Debye length

John "Jack" Brooks

March 20, 2021

1 Debye length derivation

The Debye length is the scale length associated with plasma screening out external electric fields, $\mathbf{E}$. To show this, we start with Gauss's law and convert $\mathbf{E}$ to potential, ϕ with $\mathbf{E} = -\nabla\phi$.

$$\begin{aligned}\nabla^2\phi &= \frac{1}{\epsilon_0}\left(\rho_{plasma}\right) \\ &= \frac{1}{\epsilon_0}\left[qn_0 \exp\left(\frac{-q\phi(r)}{k_b T}\right)\right]\end{aligned} \tag{1}$$

where and the plasma charge is assuming a Boltzmann distribution.

If we assume that $k_b T \gg q\phi$, then we can Taylor expand the exponential term to get

$$\nabla^2\phi = \frac{1}{\epsilon_0}\left[qn_0\left(1 - \frac{q\phi(r)}{k_b T}\right)\right] \tag{2}$$

Assuming that we have equal ion and electron charge density, $q_e n_{oe}$ cancels with $q_i n_{oi}$ leaving

$$\nabla^2\phi = \frac{1}{\epsilon_0}\left[-\frac{q^2 n_0 \phi(r)}{k_b T}\right]. \tag{3}$$

The solution to this is

$$\begin{aligned}\phi &= \phi_0 \exp\left(-r/\left(\frac{q^2 n_0}{\epsilon_0 k_b T}\right)^{1/2}\right) \\ &= \phi_0 \exp\left(-r/\lambda_D^{1/2}\right)\end{aligned} \tag{4}$$

where the scale length,

$$\lambda_D = \left(\frac{q^2 n_0}{\epsilon_0 k_b T}\right)^{1/2}, \tag{5}$$

is called the Debye length.

2 Debye length screening

If we now include a test charge, Q_{tc}, to our Gauss's law equation,

$$\begin{aligned}\nabla^2\phi &= \frac{1}{\epsilon_0}\left(\rho_{plasma} - \rho_{tc}\right) \\ &= \frac{1}{\lambda_D^2}\phi - \frac{Q_{tc}}{\epsilon_0}\delta(r - r_0)\end{aligned} \tag{6}$$

we can now investigate how the plasma screens out this charge.

TODO