Chapter 10 - Problem 5

10.5 Consider a system of fermions at $T \neq 0$. The fermions can be in only one of two levels. The energies of these levels are respectively ε_1 and ε_2, and the degeneracy of these levels are respectively g_1 and g_2. Assume that the total number of electrons N is equal to g_1. Denoting

$x = \exp(\beta(\varepsilon_1 - \mu))$, $\beta = 1/(k_B T)$, $g = g_2/g_1$, and $\Delta\varepsilon = \varepsilon_2 - \varepsilon_1$.

a) Find x when the system is in thermal equilibrium
b) Find μ (in eV), if $g_1 = 2$, $g_2 = 5$, $\varepsilon_1 = 0.5$ eV, $\varepsilon_2 = 1$ eV, $k_B T = 26$ meV.

Part A.

In[129]:=
```
Clear["Global`*"];
(* linear density is given *)
```
$$n = \frac{1}{1+x} + \frac{g}{1 + x * \mathrm{Exp}[\beta * d\epsilon]};$$
```
Solve[n == 1, x]
```

Out[131]=
$$\left\{\left\{x \to -\frac{1}{2} e^{-d\epsilon\,\beta}\left(1 - g - \sqrt{1 - 2\,g + 4\,e^{d\epsilon\,\beta}\,g + g^2}\right)\right\}, \left\{x \to -\frac{1}{2} e^{-d\epsilon\,\beta}\left(1 - g + \sqrt{1 - 2\,g + 4\,e^{d\epsilon\,\beta}\,g + g^2}\right)\right\}\right\}$$

The above equations are the two roots of x.
One of which will give you a complex chemical potential. You want the real one.

Part B.

In[132]:=
```
g1 = 2;
g2 = 5;
ϵ1 = 0.5; (* eV *)
ϵ2 = 1; (* eV *)
kbT = 0.026; (* eV *)
```
$$g = \frac{g2}{g1};$$
$$\beta = \frac{1}{kbT};$$
```
dϵ = ϵ2 - ϵ1;
x = Exp[β * (ϵ1 - μ)];
```

μ, the chemical potential, is the only unknown.
We have two roots for x.

In[141]:=
$$\text{xroot1} = -\frac{1}{2} e^{-d\epsilon\,\beta} \left(1 - g - \sqrt{1 - 2\,g + 4\,e^{d\epsilon\,\beta}\,g + g^2}\right);$$
$$\text{xroot2} = -\frac{1}{2} e^{-d\epsilon\,\beta} \left(1 - g + \sqrt{1 - 2\,g + 4\,e^{d\epsilon\,\beta}\,g + g^2}\right);$$

In[143]:=
```
Solve[x == xroot1, μ]
Solve[x == xroot2, μ]
```

Out[143]= $\{\{\mu \to 0.738087\}\}$

Out[144]= $\{\{\mu \to 0.738089 - 0.0816814\, i\}\}$

Solution

So our solution is $\mu = 0.738087$ eV