

Chapter 2

2.7

Problem

Express $1 + i$ in polar form, and hence calculate $(1 + i)^{16}$.

Solution

$$1 + i = \sqrt{2}e^{i\pi/4}$$

Therefore

$$(1 + i)^{16} = \sqrt{2}^{16}(e^{i\pi/4})^{16} = 2^8 e^{i2\pi} = 256$$

2.10

Problem

Calculate $\sum_{r=0}^{100} i^r$.

Solution

$$\sum_{r=k}^{k+3} i^r = 0, k \in \mathbb{Z}$$

and

$$100 \mod 4 = 0$$

Therefore the sum is 0.

2.13

Problem

Prove that if $z_1, z_2 \in \mathbb{C}$, then

$$|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2(|z_1|^2 + |z_2|^2)$$

Solution

Let $z_1 = a + ib$ and $z_2 = c + id$.

$$\begin{aligned}(a + c)^2 + (b + d)^2 + (a - c)^2 + (b - d)^2 &= 2a^2 + 2c^2 + 2b^2 + 2d^2 \\ &= 2(a^2 + b^2 + c^2 + d^2)\end{aligned}$$