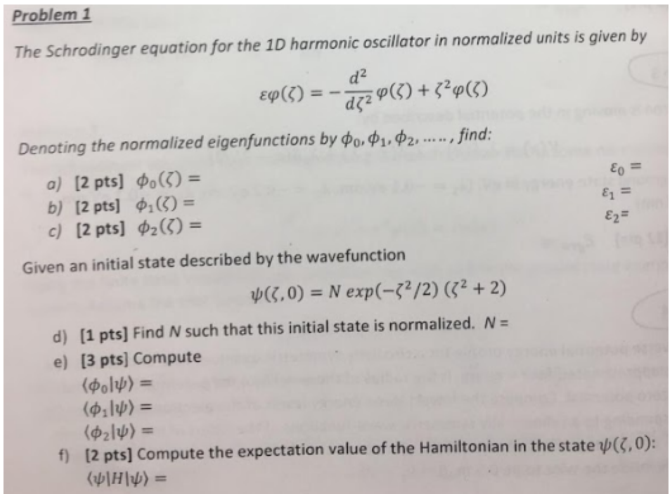
Problem 1



Problem 1

The Schrodinger equation for the 1D harmonic oscillator in normalized units is given by

$$\varepsilon\varphi(\zeta) = -\frac{d^2}{d\zeta^2}\varphi(\zeta) + \zeta^2\varphi(\zeta)$$

Denoting the normalized eigenfunctions by $\phi_0, \phi_1, \phi_2, \ldots\ldots$, find:

a) [2 pts] $\phi_0(\zeta) =$ $\varepsilon_0 =$
b) [2 pts] $\phi_1(\zeta) =$ $\varepsilon_1 =$
c) [2 pts] $\phi_2(\zeta) =$ $\varepsilon_2 =$

Given an initial state described by the wavefunction

$$\psi(\zeta, 0) = N\, exp(-\zeta^2/2)\,(\zeta^2 + 2)$$

d) [1 pts] Find N such that this initial state is normalized. $N =$
e) [3 pts] Compute
$\langle\phi_0|\psi\rangle =$
$\langle\phi_1|\psi\rangle =$
$\langle\phi_2|\psi\rangle =$
f) [2 pts] Compute the expectation value of the Hamiltonian in the state $\psi(\zeta, 0)$:
$\langle\psi|H|\psi\rangle =$

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The energy equation for the energy when the Schrodinger equation is written this way is:
$E_n = 2n + 1$

```
en[n_] = (2 * n) + 1;
```

The wave equations is given by:
$\phi(\zeta) = A_n\, H(n, \zeta)\, e^{\frac{-\zeta^2}{2}}$ where $A_n = (2^n\, n! * \text{Sqrt(Pi)})^{-1/2}$
and H is the hermite polynomial(s).

In[6]:= $\phi[n\_] = \dfrac{1}{\text{Sqrt}[2^n * n! * \text{Sqrt}[\text{Pi}]]} * \text{HermiteH}[n, \xi] * \text{Exp}\left[\dfrac{-\xi^2}{2}\right];$

Parts a - c:

```
In[7]:= ϕ[0]
        ϕ[1]
        ϕ[2]
```

Out[7]= $\dfrac{e^{-\frac{\xi^2}{2}}}{\pi^{1/4}}$

Out[8]= $\dfrac{\sqrt{2}\, e^{-\frac{\xi^2}{2}}\, \xi}{\pi^{1/4}}$

Out[9]= $\dfrac{e^{-\frac{\xi^2}{2}}\,(-2 + 4\,\xi^2)}{2\sqrt{2}\,\pi^{1/4}}$

Part D.

So for this part, all we have to do is the find the normalization constant of the given wave equation.

In[10]:=
```
ψ[ζ_] = Exp[-ζ^2/2] * (ζ^2 + 2);
```

In[13]:=
```
normconst = 1 / Sqrt[Integrate[Abs[ψ[ζ]]^2, {ζ, -Infinity, Infinity}]]
```

Out[13]= $\frac{2}{3\sqrt{3}\,\pi^{1/4}}$

Part E.

So this is very much like Chapter 7 - Problem 3. This is just another way of writing Cn. In fact, this is just the first three Cn's. Not only that but since the wave equation is literally the same, our solution will also be ... literally be the same.

Anyway, here is the expression you need to find your Cn

$$= C_n = \int \phi_n(\zeta)\, \psi(\zeta, 0)\, d\zeta$$

In[14]:=
```
c0 = Integrate[normconst * Exp[-ζ^2/2] * (ζ^2 + 2) * ϕ[0], {ζ, -Infinity, Infinity}]
```

Out[14]= $\frac{5}{3\sqrt{3}}$

In[15]:=
```
c1 = Integrate[normconst * Exp[-ζ^2/2] * (ζ^2 + 2) * ϕ[1], {ζ, -Infinity, Infinity}]
```

Out[15]= 0

In[16]:=
```
c2 = Integrate[normconst * Exp[-ζ^2/2] * (ζ^2 + 2) * ϕ[2], {ζ, -Infinity, Infinity}]
```

Out[16]= $\frac{\sqrt{\frac{2}{3}}}{3}$

Part F.

Alright so now you're computing the expectation value of the Hamiltonian in the state $\psi(\zeta,0)$.

I think Professor Mannasah wants to see if you really want to understand the language used in quantum mechanics. Computing the expectation value of the Hamiltonian is the same as computing the average energy. This comes out to the same exact thing as Chapter 7 - Problem 1.

But just in case the numbers change in any future exams, there is a requisite step we must perform. We have to make sure that our probabilities add up to 1, or else we are missing a state.

How do we do that?

Well, the $|C_n|^2$ is the probability of finding the particle in state n. Which means that $\Sigma |C_n|^2$ must be 1, or else we are missing a state that might exist.

After we check that we have all our C_n's, the expectation of
the energy is:

$<E> = \langle \psi | H | \psi \rangle = \Sigma |Cn|^2 En$

The equation above should look familiar from EE 311.

The equation of the harmonic oscillator energy is given by the first equation on this page.

In[17]:=
```
Abs[c0]^2 + Abs[c1]^2 + Abs[c2]^2
```
Out[17]= 1

```
(* Cool, we have all our Cn's*)
```

In[33]:=
```
en[0];
en[2];
avgenergy = (c0^2 * en[0]) + (c2^2 * en[2])
```
Out[35]= $\frac{35}{27}$