

# Debye length

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## 1 Debye length derivation

The Debye length is the scale length associated with plasma screening out external electric fields,  $\mathbf{E}$ . To show this, we start with Gauss’s law and convert  $\mathbf{E}$  to potential,  $\phi$  with  $\mathbf{E} = -\nabla\phi$ .

$$\begin{aligned}\nabla^2\phi &= \frac{1}{\epsilon_0} (\rho_{plasma}) \\ &= \frac{1}{\epsilon_0} \left[ qn_0 \exp\left(\frac{-q\phi(r)}{k_bT}\right) \right]\end{aligned}\tag{1}$$

where and the plasma charge is assuming a Boltzmann distribution.

If we assume that  $k_bT \gg q\phi$ , then we can Taylor expand the exponential term to get

$$\nabla^2\phi = \frac{1}{\epsilon_0} \left[ qn_0 \left( 1 - \frac{q\phi(r)}{k_bT} \right) \right]\tag{2}$$

Assuming that we have equal ion and electron charge density,  $q_en_{oe}$  cancels with  $q_in_{oi}$  leaving

$$\nabla^2\phi = \frac{1}{\epsilon_0} \left[ -\frac{q^2n_0\phi(r)}{k_bT} \right].\tag{3}$$

The solution to this is

$$\begin{aligned}\phi &= \phi_0 \exp\left(-r/\left(\frac{q^2n_0}{\epsilon_0k_bT}\right)^{1/2}\right) \\ &= \phi_0 \exp\left(-r/\lambda_D^{1/2}\right)\end{aligned}\tag{4}$$

where the scale length,

$$\lambda_D = \left(\frac{q^2n_0}{\epsilon_0k_bT}\right)^{1/2},\tag{5}$$

is called the Debye length.

## 2 Debye length screening

If we now include a test charge,  $Q_{tc}$ , to our Gauss’s law equation,

$$\begin{aligned}\nabla^2\phi &= \frac{1}{\epsilon_0} (\rho_{plasma} - \rho_{tc}) \\ &= \frac{1}{\lambda_D^2}\phi - \frac{Q_{tc}}{\epsilon_0}\delta(r-r_0)\end{aligned}\tag{6}$$

we can now investigate how the plasma screens out this charge.

TODO