# Practical Week 7

## Solutions

## Stepwise Method (Similar final results for Forward Selection):

**Variables Entered/Removed[a]**

| Model | Variables Entered | Variables Removed | Method |
|---|---|---|---|
| 1 | Abdomen | . | Stepwise (Criteria: Probability-of-F-to-enter <= .050, Probability-of-F-to-remove >= .100). |
| 2 | Weight | . | Stepwise (Criteria: Probability-of-F-to-enter <= .050, Probability-of-F-to-remove >= .100). |
| 3 | Wrist | . | Stepwise (Criteria: Probability-of-F-to-enter <= .050, Probability-of-F-to-remove >= .100). |
| 4 | Forearm | . | Stepwise (Criteria: Probability-of-F-to-enter <= .050, Probability-of-F-to-remove >= .100). |

a. Dependent Variable: Percentage of body fat

**Model Summary**

| Model | R | R Square | Adjusted R Square | Std. Error of the Estimate | Akaike Information Criterion | Amemiya Prediction Criterion | Mallows' Prediction Criterion | Schwarz Bayesian Criterion |
|---|---|---|---|---|---|---|---|---|
| | | | | | Selection Criteria | | | |
| 1 | .813[a] | .662 | .660 | 4.877 | 800.645 | .344 | 72.243 | 807.704 |
| 2 | .848[b] | .719 | .717 | 4.456 | 756.040 | .288 | 20.171 | 766.628 |
| 3 | .853[c] | .728 | .724 | 4.393 | 749.896 | .281 | 13.707 | 764.014 |
| 4 | .857[d] | .735 | .731 | 4.343 | 745.075 | .276 | 8.824 | 762.722 |

a. Predictors: (Constant), Abdomen

b. Predictors: (Constant), Abdomen, Weight

c. Predictors: (Constant), Abdomen, Weight, Wrist

d. Predictors: (Constant), Abdomen, Weight, Wrist, Forearm

**ANOVA**

| Model | | Sum of Squares | df | Mean Square | F | Sig. |
|---|---|---|---|---|---|---|
| 1 | Regression | 11631.527 | 1 | 11631.527 | 488.928 | .000 |
| | Residual | 5947.463 | 250 | 23.790 | | |
| | Total | 17578.990 | 251 | | | |
| 2 | Regression | 12635.745 | 2 | 6317.872 | 318.242 | .000 |
| | Residual | 4943.245 | 249 | 19.852 | | |
| | Total | 17578.990 | 251 | | | |
| 3 | Regression | 12792.936 | 3 | 4264.312 | 220.965 | .000 |
| | Residual | 4786.054 | 248 | 19.299 | | |
| | Total | 17578.990 | 251 | | | |
| 4 | Regression | 12920.754 | 4 | 3230.189 | 171.279 | .000 |
| | Residual | 4658.236 | 247 | 18.859 | | |
| | Total | 17578.990 | 251 | | | |

**Coefficients**

| Model | | Unstandardized Coefficients B | Unstandardized Coefficients Std. Error | Standardized Coefficients Beta | t | Sig. |
|---|---|---|---|---|---|---|
| 1 | (Constant) | -39.280 | 2.660 | | -14.765 | .000 |
| | Abdomen | .631 | .029 | .813 | 22.112 | .000 |
| 2 | (Constant) | -45.952 | 2.605 | | -17.640 | .000 |
| | Abdomen | .990 | .057 | 1.275 | 17.447 | .000 |
| | Weight | -.148 | .021 | -.520 | -7.112 | .000 |
| 3 | (Constant) | -27.930 | 6.817 | | -4.097 | .000 |
| | Abdomen | .975 | .056 | 1.256 | 17.368 | .000 |
| | Weight | -.114 | .024 | -.402 | -4.841 | .000 |
| | Wrist | -1.245 | .436 | -.139 | -2.854 | .005 |
| 4 | (Constant) | -34.854 | 7.245 | | -4.811 | .000 |
| | Abdomen | .996 | .056 | 1.283 | 17.760 | .000 |
| | Weight | -.136 | .025 | -.476 | -5.480 | .000 |
| | Wrist | -1.506 | .443 | -.168 | -3.401 | .001 |
| | Forearm | .473 | .182 | .114 | 2.603 | .010 |

$$\text{AIC} = n \ln\left(\frac{SSE}{n}\right) + 2k = 252 \ln\left(\frac{4658.236}{252}\right) + 2(5) = 745.075$$

$$\text{AICC} = \text{AIC} + \frac{2k(k+1)}{n-k-1} = 745.075 + \frac{2 \times 5 \times 6}{252-5-1} = 745.319$$

$$\text{BSC} = n \ln\left(\frac{SSE}{n}\right) + k \ln(n) = 252 \ln\left(\frac{4658.236}{252}\right) + 5 \ln(252) = 762.722$$

$$\text{AIC} = n \ln\left(\frac{SSE}{n}\right) + 2k = 252 \ln\left(\frac{4658.236}{252}\right) + 2(5) = 745.075$$

$$\text{AICC} = \text{AIC} + \frac{2k(k+1)}{n-k-1} = 745.075 + \frac{2 \times 5 \times 6}{252-5-1} = 745.319$$

**Backward Elimination:**

**Variables Entered/Removed**

| Model | Variables Entered | Variables Removed | Method |
|---|---|---|---|
| 1 | Wrist, Age, Height, Ankle, Forearm, Abdomen, Biceps, Knee, Neck, Thigh, Chest, Hip, Weight | . | Enter |
| 2 | | . Knee | Backward (criterion: Probability of F-to-remove >= .100). |
| 3 | | . Chest | Backward (criterion: Probability of F-to-remove >= .100). |
| 4 | | . Height | Backward (criterion: Probability of F-to-remove >= .100). |
| 5 | | . Ankle | Backward (criterion: Probability of F-to-remove >= .100). |
| 6 | | . Biceps | Backward (criterion: Probability of F-to-remove >= .100). |
| 7 | | . Hip | Backward (criterion: Probability of F-to-remove >= .100). |

**ANOVA**

| Model | | Sum of Squares | df | Mean Square | F | Sig. |
|---|---|---|---|---|---|---|
| 1 | Regression | 13158.926 | 13 | 1012.225 | 54.504 | .000 |
| | Residual | 4420.064 | 238 | 18.572 | | |
| | Total | 17578.990 | 251 | | | |
| 2 | Regression | 13158.867 | 12 | 1096.572 | 59.293 | .000 |
| | Residual | 4420.123 | 239 | 18.494 | | |
| | Total | 17578.990 | 251 | | | |
| 3 | Regression | 13158.344 | 11 | 1196.213 | 64.943 | .000 |
| | Residual | 4420.646 | 240 | 18.419 | | |
| | Total | 17578.990 | 251 | | | |
| 4 | Regression | 13157.659 | 10 | 1315.766 | 71.720 | .000 |
| | Residual | 4421.330 | 241 | 18.346 | | |
| | Total | 17578.990 | 251 | | | |
| 5 | Regression | 13144.377 | 9 | 1460.486 | 79.700 | .000 |
| | Residual | 4434.613 | 242 | 18.325 | | |
| | Total | 17578.990 | 251 | | | |
| 6 | Regression | 13123.666 | 8 | 1640.458 | 89.473 | .000 |
| | Residual | 4455.324 | 243 | 18.335 | | |
| | Total | 17578.990 | 251 | | | |
| 7 | Regression | 13087.141 | 7 | 1869.592 | 101.557 | .000 |
| | Residual | 4491.849 | 244 | 18.409 | | |
| | Total | 17578.990 | 251 | | | |

$$\text{AIC} = n \ln\left(\frac{SSE}{n}\right) + 2k = 252 \ln\left(\frac{4491.849}{252}\right) + 2(8) = 741.909$$

$$\text{AICC} = \text{AIC} + \frac{2k(k+1)}{n-k-1} = 741.909 + \frac{2 \times 8 \times 9}{252 - 8 - 1} = 742.543$$

$$\text{BSC} = n \ln\left(\frac{SSE}{n}\right) + k \ln(n) = 252 \ln\left(\frac{4491.849}{252}\right) + 8 \ln(252) = 770.144$$

Partial Output:

**Coefficients**

| Model 7 | Unstandardized Coefficients B | Std. Error | Standardized Coefficients Beta | t | Sig. |
|---|---|---|---|---|---|
| (Constant) | -33.258 | 9.007 | | -3.693 | .000 |
| Age | .068 | .031 | .103 | 2.214 | .028 |
| Weight | -.119 | .034 | -.419 | -3.510 | .001 |
| Neck | -.404 | .221 | -.117 | -1.830 | .068 |
| Abdomen | .918 | .069 | 1.183 | 13.207 | .000 |
| Thigh | .222 | .116 | .139 | 1.913 | .057 |
| Forearm | .553 | .185 | .134 | 2.993 | .003 |
| Wrist | -1.532 | .510 | -.171 | -3.002 | .003 |

AIC and BSC point to two different models. In such cases, generally the suggestion of BSC is used. Therefore, the final model is selected as the one suggested by Stepwise method which is also more parsimonious (besides two of the coefficients in the backward model are not significant).

Note that Mallows $C_p$ in both cases show a good fit:

Backward model

$$C_p = p + \frac{\left(MSE_p - MSE_{full}\right)(n-p)}{MSE_{full}} = 8 + \frac{(18.409\text{-}18.572)(252\text{-}8)}{18.572} = 5.89$$

Stepwise model

$$C_p = p + \frac{\left(MSE_p - MSE_{full}\right)(n-p)}{MSE_{full}} = 5 + \frac{(18.859\text{-}18.572)(252\text{-}5)}{18.572} = 8.82$$

And there is not much between the two in terms of $R^2$ (0.744 and 0.735) or adjusted $R^2$ (0.737 and 0.731).