Chapter 7 - Problem 3

7.3 Given an initial state for the 1D harmonic oscillator in normalized units

$$\psi(\zeta,0) = N\, exp(-\zeta^2/2)\,(\zeta^2+2)$$

a) Find N such that this initial state is normalized.
b) Find $\psi(\zeta,t)$ for t > 0.
c) Compute the average energy of this state.

In[ ]:=
```
Clear["Global`*"]
wavefunction[ξ_] = Exp[-ξ^2/2] * (ξ^2 + 2);
normconst = 1 / Sqrt[Integrate[wavefunction[ξ] * wavefunction[ξ], {ξ, -∞, ∞}]]
```

Out[ ]= $\frac{2}{3\sqrt{3}\,\pi^{1/4}}$

In[ ]:=
```
groundphi[ξ_] = normconst * wavefunction[ξ]
```

Out[ ]= $\frac{2\,e^{-\frac{\xi^2}{2}}\,(2+\xi^2)}{3\sqrt{3}\,\pi^{1/4}}$

In[ ]:=
```
C0 = Integrate[groundphi[ξ] * (2^0 * 0! * Sqrt[Pi])^(-1/2) * HermiteH[0, ξ] * Exp[-ξ^2/2], {ξ, -∞, ∞}]
```

Out[ ]= $\frac{5}{3\sqrt{3}}$

In[ ]:=
```
C2 = Integrate[groundphi[ξ] * (2^2 * 2! * Sqrt[Pi])^(-1/2) * HermiteH[2, ξ] * Exp[-ξ^2/2], {ξ, -∞, ∞}]
```

Out[ ]= $\frac{\sqrt{\frac{2}{3}}}{3}$

In[ ]:=
```
C0^2 + C2^2 (*double check that this totals to 1*)
```

Out[ ]= 1

Part C. Average energy of the state

In[ ]:=
```
e[n_] = (2 * n) + 1;
avgenergy = ((Abs[C0]^2) * e[0]) + ((Abs[C2]^2) * e[2])
```

Out[ ]= $\frac{35}{27}$