Kelvin Ma – Chapter 10 – Problem 3

The Sommerfield Expansion is

$$\int_0^{\infty} H(E)f(E,T)dE \;\cong \int_0^{E_F} H(E)dE + (\mu - E_F)H(E_F) + \frac{\pi}{6}(k_B T)^2 \frac{dH(E)}{DE}|_{E=E_F}$$

This is the Sommerfield expansion. In two dimension, we know that the density of states is given by

$$H(E) = \frac{n_{2D}}{E_f}$$

This is in Professor Mannasah's notes and in the document provided during tutoring

Then we know that the Sommerfield Expansion can be written as:

$$\int_0^{\infty} H(E)f(E,T)dE \;\cong \boxed{\int_0^{E_F} \frac{n_{2D}}{E_f} dE} + (\mu - E_F)H(E_F) + \frac{\pi}{6}(k_B T)^2 \frac{dH(E)}{dE}|_{E=E_F}$$

If we evaluate the boxed integral:

$$\int_0^{E_F} \frac{n_{2D}}{E_f} dE = \frac{n_{2D}}{E_f}\int_0^{E_f} dE = \frac{n_{2D}}{E_f} \cdot E_f = n_{2D}$$

And

$$(\mu - E_f)H(E_f) = (\mu - E_f) * \frac{n_{2D}}{E_f}$$

And the last piece of the Sommerfield expansion until $O(T^2)$ is:

$$\frac{\pi}{6}(k_B T)^2 \frac{dH(E)}{dE} = 0$$

Then

$$n = \int_0^{\infty} H(E)f(E,T)dE \;\cong\; n_{2D} + (\mu - E_f) * \frac{n_{2D}}{E_f}$$

$$n_{2D} = n_{2D} + \frac{\mu n_{2D}}{E_f} - n_{2D}$$

Solving for μ

$$1 = 1 + \frac{\mu}{E_f} - 1$$

$$1 = \frac{\mu}{E_f}$$

$$\mu = E_f$$

This means that we can write the chemical potential as a function of temperature:

$$\mu(T) = \mu(0)$$

Since there is no dependency on temperature in the 2 dimensional case anyway.