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# Euler Angles, Quaternions, and Transformation Matrices

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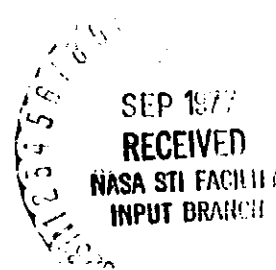
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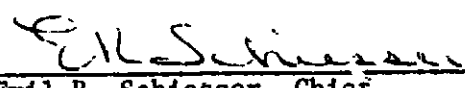
EULER ANGLES, QUATERNIONS, AND TRANSFORMATION MATRICES -

WORKING RELATIONSHIPS


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EULER ANGLES, QUATERNIONS, AND TRANSFORMATION MATRICES -  
WORKING RELATIONSHIPS

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1.0 INTRODUCTION

Due to the extensive use of the quaternion in the onboard Space Shuttle Computer System, considerable analysis is being performed using relationships between the quaternion, the transformation matrix, and the Euler angles. This Internal Note offers a brief mathematical development of the relationships between the Euler angles and the transformation matrix, the quaternion and the transformation matrix, and the Euler angles and the quaternion. The analysis and equations presented here apply directly to current Space Shuttle problems. Appendix A presents the twelve three-axis Euler transformation matrices as functions of the Euler angles, the equations for the quaternion as a function of the Euler angles, and the Euler angles as a function of the transformation matrix elements.

The equations of Appendix A are a valuable reference in Shuttle analysis work and this Internal Note is the only known document where each of the twelve Euler angle to quaternion relationships are given. Appendix B presents a group of utility subroutines to accomplish the Euler-matrix, quaternion-matrix, and Euler-quaternion relationships of Appendix A.

## 2.0 DISCUSSION

### 2.1 Euler Angle Transformation Matrices

The following analysis and utility subroutines are offered to simplify computer programs when working with coordinate transformation matrices and their relationships with the Euler Angles and the Quaternions. The coordinate transformation matrices discussed here are defined using the following figure,

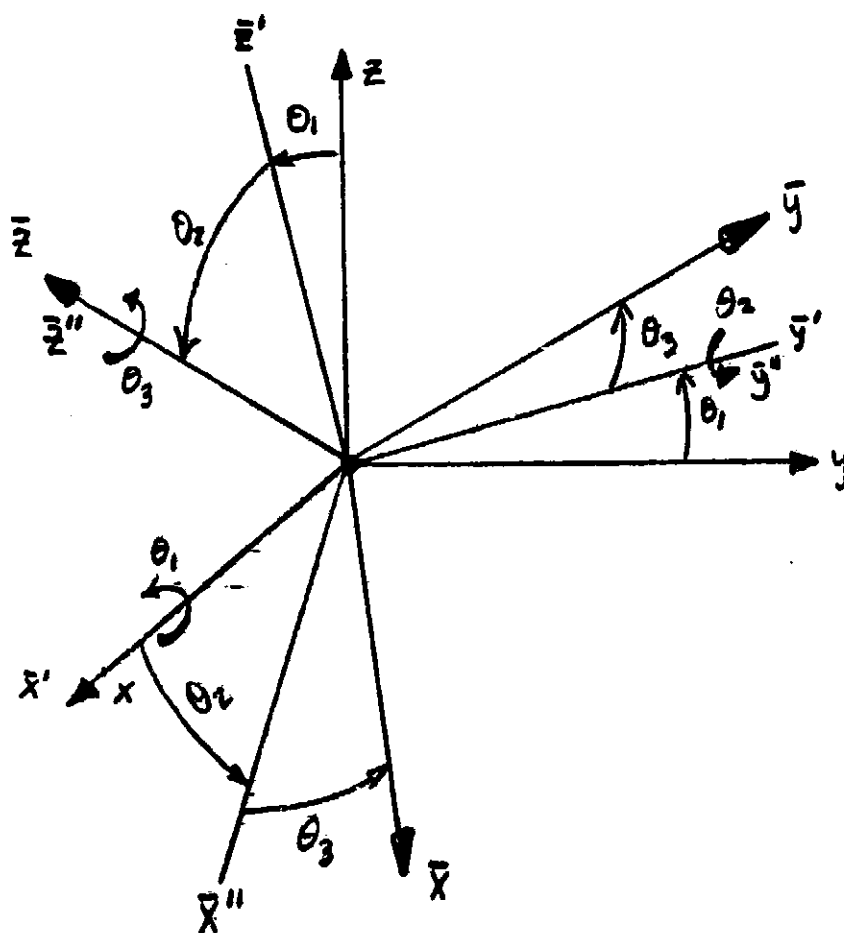


Figure 1.- Coordinate system and Euler angles.

The transformation matrix  $M$ , is defined to transform vectors in the  $\bar{x}$ - system  $(\bar{x}, \bar{y}, \bar{z})$  into the original x-system  $(x, y, z)$  and is given by the equation,

$$x = M\bar{x}$$

where

(1)

$$x = (x, y, z) \text{ and } \bar{x} = (\bar{x}, \bar{y}, \bar{z}).$$

Using the right-hand rule for positive rotations, the  $M$  matrix in (1) above is constructed by the following analysis. The first rotation in Figure 1 above is about the  $x$ -axis by the amount  $\theta_1$ . The single rotation about the  $x$ -axis results in the following transformation,

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_1 & -\sin\theta_1 \\ 0 & \sin\theta_1 & \cos\theta_1 \end{pmatrix} \begin{pmatrix} \bar{x}' \\ \bar{y}' \\ \bar{z}' \end{pmatrix} \quad (2)$$

or  $x = X\bar{x}'$  in matrix form. Rotation about the  $\bar{y}'$ -axis by the amount  $\theta_2$  yields the intermediate transformation matrix:

$$\begin{pmatrix} \bar{x}' \\ \bar{y}' \\ \bar{z}' \end{pmatrix} = \begin{pmatrix} \cos\theta_2 & 0 & \sin\theta_2 \\ 0 & 1 & 0 \\ -\sin\theta_2 & 0 & \cos\theta_2 \end{pmatrix} \begin{pmatrix} \bar{x}'' \\ \bar{y}'' \\ \bar{z}'' \end{pmatrix} \quad (3)$$

or  $\bar{x}' = Y\bar{x}''$  in matrix form. Finally rotation about the  $\bar{z}''$ -axis by the amount  $\theta_3$  yields the intermediate transformation matrix,

$$\begin{pmatrix} \bar{x}'' \\ \bar{y}'' \\ \bar{z}'' \end{pmatrix} = \begin{pmatrix} \cos\theta_3 & -\sin\theta_3 & 0 \\ \sin\theta_3 & \cos\theta_3 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \end{pmatrix} \quad (4)$$

and in matrix form  $\bar{x}'' = Z\bar{x}'$ . Now using the three equations,

$$\begin{aligned} x &= X\bar{x}' \\ \bar{x}' &= Y\bar{x}'' \\ \bar{x}'' &= Z\bar{x} \end{aligned} \quad (5)$$

by substitution

$$x = (X Y Z) \bar{x}. \quad (6)$$

Then from equation 1,

$$M = (X Y Z) \quad (7)$$

Computation for the M matrix from the indicated matrix multiplication in equation (7) yields,

$$M = \begin{pmatrix} (\cos\theta_2 \cos\theta_3) & (-\cos\theta_2 \sin\theta_3) & (\sin\theta_2) \\ (\cos\theta_1 \sin\theta_3 + \sin\theta_1 \sin\theta_2 \cos\theta_3) & (\cos\theta_1 \cos\theta_3 - \sin\theta_1 \sin\theta_2 \sin\theta_3) & (-\sin\theta_1 \cos\theta_2) \\ (\sin\theta_1 \sin\theta_3 - \cos\theta_1 \sin\theta_2 \cos\theta_3) & (\sin\theta_1 \cos\theta_3 + \cos\theta_1 \sin\theta_2 \sin\theta_3) & (\cos\theta_1 \cos\theta_2) \end{pmatrix} \quad (8)$$

The matrix M in equation (8) is a function of;

- (1) The three Euler angles  $\theta_1$ ,  $\theta_2$  and  $\theta_3$  and
- (2) The sequence of rotations used to generate the matrix.

By examination of equation (7) it is possible to show that there are twelve possible Euler rotational sequences. If the (X Y Z) notation in equation (7) represents a rotation about the X axis, then the Y axis and finally the Z axis, then the following per-

mutations of the rotational order represents the twelve possible Euler angle sets using three rotations,

$$\begin{array}{lll}
 X Y Z & Y X Z & Z X Y \\
 X Z Y & Y Z X & Z Y X \\
 X Y X & Y X Y & Z X Z \\
 X Z X & Y Z Y & Z Y Z
 \end{array} \tag{9}$$

Any repeated axis rotation such as  $XXY$  does not represent a three axis rotation but reduces to the two axis rotation  $XY$ . Hence the rotations described in (9) above represent all twelve possible sets of Euler angle defining sequences. Conversely then, for a given transformation matrix there are twelve Euler angle sets which can be extracted from the matrix. The Euler matrices corresponding to all possible rotational sequences of (9) above are presented in Appendix A.

The utility subroutines "EULMAT" generates the transformation matrix from a given Euler sequence and the Euler angles. The utility subroutine "MATEUL" extracts the Euler angles from a given Euler rotational sequence. The convention is established that the Euler angles occur in the same sequences as the axis rotations. Using this concept and functional notation, equation (7) could be expressed as

$$M = X Y Z = M(\theta_x, \theta_y, \theta_z) \tag{10}$$

and from (9)



$$M = X Z X = M(\theta_x, \theta_z, \theta'_x) \text{ etc.} \quad (11)$$

This convention is assumed in both subroutines and also used throughout this design note when Euler angles are used. A brief explanation of the use of these two utility subroutines is given in Appendix B.

It is interesting to note that a negative rotation in the single axis rotation matrices of equations (2), (3) and (4) will result in the formation of the transpose of the matrix. However the transpose of M in equation (7) is formed from reversing the order of multiplication and transposing the individual axis rotation equations, i.e.

$$M^T = (X Y Z)^T = (Y Z)^T X^T = Z^T Y^T X^T. \quad (12)$$

Hence the transposes of the matrices of (9) are easily formed by reversing the order of multiplication and transposing each single axis rotation equation. Using the notation in equations (10) and (11) above equation (12) could be written,

$$M^T(\theta_x, \theta_y, \theta_z) = M(-\theta_z, -\theta_y, -\theta_x). \quad (13)$$

It is recommended to avoid confusion that the forward transformation be computed and simply transposed to yield the reverse transformation matrix. All matrices of Appendix A are in the forward form, i.e.  $X = M\bar{x}$  and formed from (9).

## 2.2 Transformation Matrices Using the Hamilton Quaternion

The transformation matrix of equation (1) can be written as a function of the Hamilton Quaternion;

$$\begin{aligned} q_1 &= \cos \omega/2 \\ q_2 &= \cos \alpha \sin \omega/2 \\ q_3 &= \cos \beta \sin \omega/2 \\ q_4 &= \cos \gamma \sin \omega/2, \end{aligned} \quad (14)$$

where  $\omega$  is the rotation angle about the rotation axis with  $\alpha$ ,  $\beta$ , and  $\gamma$  direction angles with the x, y and z axes respectively. Notice also that  $q_1^2 + q_2^2 + q_3^2 + q_4^2 = 1$ , since  $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$ . The rotation angle,  $\omega$ , is assumed positive according to the right-hand rule of axis rotation.

The matrix M becomes

$$M = \begin{pmatrix} (q_1^2 + q_2^2 - q_3^2 - q_4^2) & 2(q_2q_3 - q_1q_4) & 2(q_2q_4 + q_1q_3) \\ 2(q_2q_3 + q_1q_4) & (q_1^2 - q_2^2 + q_3^2 - q_4^2) & 2(q_3q_4 - q_1q_2) \\ 2(q_2q_4 - q_1q_3) & 2(q_3q_4 + q_1q_2) & (q_1^2 - q_2^2 - q_3^2 + q_4^2) \end{pmatrix}. \quad (15)$$

For a more detailed discussion of the derivation of equation (15), see Reference 1. Using functional notation, equation (15) can be written,

$$M = M(q_1, q_2, q_3, q_4). \quad (16)$$

Unlike the Euler angle rotational sequences to describe the transformation matrix of equation (1), only two quaternions can be found from equation (15). The two quaternions are:

$$\begin{array}{ll}
 q_1 & -q_1 \\
 q_2 & -q_2 \\
 q_3 & -q_3 \\
 q_4 & -q_4
 \end{array}
 \quad \text{and} \quad (17)$$

These two quaternions represent a positive rotation about the rotation axis pointing in one direction and a positive rotation about the same line of rotation pointing in the opposite direction. Both quaternions of (17) satisfy equation (15).

The utility subroutine "QMAT" generates the transformation matrix from a given quaternion. The "QMAT" algorithm generates the matrix as given in equation (15) without duplicating any arithmetic operations. The subroutine "MATQ" extracts the positive quaternion, i.e.,  $q_1 > 0$ , from the transformation matrix and normalizes the results to guarantee an orthogonal matrix. In order to avoid any discontinuity in extracting the quaternion from the transformation matrix, the procedure as described in Reference 2 is used.

Early works by Hamilton (Reference 3) presented the quaternion as having a scalar and a vector part, i.e.,

$$q_1 = S \quad \vec{V} = (q_2, q_3, q_4) \quad (18)$$

and equation (16) could be expressed as,

$$M = M(q_1, q_2, q_3, q_4) = M(S, \vec{V}). \quad (19)$$

For a given quaternion the following relationship is true (from (17) above),

$$M(S, \vec{V}) = M(-S, -\vec{V}). \quad (20)$$

The transpose of the transformation matrix is given by,

$$M^T(S, \vec{V}) = M(-S, \vec{V}) = M(S, -\vec{V}). \quad (21)$$

### 2.3 Euler Angle and Quaternion Relationships

By examination of equations (10) and (16) the equality,

$$M(X(\theta_1), Y(\theta_2), Z(\theta_3)) = M(\theta_1, \theta_2, \theta_3) = M(q_1, q_2, q_3, q_4) \quad (22)$$

can be written. Based on an equality for each element of the matrix the following nine equations must be true;

$$\begin{aligned} \cos\theta_2 \cos\theta_3 &= q_1^2 + q_2^2 - q_3^2 - q_4^2 \\ -\cos\theta_2 \sin\theta_3 &= 2(q_2q_3 - q_1q_4) \\ \sin\theta_2 &= 2(q_2q_4 + q_1q_3) \\ \cos\theta_1 \sin\theta_3 + \sin\theta_1 \sin\theta_2 \cos\theta_3 &= 2(q_2q_3 + q_1q_4) \\ \cos\theta_1 \cos\theta_3 - \sin\theta_1 \sin\theta_2 \sin\theta_3 &= q_1^2 - q_2^2 + q_3^2 - q_4^2 \\ -\sin\theta_1 \cos\theta_2 &= 2(q_3q_4 - q_1q_2) \\ \sin\theta_1 \sin\theta_3 - \cos\theta_1 \sin\theta_2 \cos\theta_3 &= 2(q_3q_4 - q_1q_3) \\ \sin\theta_1 \cos\theta_3 + \cos\theta_1 \sin\theta_2 \sin\theta_3 &= 2(q_3q_4 + q_1q_2) \\ \cos\theta_1 \cos\theta_2 &= q_1^2 - q_2^2 - q_3^2 + q_4^2. \end{aligned} \quad (23)$$

It is possible to solve for the values of the quaternion using the trigonometric half angle identities. For this Euler sequence, i.e.  $X(\theta_1) Y(\theta_2) Z(\theta_3)$ , the following quaternion results;

$$\begin{aligned}
q_1 &= -\sin\frac{1}{2}\theta_1 \sin\frac{1}{2}\theta_2 \sin\frac{1}{2}\theta_3 + \cos\frac{1}{2}\theta_1 \cos\frac{1}{2}\theta_2 \cos\frac{1}{2}\theta_3 \\
q_2 &= +\sin\frac{1}{2}\theta_1 \cos\frac{1}{2}\theta_2 \cos\frac{1}{2}\theta_3 + \sin\frac{1}{2}\theta_2 \sin\frac{1}{2}\theta_3 \cos\frac{1}{2}\theta_1 \\
q_3 &= -\sin\frac{1}{2}\theta_1 \sin\frac{1}{2}\theta_3 \cos\frac{1}{2}\theta_2 + \sin\frac{1}{2}\theta_2 \cos\frac{1}{2}\theta_1 \cos\frac{1}{2}\theta_3 \\
q_4 &= +\sin\frac{1}{2}\theta_1 \sin\frac{1}{2}\theta_2 \cos\frac{1}{2}\theta_3 + \sin\frac{1}{2}\theta_3 \cos\frac{1}{2}\theta_1 \cos\frac{1}{2}\theta_2
\end{aligned}
\tag{24}$$

Appendix A gives the quaternion as a function of the Euler angles for each of the twelve Euler rotational sequence presented in Section 2.1. The special equations for the quaternion as functions of the Euler angles, like equations (24) above, are sometimes cumbersome to use, especially when multiple Euler sequences are utilized. Less coding, but perhaps more computer operations, are required by using the more general method of first generating the matrix M from the given Euler angle set and then simply extracting the quaternion from the matrix. This method is effected by first a call to "EULMAT" and then a call to "MATQ".

### 3.0 REFERENCES

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## APPENDIX A

### RELATIONSHIPS FOR THE THREE-AXIS EULER ANGLE ROTATION SEQUENCES

The twelve Euler matrices for each rotation sequence are given based on the single axis rotation equations (2), (3) and (4). The Euler matrices transform vectors from the system that has been rotated into vectors in the stationary system. Also presented here are the equations for the quaternion as a function of the Euler angles and the Euler angles as a function of the matrix elements for each rotation sequence.

$$(1) \quad M = M(X(\theta_1), Y(\theta_2), Z(\theta_3)) = XYZ$$

Axis Rotation Sequence: 1, 2, 3

$$M = \begin{bmatrix} \cos\theta_2\cos\theta_3 & -\cos\theta_2\sin\theta_3 & \sin\theta_2 \\ \sin\theta_1\sin\theta_2\cos\theta_3 + \cos\theta_1\sin\theta_3 & -\sin\theta_1\sin\theta_2\sin\theta_3 + \cos\theta_1\cos\theta_3 & -\sin\theta_1\cos\theta_2 \\ -\cos\theta_1\sin\theta_2\cos\theta_3 + \sin\theta_1\sin\theta_3 & \cos\theta_1\sin\theta_2\sin\theta_3 + \sin\theta_1\cos\theta_3 & \cos\theta_1\cos\theta_2 \end{bmatrix}$$

$$q_1 = -\sin\frac{1}{2}\theta_1\sin\frac{1}{2}\theta_2\sin\frac{1}{2}\theta_3 + \cos\frac{1}{2}\theta_1\cos\frac{1}{2}\theta_2\cos\frac{1}{2}\theta_3$$

$$q_2 = \sin\frac{1}{2}\theta_1\cos\frac{1}{2}\theta_2\cos\frac{1}{2}\theta_3 + \sin\frac{1}{2}\theta_2\sin\frac{1}{2}\theta_3\cos\frac{1}{2}\theta_1$$

$$q_3 = -\sin\frac{1}{2}\theta_1\sin\frac{1}{2}\theta_3\cos\frac{1}{2}\theta_2 + \sin\frac{1}{2}\theta_2\cos\frac{1}{2}\theta_1\cos\frac{1}{2}\theta_3$$

$$q_4 = \sin\frac{1}{2}\theta_1\sin\frac{1}{2}\theta_2\cos\frac{1}{2}\theta_3 + \sin\frac{1}{2}\theta_3\cos\frac{1}{2}\theta_1\cos\frac{1}{2}\theta_2$$

$$\theta_1 = \tan^{-1}\left(\frac{-m_{23}}{m_{33}}\right)$$

$$\theta_2 = \tan^{-1}\left(\frac{m_{13}}{\sqrt{1-m_{13}^2}}\right)$$

$$\theta_3 = \tan^{-1}\left(\frac{-m_{12}}{m_{11}}\right)$$



$$(2) M = M(X(\theta_1), Z(\theta_2), Y(\theta_3)) = XZY$$

Axis Rotation Sequence: 1, 3, 2

$$M = \begin{bmatrix} \cos\theta_2\cos\theta_3 & -\sin\theta_2 & \cos\theta_2\sin\theta_3 \\ \cos\theta_1\sin\theta_2\cos\theta_3 & \cos\theta_1\cos\theta_2 & \cos\theta_1\sin\theta_2\sin\theta_3 \\ +\sin\theta_1\sin\theta_3 & & -\sin\theta_1\cos\theta_3 \\ \sin\theta_1\sin\theta_2\cos\theta_3 & \sin\theta_1\cos\theta_2 & \sin\theta_1\sin\theta_2\sin\theta_3 \\ -\cos\theta_1\sin\theta_3 & & +\cos\theta_1\cos\theta_3 \end{bmatrix}$$

$$q_1 = +\sin\frac{1}{2}\theta_1\sin\frac{1}{2}\theta_2\sin\frac{1}{2}\theta_3 + \cos\frac{1}{2}\theta_1\cos\frac{1}{2}\theta_2\cos\frac{1}{2}\theta_3$$

$$q_2 = +\sin\frac{1}{2}\theta_1\cos\frac{1}{2}\theta_2\cos\frac{1}{2}\theta_3 - \sin\frac{1}{2}\theta_2\sin\frac{1}{2}\theta_3\cos\frac{1}{2}\theta_1$$

$$q_3 = -\sin\frac{1}{2}\theta_1\sin\frac{1}{2}\theta_2\cos\frac{1}{2}\theta_3 + \sin\frac{1}{2}\theta_3\cos\frac{1}{2}\theta_1\cos\frac{1}{2}\theta_2$$

$$q_4 = +\sin\frac{1}{2}\theta_1\sin\frac{1}{2}\theta_3\cos\frac{1}{2}\theta_2 + \sin\frac{1}{2}\theta_2\cos\frac{1}{2}\theta_1\cos\frac{1}{2}\theta_3$$

$$\theta_1 = \tan^{-1}\left(\frac{m_{32}}{m_{22}}\right)$$

$$\theta_2 = \tan^{-1}\left(\frac{-m_{12}}{\sqrt{1-m_{12}^2}}\right)$$

$$\theta_3 = \tan^{-1}\left(\frac{m_{13}}{m_{11}}\right)$$

Axis Rotation Sequence: 1, 2, 1

$$M = \begin{bmatrix} \cos\theta_2 & \sin\theta_2\sin\theta_3 & \sin\theta_2\cos\theta_3 \\ \sin\theta_1\sin\theta_2 & \cos\theta_1\cos\theta_3 & -\cos\theta_1\sin\theta_3 \\ -\cos\theta_1\sin\theta_2 & -\sin\theta_1\cos\theta_2\sin\theta_3 & -\sin\theta_1\cos\theta_2\cos\theta_3 \\ \sin\theta_1\cos\theta_2 & \cos\theta_1\sin\theta_2\sin\theta_3 & \cos\theta_1\sin\theta_2\cos\theta_3 \end{bmatrix}$$

$$q_1 = \cos\frac{1}{2}\theta_2 \cos(\frac{1}{2}(\theta_1 + \theta_3))$$

$$q_2 = \cos\frac{1}{2}\theta_2 \sin(\frac{1}{2}(\theta_1 + \theta_3))$$

$$q_3 = \sin\frac{1}{2}\theta_2 \cos(\frac{1}{2}(\theta_1 - \theta_3))$$

$$q_4 = \sin\frac{1}{2}\theta_2 \sin(\frac{1}{2}(\theta_1 - \theta_3))$$

$$\theta_1 = \tan^{-1} \left( \frac{m_{21}}{-m_{31}} \right)$$

$$\theta_2 = \tan^{-1} \left( \frac{\sqrt{1 - m_{11}^2}}{m_{11}} \right)$$

$$\theta_3 = \tan^{-1} \left( \frac{m_{12}}{m_{13}} \right)$$

$$(4) \quad M = M(X(\theta_1), Z(\theta_2), X(\theta_3)) = XZX$$

Axis Rotation Sequence: 1, 3, 1

$$M = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2\cos\theta_3 & \sin\theta_2\sin\theta_3 \\ \cos\theta_1\sin\theta_2 & \cos\theta_1\cos\theta_2\cos\theta_3 & -\cos\theta_1\cos\theta_2\sin\theta_3 \\ \sin\theta_1\sin\theta_2 & \sin\theta_1\cos\theta_2\cos\theta_3 & -\sin\theta_1\cos\theta_2\sin\theta_3 \end{bmatrix}$$

$$q_1 = \cos \frac{1}{2} \theta_2 \cos (\frac{1}{2} (\theta_1 + \theta_3))$$

$$q_2 = \cos \frac{1}{2} \theta_2 \sin(\frac{1}{2}(\theta_1 + \theta_3))$$

$$q_3 = -\sin\frac{1}{2}\theta_2 \sin(\frac{1}{2}(\theta_1 - \theta_3))$$

$$q_4 = \sin \frac{1}{2} \theta_2 \cos (\frac{1}{2} (\theta_1 - \theta_3))$$

$$\theta_1 = \tan^{-1} \left( \frac{m_{31}}{m_{21}} \right)$$

$$\theta_2 = \tan^{-1} \left( \frac{\sqrt{1 - m_{11}^2}}{m_{11}} \right)$$

$$\theta_3 = \tan^{-1} \left( \frac{m_{13}}{-m_{12}} \right)$$

$$(5) \quad M = M(Y(\theta_1), X(\theta_2), Z(\theta_3)) = YXZ$$

Axis Rotation Sequence: 2, 1, 3

$$M = \begin{bmatrix} \sin\theta_1 \sin\theta_2 \sin\theta_3 & \sin\theta_1 \sin\theta_2 \cos\theta_3 & \sin\theta_1 \cos\theta_2 \\ +\cos\theta_1 \cos\theta_3 & -\cos\theta_1 \sin\theta_3 & \\ \cos\theta_2 \sin\theta_3 & \cos\theta_2 \cos\theta_3 & -\sin\theta_2 \\ \cos\theta_1 \sin\theta_2 \sin\theta_3 & \cos\theta_1 \sin\theta_2 \cos\theta_3 & \cos\theta_1 \cos\theta_2 \\ -\sin\theta_1 \cos\theta_3 & +\sin\theta_1 \sin\theta_3 & \end{bmatrix}$$

$$q_1 = \sin^{\frac{1}{2}}\theta_1 \sin^{\frac{1}{2}}\theta_2 \sin^{\frac{1}{2}}\theta_3 + \cos^{\frac{1}{2}}\theta_1 \cos^{\frac{1}{2}}\theta_2 \cos^{\frac{1}{2}}\theta_3$$

$$q_2 = \sin^{\frac{1}{2}}\theta_1 \sin^{\frac{1}{2}}\theta_3 \cos^{\frac{1}{2}}\theta_2 + \sin^{\frac{1}{2}}\theta_2 \cos^{\frac{1}{2}}\theta_1 \cos^{\frac{1}{2}}\theta_3$$

$$q_3 = \sin^{\frac{1}{2}}\theta_1 \cos^{\frac{1}{2}}\theta_2 \cos^{\frac{1}{2}}\theta_3 - \sin^{\frac{1}{2}}\theta_2 \sin^{\frac{1}{2}}\theta_3 \cos^{\frac{1}{2}}\theta_1$$

$$q_4 = -\sin^{\frac{1}{2}}\theta_1 \sin^{\frac{1}{2}}\theta_2 \cos^{\frac{1}{2}}\theta_3 + \sin^{\frac{1}{2}}\theta_3 \cos^{\frac{1}{2}}\theta_1 \cos^{\frac{1}{2}}\theta_2$$

$$\theta_1 = \tan^{-1} \left( \frac{m_{31}}{m_{33}} \right)$$

$$\theta_2 = \tan^{-1} \left( \sqrt{\frac{-m_{23}}{1-m_{23}^2}} \right)$$

$$\theta_3 = \tan^{-1} \left( \frac{m_{21}}{m_{22}} \right)$$

$$(6) \quad M = M(Y(\theta_1), Z(\theta_2), X(\theta_3)) = YZX$$

Axis Rotation Sequence: 2, 3, 1

$$M = \begin{bmatrix} \cos\theta_1 \cos\theta_2 & -\cos\theta_1 \sin\theta_2 \cos\theta_3 & \cos\theta_1 \sin\theta_2 \sin\theta_3 \\ \sin\theta_2 & \cos\theta_2 \cos\theta_3 & -\cos\theta_2 \sin\theta_3 \\ -\sin\theta_1 \cos\theta_2 & \sin\theta_1 \sin\theta_2 \cos\theta_3 & -\sin\theta_1 \sin\theta_2 \sin\theta_3 \end{bmatrix}$$

$$q_1 = -\sin^{\frac{1}{2}}\theta_1 \sin^{\frac{1}{2}}\theta_2 \sin^{\frac{1}{2}}\theta_3 + \cos^{\frac{1}{2}}\theta_1 \cos^{\frac{1}{2}}\theta_2 \cos^{\frac{1}{2}}\theta_3$$

$$q_2 = +\sin^{\frac{1}{2}}\theta_1 \sin^{\frac{1}{2}}\theta_2 \cos^{\frac{1}{2}}\theta_3 + \sin^{\frac{1}{2}}\theta_3 \cos^{\frac{1}{2}}\theta_1 \cos^{\frac{1}{2}}\theta_2$$

$$q_3 = +\sin^{\frac{1}{2}}\theta_1 \cos^{\frac{1}{2}}\theta_2 \cos^{\frac{1}{2}}\theta_3 + \sin^{\frac{1}{2}}\theta_2 \sin^{\frac{1}{2}}\theta_3 \cos^{\frac{1}{2}}\theta_1$$

$$q_4 = -\sin^{\frac{1}{2}}\theta_1 \sin^{\frac{1}{2}}\theta_3 \cos^{\frac{1}{2}}\theta_2 + \sin^{\frac{1}{2}}\theta_2 \cos^{\frac{1}{2}}\theta_1 \cos^{\frac{1}{2}}\theta_3$$

$$\theta_1 = \tan^{-1} \left( \frac{-m_{31}}{m_{11}} \right)$$

$$\theta_2 = \tan^{-1} \left( \frac{m_{21}}{\sqrt{1-m_{21}^2}} \right)$$

$$\theta_3 = \tan^{-1} \left( \frac{-m_{23}}{m_{22}} \right)$$

$$(7) \quad M = M(Y(\theta_1), X(\theta_2), Y(\theta_3)) = YXY$$

Axis Rotation Sequence: 2, 1, 2

$$M = \begin{bmatrix} -\sin\theta_1 \cos\theta_2 \sin\theta_3 & \sin\theta_1 \sin\theta_2 & \sin\theta_1 \cos\theta_2 \cos\theta_3 \\ +\cos\theta_1 \cos\theta_3 & & +\cos\theta_1 \sin\theta_3 \\ \sin\theta_2 \sin\theta_3 & \cos\theta_2 & -\sin\theta_2 \cos\theta_3 \\ -\cos\theta_1 \cos\theta_2 \sin\theta_3 & \cos\theta_1 \sin\theta_2 & \cos\theta_1 \cos\theta_2 \cos\theta_3 \\ -\sin\theta_1 \cos\theta_3 & & -\sin\theta_1 \sin\theta_3 \end{bmatrix}$$

$$q_1 = +\cos\frac{1}{2}\theta_2 \cos(\frac{1}{2}(\theta_1 + \theta_3))$$

$$q_2 = +\sin\frac{1}{2}\theta_2 \cos(\frac{1}{2}(\theta_1 - \theta_3))$$

$$q_3 = +\cos\frac{1}{2}\theta_2 \sin(\frac{1}{2}(\theta_1 + \theta_3))$$

$$q_4 = -\sin\frac{1}{2}\theta_2 \sin(\frac{1}{2}(\theta_1 - \theta_3))$$

$$\theta_1 = \tan^{-1} \left( \frac{m_{12}}{m_{32}} \right)$$

$$\theta_2 = \tan^{-1} \left( \frac{\sqrt{1-m_{22}^2}}{m_{22}} \right)$$

$$\theta_3 = \tan^{-1} \left( \frac{m_{21}}{-m_{23}} \right)$$

$$(8) \quad M = M(Y(\theta_1), Z(\theta_2), Y(\theta_3)) = YZY$$

Axis Rotation Sequence: 2, 3, 2

$$M = \begin{bmatrix} \cos\theta_1 \cos\theta_2 \cos\theta_3 & -\cos\theta_1 \sin\theta_2 & \cos\theta_1 \cos\theta_2 \sin\theta_3 \\ -\sin\theta_1 \sin\theta_3 & & +\sin\theta_1 \cos\theta_3 \\ \sin\theta_2 \cos\theta_3 & \cos\theta_2 & \sin\theta_2 \sin\theta_3 \\ -\sin\theta_1 \cos\theta_2 \cos\theta_3 & \sin\theta_1 \sin\theta_2 & -\sin\theta_1 \cos\theta_2 \sin\theta_3 \\ -\cos\theta_1 \sin\theta_3 & & +\cos\theta_1 \cos\theta_3 \end{bmatrix}$$

$$q_1 = +\cos\frac{1}{2}\theta_2 \cos(\frac{1}{2}(\theta_1 + \theta_3))$$

$$q_2 = +\sin\frac{1}{2}\theta_2 \sin(\frac{1}{2}(\theta_1 - \theta_3))$$

$$q_3 = +\cos\frac{1}{2}\theta_2 \sin(\frac{1}{2}(\theta_1 + \theta_3))$$

$$q_4 = +\sin\frac{1}{2}\theta_2 \cos(\frac{1}{2}(\theta_1 - \theta_3))$$

$$\theta_1 = \tan^{-1} \left( \frac{m_{32}}{-m_{12}} \right)$$

$$\theta_2 = \tan^{-1} \left( \frac{\sqrt{1-m_{22}^2}}{m_{22}} \right)$$

$$\theta_3 = \tan^{-1} \left( \frac{m_{23}}{m_{21}} \right)$$

$$(9) \quad M = M(Z(\theta_1), X(\theta_2), Y(\theta_3)) = ZXY$$

Axis Rotation Sequence: 3, 1, 2

$$M = \begin{bmatrix} -\sin\theta_1 \sin\theta_2 \sin\theta_3 & -\sin\theta_1 \cos\theta_2 & \sin\theta_1 \sin\theta_2 \cos\theta_3 \\ +\cos\theta_1 \cos\theta_3 & & +\cos\theta_1 \sin\theta_3 \\ \cos\theta_1 \sin\theta_2 \sin\theta_3 & \cos\theta_1 \cos\theta_2 & -\cos\theta_1 \sin\theta_2 \cos\theta_3 \\ +\sin\theta_1 \cos\theta_3 & & +\sin\theta_1 \sin\theta_3 \\ -\cos\theta_2 \sin\theta_3 & \sin\theta_2 & \cos\theta_2 \cos\theta_3 \end{bmatrix}$$

$$q_1 = -\sin\frac{1}{2}\theta_1 \sin\frac{1}{2}\theta_2 \sin\frac{1}{2}\theta_3 + \cos\frac{1}{2}\theta_1 \cos\frac{1}{2}\theta_2 \cos\frac{1}{2}\theta_3$$

$$q_2 = -\sin\frac{1}{2}\theta_1 \sin\frac{1}{2}\theta_3 \cos\frac{1}{2}\theta_2 + \sin\frac{1}{2}\theta_2 \cos\frac{1}{2}\theta_1 \cos\frac{1}{2}\theta_3$$

$$q_3 = +\sin\frac{1}{2}\theta_1 \sin\frac{1}{2}\theta_2 \cos\frac{1}{2}\theta_3 + \sin\frac{1}{2}\theta_3 \cos\frac{1}{2}\theta_1 \cos\frac{1}{2}\theta_2$$

$$q_4 = +\sin\frac{1}{2}\theta_1 \cos\frac{1}{2}\theta_2 \cos\frac{1}{2}\theta_3 + \sin\frac{1}{2}\theta_2 \sin\frac{1}{2}\theta_3 \cos\frac{1}{2}\theta_1$$

$$\theta_1 = \tan^{-1} \left( \frac{-m_{12}}{m_{22}} \right)$$

$$\theta_2 = \tan^{-1} \left( \frac{m_{32}}{\sqrt{1-m_{32}^2}} \right)$$

$$\theta_3 = \tan^{-1} \left( \frac{-m_{31}}{m_{33}} \right)$$



$$(10) \quad M = M(Z(\theta_1), Y(\theta_2), X(\theta_3)) = ZYX$$

Axis Rotation Sequence: 3, 2, 1

$$M = \begin{bmatrix} \cos\theta_1 \cos\theta_2 & \cos\theta_1 \sin\theta_2 \sin\theta_3 & \cos\theta_1 \sin\theta_2 \cos\theta_3 \\ \sin\theta_1 \cos\theta_2 & \sin\theta_1 \sin\theta_2 \sin\theta_3 & \sin\theta_1 \sin\theta_2 \cos\theta_3 \\ -\sin\theta_2 & \cos\theta_2 \sin\theta_3 & \cos\theta_2 \cos\theta_3 \end{bmatrix}$$

$$q_1 = +\sin^{\frac{1}{2}}\theta_1 \sin^{\frac{1}{2}}\theta_2 \sin^{\frac{1}{2}}\theta_3 + \cos^{\frac{1}{2}}\theta_1 \cos^{\frac{1}{2}}\theta_2 \cos^{\frac{1}{2}}\theta_3$$

$$q_2 = -\sin^{\frac{1}{2}}\theta_1 \sin^{\frac{1}{2}}\theta_2 \cos^{\frac{1}{2}}\theta_3 + \sin^{\frac{1}{2}}\theta_3 \cos^{\frac{1}{2}}\theta_1 \cos^{\frac{1}{2}}\theta_2$$

$$q_3 = +\sin^{\frac{1}{2}}\theta_1 \sin^{\frac{1}{2}}\theta_3 \cos^{\frac{1}{2}}\theta_2 + \sin^{\frac{1}{2}}\theta_2 \cos^{\frac{1}{2}}\theta_1 \cos^{\frac{1}{2}}\theta_3$$

$$q_4 = +\sin^{\frac{1}{2}}\theta_1 \cos^{\frac{1}{2}}\theta_2 \cos^{\frac{1}{2}}\theta_3 - \sin^{\frac{1}{2}}\theta_2 \sin^{\frac{1}{2}}\theta_3 \cos^{\frac{1}{2}}\theta_1$$

$$\theta_1 = \tan^{-1} \left( \frac{m_{21}}{m_{11}} \right)$$

$$\theta_2 = \tan^{-1} \left( \frac{-m_{31}}{\sqrt{1-m_{31}^2}} \right)$$

$$\theta_3 = \tan^{-1} \left( \frac{m_{32}}{m_{33}} \right)$$

$$(11) \quad M = M(Z(\theta_1), X(\theta_2), Z(\theta_3)) = ZXZ$$

Axis Rotation Sequence: 3, 1, 3

$$M = \begin{bmatrix} -\sin\theta_1 \cos\theta_2 \sin\theta_3 & -\sin\theta_1 \cos\theta_2 \cos\theta_3 & \sin\theta_1 \sin\theta_2 \\ +\cos\theta_1 \cos\theta_3 & -\cos\theta_1 \cos\theta_3 & -\cos\theta_1 \sin\theta_2 \\ \cos\theta_1 \cos\theta_2 \sin\theta_3 & \cos\theta_1 \cos\theta_2 \cos\theta_3 & \cos\theta_1 \sin\theta_2 \\ +\sin\theta_1 \cos\theta_3 & -\sin\theta_1 \sin\theta_3 & \cos\theta_2 \\ \sin\theta_2 \sin\theta_3 & \sin\theta_2 \cos\theta_3 & \cos\theta_2 \end{bmatrix}$$

$$q_1 = +\cos\frac{1}{2}\theta_2 \cos(\frac{1}{2}(\theta_1 + \theta_3))$$

$$q_2 = +\sin\frac{1}{2}\theta_2 \cos(\frac{1}{2}(\theta_1 - \theta_3))$$

$$q_3 = +\sin\frac{1}{2}\theta_2 \sin(\frac{1}{2}(\theta_1 - \theta_3))$$

$$q_4 = +\cos\frac{1}{2}\theta_2 \sin(\frac{1}{2}(\theta_1 + \theta_3))$$

$$\theta_1 = \tan^{-1} \left( \frac{m_{13}}{-m_{23}} \right)$$

$$\theta_2 = \tan^{-1} \left( \frac{\sqrt{1-m_{33}^2}}{m_{33}} \right)$$

$$\theta_3 = \tan^{-1} \left( \frac{m_{31}}{m_{32}} \right)$$

$$12) M = M(Z(\theta_1), Y(\theta_2), Z(\theta_3)) = ZYZ$$

Axis Rotation Sequence: 3, 2, 3

$$M = \begin{bmatrix} \cos\theta_1 \cos\theta_2 \cos\theta_3 & -\cos\theta_1 \cos\theta_2 \sin\theta_3 & \cos\theta_1 \sin\theta_2 \\ -\sin\theta_1 \sin\theta_3 & -\sin\theta_1 \cos\theta_3 & \\ \sin\theta_1 \cos\theta_2 \cos\theta_3 & -\sin\theta_1 \cos\theta_2 \sin\theta_3 & \sin\theta_1 \sin\theta_2 \\ +\cos\theta_1 \sin\theta_3 & +\cos\theta_1 \cos\theta_3 & \\ -\sin\theta_2 \cos\theta_3 & \sin\theta_2 \sin\theta_3 & \cos\theta_2 \end{bmatrix}$$

$$q_1 = +\cos\frac{1}{2}\theta_2 \cos(\frac{1}{2}(\theta_1 + \theta_3))$$

$$q_2 = -\sin\frac{1}{2}\theta_2 \sin(\frac{1}{2}(\theta_1 - \theta_3))$$

$$q_3 = +\sin\frac{1}{2}\theta_2 \cos(\frac{1}{2}(\theta_1 - \theta_3))$$

$$q_4 = +\cos\frac{1}{2}\theta_2 \sin(\frac{1}{2}(\theta_1 + \theta_3))$$

$$\theta_1 = \tan^{-1} \left( \frac{m_{23}}{m_{13}} \right)$$

$$\theta_2 = \tan^{-1} \left( \frac{\sqrt{1-m_{33}^2}}{m_{33}} \right)$$

$$\theta_3 = \tan^{-1} \left( \frac{m_{32}}{-m_{31}} \right)$$

APPENDIX B  
COMPUTER SUBROUTINES FOR THE RELATIONSHIPS

The following subroutines with a brief description of their use are presented in this appendix.

- (1) "EULMAT" - Generates the transformation matrix from a given set of Euler angles and an axis rotation sequence.
- (2) "MATEUL" - Extracts the Euler angles from the given transformation matrix and an axis rotation sequence.
- (3) "QMAT" - Generates the transformation matrix from a given quaternion.
- (4) "MATQ" - Extracts the quaternion from a given transformation matrix.
- (5) "YPRQ" - Generates the quaternion directly from the yaw-pitch-roll Euler angles.
- (6) "POSNOR" - Computes the positive-normalized quaternion from the given quaternion.

NAME: EULMAT

PURPOSE: Generates a 3 x 3 transformation matrix from a given sequence and Euler angle set.

INPUT: ISEQ - Rotation Sequence (Integer Array (3); i.e., 1, 2, 3)

EUL - Euler Angles in radians, in "ISEQ" Order; ARRAY (3)

OUTPUT: A - The 3 x 3 transformation matrix

ALGORITHM REFERENCE: Appendix A; Euler Sequences (1) thru (12).

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OF POOR QUALITY

# EULER ANGLES TO THE TRANSFORMATION MATRIX

FOR, IS FULMAT, EULMAT  
FOR SCE3-02/19/77-06:24:23 (,0)

SUBROUTINE LULMAT ENTRY POINT 000237

STORAGE USED: CODE(1) 000230; DATA(0) 000124; BLANK COMMON(2)

EXTERNAL REFERENCES (BLOCK, NAME)

0003 SIN  
0004 COS  
0005 NEPR35

STORAGE ASSIGNMENT (BLOCK, TYPE, RELATIVE LOCATION, NAME)

0001	000124	1001	0001	000123	1000	0001	000124
0001	000142	1620	0001	000144	1650	0001	000146
0001	000177	301	0001	000173	P	0001	000175
0001	000157	INJ01	0001	000146	J	0001	000144
0000	R	000047	SINA	0000	P	000053	TEMP

```

00101      1*      SUBROUTINE EULMAT(ISEC,EUL,A)
00102      2*      DIMENSION ISEC(3),EUL(3),A(3,3)
00103      3*      DIMENSION X(3,3,3),R(3,3)
00104      4*      DO 100 K=1,3
00105      5*      DO 10 I=1,3
00106      6*      DO 5 J=1,3
00107      7*      X(I,J,K)=0.0
00108      8*      IF(I.EQ.J) X(I,J,K)=1.0
00109      9*      CONTINUE
00110     10*      CONTINUE
00111     11*      IF(ISEC(K).LE.0) GO TO 100
00112     12*      SINA=SIN(FUL(K))
00113     13*      COSA=COS(FUL(K))
00114     14*      IF(ISEC(K).EQ.2) GO TO 20
00115     15*      IF(ISEC(K).EQ.3) GO TO 30
00116     16*      X(1,2,K)=COSA
00117     17*      X(2,3,K)=-SINA
00118     18*      X(3,2,K)=SINA
00119     19*      X(3,3,K)=COSA
00120     20*      GO TO 100
00121     21*      X(1,1,K)=COSA
00122     22*      X(1,2,K)=SINA
00123     23*      X(1,3,K)=-SINA
00124     24*      X(3,1,K)=SINA
00125     25*      X(3,2,K)=COSA
00126     26*      GO TO 100
00127     27*      X(1,1,K)=COSA
00128     28*      X(1,2,K)=-SINA
00129     29*      X(1,3,K)=SINA
00130     30*      X(3,1,K)=SINA
00131     31*      X(3,2,K)=COSA
00132     32*      GO TO 100
00133     33*      X(1,1,K)=COSA
00134     34*      X(1,2,K)=SINA
00135     35*      X(1,3,K)=SINA
00136     36*      X(3,1,K)=SINA
00137     37*      X(3,2,K)=COSA
00138     38*      GO TO 100
00139     39*      X(1,1,K)=COSA
00140     40*      X(1,2,K)=SINA
00141     41*      X(1,3,K)=SINA
00142     42*      X(3,1,K)=SINA
00143     43*      X(3,2,K)=COSA
00144     44*      GO TO 100
00145     45*      X(1,1,K)=COSA
00146     46*      X(1,2,K)=SINA
00147     47*      X(1,3,K)=SINA
00148     48*      X(3,1,K)=SINA
00149     49*      X(3,2,K)=COSA
00150     50*      GO TO 100
00151     51*      X(1,1,K)=COSA
00152     52*      X(1,2,K)=SINA
00153     53*      X(1,3,K)=SINA
00154     54*      X(3,1,K)=SINA
00155     55*      X(3,2,K)=COSA
00156     56*      GO TO 100
00157     57*      X(1,1,K)=COSA
00158     58*      X(1,2,K)=SINA
00159     59*      X(1,3,K)=SINA
00160     60*      X(3,1,K)=SINA
00161     61*      X(3,2,K)=COSA
00162     62*      GO TO 100
00163     63*      X(1,1,K)=COSA
00164     64*      X(1,2,K)=SINA
00165     65*      X(1,3,K)=SINA
00166     66*      X(3,1,K)=SINA
00167     67*      X(3,2,K)=COSA
00168     68*      GO TO 100
00169     69*      X(1,1,K)=COSA
00170     70*      X(1,2,K)=SINA
00171     71*      X(1,3,K)=SINA
00172     72*      X(3,1,K)=SINA
00173     73*      X(3,2,K)=COSA
00174     74*      GO TO 100
00175     75*      X(1,1,K)=COSA
00176     76*      X(1,2,K)=SINA
00177     77*      X(1,3,K)=SINA
00178     78*      X(3,1,K)=SINA
00179     79*      X(3,2,K)=COSA
00180     80*      GO TO 100
00181     81*      X(1,1,K)=COSA
00182     82*      X(1,2,K)=SINA
00183     83*      X(1,3,K)=SINA
00184     84*      X(3,1,K)=SINA
00185     85*      X(3,2,K)=COSA
00186     86*      GO TO 100
00187     87*      X(1,1,K)=COSA
00188     88*      X(1,2,K)=SINA
00189     89*      X(1,3,K)=SINA
00190     90*      X(3,1,K)=SINA
00191     91*      X(3,2,K)=COSA
00192     92*      GO TO 100
00193     93*      X(1,1,K)=COSA
00194     94*      X(1,2,K)=SINA
00195     95*      X(1,3,K)=SINA
00196     96*      X(3,1,K)=SINA
00197     97*      X(3,2,K)=COSA
00198     98*      GO TO 100
00199     99*      X(1,1,K)=COSA
00200    100*      X(1,2,K)=SINA
00201    101*      X(1,3,K)=SINA
00202    102*      X(3,1,K)=SINA
00203    103*      X(3,2,K)=COSA
00204    104*      GO TO 100
00205    105*      X(1,1,K)=COSA
00206    106*      X(1,2,K)=SINA
00207    107*      X(1,3,K)=SINA
00208    108*      X(3,1,K)=SINA
00209    109*      X(3,2,K)=COSA
00210    110*      GO TO 100
00211    111*      X(1,1,K)=COSA
00212    112*      X(1,2,K)=SINA
00213    113*      X(1,3,K)=SINA
00214    114*      X(3,1,K)=SINA
00215    115*      X(3,2,K)=COSA
00216    116*      GO TO 100
00217    117*      X(1,1,K)=COSA
00218    118*      X(1,2,K)=SINA
00219    119*      X(1,3,K)=SINA
00220    120*      X(3,1,K)=SINA
00221    121*      X(3,2,K)=COSA
00222    122*      GO TO 100
00223    123*      X(1,1,K)=COSA
00224    124*      X(1,2,K)=SINA
00225    125*      X(1,3,K)=SINA
00226    126*      X(3,1,K)=SINA
00227    127*      X(3,2,K)=COSA
00228    128*      GO TO 100
00229    129*      X(1,1,K)=COSA
00230    130*      X(1,2,K)=SINA
00231    131*      X(1,3,K)=SINA
00232    132*      X(3,1,K)=SINA
00233    133*      X(3,2,K)=COSA
00234    134*      GO TO 100
00235    135*      X(1,1,K)=COSA
00236    136*      X(1,2,K)=SINA
00237    137*      X(1,3,K)=SINA
00238    138*      X(3,1,K)=SINA
00239    139*      X(3,2,K)=COSA
00240    140*      GO TO 100
00241    141*      X(1,1,K)=COSA
00242    142*      X(1,2,K)=SINA
00243    143*      X(1,3,K)=SINA
00244    144*      X(3,1,K)=SINA
00245    145*      X(3,2,K)=COSA
00246    146*      GO TO 100
00247    147*      X(1,1,K)=COSA
00248    148*      X(1,2,K)=SINA
00249    149*      X(1,3,K)=SINA
00250    150*      X(3,1,K)=SINA
00251    151*      X(3,2,K)=COSA
00252    152*      GO TO 100
00253    153*      X(1,1,K)=COSA
00254    154*      X(1,2,K)=SINA
00255    155*      X(1,3,K)=SINA
00256    156*      X(3,1,K)=SINA
00257    157*      X(3,2,K)=COSA
00258    158*      GO TO 100
00259    159*      X(1,1,K)=COSA
00260    160*      X(1,2,K)=SINA
00261    161*      X(1,3,K)=SINA
00262    162*      X(3,1,K)=SINA
00263    163*      X(3,2,K)=COSA
00264    164*      GO TO 100
00265    165*      X(1,1,K)=COSA
00266    166*      X(1,2,K)=SINA
00267    167*      X(1,3,K)=SINA
00268    168*      X(3,1,K)=SINA
00269    169*      X(3,2,K)=COSA
00270    170*      GO TO 100
00271    171*      X(1,1,K)=COSA
00272    172*      X(1,2,K)=SINA
00273    173*      X(1,3,K)=SINA
00274    174*      X(3,1,K)=SINA
00275    175*      X(3,2,K)=COSA
00276    176*      GO TO 100
00277    177*      X(1,1,K)=COSA
00278    178*      X(1,2,K)=SINA
00279    179*      X(1,3,K)=SINA
00280    180*      X(3,1,K)=SINA
00281    181*      X(3,2,K)=COSA
00282    182*      GO TO 100
00283    183*      X(1,1,K)=COSA
00284    184*      X(1,2,K)=SINA
00285    185*      X(1,3,K)=SINA
00286    186*      X(3,1,K)=SINA
00287    187*      X(3,2,K)=COSA
00288    188*      GO TO 100
00289    189*      X(1,1,K)=COSA
00290    190*      X(1,2,K)=SINA
00291    191*      X(1,3,K)=SINA
00292    192*      X(3,1,K)=SINA
00293    193*      X(3,2,K)=COSA
00294    194*      GO TO 100
00295    195*      X(1,1,K)=COSA
00296    196*      X(1,2,K)=SINA
00297    197*      X(1,3,K)=SINA
00298    198*      X(3,1,K)=SINA
00299    199*      X(3,2,K)=COSA
00300    200*      GO TO 100
00301    201*      X(1,1,K)=COSA
00302    202*      X(1,2,K)=SINA
00303    203*      X(1,3,K)=SINA
00304    204*      X(3,1,K)=SINA
00305    205*      X(3,2,K)=COSA
00306    206*      GO TO 100
00307    207*      X(1,1,K)=COSA
00308    208*      X(1,2,K)=SINA
00309    209*      X(1,3,K)=SINA
00310    210*      X(3,1,K)=SINA
00311    211*      X(3,2,K)=COSA
00312    212*      GO TO 100
00313    213*      X(1,1,K)=COSA
00314    214*      X(1,2,K)=SINA
00315    215*      X(1,3,K)=SINA
00316    216*      X(3,1,K)=SINA
00317    217*      X(3,2,K)=COSA
00318    218*      GO TO 100
00319    219*      X(1,1,K)=COSA
00320    220*      X(1,2,K)=SINA
00321    221*      X(1,3,K)=SINA
00322    222*      X(3,1,K)=SINA
00323    223*      X(3,2,K)=COSA
00324    224*      GO TO 100
00325    225*      X(1,1,K)=COSA
00326    226*      X(1,2,K)=SINA
00327    227*      X(1,3,K)=SINA
00328    228*      X(3,1,K)=SINA
00329    229*      X(3,2,K)=COSA
00330    230*      GO TO 100
00331    231*      X(1,1,K)=COSA
00332    232*      X(1,2,K)=SINA
00333    233*      X(1,3,K)=SINA
00334    234*      X(3,1,K)=SINA
00335    235*      X(3,2,K)=COSA
00336    236*      GO TO 100
00337    237*      X(1,1,K)=COSA
00338    238*      X(1,2,K)=SINA
00339    239*      X(1,3,K)=SINA
00340    240*      X(3,1,K)=SINA
00341    241*      X(3,2,K)=COSA
00342    242*      GO TO 100
00343    243*      X(1,1,K)=COSA
00344    244*      X(1,2,K)=SINA
00345    245*      X(1,3,K)=SINA
00346    246*      X(3,1,K)=SINA
00347    247*      X(3,2,K)=COSA
00348    248*      GO TO 100
00349    249*      X(1,1,K)=COSA
00350    250*      X(1,2,K)=SINA
00351    251*      X(1,3,K)=SINA
00352    252*      X(3,1,K)=SINA
00353    253*      X(3,2,K)=COSA
00354    254*      GO TO 100
00355    255*      X(1,1,K)=COSA
00356    256*      X(1,2,K)=SINA
00357    257*      X(1,3,K)=SINA
00358    258*      X(3,1,K)=SINA
00359    259*      X(3,2,K)=COSA
00360    260*      GO TO 100
00361    261*      X(1,1,K)=COSA
00362    262*      X(1,2,K)=SINA
00363    263*      X(1,3,K)=SINA
00364    264*      X(3,1,K)=SINA
00365    265*      X(3,2,K)=COSA
00366    266*      GO TO 100
00367    267*      X(1,1,K)=COSA
00368    268*      X(1,2,K)=SINA
00369    269*      X(1,3,K)=SINA
00370    270*      X(3,1,K)=SINA
00371    271*      X(3,2,K)=COSA
00372    272*      GO TO 100
00373    273*      X(1,1,K)=COSA
00374    274*      X(1,2,K)=SINA
00375    275*      X(1,3,K)=SINA
00376    276*      X(3,1,K)=SINA
00377    277*      X(3,2,K)=COSA
00378    278*      GO TO 100
00379    279*      X(1,1,K)=COSA
00380    280*      X(1,2,K)=SINA
00381    281*      X(1,3,K)=SINA
00382    282*      X(3,1,K)=SINA
00383    283*      X(3,2,K)=COSA
00384    284*      GO TO 100
00385    285*      X(1,1,K)=COSA
00386    286*      X(1,2,K)=SINA
00387    287*      X(1,3,K)=SINA
00388    288*      X(3,1,K)=SINA
00389    289*      X(3,2,K)=COSA
00390    290*      GO TO 100
00391    291*      X(1,1,K)=COSA
00392    292*      X(1,2,K)=SINA
00393    293*      X(1,3,K)=SINA
00394    294*      X(3,1,K)=SINA
00395    295*      X(3,2,K)=COSA
00396    296*      GO TO 100
00397    297*      X(1,1,K)=COSA
00398    298*      X(1,2,K)=SINA
00399    299*      X(1,3,K)=SINA
00400    300*      X(3,1,K)=SINA
00401    301*      X(3,2,K)=COSA
00402    302*      GO TO 100
00403    303*      X(1,1,K)=COSA
00404    304*      X(1,2,K)=SINA
00405    305*      X(1,3,K)=SINA
00406    306*      X(3,1,K)=SINA
00407    307*      X(3,2,K)=COSA
00408    308*      GO TO 100
00409    309*      X(1,1,K)=COSA
00410    310*      X(1,2,K)=SINA
00411    311*      X(1,3,K)=SINA
00412    312*      X(3,1,K)=SINA
00413    313*      X(3,2,K)=COSA
00414    314*      GO TO 100
00415    315*      X(1,1,K)=COSA
00416    316*      X(1,2,K)=SINA
00417    317*      X(1,3,K)=SINA
00418    318*      X(3,1,K)=SINA
00419    319*      X(3,2,K)=COSA
00420    320*      GO TO 100
00421    321*      X(1,1,K)=COSA
00422    322*      X(1,2,K)=SINA
00423    323*      X(1,3,K)=SINA
00424    324*      X(3,1,K)=SINA
00425    325*      X(3,2,K)=COSA
00426    326*      GO TO 100
00427    327*      X(1,1,K)=COSA
00428    328*      X(1,2,K)=SINA
00429    329*      X(1,3,K)=SINA
00430    330*      X(3,1,K)=SINA
00431    331*      X(3,2,K)=COSA
00432    332*      GO TO 100
00433    333*      X(1,1,K)=COSA
00434    334*      X(1,2,K)=SINA
00435    335*      X(1,3,K)=SINA
00436    336*      X(3,1,K)=SINA
00437    337*      X(3,2,K)=COSA
00438    338*      GO TO 100
00439    339*      X(1,1,K)=COSA
00440    340*      X(1,2,K)=SINA
00441    341*      X(1,3,K)=SINA
00442    342*      X(3,1,K)=SINA
00443    343*      X(3,2,K)=COSA
00444    344*      GO TO 100
00445    345*      X(1,1,K)=COSA
00446    346*      X(1,2,K)=SINA
00447    347*      X(1,3,K)=SINA
00448    348*      X(3,1,K)=SINA
00449    349*      X(3,2,K)=COSA
00450    350*      GO TO 100
00451    351*      X(1,1,K)=COSA
00452    352*      X(1,2,K)=SINA
00453    353*      X(1,3,K)=SINA
00454    354*      X(3,1,K)=SINA
00455    355*      X(3,2,K)=COSA
00456    356*      GO TO 100
00457    357*      X(1,1,K)=COSA
00458    358*      X(1,2,K)=SINA
00459    359*      X(1,3,K)=SINA
00460    360*      X(3,1,K)=SINA
00461    361*      X(3,2,K)=COSA
00462    362*      GO TO 100
00463    363*      X(1,1,K)=COSA
00464    364*      X(1,2,K)=SINA
00465    365*      X(1,3,K)=SINA
00466    366*      X(3,1,K)=SINA
00467    367*      X(3,2,K)=COSA
00468    368*      GO TO 100
00469    369*      X(1,1,K)=COSA
00470    370*      X(1,2,K)=SINA
00471    371*      X(1,3,K)=SINA
00472    372*      X(3,1,K)=SINA
00473    373*      X(3,2,K)=COSA
00474    374*      GO TO 100
00475    375*      X(1,1,K)=COSA
00476    376*      X(1,2,K)=SINA
00477    377*      X(1,3,K)=SINA
00478    378*      X(3,1,K)=SINA
00479    379*      X(3,2,K)=COSA
00480    380*      GO TO 100
00481    381*      X(1,1,K)=COSA
00482    382*      X(1,2,K)=SINA
00483    383*      X(1,3,K)=SINA
00484    384*      X(3,1,K)=SINA
00485    385*      X(3,2,K)=COSA
00486    386*      GO TO 100
00487    387*      X(1,1,K)=COSA
00488    388*      X(1,2,K)=SINA
00489    389*      X(1,3,K)=SINA
00490    390*      X(3,1,K)=SINA
00491    391*      X(3,2,K)=COSA
00492    392*      GO TO 100
00493    393*      X(1,1,K)=COSA
00494    394*      X(1,2,K)=SINA
00495    395*      X(1,3,K)=SINA
00496    396*      X(3,1,K)=SINA
00497    397*      X(3,2,K)=COSA
00498    398*      GO TO 100
00499    399*      X(1,1,K)=COSA
00500    400*      X(1,2,K)=SINA
00501    401*      X(1,3,K)=SINA
00502    402*      X(3,1,K)=SINA
00503    403*      X(3,2,K)=COSA
00504    404*      GO TO 100
00505    405*      X(1,1,K)=COSA
00506    406*      X(1,2,K)=SINA
00507    407*      X(1,3,K)=SINA
00508    408*      X(3,1,K)=SINA
00509    409*      X(3,2,K)=COSA
00510    410*      GO TO 100
00511    411*      X(1,1,K)=COSA
00512    412*      X(1,2,K)=SINA
00513    413*      X(1,3,K)=SINA
00514    414*      X(3,1,K)=SINA
00515    415*      X(3,2,K)=COSA
00516    416*      GO TO 100
00517    417*      X(1,1,K)=COSA
00518    418*      X(1,2,K)=SINA
00519    419*      X(1,3,K)=SINA
00520    420*      X(3,1,K)=SINA
00521    421*      X(3,2,K)=COSA
00522    422*      GO TO 100
00523    423*      X(1,1,K)=COSA
00524    424*      X(1,2,K)=SINA
00525    425*      X(1,3,K)=SINA
00526    426*      X(3,1,K)=SINA
00527    427*      X(3,2,K)=COSA
00528    428*      GO TO 100
00529    429*      X(1,1,K)=COSA
00530    430*      X(1,2,K)=SINA
00531    431*      X(1,3,K)=SINA
00532    432*      X(3,1,K)=SINA
00533    433*      X(3,2,K)=COSA
00534    434*      GO TO 100
00535    435*      X(1,1,K)=COSA
00536    436*      X(1,2,K)=SINA
00537    437*      X(1,3,K)=SINA
00538    438*      X(3,1,K)=SINA
00539    439*      X(3,2,K)=COSA
00540    440*      GO TO 100
00541    441*      X(1,1,K)=COSA
00542    442*      X(1,2,K)=SINA
00543    443*      X(1,3,K)=SINA
00544    444*      X(3,1,K)=SINA
00545    445*      X(3,2,K)=COSA
00546    446*      GO TO 100
00547    447*      X(1,1,K)=COSA
00548    448*      X(1,2,K)=SINA
00549    449*      X(1,3,K)=SINA
00550    450*      X(3,1,K)=SINA
00551    451*      X(3,2,K)=COSA
00552    452*      GO TO 100
00553    453*      X(1,1,K)=COSA
00554    454*      X(1,2,K)=SINA
00555    455*      X(1,3,K)=SINA
00556    456*      X(3,1,K)=SINA
00557    457*      X(3,2,K)=COSA
00558    458*      GO TO 100
00559    459*      X(1,1,K)=COSA
00560    460*      X(1,2,K)=SINA
00561    461*      X(1,3,K)=SINA
00562    462*      X(3,1,K)=SINA
00563    463*      X(3,2,K)=COSA
00564    464*      GO TO 100
00565    465*      X(1,1,K)=COSA
00566    466*      X(1,2,K)=SINA
00567    467*      X(1,3,K)=SINA
00568    468*      X(3,1,K)=SINA
00569    469*      X(3,2,K)=COSA
00570    470*      GO TO 100
00571    471*      X(1,1,K)=COSA
00572    472*      X(1,2,K)=SINA
00573    473*      X(1,3,K)=SINA
00574    474*      X(3,1,K)=SINA
00575    475*      X(3,2,K)=COSA
00576    476*      GO TO 100
00577    477*      X(1,1,K)=COSA
00578    478*      X(1,2,K)=SINA
00579    479*      X(1,3,K)=SINA
00580    480*      X(3,1,K)=SINA
00581    481*      X(3,2,K)=COSA
00582    482*      GO TO 100
00583    483*      X(1,1,K)=COSA
00584    484*      X(1,2,K)=SINA
00585    485*      X(1,3,K)=SINA
00586    486*      X(3,1,K)=SINA
00587    487*      X(3,2,K)=COSA
00588    488*      GO TO 100
00589    489*      X(1,1,K)=COSA
00590    490*      X(1,2,K)=SINA
00591    491*      X(1,3,K)=SINA
00592    492*      X(3,1,K)=SINA
00593    493*      X(3,2,K)=COSA
00594    494*      GO TO 100
00595    495*      X(1,1,K)=COSA
00596    496*      X(1,2,K)=SINA
00597    497*      X(1,3,K)=SINA
00598    498*      X(3,1,K)=SINA
00599    499*      X(3,2,K)=COSA
00600    500*      GO TO 100
00601    501*      X(1,1,K)=COSA
00602    502*      X(1,2,K)=SINA
00603    503*      X(1,3,K)=SINA
00604    504*      X(3,1,K)=SINA
00605    505*      X(3,2,K)=COSA
00606    506*      GO TO 100
00607    507*      X(1,1,K)=COSA
00608    508*      X(1,2,K)=SINA
00609    509*      X(1,3,K)=SINA
00610    510*      X(3,1,K)=SINA
00611    511*      X(3,2,K)=COSA
00612    512*      GO TO 100
00613    513*      X(1,1,K)=COSA
00614    514*      X(1,2,K)=SINA
00615    515*      X(1,3,K)=SINA
00616    516*      X(3,1,K)=SINA
00617    517*      X(3,2,K)=COSA
00618    518*      GO TO 100
00619    519*      X(1,1,K)=COSA
00620    520*      X(1,2,K)=SINA
00621    521*      X(1,3,K)=SINA
00622    522*      X(3,1,K)=SINA
00623    523*      X(3,2,K)=COSA
00624    524*      GO TO 100
00625    525*      X(1,1,K)=COSA
00626    526*      X(1,2,K)=SINA
00627    527*      X(1,3,K)=SINA
00628    528*      X(3,1,K)=SINA
00629    529*      X(3,2,K)=COSA
00630    530*      GO TO 100
00631    531*      X(1,1,K)=COSA
00632    532*      X(1,2,K)=SINA
00633    533*      X(1,3,K)=SINA
00634    534*      X(3,1,K)=SINA
006
```

# EULER ANGLES TO THE TRANSFORMATION MATRIX

(CONTINUED)

00153	20*	100	CONTINUE
00155	1*	00	400 L=1,2
00160	2*	M=3-L	
00161	3*	00	300 I=1,3
00164	34*	00	300 J=1,3
00167	25*	TEMP=0.0	
00170	12*	00	350 K=1,3
00173	37*	IF(L.EQ.1)	HOLD=X(K,J,3)
00175	28*	IF(L.EQ.2)	HOLD=X(K,J,1)
00177	26*	IF(ABS(HOLD)-(1.1,JE-L))	GO TO 250
00201	41*	IF(ABS(X(I,K,M))-(1.1,JE-10))	GO TO 250
00203	41*	TEMP=TEMP+X(I,K,M)*HOLD	
00204	42*	250	CONTINUE
00206	42*	IF(L.EQ.1)	B(I,J)=TEMP
00210	44*	IF(L.EQ.2)	A(I,J)=TEMP
00213	45*	300	CONTINUE
00215	46*	400	CONTINUE
00217	47*		RETURN
00220	48*		END

END OF COMPILATION:

NO DIAGNOSTICS.

ORIGINAL PAGE IS  
OF POOR QUALITY

NAME: MATEUL

PURPOSE: Extracts the Euler angles from the given transformation matrix and the required Euler rotational sequence.

INPUT: ISEQ - Rotation sequence, (Integer Array (3), i.e., 1,2,3.)  
A - The 3 x 3 transformation

OUTPUT: EUL - The Euler angles, in "ISEQ" order; ARRAY(3).

ALGORITHM REFERENCE: Appendix A; Euler angles as a function of the matrix elements, sequences (1) thru (12).



# TRANSFORMATION MATRIX TO THE EULER ANGLES

T.

JFOR,15 MATEUL,MATEUL  
FOR 5003-D2/19/77-06:24:25 (,0)

ORIGINAL PAGE IS  
OF POOR QUALITY

SUBROUTINE MATEUL ENTRY POINT 070335

STORAGE USED: CODE(1) 000353; DATA(5) 000352; BLANK COMMON(2)

EXTERNAL REFERENCES (BLOCK, NAME)

003 SORT  
004 ATAN2  
005 MERR3

STORAGE ASSIGNMENT (BLOCK, TYPE, RELATIVE LOCATION, NAME)

001	000054	10L	001	000066	10L	001	000077
001	000110	30L	001	000220	40L	001	000237
001	000251	00L	001	000257	40L	001	000257
000	000254	0510N	000	000255	0510N	000	000251
000	000255	FNUN	000	000256	1	000	000250
000	000256	1	000	000257	00	000	000250

```

00101 1*
00102 2*
00103 3*
00104 4*
00105 5*
00106 6*
00107 7*
00108 8*
00109 9*
00110 10*
00111 11*
00112 12*
00113 13*
00114 14*
00115 15*
00116 16*
00117 17*
00118 18*
00119 19*
00120 20*
00121 21*
00122 22*
00123 23*
00124 24*
00125 25*
00126 26*
00127 27*
00128 28*
00129 29*
00130 30*
00131 31*
00132 32*
00133 33*
00134 34*
00135 35*
00136 36*
00137 37*
00138 38*
00139 39*
00140 40*
00141 41*
00142 42*
00143 43*
00144 44*
00145 45*
00146 46*
00147 47*
00148 48*
00149 49*
00150 50*

SUBROUTINE MATEUL(ISEQ,A,EUL)
DIMENSION A(3,3),EUL(3)
DIMENSION ISEQ(3)
I=ISEQ(1)
J=ISEQ(2)
K=ISEQ(3)
IECK=1
IF(I.EQ.A) IECK=4.95
CSIGN=1.0
IF(I.EQ.1) GO TO 10
IF(I.EQ.2) GO TO 20
IF(J.EQ.1) GO TO 5
CSIGN=-1.0
IF(I.EQ.A) L=1
GO TO 3
CSIGN=-1.0
IF(I.EQ.A) L=1
GO TO 3
IF(J.EQ.2) GO TO 15
CSIGN=-1.0
IF(I.EQ.A) L=3
GO TO 3
CSIGN=-1.0
IF(I.EQ.A) L=1
GO TO 3
IF(J.EQ.3) GO TO 25
CSIGN=-1.0

```

TRANSFORMATION MATRIX TO THE EULER ANGLES  
(CONTINUED)

00150	29*	IF (IEOK.NE.O) L=1
00152	30*	GO TO 30
00153	31*	25 CSIGN=-1.0
00154	32*	IF (IEOK.NE.O) L=3
00156	33*	30 DO I=1,N=1,3
00161	34*	FNSGN=1.0
00162	35*	FDSGN=1.0
00163	36*	IF (N.EQ.2) GO TO 70
00165	37*	IF (N.EQ.1) GO TO 50
00167	38*	IF (IEOK.NE.O) GO TO 40
00171	39*	FNSGN=BSIGN
00172	40*	JJ=1
00173	41*	GO TO 45
00174	42*	40 JJ=L
00175	43*	IF (BSIGN.GT.0.0) FDSGN=-1.0
00177	44*	45 FNUM=FNSGN*A(I,J)
00200	45*	FDEN=FDSGN*A(I,JJ)
00201	46*	GO TO 90
00202	47*	50 IF (IEOK.NE.O) GO TO 55
00204	48*	FNSGN=BSIGN
00205	49*	II=K
00206	50*	JJ=K
00207	51*	GO TO 60
00210	52*	55 FDSGN=BSIGN
00211	53*	II=L
00212	54*	JJ=I
00213	55*	60 FNUM=FNSGN*A(I,K)
00214	56*	FDEN=FDSGN*A(I,JJ)
00215	57*	GO TO 90
00216	58*	70 IF (IEOK.NE.O) GO TO 80
00220	59*	FNUM=CSIGN*A(I,K)
00221	60*	FDEN=SQRT(1.0-A(I,K)**2)
00222	61*	GO TO 90
00223	62*	80 FNUM=SQRT(1.0-A(I,I)**2)
00224	63*	FDEN=A(I,I)
00225	64*	90 CUL(I)=ATAN2(FNUM,FDEN)
00226	65*	100 CONTINUE
00231	66*	RETURN
	67*	END

END OF COMPILATION:

NO DIAGNOSTICS.

ORIGINAL PAGE IS  
OF POOR QUALITY

NAME: QMAT

PURPOSE: Generates the transformation matrix from the given quaternion.

INPUT: Q - The quaternion; ARRAY(4).

OUTPUT: A - The 3 x 3 transformation matrix

ALGORITHM REFERENCE: Equation (15) from Section 2.2.

# QUATERNION TO THE TRANSFORMATION MATRIX

ORIGINAL PAGE IS  
OF POOR QUALITY

FOR IS QMAT, QMAT  
FOR SPEC-02/19/77-06:24:19 (,0)

SUBROUTINE QMAT ENTRY POINT 000077

STORAGE USED: CODE(11) 000103: DATA(0) 000010: BLANK COMMON(2)

EXTERNAL REFERENCES (BLOCK, NAME)

0003 NERR31

STORAGE ASSIGNMENT (BLOCK, TYPE, RELATIVE LOCATION, NAME)

0000 000007 INJPL 0000 P 000000 P2 0000 P 000001  
0000 P 000004 P5 0000 R 000000 TEMP

00101	1*	SUBROUTINE QMAT(C,A)
00103	2*	DIMENSION Q(4),A(3,3)
00104	3*	P2=Q(2)+Q(2)
00105	4*	P3=Q(3)+Q(3)
00106	5*	P4=Q(4)+Q(4)
00107	6*	P5=P2*Q(2)
00110	7*	P6=P4*Q(4)
00111	8*	TEMP=1.0-P3*Q(3)
00112	9*	A(1,1)=TEMP-P6
00113	10*	A(2,2)=1.0-P5-P6
00114	11*	A(3,3)=TEMP-P5
00115	12*	P5=P2*Q(3)
00116	13*	P5=P4*Q(1)
00117	14*	A(1,2)=P5-P6
00120	15*	A(2,1)=P5-P6
00121	16*	P5=P2*Q(4)
00122	17*	P6=P3*Q(1)
00123	18*	A(1,3)=P5-P6
00124	19*	A(3,1)=P5-P6
00125	20*	P5=P3*Q(4)
00126	21*	P5=P2*Q(1)
00127	22*	A(2,3)=P5-P6
00130	23*	A(3,2)=P5-P6
00131	24*	RETURN
00132	25*	END

END OF COMPILATION. NO DIAGNOSTICS.

NAME: MATQ

PURPOSE: Extracts the positive quaternion from the given transformation matrix.

INPUT: A - The 3 x 3 transformation matrix

OUTPUT: Q - The positive quaternion; ARRAY(4).

ALGORITHM REFERENCE: See Reference 2.

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# TRANSFORMATION MATRIX TO THE QUATERNION

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FOR SI MATQ,MATQ  
FOR SRE3-02/19/77-06:24:21 (C)

SUBROUTINE MATQ ENTRY POINT 000003

STORAGE USED: CODE(1) 000220; DATAT(1) 000050; BLANK COMMON(2)

EXTERNAL REFERENCES (BLOCK, NAME)

0003 SORT  
0004 NERN29

STORAGE ASSIGNMENT (BLOCK, TYPE, RELATIVE LOCATION, NAME)

0001	000473	10L	0001	000052	1076	0001	000157
0001	000111	35L	0001	000121	40L	0001	000116
0001	000018	INJPL	0001	000000	0	0000	R 000000

00101 1\*  
00102 2\*  
00103 3\*  
00104 4\*  
00105 5\*  
00106 6\*  
00107 7\*  
00108 8\*  
00109 9\*  
00110 10\*  
00111 11\*  
00112 12\*  
00113 13\*  
00114 14\*  
00115 15\*  
00116 16\*  
00117 17\*  
00118 18\*  
00119 19\*  
00120 20\*  
00121 21\*  
00122 22\*  
00123 23\*  
00124 24\*  
00125 25\*  
00126 26\*  
00127 27\*  
00128 28\*  
00129 29\*  
00130 30\*  
00131 31\*  
00132 32\*  
00133 33\*  
00134 34\*  
00135 35\*  
00136 36\*  
00137 37\*  
00138 38\*  
00139 39\*  
00140 40\*  
00141 41\*  
00142 42\*  
00143 43\*  
00144 44\*  
00145 45\*  
00146 46\*  
00147 47\*  
00148 48\*  
00149 49\*  
00150 50\*  
00151 51\*  
00152 52\*  
00153 53\*  
00154 54\*  
00155 55\*  
00156 56\*  
00157 57\*

```

SUBROUTINE MATQ(A,Q)
DIMENSION A(3,3),Q(4,16)
I=1
BIG=0.0
DO 40 J=1,4
  Q(J)=0.0
  IF(J.EQ.2) GO TO 10
  IF(J.EQ.3) GO TO 20
  IF(J.EQ.4) GO TO 30
  Q(J)=1.0
  TEMP=A(1,1)+A(2,2)+A(3,3)+1.0
  T(J)=0.0
  GO TO 35
10 TEMP=A(1,1)-A(2,2)-A(3,3)+1.0
  T(J)=A(3,2)-A(2,3)
  GO TO 35
20 TEMP=-A(1,1)+A(2,2)-A(3,3)+1.0
  T(J)=A(3,1)-A(1,3)
  GO TO 35
30 TEMP=-A(1,1)-A(2,2)+A(3,3)+1.0
  T(J)=A(2,1)-A(1,2)
  IF(TEMP.LT.BIG) GO TO 40
  BIG=TEMP
  I=J
40 CONTINUE
  IF(I.EQ.0) GO TO 60
  A(1)=.5*(BIG+T(I)*T(I))
  IF(I.EQ.1) Q(1)=ABS(.25*T(I)/Q(1))
  TEMP=.25/Q(1)
  DO 50 J=2,4
    Q(J)=TEMP*T(J)
  CONTINUE
50 RETURN
END

```

END OF COMPILATION:

NO DIAGNOSTICS.

NAME:

YPRQ

PURPOSE:

Generates the quaternion directly from the yaw-pitch-roll Euler angles, i.e., a 3, 2, 1 Euler sequence.

INPUT:

YPR - The yaw-pitch-roll Euler angles; ARRAY (3).

OUTPUT:

Q0 - The positive quaternion, ARRAY (4).

ALGORITHM REFERENCE:

Appendix A, the quaternion equations for Euler sequence (10), a 3, 2, 1 sequence.

NOTE: This subroutine calls "POSNOR" to take the normal of the positive quaternion.

# Yaw-PITCH-ROLL EULER ANGLES TO THE QUATERNION

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2FOR,S YPRQ,YPRO  
FOR SOE3-02/19/77-06:24:03 (,0)

SUBROUTINE YPRO ENTRY POINT DT0114

STORAGE USED: CODE(1) 000101; DATA(0) 000020; BLANK COMMON(2) 00

EXTERNAL REFERENCES (BLOCK, NAME)

0003 POSNR  
0004 COS  
0005 SIN  
0006 NEPR31

STORAGE ASSIGNMENT (BLOCK, TYPE, RELATIVE LOCATION, NAME)

0000 R 000011 CP 0000 R 000011 CR 0000 R 000007 C  
0000 R 000011 HY 0000 000016 INOPS 0000 R 000000 C  
0000 R 000011 SY

00101	1*	SUBROUTINE YPRQ(YPR,Q0)
00103	2*	DIMENSION YPR(3),Q(4),Q0(4)
00104	3*	HY=0.50*YPR(1)
00105	4*	HP=0.50*YPR(2)
00106	5*	HR=0.50*YPR(3)
00107	6*	CY=COS(HY)
00110	7*	CP=COS(HP)
00111	8*	CR=COS(HR)
00112	9*	SY=SIN(HY)
00113	10*	SP=SIN(HP)
00114	11*	SR=SIN(HR)
00115	12*	Q(1)=CY*CP*CR+SY*SP*SR
00116	13*	Q(2)=CY*CP*SR-SY*SP*CR
00117	14*	Q(3)=CY*SP*CR+SY*CP*SR
00120	15*	Q(4)=-CY*SP*SR+SY*CP*CR
00121	16*	CALL POSNR(Q,Q0)
00122	17*	RETURN
00123	18*	END

END OF COMPILATION: NO DIAGNOSTICS.



NAME: POSNOR

PURPOSE: To output the positive and normalized quaternion  
from the given quaternion.

INPUT: Q - The quaternion; ARRAY (4).

OUTPUT: Q0 - The positive-normalized quaternion;  
ARRAY (4).

ALGORITHM REFERENCE:

1. If the sign of Q(1) is negative:

Set Q0(I) = -Q(I) for I = 1, 2, 3, 4.

2. Set Q0(I) = Q0(I)/TEMP

where TEMP =  $\sqrt{Q0_1^2 + Q0_2^2 + Q0_3^2 + Q0_4^2}$

# SELECTS THE POSITIVE QUATERNION AND NORMALIZES

3FOR, IS POSNOR, POSNOR  
FOR SDE3-02/1977-06:24:14 (, 0)

SUBROUTINE POSNOR ENTRY POINT 000055

STORAGE USED: CODE(1) 000067; DATA(1) 000017; BLANK COMMON(2) 0

EXTERNAL REFERENCES (BLOCK, NAME)

0003 SQR  
0004 NLPDS

STORAGE ASSIGNMENT (BLOCK, TYPE, RELATIVE LOCATION, NAME)

0001 00016 1110 0001 00036 1210 0000 1 000002  
0000 R 000000 TEMP

00101	1*	SUBROUTINE POSNOR(0,00)
00103	2*	DIMENSION Q(4),00(4)
00104	3*	TEMP=1.0
00105	4*	IF(Q(1).LT.0.0) TEMP=-1.0
00107	5*	SUM=0.0
00110	6*	DO 100 I=1,4
00113	7*	Q(I)=TEMP*Q(I)
00114	8*	SUM=SUM+Q(I)*Q(I)
00115	9*	50 CONTINUE
00117	10*	TEMP=1.0/SQR(SUM)
00120	11*	DO 100 I=1,4
00123	12*	Q(I)=TEMP*Q(I)
00124	13*	100 CONTINUE
00126	14*	RETURN
00127	15*	END

END OF COMPILATION:

NO DIAGNOSTICS.

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