Example worked in proof assistant taken from Baronett *Intro to Logic* p422

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Implementation of propositional logic uses Python objects to represent logical expressions. ¶

Logical connectives are implemented using functions that overload standard Python operators: AND is &, OR is |, IF/THEN is >>, IFF is **, and NOT is ~.

Replacement rules are functions that check for valid input before returning a result.

Proof object adds another layer of validity checking by doing such things as preventing extra assumptions being added after proof has begun and limiting deletion to last entry (entries can be sequentially deleted by calling the proof.remove() method repeatedly).

Ideally, this would allow any Python environment to act as an interactive proof assistant.

Tell Python where to find class files and load.

In [1]:
```
import sys
sys.path.append("C:\Users\John\Documents\Code\pyproof")
```

In [2]:
```
from pyproof import *
```

Create logical atoms.

In [3]:
```
P = atom("P")
Q = atom("Q")
R = atom("R")
```

Once atoms have been created, they can be directly typed in and connected using overloaded operators.

In [4]:
```
(P&Q)>>R
```

Out[4]:
```
((P&Q)>>R)
```

These expressions are represented internally in Python as objects. The example expression is a "conditional" object because the conditional operator is the main operator. The "conditional" object has the "conjunction" object P&Q and the "atom" object R as its contents.

In [5]:
```
((P&Q)>>R).__class__
```

Out[5]:
```
<class pyproof.objects.conditional at 0x0312CF48>
```

In [6]:
```
((P&Q)>>R).contents
```

Out[6]:
```
[(P&Q), R]
```

Create proof object.

In [7]:
```
one = Proof()
```

Assume hypotheses.

In [8]:
```
one.assume(~(P&~Q)|(P>>R))
```

```
1.      ((~(P&(~Q)))|(P>>R))
```

Set goal of proof (optional).

In [9]:
```
one.goal(P>>(Q|R))
```

```
1.      ((~(P&(~Q)))|(P>>R))            /(P>>(Q|R))
```

Begin scope for conditional section of proof.

In [10]:
```
one.cp(P)
```

```
1.      ((~(P&(~Q)))|(P>>R))            /(P>>(Q|R))
==============================
|2.     P                               Assumption (CP)
```

Begin scope for indirect section of proof.

In [11]:
```
one.ip(~(Q|R))
```

```
1.      ((~(P&(~Q)))|(P>>R))            /(P>>(Q|R))
==============================
|2.     P                               Assumption (CP)
||3.    (~(Q|R))                                Assumption (IP)
```

In [12]:
```
one.dm(3)
```

```
1.      ((~(P&(~Q)))|(P>>R))            /(P>>(Q|R))
==============================
|2.     P                               Assumption (CP)
||3.    (~(Q|R))                                Assumption (IP)
||4.    ((~Q)&(~R))                             DM 3
```

In [13]:
```
one.simp(4)
```

```
1.      ((~(P&(~Q)))|(P>>R))            /(P>>(Q|R))
==============================
|2.     P                               Assumption (CP)
||3.    (~(Q|R))                                Assumption (IP)
||4.    ((~Q)&(~R))                             DM 3
||5.    (~Q)                            Simp 4
```

In [14]:
```
one.conj(2,5)
```

```
1.      ((~(P&(~Q)))|(P>>R))            /(P>>(Q|R))
==============================
|2.     P                               Assumption (CP)
||3.    (~(Q|R))                                Assumption (IP)
||4.    ((~Q)&(~R))                             DM 3
||5.    (~Q)                            Simp 4
||6.    (P&(~Q))                                Conj 2, 5
```

If application of replacement rule has ambiguous cases, requires explicitly noting the case.

In [15]:
```
one.dn(6)
```

```
1.      p => ~~p
2.      ~~p => p
```

Out[15]:
```
'INVALID INPUT: must select case'
```

In [16]:
```
one.dn(6,1)
```

```
1.      ((~(P&(~Q)))|(P>>R))            /(P>>(Q|R))
================================
|2.     P                               Assumption (CP)
||3.    (~(Q|R))                               Assumption (IP)
||4.    ((~Q)&(~R))                            DM 3
||5.    (~Q)                            Simp 4
||6.    (P&(~Q))                               Conj 2, 5
||7.    (~(~(P&(~Q))))                         DN 6
```

In [17]:
```
one.ds(1,7)
```

```
1.      ((~(P&(~Q)))|(P>>R))            /(P>>(Q|R))
================================
|2.     P                               Assumption (CP)
||3.    (~(Q|R))                               Assumption (IP)
||4.    ((~Q)&(~R))                            DM 3
||5.    (~Q)                            Simp 4
||6.    (P&(~Q))                               Conj 2, 5
||7.    (~(~(P&(~Q))))                         DN 6
||8.    (P>>R)                          DS 1, 7
```

In [18]:
```
one.mp(8,2)
```

```
1.      ((~(P&(~Q)))|(P>>R))            /(P>>(Q|R))
================================
|2.     P                               Assumption (CP)
||3.    (~(Q|R))                               Assumption (IP)
||4.    ((~Q)&(~R))                            DM 3
||5.    (~Q)                            Simp 4
||6.    (P&(~Q))                               Conj 2, 5
||7.    (~(~(P&(~Q))))                         DN 6
||8.    (P>>R)                          DS 1, 7
||9.    R                               MP 8, 2
```

In [19]:
```
one.com(4)
```

```
1.      ((~(P&(~Q)))|(P>>R))            /(P>>(Q|R))
================================
|2.     P                               Assumption (CP)
||3.    (~(Q|R))                               Assumption (IP)
||4.    ((~Q)&(~R))                            DM 3
||5.    (~Q)                            Simp 4
||6.    (P&(~Q))                               Conj 2, 5
||7.    (~(~(P&(~Q))))                         DN 6
||8.    (P>>R)                          DS 1, 7
||9.    R                               MP 8, 2
||10.   ((~R)&(~Q))                            Com 4
```

In [20]:
```
one.simp(10)
```

```
1.      ((~(P&(~Q)))|(P>>R))            /(P>>(Q|R))
==============================
|2.     P                               Assumption (CP)
||3.    (~(Q|R))                               Assumption (IP)
||4.    ((~Q)&(~R))                            DM 3
||5.    (~Q)                            Simp 4
||6.    (P&(~Q))                               Conj 2, 5
||7.    (~(~(P&(~Q))))                         DN 6
||8.    (P>>R)                          DS 1, 7
||9.    R                               MP 8, 2
||10.   ((~R)&(~Q))                            Com 4
||11.   (~R)                            Simp 10
```

In [21]:
```
one.conj(9,11)
```

```
1.      ((~(P&(~Q)))|(P>>R))            /(P>>(Q|R))
==============================
|2.     P                               Assumption (CP)
||3.    (~(Q|R))                               Assumption (IP)
||4.    ((~Q)&(~R))                            DM 3
||5.    (~Q)                            Simp 4
||6.    (P&(~Q))                               Conj 2, 5
||7.    (~(~(P&(~Q))))                         DN 6
||8.    (P>>R)                          DS 1, 7
||9.    R                               MP 8, 2
||10.   ((~R)&(~Q))                            Com 4
||11.   (~R)                            Simp 10
||12.   (R&(~R))                               Conj 9, 11
```

Discharge result of indirect section of proof.

In [22]:
```
one.discharge()
```

```
1.      ((~(P&(~Q)))|(P>>R))            /(P>>(Q|R))
==============================
|2.     P                               Assumption (CP)
||3.    (~(Q|R))                               Assumption (IP)
||4.    ((~Q)&(~R))                            DM 3
||5.    (~Q)                            Simp 4
||6.    (P&(~Q))                               Conj 2, 5
||7.    (~(~(P&(~Q))))                         DN 6
||8.    (P>>R)                          DS 1, 7
||9.    R                               MP 8, 2
||10.   ((~R)&(~Q))                            Com 4
||11.   (~R)                            Simp 10
||12.   (R&(~R))                               Conj 9, 11
|13.    (~(~(Q|R)))                            IP 3 - 12
```

In [23]:
```
one.dn(13,2)
```

```
1.      ((~(P&(~Q)))|(P>>R))          /(P>>(Q|R))
==============================
|2.     P                             Assumption (CP)
||3.    (~(Q|R))                              Assumption (IP)
||4.    ((~Q)&(~R))                           DM 3
||5.    (~Q)                          Simp 4
||6.    (P&(~Q))                              Conj 2, 5
||7.    (~(~(P&(~Q))))                        DN 6
||8.    (P>>R)                        DS 1, 7
||9.    R                             MP 8, 2
||10.   ((~R)&(~Q))                           Com 4
||11.   (~R)                          Simp 10
||12.   (R&(~R))                              Conj 9, 11
|13.    (~(~(Q|R)))                           IP 3 - 12
|14.    (Q|R)                         DN 13
```

Discharge result of conditional section of proof.

In [24]:
```
one.discharge()
```

```
1.      ((~(P&(~Q)))|(P>>R))          /(P>>(Q|R))
==============================
|2.     P                             Assumption (CP)
||3.    (~(Q|R))                              Assumption (IP)
||4.    ((~Q)&(~R))                           DM 3
||5.    (~Q)                          Simp 4
||6.    (P&(~Q))                              Conj 2, 5
||7.    (~(~(P&(~Q))))                        DN 6
||8.    (P>>R)                        DS 1, 7
||9.    R                             MP 8, 2
||10.   ((~R)&(~Q))                           Com 4
||11.   (~R)                          Simp 10
||12.   (R&(~R))                              Conj 9, 11
|13.    (~(~(Q|R)))                           IP 3 - 12
|14.    (Q|R)                         DN 13
15.     (P>>(Q|R))                            CP 2 - 14
QED
```

In [ ]:
```

```