# Nonparametric Noise models for the Gaussian Process

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## Abstract

Notes while developing a Gaussian Process with nonparametric noise model.

## 1. Introduction

It is sometimes really hard to tell what noise model we want to use for a GP. A specific example is quantile regression for product demand forecasting. We've got an underlying trend, perhaps with cyclical component, which we can easily model with a GP by encoding prior knowledge in the covariance matrix. Unfortunately sales data might have spike and other irregularities which make choosing a noise model quite tricky. One option is to use a robust noise model like student-t or Laplace. In this work, we learn a noise model by using a non-parametric mixture of Gaussians.

## 2. Model

Imagine we have a time series with observations $y_t$ at times $x_t$ with $t \in [0, T]$. We model this data by assuming a latent Gaussian process

$$\boldsymbol{f} \sim \mathcal{GP}\left(0, \boldsymbol{K}(\boldsymbol{x}, \boldsymbol{x})\right) \tag{1}$$

We model the noise as a non-parametric mixture model using the Dirichlet process. Let

$$\boldsymbol{G} \sim \mathcal{DP}\left(\alpha, H\right) \tag{2}$$

be a Dirichlet process with concentration parameter $\alpha$ and base measure $H$. For each time $t$ we introduce a noise variable $\epsilon_t \sim G$. We then model the observation $y_t = \boldsymbol{f}(x_t) + \epsilon_t$.

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**Algorithm 1** Collapsed Gibbs Sampling

**Input:** data $\boldsymbol{x}, \boldsymbol{y}$ and kernel $\boldsymbol{K}(\boldsymbol{x}, \boldsymbol{x})$
**Initialisation:** $z_t \sim \text{CRP}(\alpha)$, $\theta_n \sim H$
**repeat**
  Sample $z_t | \boldsymbol{K}, \boldsymbol{y}, \boldsymbol{z}_{\neg t}, \boldsymbol{\theta}$
  Sample $\theta_n | \boldsymbol{x}, \boldsymbol{y}, \boldsymbol{\theta}$
**until** convergence

## 3. Inference

We can perform inference in this model using a collapsed Gibbs sampler. In order to work with the CRP representation of the Dirichlet process we introduce a new variable $z_t$ which will represent CRP partition that datapoint $t$ belongs to. For each CRP partition $n$ we represent the cluster parameters using $\theta_n$.

In algorithm 1 we integrate out the Gaussian process $\boldsymbol{f}$. In what follows we derive the resampling steps for $z_t$ and $\theta_n$.

**Sampling** $z_t$ The conditional distribution of $z_t$ can be written as follows

$$\begin{aligned} p(z_t | \boldsymbol{K}, \boldsymbol{y}, \boldsymbol{z}_{\neg t}, \boldsymbol{\theta}) &\propto \int p(\boldsymbol{y} | \boldsymbol{f}, \boldsymbol{z}_{\neg t}, z_t, \boldsymbol{\theta}) p(\boldsymbol{f} | \boldsymbol{K}) d\boldsymbol{f}, \\ &= \int p(y_t | f_t, \theta_{z_t}) p(f_t | \boldsymbol{y}, \boldsymbol{z}_{\neg t}, \boldsymbol{\theta}, \boldsymbol{K}) df_t, \\ &= \text{WHAT THEN?} \end{aligned}$$

Explain why we can make the jump above.

The key bit is that $p(f_t | \boldsymbol{y}, \boldsymbol{z}_{\neg t}, \boldsymbol{\theta}, \boldsymbol{K})$ is a Gaussian process where every datapoint contributes noise that is dependent on the CRP partition it belongs to.

$$\begin{pmatrix} \boldsymbol{y} \\ f_t \end{pmatrix} \sim \begin{pmatrix} K(x_1, x_1) & K(x_1, x_2) & ... \\ K(x_1, x_1) & K(x_1, x_2) & ... \\ K(x_1, x_1) & K(x_1, x_2) & ... \end{pmatrix} \tag{3}$$

We know that [CITE Carl] $p(\boldsymbol{f} | \boldsymbol{K})$ can be represented as a multivariate Gaussian distribution.

**Sampling $\theta_n$**

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**Algorithm 2** Bubble Sort

**Input:** data $x_i$, size $m$
**repeat**
  Initialize $noChange = true$.
  **for** $i = 1$ **to** $m - 1$ **do**
    **if** $x_i > x_{i+1}$ **then**
      Swap $x_i$ and $x_{i+1}$
      $noChange = false$
    **end if**
  **end for**
**until** $noChange$ is $true$

*Table 1.* Classification accuracies for naive Bayes and flexible Bayes on various data sets.

| Data set | Naive | Flexible | Better? |
|---|---|---|---|
| Breast | 95.9± 0.2 | 96.7± 0.2 | √ |
| Cleveland | 83.3± 0.6 | 80.0± 0.6 | × |
| Glass2 | 61.9± 1.4 | 83.8± 0.7 | √ |
| Credit | 74.8± 0.5 | 78.3± 0.6 | |
| Horse | 73.3± 0.9 | 69.7± 1.0 | × |
| Meta | 67.1± 0.6 | 76.5± 0.5 | √ |
| Pima | 75.1± 0.6 | 73.9± 0.5 | |
| Vehicle | 44.9± 0.6 | 61.5± 0.4 | √ |

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