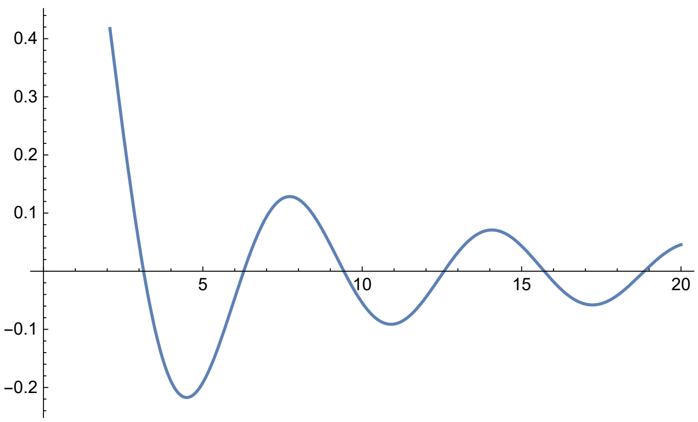
Chapter 9 - Problem 3 

9.3 Find the lowest five eigenvalues corresponding to spherically symmetric states of an electron entrapped in a metalized surface sphere of radius *R*, where *R* = 3 nm.

For this problem, Spherical coordinates and Spherical Bessel Functions :(. In the Pillbox.pdf, it is covered from page 88 to 91. First let's find the zeros of the spherical bessel function.

Part I. Spherical Bessel Roots

In[28]:=
```
ClearAll["Global`*"];
n = 0;
Plot[SphericalBesselJ[n, x], {x, 0, 20}]
x = {3, 6, 9, 12, 15, 19};
besselzeros = roots /. FindRoot[SphericalBesselJ[n, roots], {roots, x}]
```

Out[30]=


Out[32]=
```
{3.14159, 6.28319, 9.42478, 12.5664, 15.708, 18.8496}
```

In[ ]:=
```
(* Alternative way of finding the bessel roots*)
(*Clear[besselzeros];
nRoots=5;
nBessel=3;
statenumber = 0;
SphBesselRoot[l_,k_]:=N[BesselJZero[l+1/2,k]];
n = Table[SphBesselRoot[l,i],{l,0,nBessel},{i,1,nRoots}];
besselzeros = n[[1]]*)
```

Part II. Solving the god damn problem

Now that we have the roots, we can apply the same method as in Chapter 9 Problem 1.

In[33]:=
```
h = 6.62607004 * 10^-34; (* Plankton's Constant *)
mo = 9.10938 * 10^-31; (* Effective mass of an electron *)
ħ = h/(2 * π);
nano = 1 * 10^-9;
e = 1.60217 * 10^-19;
r = 3 * nano; (* Radius of sphere *)
NumberOfEnergies = 5;

En = N[ħ^2/(2 * mo) * ((besselzeros/r)^2)]; (* Energy values in Joules*)
EnEv = En / e; (* Energy values in electron volts *)

For[i = 1, i <= NumberOfEnergies, i++,
 Print[i, " lowest Energy Level is : \n\t", EnEv[[i]], " eV \t ", En[[i]], "J"]]
```

1 lowest Energy Level is :
    0.0417813 eV 6.69408×10^{-21}J

2 lowest Energy Level is :
    0.167125 eV 2.67763×10^{-20}J

3 lowest Energy Level is :
    0.376032 eV 6.02467×10^{-20}J

4 lowest Energy Level is :
    0.668501 eV 1.07105×10^{-19}J

5 lowest Energy Level is :
    1.04453 eV 1.67352×10^{-19}J

Faster way:

Instead of finding the roots of the Bessel functions, we can actually see that for BesselJo, the zeros are $n\pi$...thus our formula can change and we can create a function to find the energy at ANY state (ground, first excited, second excited, etc...).

In[51]:=
```
Clear[n];
n = 1; (* ground state *)
```

In[57]:=
```
EnJ[n_] = N[ħ^2/(2 * mo) * ((n * π/r)^2)] (* energy in joules *)
Enev[n_] = N[ħ^2/(2 * mo * e) * ((n * π/r)^2)] (* energy in electron volts *)
```

Out[57]= 6.69408×10^{-21}

Out[58]= 0.0417813

The above is a demonstration that the formula works, this matches our ground state, so let’s write our for loop again.

In[59]:=
```
Clear[n];
For[n = 1, n ≤ NumberOfEnergies, n++,
 Print[n, " lowest Energy Level is : \n\t", Enev[n], " eV \t ", EnJ[n], "J"]]
```

```
1 lowest Energy Level is :
    0.0417813 eV      6.69408×10^-21J
2 lowest Energy Level is :
    0.0417813 eV      6.69408×10^-21J
3 lowest Energy Level is :
    0.0417813 eV      6.69408×10^-21J
4 lowest Energy Level is :
    0.0417813 eV      6.69408×10^-21J
5 lowest Energy Level is :
    0.0417813 eV      6.69408×10^-21J
```

Same answers as above, except this is only one line of code (after writing constants).