a(\theta_1), \ldots, a(\theta_{L_p})]$$

$$\underline{\alpha} = [\alpha_1, \ldots, \alpha_{L_p}]$$

* TDD operation: UL & DL transmission share the same band

* UL channel estimation: $X_{K\times L} = \sqrt{\frac{P}{K}} \phi_{K\times L}$ $\quad L \geq K$

$$Y = HX + N \qquad \phi\phi^H = I_K$$

$$Z = Y\phi^H \cdot \sqrt{\frac{K}{P}} = H + N' \quad N' \sim \mathcal{CN}\left(0, \frac{\sigma^2 K}{P}\right)$$

$$z_k = \underbrace{h_k}_{\text{sparse representation}} + n'_k$$

$$\min_{\hat{h}_k} \|\hat{h}_k\|_A \quad \text{s.t. } \|z_k - h_k\|_2^2 \leq \xi$$

$$\min_{\hat{h}_k} \tau \|\hat{h}_k\|_A + \|z_k - h_k\|_2^2$$

$$\tau = f(\alpha)$$

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* Original off-grid recovery formulation

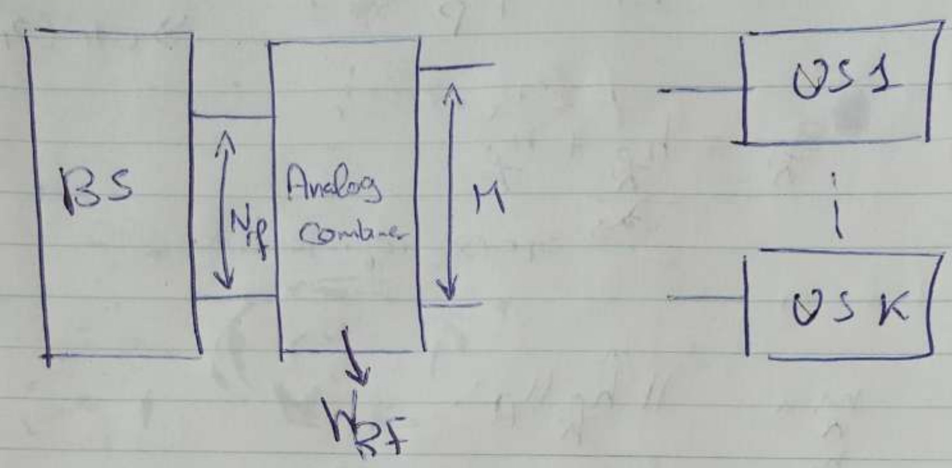
$$h = \begin{bmatrix} h_1 \\ \vdots \\ h_M \end{bmatrix} \qquad h_i = \sum_{l=1}^{L_p} c_i e^{j 2\pi f_l i} \qquad i = 1, \dots, M$$

we observe $h_i^* = h_i \quad i \in B \subseteq [M]$

$$|B| < M$$

$$\min_h \|h\|_A \quad \text{s.t.} \quad h_i^* = h_i \quad i \in B$$

Formulation 2



$$W_{RF} = [\omega_{ij}] \qquad |\omega_{ij}| = 1$$
$$N_{rf} \times M \qquad \angle \omega_{ij} = \theta_{ij}$$

Consider L pilot transmissions per user

$$X = C \begin{bmatrix} \overbrace{1 \cdots 1}^{L} & 0 \cdots & \cdots & 0 \\ & 1 \cdots 1 & & \\ & & \ddots & \\ & & & 1 \cdots 1 \end{bmatrix}$$

$$y_k^{(\ell)} = C W_{RF}^{(\ell)} h_k + n_k^{(\ell)} \qquad \ell = 0, \ldots, L-1$$
$$k = 1, \ldots, K$$

$$y_k = C \underbrace{\begin{bmatrix} W_{RF}^{(0)} \\ \vdots \\ W_{RF}^{(L-1)} \end{bmatrix}}_{\to\ W_{RF}\ L \cdot N_{RF} \times M} h_k + \begin{bmatrix} n_k^{(0)} \\ \vdots \\ n_k^{(L-1)} \end{bmatrix}$$

Now, consider a switched analog phase

$$A \; = \; B \odot W_{RF}$$

$$y_k = C B \odot W_{RF}\, h_k + \begin{bmatrix} n_k^{(0)} \\ \vdots \\ n_k^{(L-1)} \end{bmatrix}$$

B has an entry of one in each row, and all other entries are zeros.

$$\min_{\hat{h}_k} \; \|\hat{h}_k\|_A$$

$$\text{s.t.} \quad \|y_k - C(B \odot W_{RF} h_k)\|_2^2 \leq \xi$$