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COMPUTING VARIANCES AND COVARIANCES OF NORMAL ORDER STATISTICS

Rudolph S. Parrish

Computer Sciences Corporation
c/o U. S. Environmental Protection Agency
College Station Road
Athens, GA 30613-7799

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ABSTRACT

A technique for computation of variances and covariances of normal order statistics is presented. This method provides the means to extend the precision of and correct errors in current tables. A numerical integration approach is employed for the calculations and associated error bounds are developed. Tables were constructed for samples sizes up to 50 with precision as follows: 25 decimal places (d.p.) for samples sizes of 2(1)20; 20 d.p. for 21(1)30; 15 d.p. for 31(1)40; 10 d.p. for 41(1)50. A table of variances and covariances for sample sizes up to 20 and a table of product moments of normal order statistics for samples sizes of 20(10)50 are presented.

1. INTRODUCTION

Much previous research effort has been directed toward evaluation of the moments of order statistics for normal distributions. Order statistics form the basis for many inferential techniques, and a knowledge of associated moments provides information about performance characteristics (see David, 1981). Applications are found in methods associated with trimmed means, quasi-ranges, quantile estimation, and, more generally, L-statistics. The W test for departure from normality presented by Shapiro and Wilk (1965), for example, relies upon a table of coefficients that are defined in terms of the expected values, variances, and covariances of normal order statistics.

As cited by Parrish (1991), expected values have been reported by several authors to varying degrees of accuracy and precision. Exact product moments of normal order statistics for small sample sizes were given by Jones (1948) and extended by Godwin (1949) to include sample sizes of six and less. Variances and covariances were reported by Godwin to five decimal places (d.p.) for sample sizes of 2(1)10 and by Teichroew (1956) to 10 d.p. for sample sizes of 2(1)20. Yamauti (1972) provided 8-decimal-place tables of product moments for sample sizes of 30 and less. Tietjen *et al.* (1977) presented tables for sample sizes up to 50, although the present effort has found these to be of limited accuracy. Approximations to covariances have been discussed by David and Johnson (1954), Davis and Stephens (1978), and others.

Parrish (1991) used a numerical integration technique to provide high-precision tables of expected values and standard deviations of normal order statistics. A related method can be applied to obtain the covariances, although the computation of covariances involves the numerical evaluation of double integrals and, thus, is more complex and computationally intensive. The precision with which covariances can be practically computed is more limited, especially for larger sample sizes. With respect to all other known tables, the present results extend the accuracy and precision of variances and covariances of normal order statistics for sample sizes up to 50.

Reported here are tables of variances and covariances (Table 1) for pairs of normal order statistics for sample sizes of 2(1)20 and product moments (Table 2) for samples sizes 20(10)50. Values were computed to 25 d.p. for sample sizes of 2(1)20, to 20 d.p. for sample sizes of 21(1)30, to 15 d.p. for sample sizes of 31(1)40, and to 10 d.p. for sample sizes of 41(1)50. The numbers of decimal places reported correspond generally to the indications of precision from several different numerical checks that were applied in an attempt to verify the tabled values. Tabled values of product moments may be used in conjunction with expected values to produce variance and covariances.

TABLE 1. VARIANCES AND COVARIANCES OF NORMAL ORDER STATISTICS

n	i	j	Cov[$X_{i:n}$,$X_{j:n}$]
2	1	1	0.68169 01138 16209 32846 22325
2	1	2	0.31830 98861 83790 67153 77675
3	1	1	0.55946 72037 97367 01379 56863
3	1	2	0.27566 44477 10896 02475 56632
3	1	3	0.16486 83484 91736 96144 86504
3	2	2	0.44867 11045 78207 95048 86735
4	1	1	0.49171 52368 74741 76068 17470
4	1	2	0.24559 26930 06406 03677 22614
4	1	3	0.15800 80701 23173 92832 97147
4	1	4	0.10468 39999 95678 27421 62769
4	2	2	0.36045 53433 77512 45102 96484
4	2	3	0.23594 38934 92907 58386 83755
5	1	1	0.44753 40690 20661 98876 56847
5	1	2	0.22433 09595 50172 72964 38391
5	1	3	0.14814 77252 38938 25307 10913
5	1	4	0.10577 19776 36708 45419 10027
5	1	5	0.07421 52685 53518 57432 83823
5	2	2	0.31151 89521 13385 88948 90672
5	2	3	0.20843 54439 58123 51647 45028
5	2	4	0.14994 26667 41609 41020 15882
5	3	3	0.28683 36616 05876 46090 88117
6	1	1	0.41592 71089 83248 11918 14091
6	1	2	0.20850 30022 53640 31252 83929
6	1	3	0.13943 52565 06533 28673 26912
6	1	4	0.10242 93939 61934 70506 09626
6	1	5	0.07736 37839 26525 42991 49707
6	1	6	0.05634 14543 68118 14658 15735
6	2	2	0.27957 77392 29791 33761 67720
6	2	3	0.18898 59559 89407 46729 68518
6	2	4	0.13966 40603 79097 61422 37937
6	2	5	0.10590 54582 21537 83841 92189
6	3	3	0.24621 25353 90384 66575 77410
6	3	4	0.18327 27977 72642 26092 79597
7	1	1	0.39191 77761 26750 45281 96850
7	1	2	0.19619 90245 86742 22680 97464
7	1	3	0.13211 55811 11366 25079 14048
7	1	4	0.09848 68606 91604 97284 01394
7	1	5	0.07655 98345 66498 37466 91458
7	1	6	0.05991 87124 45016 77980 77829
7	1	7	0.04480 22104 72020 94226 20957
7	2	2	0.25673 28861 62101 58648 19316
7	2	3	0.17448 33274 31701 48264 96004
7	2	4	0.13072 98656 28494 72132 14255
7	2	5	0.10195 50088 92810 39017 77472
7	2	6	0.07998 11748 53132 81275 17661
7	3	3	0.21972 15626 23859 62799 35661
7	3	4	0.16555 98429 12246 60187 39682
7	3	5	0.12960 48424 61517 27184 45676
7	4	4	0.21044 68615 35307 40792 89338
8	1	1	0.37289 71432 86728 99422 02112
8	1	2	0.18630 73995 30031 75592 43840
8	1	3	0.12596 60298 39518 59943 93660
8	1	4	0.09472 30277 22263 02876 74445
8	1	5	0.07476 50242 15114 05064 73258
8	1	6	0.06020 75170 27414 84715 22744
8	1	7	0.04829 85509 19438 88560 36311
8	1	8	0.03683 53074 59489 83824 53630
8	2	2	0.23940 10457 44445 50109 47216
8	2	3	0.16319 58726 33937 76166 12041
8	2	4	0.12326 33316 94244 93600 14884
8	2	5	0.09756 47193 38975 50207 54380
8	2	6	0.07872 24682 44245 84744 32724
8	2	7	0.06324 66118 94679 81019 58604
8	3	3	0.20076 87900 11030 03545 71653
8	3	4	0.15235 84311 89685 82374 56914
8	3	5	0.12096 37555 20849 48948 74766
8	3	6	0.09781 71355 33317 59561 35497
8	4	4	0.18718 62194 78350 03410 72443
8	4	5	0.14917 54908 40517 13516 78910
9	1	1	0.35735 33263 57813 34373 26239
9	1	2	0.17814 34239 48892 81257 10488
9	1	3	0.12074 54441 77061 18539 43433
9	1	4	0.09130 71399 75589 70575 24664
9	1	5	0.07274 22354 49847 96223 98691
9	1	6	0.05948 31124 61662 52199 41253
9	1	7	0.04907 64060 87063 75589 24152
9	1	8	0.04009 36927 55801 75502 29633
9	1	9	0.03105 52187 86266 95740 01447
9	2	2	0.22569 68777 58563 53923 02924
9	2	3	0.15411 63525 86232 47624 28554
9	2	4	0.11700 56917 39859 08743 09568
9	2	5	0.09344 77393 54213 21393 36724
9	2	6	0.07654 61431 55055 21529 68431
9	2	7	0.06323 54695 25296 38709 98456
9	2	8	0.05171 46091 76085 51317 15223
9	3	3	0.18638 26133 21648 30698 51619
9	3	4	0.14207 79776 14356 82641 70420
9	3	5	0.11376 80176 27272 73610 85431
9	3	6	0.09336 25385 50005 67381 71893
9	3	7	0.07723 51805 11062 65204 26041
9	4	4	0.17055 88454 12035 91807 38390
9	4	5	0.13699 13668 89306 38458 29355
9	4	6	0.11266 71842 02128 66663 46027
9	5	5	0.16610 12813 58719 40626 99597
10	1	1	0.34434 38232 60690 25506 82754
10	1	2	0.17126 29030 31319 92124 46894
10	1	3	0.11625 90988 54684 17537 85485
10	1	4	0.08824 94247 31749 44970 86052
10	1	5	0.07074 13676 78926 26176 67183
10	1	6	0.05839 87134 42538 05551 17401
10	1	7	0.04892 06279 38933 40123 80936
10	1	8	0.04108 44588 55782 16030 74653
10	1	9	0.03404 06470 23559 61189 13744
10	1	10	0.02669 89351 81816 70788 44897
10	2	2	0.21452 41429 82770 95742 67343
10	2	3	0.14662 26179 78671 64928 38817
10	2	4	0.11170 15961 67036 10088 92372
10	2	5	0.08974 28245 51933 89614 68374
10	2	6	0.07419 95414 12961 24651 48901
10	2	7	0.06222 78486 34014 20648 64642
10	2	8	0.05230 67221 37449 69651 34455
10	2	9	0.04337 11560 80282 71360 24458
10	3	3	0.17500 32834 03013 73835 09923
10	3	4	0.13380 22448 15267 14528 00892

(continued)

TABLE 1. VARIANCES AND COVARIANCES OF NORMAL ORDER STATISTICS (cont'd)

n	i	j	$Cov[X_{i:n}, X_{j:n}]$
10	3	5	0.10774 45335 88133 81737 57556
10	3	6	0.08922 54012 00355 56780 27096
10	3	7	0.07491 83943 09245 55650 51872
10	3	8	0.06303 32448 57396 49320 19251
10	4	4	0.15793 89143 78576 69378 59442
10	4	5	0.12750 89295 27842 07833 81847
10	4	6	0.10578 58169 24881 73308 62690
10	4	7	0.08894 62025 72453 63468 19255
10	5	5	0.15105 39039 08227 67989 61722
10	5	6	0.12559 89677 64199 66356 07230
11	1	1	0.33324 74427 02957 43511 96030
11	1	2	0.16536 47711 68893 07416 46265
11	1	3	0.11235 84351 34182 09463 62640
11	1	4	0.08551 70596 23221 83880 01810
11	1	5	0.06884 83064 83730 17732 15875
11	1	6	0.05720 07585 83488 55515 46316
11	1	7	0.04837 54062 79792 21123 13519
11	1	8	0.04124 23472 08034 11125 31056
11	1	9	0.03511 03356 96915 34431 76022
11	1	10	0.02941 98502 81981 34658 09577
11	1	11	0.02331 52868 36803 81142 00890
11	2	2	0.20519 75797 90150 54668 82969
11	2	3	0.14030 96510 52424 47293 35731
11	2	4	0.10714 92594 59296 67458 99536
11	2	5	0.08644 30256 94649 11331 75739
11	2	6	0.07192 05024 36253 53890 51236
11	2	7	0.06088 69662 21848 06538 19518
11	2	8	0.05195 04506 51835 94122 58143
11	2	9	0.04425 49455 52720 71194 32340
11	2	10	0.03710 29976 89946 51426 88947
11	3	3	0.16572 42879 53709 65131 33649
11	3	4	0.12696 72925 23695 26515 51880
11	3	5	0.10264 07290 87832 55625 65211
11	3	6	0.08551 78832 11267 41288 29602
11	3	7	0.07247 41049 98589 07145 99844
11	3	8	0.06188 73278 25975 85227 46157
11	3	9	0.05275 50069 62687 56682 66925
11	4	4	0.14795 46564 57097 10161 57594
11	4	5	0.11987 52861 31655 73127 10855
11	4	6	0.10003 46585 02847 10912 26502
11	4	7	0.08487 65182 14102 18332 82633
11	4	8	0.07254 51434 02238 19136 33835
11	5	5	0.13964 10803 26028 13099 03635
11	5	6	0.11674 49804 92327 23048 57016
11	5	7	0.09919 35960 69445 52895 56155
11	6	6	0.13716 24335 47632 30689 78657
12	1	1	0.32363 63870 47645 11498 03031
12	1	2	0.16023 73762 05946 11030 66774
12	1	3	0.10893 09641 56025 80859 03760
12	1	4	0.08306 86766 37065 73468 80615
12	1	5	0.06708 84463 63526 41645 43026
12	1	6	0.05599 33693 57472 12109 00140
12	1	7	0.04766 20974 51179 91381 64302
12	1	8	0.04102 08554 19708 33846 68038
12	1	9	0.03544 39059 80809 43131 32258
12	1	10	0.03050 12590 58495 76716 52513
12	1	11	0.02579 45391 35866 60221 58005
12	1	12	0.02062 21231 86258 64091 27537
12	2	2	0.19726 46039 30805 59835 06671
12	2	3	0.13490 20327 91126 35889 39595
12	2	4	0.10319 59206 26079 39303 90520
12	2	5	0.08350 45822 24072 72559 01834
12	2	6	0.06978 59657 43914 60493 19587
12	2	7	0.05945 90652 25933 72177 88826
12	2	8	0.05121 13198 07446 84675 97359
12	2	9	0.04427 47124 18167 85540 51771
12	2	10	0.03811 91478 50675 70191 40023
12	2	11	0.03225 07340 39964 48081 39035
12	3	3	0.15797 86876 94526 49572 76991
12	3	4	0.12120 63210 98527 83191 20229
12	3	5	0.09826 05601 79110 73727 59025
12	3	6	0.08222 28461 10256 09548 17250
12	3	7	0.07012 13963 77910 71987 68395
12	3	8	0.06043 84621 35128 30133 07806
12	3	9	0.05228 25611 17478 88136 11687
12	3	10	0.04503 57614 30737 30047 02726
12	4	4	0.13981 09404 68305 40713 87002
12	4	5	0.11356 87821 29067 00868 61448
12	4	6	0.09516 45279 25116 70043 46205
12	4	7	0.08124 19809 28832 42768 16732
12	4	8	0.07007 95832 30916 58382 33015
12	4	9	0.06066 20874 39632 74451 68517
12	5	5	0.13061 37358 24183 16671 64861
12	5	6	0.10962 12246 69682 74246 68816
12	5	7	0.09369 51519 72034 27708 35166
12	5	8	0.08089 72960 45122 85534 59607
12	6	6	0.12663 77911 42238 87851 05082
12	6	7	0.10839 45830 95427 79684 69499
13	1	1	0.31520 53842 12311 31148 12179
13	1	2	0.15572 72904 50551 68871 08060
13	1	3	0.10589 08841 50934 98522 40473
13	1	4	0.08086 49736 10706 13408 02748
13	1	5	0.06546 34498 24451 90079 46535
13	1	6	0.05482 21796 17225 02270 74015
13	1	7	0.04688 33088 48644 69407 23773
13	1	8	0.04061 32548 73561 86417 76424
13	1	9	0.03542 26461 98171 67557 21950
13	1	10	0.03093 22743 93281 52650 13417
13	1	11	0.02685 37250 34310 43658 42566
13	1	12	0.02288 58067 74707 57870 40847
13	1	13	0.01843 48220 11141 18138 97013
13	2	2	0.19041 30720 78920 60691 83469
13	2	3	0.13020 55829 28062 12943 64920
13	2	4	0.09972 62695 47972 45306 35239
13	2	5	0.08087 85938 84091 17219 61169
13	2	6	0.06781 45832 12215 12383 67851
13	2	7	0.05804 57284 69496 20291 27488
13	2	8	0.05031 67945 77377 17987 59866
13	2	9	0.04390 95086 70501 72962 34270
13	2	10	0.03836 01798 43560 37293 14226
13	2	11	0.03331 47765 46926 18292 28307
13	2	12	0.02840 18130 15617 57886 74288
13	3	3	0.15139 17013 40165 95125 13792
13	3	4	0.11626 98131 34301 18672 88078
13	3	5	0.09445 66602 81384 77011 34382
13	3	6	0.07929 22993 54075 80045 39197
13	3	7	0.06792 82353 97977 85551 18976
13	3	8	0.05892 21431 83161 85831 31621
13	3	9	0.05144 60445 76137 90203 88937

(continued)

TABLE 1. VARIANCES AND COVARIANCES OF NORMAL ORDER STATISTICS (cont'd)

n	i	j	$Cov[X_{i:n},X_{j:n}]$
13	3	10	0.04496 37541 74331 15177 86290
13	3	11	0.03906 43798 98229 78964 22462
13	4	4	0.13301 11819 13517 79070 87139
13	4	5	0.10825 12666 42208 70112 88454
13	4	6	0.09098 55604 74007 23355 33718
13	4	7	0.07801 73339 48604 62138 44601
13	4	8	0.06772 17142 40760 30000 70794
13	4	9	0.05916 28729 97725 51307 34410
13	4	10	0.05173 28050 79023 01506 00887
13	5	5	0.12325 03255 88780 13851 36756
13	5	6	0.10373 67700 78365 41495 15466
13	5	7	0.08904 34754 32460 46254 97114
13	5	8	0.07735 52863 97782 08049 58600
13	5	9	0.06762 30994 27938 53894 81958
13	6	6	0.11831 75325 76840 58857 88063
13	6	7	0.10168 24204 02127 33392 73660
13	6	8	0.08841 94610 12500 19912 10725
13	7	7	0.11679 89950 01377 65928 28775
14	1	1	0.30773 01024 70513 52042 40323
14	1	2	0.15172 03662 67101 86755 13087
14	1	3	0.10317 19530 51956 46949 36683
14	1	4	0.07887 15915 09936 35070 36071
14	1	5	0.06396 57428 06609 23416 20057
14	1	6	0.05370 64713 65928 19056 91307
14	1	7	0.04608 99189 82596 08781 08840
14	1	8	0.04011 41687 48551 74861 96426
14	1	9	0.03521 41760 21545 31310 48211
14	1	10	0.03103 71162 77343 58083 85400
14	1	11	0.02733 62865 20257 96578 03859
14	1	12	0.02390 61000 97031 81005 88472
14	1	13	0.02050 80256 38533 28720 38172
14	1	14	0.01662 79802 42094 57367 93093
14	2	2	0.18442 00251 96606 54357 05017
14	2	3	0.12607 91989 99505 10687 69505
14	2	4	0.09665 24633 45852 30712 55071
14	2	5	0.07852 02979 94498 21784 22972
14	2	6	0.06600 28339 71940 76990 48345
14	2	7	0.05668 96715 50891 13254 79927
14	2	8	0.04937 08147 21265 30011 99383
14	2	9	0.04336 17156 50348 49872 42680
14	2	10	0.03823 37404 21711 59495 28932
14	2	11	0.03368 63220 99766 77573 73652
14	2	12	0.02946 81313 55842 21783 89837
14	2	13	0.02528 63927 86136 38000 33421
14	3	3	0.14570 45665 71064 68874 14730
14	3	4	0.11198 16876 44283 51303 93360
14	3	5	0.09111 81271 06229 70402 22214
14	3	6	0.07667 54957 05635 28469 40160
14	3	7	0.06590 84825 50572 20504 15077
14	3	8	0.05743 41187 81472 33761 20406
14	3	9	0.05046 77802 17757 62477 37886
14	3	10	0.04451 69192 30622 61958 97407
14	3	11	0.03923 52316 60105 23208 36657
14	3	12	0.03433 22070 27921 18613 37606
14	4	4	0.12722 73070 15384 14627 51423
14	4	5	0.10369 31108 10372 75324 54761
14	4	6	0.08735 62483 22265 65955 89240
14	4	7	0.07515 19908 65221 70991 90125
14	4	8	0.06553 10935 45637 81122 94243
14	4	9	0.05761 20956 62731 99296 73978
14	4	10	0.05084 02240 85718 57768 95347
14	4	11	0.04482 43469 12465 20464 52212
14	5	5	0.11710 12460 67737 92588 41199
14	5	6	0.09877 47549 59807 92377 42217
14	5	7	0.08505 36546 09702 79954 73652
14	5	8	0.07421 81415 70841 58170 85712
14	5	9	0.06528 67776 32337 08235 41824
14	5	10	0.05764 01464 26466 40438 88307
14	6	6	0.11153 24579 40371 00374 29422
14	6	7	0.09614 05595 20266 09686 81710
14	6	8	0.08396 17109 60829 75132 36377
14	6	9	0.07390 69220 68234 80763 96643
14	7	7	0.10902 69479 79116 44468 61732
14	7	8	0.09530 87256 13034 99296 56390
15	1	1	0.30104 15703 13893 97523 47570
15	1	2	0.14812 97708 19171 45125 31803
15	1	3	0.10072 23448 56814 83849 43616
15	1	4	0.07705 94059 92853 14762 59473
15	1	5	0.06258 45850 36391 69687 27829
15	1	6	0.05265 30128 35834 53103 66298
15	1	7	0.04530 78885 82841 59673 13092
15	1	8	0.03957 36673 08569 13288 36250
15	1	9	0.03490 35904 94140 88926 08037
15	1	10	0.03096 14122 13575 30066 17646
15	1	11	0.02752 11039 53074 11887 54621
15	1	12	0.02441 26313 47049 89882 55615
15	1	13	0.02148 19828 28459 31756 95420
15	1	14	0.01853 33263 29026 66350 27865
15	1	15	0.01511 37070 88303 44117 14865
15	2	2	0.17912 15291 07299 55170 75052
15	2	3	0.12241 76952 30142 23746 66888
15	2	4	0.09390 67143 10152 57794 83240
15	2	5	0.07639 12337 08756 12217 39266
15	2	6	0.06433 90895 06850 65302 55846
15	2	7	0.05540 74400 40832 63962 28786
15	2	8	0.04842 38833 02207 16151 81440
15	2	9	0.04272 94113 02042 81285 23570
15	2	10	0.03791 77516 42150 98509 47985
15	2	11	0.03371 51720 72093 75612 50424
15	2	12	0.02991 52347 35717 86881 14491
15	2	13	0.02633 03885 00847 43432 29436
15	2	14	0.02272 13593 92708 08457 43908
15	3	3	0.14073 22502 53284 13524 97909
15	3	4	0.10821 38452 50790 82513 98453
15	3	5	0.08816 05755 15738 85244 79552
15	3	6	0.07432 68436 53112 19864 86326
15	3	7	0.06405 58182 26034 06262 74660
15	3	8	0.05601 36122 43861 29452 85588
15	3	9	0.04944 85109 72784 22689 62642
15	3	10	0.04389 60669 23271 19151 12079
15	3	11	0.03904 26915 18300 55252 92396
15	3	12	0.03465 13381 42948 11427 27645
15	3	13	0.03050 60358 83610 71829 47388
15	4	4	0.12223 28270 30676 71375 19714
15	4	5	0.09973 23940 91950 61048 67267
15	4	6	0.08417 05696 28769 46913 87329
15	4	7	0.07259 46868 47634 84667 07778
15	4	8	0.06351 75906 55246 98430 88691
15	4	9	0.05609 90511 93550 32662 75288
15	4	10	0.04981 87836 33230 00823 67064

(continued)

TABLE 1. VARIANCES AND COVARIANCES OF NORMAL ORDER STATISTICS (cont'd)

n	i	j	$Cov[X_{i:n},X_{j:n}]$
15	4	11	0.04432 47451 97705 78433 84136
15	4	12	0.03935 01819 41722 82381 63816
15	5	5	0.11186 98986 20455 06628 76297
15	5	6	0.09452 06004 28910 75126 94745
15	5	7	0.08158 91121 64805 83182 63071
15	5	8	0.07143 31681 23398 72701 06638
15	5	9	0.06312 24388 88241 20853 14192
15	5	10	0.05607 95064 33287 53164 24854
15	5	11	0.04991 27742 46889 38958 24712
15	6	6	0.10586 66366 30434 12447 90000
15	6	7	0.09146 83203 46047 24161 39203
15	6	8	0.08014 07559 44185 90779 29769
15	6	9	0.07085 82099 81432 34170 95160
15	6	10	0.06298 24401 98907 76413 85695
15	7	7	0.10269 16922 42873 64222 25892
15	7	8	0.09004 99963 81346 57117 64381
15	7	9	0.07967 38323 35391 76163 04247
15	8	8	0.10169 46520 82368 44156 14485
16	1	1	0.29500 98090 10319 79787 70853
16	1	2	0.14488 81688 44430 78906 45628
16	1	3	0.09850 09764 55232 15430 78939
16	1	4	0.07540 40023 89649 46029 48995
16	1	5	0.06130 86724 25467 25544 82173
16	1	6	0.05166 24962 82326 45018 39098
16	1	7	0.04455 03704 63355 47613 90335
16	1	8	0.03901 94715 44070 77380 03861
16	1	9	0.03453 78157 86003 33553 11556
16	1	10	0.03078 10093 41185 02017 81591
16	1	11	0.02753 53611 49206 59868 10838
16	1	12	0.02464 79005 92788 24806 56771
16	1	13	0.02199 56754 01860 96317 51229
16	1	14	0.01945 85036 84171 39276 37628
16	1	15	0.01687 10289 00827 70271 92269
16	1	16	0.01382 87377 29104 58176 98237
16	2	2	0.17439 40788 11474 00768 28470
16	2	3	0.11914 09286 25536 19019 73640
16	2	4	0.09143 59918 14202 09986 13070
16	2	5	0.07445 91144 60823 65840 29151
16	2	6	0.06280 93908 67731 47634 04644
16	2	7	0.05420 33940 22411 36130 23366
16	2	8	0.04750 09769 66241 43417 04506
16	2	9	0.04206 38230 26990 58743 30018
16	2	10	0.03750 18250 66728 35154 74805
16	2	11	0.03355 74913 00695 74973 98398
16	2	12	0.03004 61298 00229 49023 70060
16	2	13	0.02681 89579 36964 32277 69918
16	2	14	0.02373 01562 55666 77316 30602
16	2	15	0.02057 85432 99046 00536 11456
16	3	3	0.13633 85613 25692 67316 51887
16	3	4	0.10487 06756 90935 76703 90774
16	3	5	0.08551 89036 04128 05301 18515
16	3	6	0.07220 75087 55333 15594 99382
16	3	7	0.06235 68514 97020 78141 56773
16	3	8	0.05467 49106 64688 35328 49046
16	3	9	0.04843 66096 29385 46613 76925
16	3	10	0.04319 79377 52923 28673 37930
16	3	11	0.03866 52994 29657 22406 73842
16	3	12	0.03462 77255 51892 29325 93289
16	3	13	0.03091 49134 23443 58513 12941
16	3	14	0.02735 95376 54292 85037 17887

n	i	j	$Cov[X_{i:n},X_{j:n}]$
16	4	4	0.11786 57554 16807 22464 61546
16	4	5	0.09625 13413 73747 77865 45986
16	4	6	0.08134 80447 64348 17301 19915
16	4	7	0.07030 00910 95949 97715 62711
16	4	8	0.06167 28989 72689 10738 75220
16	4	9	0.05465 95025 88201 66092 79739
16	4	10	0.04876 47746 22445 41425 14580
16	4	11	0.04366 07327 92222 65646 90472
16	4	12	0.03911 12668 48779 30270 04172
16	4	13	0.03492 53748 67752 50651 58734
16	5	5	0.10735 17088 70802 15057 05865
16	5	6	0.09082 32621 39839 39539 94559
16	5	7	0.07854 80532 80716 23316 19284
16	5	8	0.06894 88801 89671 84953 85163
16	5	9	0.06113 64181 62612 25122 29761
16	5	10	0.05456 38940 73653 54251 44603
16	5	11	0.04886 84327 45608 19349 51345
16	5	12	0.04378 82958 79240 30431 69303
16	6	6	0.10104 61905 73460 63297 98149
16	6	7	0.08746 27155 11249 05537 10088
16	6	8	0.07682 39667 92666 86394 74929
16	6	9	0.06815 45539 73142 69710 23549
16	6	10	0.06085 34805 25250 00727 36197
16	6	11	0.05452 10723 97261 66998 74593
16	7	7	0.09740 26613 66923 98281 54645
16	7	8	0.08561 81915 71175 83633 91506
16	7	9	0.07600 15576 82604 82761 82055
16	7	10	0.06789 31921 26406 84618 19531
16	8	8	0.09572 13007 15770 50710 08902
16	8	9	0.08502 91217 34084 44845 73266
17	1	1	0.28953 30036 87695 81952 00456
17	1	2	0.14194 24628 99699 87035 76295
17	1	3	0.09647 48736 60462 14754 16425
17	1	4	0.07388 49614 67550 52624 73378
17	1	5	0.06012 72301 97931 72918 70718
17	1	6	0.05073 26947 12792 78523 59010
17	1	7	0.04382 36490 64893 59806 65156
17	1	8	0.03846 72833 29430 42925 40925
17	1	9	0.03414 41054 99724 77898 35692
17	1	10	0.03053 89548 76300 58135 46284
17	1	11	0.02744 65527 42181 56875 90271
17	1	12	0.02472 37144 69418 70091 45378
17	1	13	0.02226 20771 00909 17352 22869
17	1	14	0.01996 90650 53339 84952 19723
17	1	15	0.01774 76891 20439 78672 40348
17	1	16	0.01545 52070 37106 32860 14905
17	1	17	0.01272 64750 80122 32620 82167
17	2	2	0.17014 26762 72618 01541 73860
17	2	3	0.11618 66733 56562 66091 57962
17	2	4	0.08919 82556 08134 31368 98304
17	2	5	0.07269 70385 34772 68908 63106
17	2	6	0.06139 98459 11010 30026 37370
17	2	7	0.05307 61572 77870 90458 69761
17	2	8	0.04661 40918 04897 27672 11329
17	2	9	0.04139 28191 55772 86645 78740
17	2	10	0.03703 49110 15646 73512 24002
17	2	11	0.03329 40891 73768 07169 81530
17	2	12	0.02999 82825 58422 96281 77931
17	2	13	0.02701 70379 13813 65747 88784
17	2	14	0.02423 86812 74901 14496 26200

(continued)

TABLE 1. VARIANCES AND COVARIANCES OF NORMAL ORDER STATISTICS (cont'd)

n	i	j	$Cov[X_{i:n}, X_{j:n}]$
17	2	15	0.02154 59396 30339 50283 41636
17	2	16	0.01876 58305 74662 69898 78285
17	3	3	0.13242 07975 08610 22606 98629
17	3	4	0.10187 92434 36739 05384 04746
17	3	5	0.08314 21716 22263 24822 22864
17	3	6	0.07028 50403 09334 81878 38480
17	3	7	0.06079 64413 57432 44657 35954
17	3	8	0.05342 08201 98599 47538 11064
17	3	9	0.04745 55486 95588 62518 26061
17	3	10	0.04247 26883 93247 42495 61699
17	3	11	0.03819 25586 54145 56176 68422
17	3	12	0.03441 94566 84259 06590 72856
17	3	13	0.03100 47771 14943 85060 59007
17	3	14	0.02782 10707 59748 86288 19200
17	3	15	0.02473 42094 97283 24181 24647
17	4	4	0.11400 68196 58613 57435 61674
17	4	5	0.09316 20339 28604 39319 09104
17	4	6	0.07882 66620 85420 68271 84032
17	4	7	0.06822 98908 09483 46511 13332
17	4	8	0.05998 26091 81209 36664 24920
17	4	9	0.05330 57575 45429 45058 66271
17	4	10	0.04772 39972 88653 05038 37801
17	4	11	0.04292 61816 64014 33186 61119
17	4	12	0.03869 42630 40427 22931 26398
17	4	13	0.03486 24030 13225 46724 92506
17	4	14	0.03128 81041 84505 23743 81295
17	5	5	0.10340 04377 05265 17772 45623
17	5	6	0.08757 29930 35493 49709 58118
17	5	7	0.07585 34533 41594 73099 65450
17	5	8	0.06672 04244 61663 20744 72298
17	5	9	0.05931 87706 08815 41089 61268
17	5	10	0.05312 57771 23103 82704 10364
17	5	11	0.04779 87292 43672 71920 32090
17	5	12	0.04309 70793 13502 74402 09027
17	5	13	0.03883 75657 40424 47703 16803
17	6	6	0.09688 24668 86129 41630 16727
17	6	7	0.08398 11737 71714 41781 02891
17	6	8	0.07391 30258 93418 65727 05407
17	6	9	0.06574 42736 41398 71535 44048
17	6	10	0.05890 30403 18336 11008 36102
17	6	11	0.05301 37274 79190 71413 28810
17	6	12	0.04781 22598 89729 18197 57416
17	7	7	0.09290 31779 69688 61318 28307
17	7	8	0.08181 94606 71015 45029 60543
17	7	9	0.07281 54074 14409 39164 19925
17	7	10	0.06526 67274 40602 41203 68832
17	7	11	0.05876 26219 24321 60227 07608
17	8	8	0.09073 61649 53276 99866 52362
17	8	9	0.08080 00267 27470 95806 40486
17	8	10	0.07245 99963 23128 03927 95584
17	9	9	0.09004 65814 22779 60566 55018
18	1	1	0.28453 01297 41373 23776 62106
18	1	2	0.13925 01619 82567 22274 53399
18	1	3	0.09461 72635 93836 30683 43254
18	1	4	0.07248 51730 41042 92320 33851
18	1	5	0.05903 04273 94951 95505 60921
18	1	6	0.04986 00635 42274 06154 85795
18	1	7	0.04313 02309 99993 48238 12204
18	1	8	0.03792 60194 78895 41720 86930
18	1	9	0.03373 88140 56592 69760 56760
18	1	10	0.03026 10666 57718 99669 36375
18	1	11	0.02729 38041 17200 64316 04400
18	1	12	0.02470 02470 55015 19649 28255
18	1	13	0.02238 01572 23538 78164 62717
18	1	14	0.02025 37420 87120 86497 21619
18	1	15	0.01824 88619 23496 38297 71695
18	1	16	0.01628 50441 83481 66324 12818
18	1	17	0.01423 68875 26479 40867 33007
18	1	18	0.01177 19053 94420 75779 33895
18	2	2	0.16629 29294 40493 90431 50911
18	2	3	0.11350 58132 40024 48599 78038
18	2	4	0.08715 97603 64568 00193 64953
18	2	5	0.07108 25990 22816 57900 12813
18	2	6	0.06009 75753 43713 99855 64021
18	2	7	0.05202 17422 37515 53300 80605
18	2	8	0.04576 83625 20454 04084 93104
18	2	9	0.04073 17967 38712 32849 38036
18	2	10	0.03654 51033 81184 41906 87459
18	2	11	0.03297 04894 52106 82818 76911
18	2	12	0.02984 42464 05047 27930 96340
18	2	13	0.02704 62261 17769 61402 11886
18	2	14	0.02448 06359 16457 19516 25653
18	2	15	0.02206 07111 39427 68039 36136
18	2	16	0.01968 94667 21859 68574 48884
18	2	17	0.01721 54924 48801 79453 47844
18	3	3	0.12889 98942 36552 04297 82395
18	3	4	0.09918 28539 34689 94748 49866
18	3	5	0.08098 99791 37595 87358 78653
18	3	6	0.06853 24700 43169 82745 75428
18	3	7	0.05935 98602 27959 25065 34543
18	3	8	0.05224 88412 92760 31706 07098
18	3	9	0.04651 62120 37361 29791 19874
18	3	10	0.04174 73296 27716 72620 39532
18	3	11	0.03767 30986 97010 02479 56573
18	3	12	0.03410 80170 94803 16847 11716
18	3	13	0.03091 57650 04907 56014 24705
18	3	14	0.02798 75014 25574 45771 80862
18	3	15	0.02522 44785 87252 58710 13791
18	3	16	0.02251 61109 13444 77661 41972
18	4	4	0.11056 60330 91825 29045 17750
18	4	5	0.09039 73786 41099 40372 68675
18	4	6	0.07655 79277 69038 37668 43163
18	4	7	0.06635 22085 25003 43514 40809
18	4	8	0.05843 10521 09551 75929 85572
18	4	9	0.05203 94281 71552 36691 88418
18	4	10	0.04671 83402 91747 86188 24697
18	4	11	0.04216 94861 40356 87642 02490
18	4	12	0.03818 69632 27958 19242 78360
18	4	13	0.03461 92644 36041 91953 62682
18	4	14	0.03134 52499 78474 44864 65288
18	4	15	0.02825 48286 26872 54576 51801
18	5	5	0.09990 84320 70870 99434 20300
18	5	6	0.08468 79168 17757 17532 44170
18	5	7	0.07344 60810 75888 84867 29345
18	5	8	0.06471 01857 36487 32035 88267
18	5	9	0.05765 43520 44761 99493 29443
18	5	10	0.05177 56674 28444 33834 45767
18	5	11	0.04674 68133 44708 97649 72697
18	5	12	0.04234 15563 06071 75512 66072
18	5	13	0.03839 32045 98457 71622 51432
18	5	14	0.03476 82769 72460 10230 38024

(continued)

TABLE 1. VARIANCES AND COVARIANCES OF NORMAL ORDER STATISTICS (cont'd)

n	i	j	$Cov[X_{i:n}, X_{j:n}]$
18	6	6	0.09324 07331 41731 09945 71809
18	6	7	0.08092 02644 64696 45145 47482
18	6	8	0.07133 38045 10218 43642 88467
18	6	9	0.06358 29688 59453 88934 22151
18	6	10	0.05711 97287 63328 39494 47921
18	6	11	0.05158 68552 10792 40366 68250
18	6	12	0.04673 70895 72269 89774 31645
18	6	13	0.04238 79845 80840 39581 96276
18	7	7	0.08901 67024 87311 34074 81197
18	7	8	0.07851 79676 66481 01411 66471
18	7	9	0.07001 99026 22799 10816 61289
18	7	10	0.06292 69074 00005 47875 51333
18	7	11	0.05685 01034 60222 32935 98826
18	7	12	0.05151 99091 70958 23796 83509
18	8	8	0.08649 60638 37520 73399 69420
18	8	9	0.07717 62286 00631 58996 89337
18	8	10	0.06938 91332 13010 65754 27887
18	8	11	0.06271 16906 11590 63108 17301
18	9	9	0.08531 27880 37823 43509 68582
18	9	10	0.07674 42320 67154 41812 65139
19	1	1	0.27993 58049 28328 91811 38428
19	1	2	0.13677 68167 86419 96855 67575
19	1	3	0.09290 61762 76690 46661 51178
19	1	4	0.07119 02424 60449 36399 26855
19	1	5	0.05800 94834 87105 05967 78070
19	1	6	0.04904 05677 97438 72642 42420
19	1	7	0.04247 05246 47362 73244 21206
19	1	8	0.03740 06328 84156 37301 52757
19	1	9	0.03333 19394 82390 32353 12024
19	1	10	0.02996 34144 31696 23941 77753
19	1	11	0.02710 11338 53900 66765 66285
19	1	12	0.02461 29451 76184 42238 68499
19	1	13	0.02240 37539 75929 45381 36871
19	1	14	0.02040 07370 65679 61474 95457
19	1	15	0.01854 31530 55471 39386 99502
19	1	16	0.01677 31147 35339 08151 17157
19	1	17	0.01502 23067 55611 63279 25478
19	1	18	0.01317 89994 05814 54074 01215
19	1	19	0.01093 82527 94031 02069 21274
19	2	2	0.16278 56650 67087 62460 49640
19	2	3	0.11105 90144 81207 19567 73019
19	2	4	0.08529 31052 33350 56390 40378
19	2	5	0.06959 70758 16590 35880 47722
19	2	6	0.05889 10196 33274 99272 74965
19	2	7	0.05103 51092 24273 52837 52732
19	2	8	0.04496 52247 80844 00633 05786
19	2	9	0.04008 91753 68208 76049 52497
19	2	10	0.03604 90039 97916 60050 38646
19	2	11	0.03261 37544 06404 42049 47009
19	2	12	0.02962 58235 40415 89232 06757
19	2	13	0.02697 16592 41758 12465 44629
19	2	14	0.02456 41908 45620 19663 84969
19	2	15	0.02233 06885 83775 09321 68563
19	2	16	0.02020 17248 04420 01508 61756
19	2	17	0.01809 52193 76911 16413 83241
19	2	18	0.01587 67294 05706 95272 98901
19	3	3	0.12571 38903 95010 40004 93291
19	3	4	0.09673 67096 74731 14795 96909
19	3	5	0.07902 98792 45212 07468 98475
19	3	6	0.06692 73696 57008 15443 81812
19	3	7	0.05803 36124 55639 33063 49270
19	3	8	0.05115 41417 93525 20267 20554
19	3	9	0.04562 28815 54497 44586 79556
19	3	10	0.04103 65628 76492 91556 36341
19	3	11	0.03713 46427 28239 52610 79382
19	3	12	0.03373 91171 53491 03334 55396
19	3	13	0.03072 15918 18048 88839 80915
19	3	14	0.02798 35020 14252 42564 69607
19	3	15	0.02544 24108 41890 75123 65112
19	3	16	0.02301 95063 21225 51774 63285
19	3	17	0.02062 14645 80314 72641 97181
19	4	4	0.10747 40838 19874 21729 16459
19	4	5	0.08790 51966 45651 38755 71348
19	4	6	0.07450 33877 68877 51542 88504
19	4	7	0.06464 06187 78990 99954 96295
19	4	8	0.05700 32284 73977 25335 15549
19	4	9	0.05085 72608 17359 55970 02783
19	4	10	0.04575 76598 10645 80374 27368
19	4	11	0.04141 65090 37969 70257 08219
19	4	12	0.03763 68751 88181 29878 25524
19	4	13	0.03427 65540 13887 41930 64598
19	4	14	0.03122 62549 51286 16372 75504
19	4	15	0.02839 44526 36915 28048 94678
19	4	16	0.02569 35148 26867 70830 06832
19	5	5	0.09679 44743 74412 43888 17913
19	5	6	0.08210 55694 49972 65293 08152
19	5	7	0.07127 96742 48691 81530 24753
19	5	8	0.06288 70095 17246 40370 05936
19	5	9	0.05612 72025 31554 35188 07481
19	5	10	0.05051 41638 64061 90540 71350
19	5	11	0.04573 30144 25199 61648 89183
19	5	12	0.04156 81234 25606 12253 39123
19	5	13	0.03786 36088 10005 31738 32804
19	5	14	0.03449 95261 71869 63628 31581
19	5	15	0.03137 52928 68768 33966 48255
19	6	6	0.09002 18692 55041 85621 46468
19	6	7	0.07820 29062 80613 04786 40616
19	6	8	0.06902 94360 10898 73666 05298
19	6	9	0.06163 36895 92698 66142 21558
19	6	10	0.05548 77905 01874 36501 86225
19	6	11	0.05024 93168 45100 75061 22046
19	6	12	0.04568 34840 64972 13605 09291
19	6	13	0.04162 03596 23546 67104 42790
19	6	14	0.03792 90224 69973 69611 72738
19	7	7	0.08561 72980 78816 38376 41000
19	7	8	0.07561 53412 92293 99597 97642
19	7	9	0.06754 33161 58405 23520 48594
19	7	10	0.06082 97030 32429 26389 59944
19	7	11	0.05510 32223 62295 41199 51339
19	7	12	0.05010 89625 09178 71501 84998
19	7	13	0.04566 21834 47833 66537 29006
19	8	8	0.08283 39961 14804 03820 43580
19	8	9	0.07402 73545 63836 34478 35881
19	8	10	0.06669 58228 79903 15312 33076
19	8	11	0.06043 72723 42699 91317 01057
19	8	12	0.05497 52082 87784 95856 93295
19	9	9	0.08128 76330 18594 41656 55716
19	9	10	0.07327 03910 68871 38752 35529
19	9	11	0.06642 02898 41773 50392 83861
19	10	10	0.08079 09750 72216 73160 67539

(continued)

TABLE 1. VARIANCES AND COVARIANCES OF NORMAL ORDER STATISTICS (cont'd)

n	i	j	$Cov[X_{i:n},X_{j:n}]$
20	1	1	0.27569 66156 18531 23248 78726
20	1	2	0.13449 41714 08364 38954 33553
20	1	3	0.09132 34063 91423 10287 59795
20	1	4	0.06998 79991 08590 47208 09781
20	1	5	0.05705 66384 55343 47381 22265
20	1	6	0.04827 01092 64575 32359 47352
20	1	7	0.04184 37825 66325 12849 28125
20	1	8	0.03689 37056 82272 16168 79641
20	1	9	0.03292 96301 78094 84915 03580
20	1	10	0.02965 62522 46077 88497 33508
20	1	11	0.02688 38808 23701 81770 95084
20	1	12	0.02448 39566 50398 51075 23130
20	1	13	0.02236 49803 54530 91341 55731
20	1	14	0.02045 84276 65231 50072 00976
20	1	15	0.01870 96782 08731 20874 65719
20	1	16	0.01707 11407 22820 66695 43328
20	1	17	0.01549 51854 19076 48176 12983
20	1	18	0.01392 27071 48511 13452 05896
20	1	19	0.01225 30116 79001 69205 57993
20	1	20	0.01020 47204 08398 05466 42843
20	2	2	0.15957 31635 56896 07530 44706
20	2	3	0.10881 43706 46033 01357 35034
20	2	4	0.08357 58043 76617 67995 46046
20	2	5	0.06822 47553 47398 10977 79612
20	2	6	0.05776 99655 57824 75078 42552
20	2	7	0.05011 09522 49017 61429 29431
20	2	8	0.04420 41191 33685 60635 11349
20	2	9	0.03946 93443 12917 82651 23726
20	2	10	0.03555 65554 08858 88594 61607
20	2	11	0.03224 05467 32103 94224 33300
20	2	12	0.02936 84959 96936 38821 16591
20	2	13	0.02683 15104 63140 48115 71440
20	2	14	0.02454 79493 23020 38036 21417
20	2	15	0.02245 26609 66950 40207 85181
20	2	16	0.02048 88031 81008 14923 56394
20	2	17	0.01859 94023 39823 93716 16533
20	2	18	0.01671 36501 93092 47718 57204
20	2	19	0.01471 07671 27308 19826 76331
20	3	3	0.12281 34687 87040 68723 18501
20	3	4	0.09450 49009 68266 57162 49977
20	3	5	0.07723 55098 67497 69200 70860
20	3	6	0.06545 10178 40731 15100 82970
20	3	7	0.05680 56676 56060 31941 20335
20	3	8	0.05013 10269 01687 58801 11716
20	3	9	0.04477 63201 80566 45060 23161
20	3	10	0.04034 82353 99589 35590 74740
20	3	11	0.03659 34286 59905 95679 18462
20	3	12	0.03333 97949 03908 45228 69487
20	3	13	0.03046 45791 76039 96786 80389
20	3	14	0.02787 56579 64782 52367 84490
20	3	15	0.02549 94381 39489 35977 57541
20	3	16	0.02327 16371 16172 12776 45002
20	3	17	0.02112 77372 76999 19285 42812
20	3	18	0.01898 74447 82202 87501 91630
20	4	4	0.10467 66242 96971 54412 54106
20	4	5	0.08564 42355 52608 63654 89946
20	4	6	0.07263 21559 09769 88864 83743
20	4	7	0.06307 31775 34406 39622 70931
20	4	8	0.05568 55081 04663 73792 78556
20	4	9	0.04975 39272 49030 41359 71459
20	4	10	0.04484 55403 00384 95327 91103
20	4	11	0.04068 11668 73231 24053 94878
20	4	12	0.03707 09493 66449 07669 81517
20	4	13	0.03387 93392 17991 22293 33882
20	4	14	0.03100 45145 79705 62994 14591
20	4	15	0.02836 50517 78010 11476 93479
20	4	16	0.02588 97454 17544 88136 70457
20	4	17	0.02350 70343 29857 92795 93221
20	5	5	0.09399 60006 72784 93039 11538
20	5	6	0.07977 73754 65604 83485 30898
20	5	7	0.06931 75756 24004 97797 03074
20	5	8	0.06122 51429 10312 69026 01278
20	5	9	0.05472 22526 45141 56609 35644
20	5	10	0.04933 74275 62770 85600 15440
20	5	11	0.04476 62310 13418 20378 59041
20	5	12	0.04080 14073 40755 15363 85625
20	5	13	0.03729 48399 97543 84579 40173
20	5	14	0.03413 51570 61671 24948 22382
20	5	15	0.03123 32039 69425 03360 60133
20	5	16	0.02851 09200 76172 92065 56914
20	6	6	0.08715 11253 27662 61394 60263
20	6	7	0.07577 03359 27976 10651 92227
20	6	8	0.06695 55788 85258 73554 87193
20	6	9	0.05986 59769 13265 89311 34348
20	6	10	0.05399 10638 90112 24975 81434
20	6	11	0.04900 08080 05345 32875 73010
20	6	12	0.04467 02771 31449 21950 02299
20	6	13	0.04083 85549 04731 97659 66623
20	6	14	0.03738 45194 63186 38256 98917
20	6	15	0.03421 11024 49899 42582 54117
20	7	7	0.08261 23954 35910 45071 22959
20	7	8	0.07303 83675 46724 17669 60137
20	7	9	0.06533 07664 77698 74853 54539
20	7	10	0.05893 87427 47074 17949 66531
20	7	11	0.05350 56766 03541 26750 12235
20	7	12	0.04878 82256 40391 98960 19804
20	7	13	0.04461 21090 36337 78099 65316
20	7	14	0.04084 59988 96933 19679 11585
20	8	8	0.07963 09756 83754 18788 32663
20	8	9	0.07125 91606 43061 65770 37825
20	8	10	0.06431 03374 80764 68883 87107
20	8	11	0.05839 97309 98099 74486 28930
20	8	12	0.05326 44494 97297 42869 78570
20	8	13	0.04871 59833 82101 40676 91480
20	9	9	0.07781 18317 10653 56022 14416
20	9	10	0.07025 26463 78214 51963 12901
20	9	11	0.06381 76734 66698 06052 38943
20	9	12	0.05822 29133 17070 23492 72439
20	10	10	0.07694 74355 33134 35565 14279
20	10	11	0.06992 66198 76972 50780 07466

TABLE 2. PRODUCT MOMENTS OF NORMAL ORDER STATISTICS

n	i	j	$E[X_{i:n} X_{j:n}]$
20	1	1	3.76315 97145 87271 90279 50642
20	1	2	2.76315 97145 87271 90279 50642
20	1	3	2.20334 06877 89574 87760 17093
20	1	4	1.78989 83551 84984 17607 47125
20	1	5	1.44904 08118 16340 37553 44818
20	1	6	1.15063 48881 21689 87917 00389
20	1	7	0.87909 21501 78211 11989 04786
20	1	8	0.62502 36799 57045 82968 71172
20	1	9	0.38206 78459 67967 76378 12888
20	1	10	0.14543 27710 74280 29865 22868
20	1	11	-0.08889 26380 04500 59596 94275
20	1	12	-0.32465 42591 39474 40387 86179
20	1	13	-0.56576 49939 20242 75458 35799
20	1	14	-0.81678 99399 46654 49067 75690
20	1	15	-1.08365 51006 48383 34682 87305
20	1	16	-1.37491 30326 38176 23476 79244
20	1	17	-1.70441 51706 57317 22223 24346
20	1	18	-2.09809 45742 49640 64020 51410
20	1	19	-2.61641 25314 99905 82119 59094
20	1	20	-3.47725 83785 60342 61564 29074
20	2	2	2.14092 24543 75011 34697 73336
20	2	3	1.70074 14811 72708 37217 06885
20	2	4	1.37995 34181 53085 50816 58458
20	2	5	1.11742 89273 56108 12989 69990
20	2	6	0.88867 43301 23749 81303 57356
20	2	7	0.68118 45643 75538 02865 47035
20	2	8	0.48750 54400 11877 39263 27353
20	2	9	0.30263 12966 00008 44190 08620
20	2	10	0.12282 27822 09335 66263 20056
20	2	11	-0.05502 56800 68372 83444 25149
20	2	12	-0.23379 34562 90154 22717 68303
20	2	13	-0.41646 98104 15051 30512 44567
20	2	14	-0.60652 56628 03500 03399 96158
20	2	15	-0.80845 17035 98974 66017 29707
20	2	16	-1.02871 53688 27701 87088 33851
20	2	17	-1.27777 82114 36643 89104 95997
20	2	18	-1.57521 34603 33582 88141 14587
20	2	19	-1.96663 85236 90807 07340 52315
20	3	3	1.40185 69655 46466 00061 73068
20	3	4	1.13608 73611 70679 56614 91463
20	3	5	0.92022 49684 34238 37546 21485
20	3	6	0.73304 61714 75286 19330 02461
20	3	7	0.56384 55906 71040 89065 32039
20	3	8	0.40630 41631 11753 66495 61707
20	3	9	0.25621 53876 11208 84842 01090
20	3	10	0.11046 28148 53516 28305 06426
20	3	11	-0.03352 11507 94020 97035 13225
20	3	12	-0.17809 92725 26733 94553 08440
20	3	13	-0.32570 85570 34026 10907 69596
20	3	14	-0.47916 42650 50198 04756 27289
20	3	15	-0.64209 57154 95065 68251 61702
20	3	16	-0.81971 78214 50568 55569 06031
20	3	17	-1.02045 47229 25413 80166 98293
20	3	18	-1.26005 60519 77222 43836 63144
20	4	4	0.95288 39168 16725 82201 13485
20	4	5	0.77212 83437 18788 19150 74238
20	4	6	0.61628 48184 68654 53387 87761
20	4	7	0.47597 85179 49205 53599 24714
20	4	8	0.34573 32605 16220 63699 39093
20	4	9	0.22193 82388 81346 69103 82331
20	4	10	0.10194 29856 56002 10274 34273
20	4	11	-0.01641 62784 82385 90892 48291
20	4	12	-0.13511 33622 65867 20074 29358
20	4	13	-0.25616 84131 93565 67613 26660
20	4	14	-0.38190 08258 35093 50982 39083
20	4	15	-0.51528 76107 80874 53046 10927
20	4	16	-0.66059 43627 48634 67359 13161
20	4	17	-0.82470 02581 89896 34992 66825
20	5	5	0.64959 18260 33764 73687 88661
20	5	6	0.51977 46691 69859 38989 51226
20	5	7	0.40349 64454 25580 87153 45850
20	5	8	0.29597 10291 99913 59897 75043
20	5	9	0.19407 70950 91706 97532 51368
20	5	10	0.09554 84059 39386 43454 50888
20	5	11	-0.00144 47473 63197 37475 76408
20	5	12	-0.09855 34351 05810 25559 30096
20	5	13	-0.19745 10462 92057 06292 33595
20	5	14	-0.30004 37127 39904 64408 20474
20	5	15	-0.40876 40897 34829 52143 59852
20	5	16	-0.52708 49052 84806 88583 20855
20	6	6	0.43560 15809 10701 71808 59499
20	6	7	0.34041 91896 70365 08064 04268
20	6	8	0.25285 97019 20242 73533 73785
20	6	9	0.17022 63337 45021 60243 04469
20	6	10	0.09058 72809 54785 45121 94412
20	6	11	0.01240 45909 40672 12729 60032
20	6	12	-0.06569 00797 00306 48981 67826
20	6	13	-0.14506 55681 30252 02319 19958
20	6	14	-0.22726 43342 79202 59155 13103
20	6	15	-0.31423 93531 33139 67831 45277
20	7	7	0.28361 37563 61005 91976 11597
20	7	8	0.21423 29399 13893 58854 22135
20	7	9	0.14914 96895 58902 54468 39985
20	7	10	0.08673 36465 62791 40049 88726
20	7	11	0.02571 07727 87824 04649 90041
20	7	12	-0.03503 06974 40811 80654 65640
20	7	13	-0.09658 24633 30831 63084 96691
20	7	14	-0.16015 53620 28162 27225 77043
20	8	8	0.17881 39223 77220 07018 37887
20	8	9	0.13013 82495 52004 64973 07423
20	8	10	0.08383 50289 56407 41463 95825
20	8	11	0.03887 50395 22457 01906 20213
20	8	12	-0.00561 46394 11645 56332 91030
20	8	13	-0.05046 69633 11364 47553 13740
20	9	9	0.11276 48879 19615 75108 24287
20	9	10	0.08184 33087 21278 22041 18988
20	9	11	0.05222 70111 23634 35974 32856
20	9	12	0.02326 98571 08108 04406 62568
20	10	10	0.08079 09750 72216 73160 67539
20	10	11	0.06608 30803 37890 13184 54206

(continued)

TABLE 2. PRODUCT MOMENTS OF NORMAL ORDER STATISTICS (cont'd)

n	i	j	$E[X_{i:n} X_{j:n}]$
30	1	1	4.41870 97660 27190 34486
30	1	2	3.41870 97660 27190 34486
30	1	3	2.86808 68546 03939 65218
30	1	4	2.46886 83570 79221 78211
30	1	5	2.14615 66351 55128 33863
30	1	6	1.86954 96339 32435 74041
30	1	7	1.62356 17737 61000 22908
30	1	8	1.39914 75585 45678 18930
30	1	9	1.19050 21238 50165 41590
30	1	10	0.99362 67107 19976 29826
30	1	11	0.80560 31879 53964 94176
30	1	12	0.62419 20522 40451 54880
30	1	13	0.44759 22559 95653 02880
30	1	14	0.27428 75298 03288 71001
30	1	15	0.10294 08548 09257 89126
30	1	16	-0.06768 41525 47757 58959
30	1	17	-0.23878 79400 79373 82400
30	1	18	-0.41159 49718 81469 14404
30	1	19	-0.58741 68716 29441 33838
30	1	20	-0.76772 75259 32128 57934
30	1	21	-0.95426 21713 11463 80489
30	1	22	-1.14915 96884 09167 93323
30	1	23	-1.35518 22735 48599 66129
30	1	24	-1.57607 86936 69754 09421
30	1	25	-1.81723 10896 92801 50332
30	1	26	-2.08691 32617 37131 82634
30	1	27	-2.39903 80489 21490 87364
30	1	28	-2.78022 18515 13612 48092
30	1	29	-3.29331 00830 91342 15995
30	1	30	-4.16692 64365 39007 64309
30	2	2	2.74773 94917 08376 18770
30	2	3	2.29836 24031 31626 88039
30	2	4	1.97572 59579 55472 54762
30	2	5	1.71631 70350 64334 69090
30	2	6	1.49472 02668 86835 17062
30	2	7	1.29811 07011 67385 70329
30	2	8	1.11904 74354 35664 42691
30	2	9	0.95278 02077 52799 85908
30	2	10	0.79605 06552 43453 04446
30	2	11	0.64648 96228 74324 09777
30	2	12	0.50228 48346 45873 86936
30	2	13	0.36198 32093 14493 60570
30	2	14	0.22436 48977 43542 82278
30	2	15	0.08835 71646 26962 17890
30	2	16	-0.04702 94548 20360 88782
30	2	17	-0.18275 34106 99398 95563
30	2	18	-0.31979 02851 54687 33364
30	2	19	-0.45918 32410 80675 14855
30	2	20	-0.60210 27887 12092 45435
30	2	21	-0.74992 54315 56870 50082
30	2	22	-0.90434 64322 93870 71926
30	2	23	-1.06755 37542 23571 02813
30	2	24	-1.24251 55736 51176 57606
30	2	25	-1.43349 21111 52575 88788
30	2	26	-1.64703 12425 27595 77352
30	2	27	-1.89414 22346 47396 51209
30	2	28	-2.19588 50385 88713 49457
30	2	29	-2.60198 25673 78007 99807
30	3	3	1.96610 51505 86321 34121
30	3	4	1.68796 00932 87193 54404
30	3	5	1.46541 98393 07629 38701
30	3	6	1.27591 35115 28424 13785
30	3	7	1.10814 12273 48415 36940
30	3	8	0.95558 50775 90873 58134
30	3	9	0.81410 30844 35624 64316
30	3	10	0.68086 48935 80809 20309
30	3	11	0.55381 93130 57598 36563
30	3	12	0.43140 19403 77576 58654
30	3	13	0.31236 19030 31967 76573
30	3	14	0.19565 18651 19630 64911
30	3	15	0.08035 31200 46467 03275
30	3	16	-0.03437 96971 80089 16619
30	3	17	-0.14936 36371 05870 09408
30	3	18	-0.26542 87040 50699 62557
30	3	19	-0.38346 08886 46152 21077
30	3	20	-0.50445 29570 72669 26751
30	3	21	-0.62957 11166 17614 06444
30	3	22	-0.76025 04624 10589 02286
30	3	23	-0.89834 20862 38887 22201
30	3	24	-1.04635 61371 44199 80257
30	3	25	-1.20789 44323 23763 52982
30	3	26	-1.38849 18867 73275 67341
30	3	27	-1.59745 35300 43274 01737
30	3	28	-1.85257 78513 24688 46732
30	4	4	1.47533 42341 11612 24105
30	4	5	1.27999 51329 06407 69829
30	4	6	1.11419 54233 64740 99112
30	4	7	0.96774 59592 37380 29677
30	4	8	0.83480 29244 89761 60694
30	4	9	0.71166 95558 75794 78165
30	4	10	0.59582 93295 50432 07571
30	4	11	0.48546 45583 77972 95531
30	4	12	0.37919 31287 31202 31764
30	4	13	0.27591 30693 52017 08678
30	4	14	0.17470 42362 14897 82581
30	4	15	0.07476 16200 91580 88001
30	4	16	-0.02465 36580 03349 11142
30	4	17	-0.12425 40521 86729 51085
30	4	18	-0.22476 17833 21441 55224
30	4	19	-0.32694 63861 98141 26715
30	4	20	-0.43166 88478 94526 12086
30	4	21	-0.53993 93883 26724 12874
30	4	22	-0.65300 00547 19557 18755
30	4	23	-0.77245 19118 07388 75847
30	4	24	-0.90046 50990 70047 00811
30	4	25	-1.04015 26193 36681 65513
30	4	26	-1.19629 72818 20359 84931
30	4	27	-1.37693 86843 28581 07794
30	5	5	1.12891 52609 08439 00648
30	5	6	0.98242 50676 29503 08132
30	5	7	0.85335 80540 73916 26091
30	5	8	0.73641 37139 34256 95375
30	5	9	0.62825 48225 37611 73832
30	5	10	0.52661 87075 29903 53085
30	5	11	0.42987 68288 88844 84078
30	5	12	0.33679 49247 44850 47100
30	5	13	0.24639 19418 31212 83224
30	5	14	0.15785 11437 49248 16346
30	5	15	0.07046 00594 29790 84266
30	5	16	-0.01643 34316 05984 66905
30	5	17	-0.10345 64836 93172 59927
30	5	18	-0.19124 33452 14934 36932

(continued)

TABLE 2. PRODUCT MOMENTS OF NORMAL ORDER STATISTICS (cont'd)

n	i	j	$E[X_{i:n} X_{j:n}]$
30	5	19	-0.28046 84326 63078 59969
30	5	20	-0.37188 50372 39656 02608
30	5	21	-0.46637 58419 48913 31207
30	5	22	-0.56502 50034 18244 49903
30	5	23	-0.66922 90444 59710 02893
30	5	24	-0.78087 99033 38758 56379
30	5	25	-0.90269 05819 99469 77779
30	5	26	-1.03882 83892 50792 26899
30	6	6	0.86912 98159 16043 48125
30	6	7	0.75501 60036 69884 74310
30	6	8	0.65184 28607 34933 80791
30	6	9	0.55657 96377 38115 39890
30	6	10	0.46718 03866 86829 10026
30	6	11	0.38217 82728 16562 24210
30	6	12	0.30046 55940 55328 06934
30	6	13	0.22116 49239 91430 99753
30	6	14	0.14354 82465 26590 26948
30	6	15	0.06698 25707 67079 61359
30	6	16	-0.00910 96320 09717 89585
30	6	17	-0.08528 21102 00422 06031
30	6	18	-0.16209 33438 26572 75390
30	6	19	-0.24013 57692 80327 59553
30	6	20	-0.32006 98047 70055 53825
30	6	21	-0.40266 81017 01119 43981
30	6	22	-0.48887 85528 90607 43158
30	6	23	-0.57992 11142 09201 12967
30	6	24	-0.67744 75323 56334 85155
30	6	25	-0.78382 54204 95085 79437
30	7	7	0.66752 64442 93195 58458
30	7	8	0.57662 05928 20009 62099
30	7	9	0.49285 01850 37215 75886
30	7	10	0.41436 07515 94856 15216
30	7	11	0.33982 85208 39612 16022
30	7	12	0.26825 79000 97169 36965
30	7	13	0.19886 33662 25443 56913
30	7	14	0.13099 56752 76299 27933
30	7	15	0.06409 24809 95553 55354
30	7	16	-0.00235 73777 79649 87705
30	7	17	-0.06884 22754 94024 68275
30	7	18	-0.13585 31580 73790 64762
30	7	19	-0.20390 93395 17463 07613
30	7	20	-0.27358 83521 92053 03748
30	7	21	-0.34556 46044 44647 85427
30	7	22	-0.42066 42837 66466 54360
30	7	23	-0.49994 96057 26871 64627
30	7	24	-0.58485 77002 12099 38956
30	8	8	0.50809 97588 71395 66577
30	8	9	0.43482 03051 95246 25704
30	8	10	0.36629 35180 22009 25799
30	8	11	0.30132 52447 42829 98406
30	8	12	0.23902 15956 48534 28937
30	8	13	0.17868 05203 07064 34746
30	8	14	0.11972 44036 55246 28688
30	8	15	0.06165 52092 26021 08693
30	8	16	0.00402 22556 30900 06927
30	8	17	-0.05360 31996 33311 45068
30	8	18	-0.11165 04473 50643 53000
30	8	19	-0.17057 19959 98652 78384
30	8	20	-0.23086 96077 04458 61856
30	8	21	-0.29312 78405 02585 60254
30	8	22	-0.35806 15354 34703 72871
30	8	23	-0.42658 85580 11840 24279
30	9	9	0.38103 87958 96331 51610
30	9	10	0.32177 37653 90636 70622
30	9	11	0.26569 95566 67176 55368
30	9	12	0.21201 61838 47002 73107
30	9	13	0.16009 87637 05814 75582
30	9	14	0.10943 59785 27963 10021
30	9	15	0.05958 93835 37998 66801
30	9	16	0.01016 44012 93169 30062
30	9	17	-0.03921 23375 53108 22400
30	9	18	-0.08891 28208 10747 15179
30	9	19	-0.13932 74200 97130 38610
30	9	20	-0.19088 73790 60584 52496
30	9	21	-0.24409 35930 29768 89784
30	9	22	-0.29955 71873 94120 93411
30	10	10	0.27997 18272 24235 59320
30	10	11	0.23228 54393 24254 82090
30	10	12	0.18673 40591 09204 70110
30	10	13	0.14276 51278 62499 18216
30	10	14	0.09992 95424 70794 63070
30	10	15	0.05784 48476 84027 94893
30	10	16	0.01616 92391 92723 09081
30	10	17	-0.02541 85114 80263 31356
30	10	18	-0.06723 63834 29253 13568
30	10	19	-0.10961 61368 32192 56328
30	10	20	-0.15292 24546 91326 13172
30	10	21	-0.19757 72253 13902 58714
30	11	11	0.20060 17499 03748 84317
30	11	12	0.16281 00819 49695 85150
30	11	13	0.12642 77673 67943 38886
30	11	14	0.09106 47412 03879 98150
30	11	15	0.05639 17470 45026 12121
30	11	16	0.02211 71843 12941 35530
30	11	17	-0.01203 05175 54558 12764
30	11	18	-0.04631 78495 91833 82948
30	11	19	-0.08102 06486 90482 15267
30	11	20	-0.11644 00462 19952 09485
30	12	12	0.13997 13506 36380 78907
30	12	13	0.11090 07970 97730 03424
30	12	14	0.08274 13732 86343 66463
30	12	15	0.05521 48476 69629 06582
30	12	16	0.02807 78342 93518 12139
30	12	17	0.00110 65101 78460 80411
30	12	18	-0.02591 60556 84910 53889
30	12	19	-0.05321 18222 90304 62367
30	13	13	0.09604 36601 52633 85982
30	13	14	0.07488 76726 58179 07037
30	13	15	0.05431 06499 39390 61301
30	13	16	0.03411 58921 66775 78622
30	13	17	0.01412 58698 10556 29264
30	13	18	-0.00582 82424 19822 60433
30	14	14	0.06745 34630 61846 25800
30	14	15	0.05368 62134 28965 21562
30	14	16	0.04029 56505 34458 96973
30	14	17	0.02714 94624 30040 84841
30	15	15	0.05335 92307 02249 28773
30	15	16	0.04668 49465 22422 50364

(continued)

TABLE 2. PRODUCT MOMENTS OF NORMAL ORDER STATISTICS (cont'd)

n	i	j	$E[X_{i:n} X_{j:n}]$	n	i	j	$E[X_{i:n} X_{j:n}]$
40	1	1	4.89694 98358 54428	40	2	23	-0.26126 90121 67552
40	1	2	3.89694 98358 54428	40	2	24	-0.37308 61931 84584
40	1	3	3.35132 56499 59292	40	2	25	-0.48644 89785 21868
40	1	4	2.95951 71965 89800	40	2	26	-0.60189 20760 85349
40	1	5	2.64587 91812 78717	40	2	27	-0.72001 71950 40758
40	1	6	2.37972 95534 02096	40	2	28	-0.84151 91185 59335
40	1	7	2.14548 20968 61433	40	2	29	-0.96722 10384 97021
40	1	8	1.93407 95406 85283	40	2	30	-1.09812 43115 68272
40	1	9	1.73975 95550 78146	40	2	31	-1.23548 12365 83061
40	1	10	1.55860 61039 10474	40	2	32	-1.38090 58902 02710
40	1	11	1.38781 89290 67106	40	2	33	-1.53655 07583 29381
40	1	12	1.22531 14328 94930	40	2	34	-1.70540 37358 55701
40	1	13	1.06947 37461 46764	40	2	35	-1.89182 15301 70233
40	1	14	0.91902 52290 24420	40	2	36	-2.10257 21319 52828
40	1	15	0.77291 82949 45728	40	2	37	-2.34911 72377 28383
40	1	16	0.63027 30877 26797	40	2	38	-2.65349 35763 35925
40	1	17	0.49033 13966 49302	40	2	39	-3.06794 20449 44256
40	1	18	0.35242 28920 99486	40	3	3	2.39393 88339 90006
40	1	19	0.21593 93716 01705	40	3	4	2.10939 93401 05926
40	1	20	0.08031 42016 74708	40	3	5	1.88349 33099 24524
40	1	21	-0.05499 49862 57934	40	3	6	1.69278 41565 53724
40	1	22	-0.19052 17850 25233	40	3	7	1.52553 21790 99457
40	1	23	-0.32680 58608 51662	40	3	8	1.37498 32888 62779
40	1	24	-0.46440 82472 65301	40	3	9	1.23687 28994 86512
40	1	25	-0.60392 82314 03319	40	3	10	1.10832 01547 48536
40	1	26	-0.74602 32109 25212	40	3	11	0.98727 51364 10851
40	1	27	-0.89143 33538 39514	40	3	12	0.87221 67072 07090
40	1	28	-1.04101 36886 76954	40	3	13	0.76197 55161 79336
40	1	29	-1.19577 76149 25555	40	3	14	0.65562 43640 31198
40	1	30	-1.35695 81900 08169	40	3	15	0.55240 70315 41841
40	1	31	-1.52609 77887 44939	40	3	16	0.45169 01345 88088
40	1	32	-1.70518 46614 10575	40	3	17	0.35292 92883 36046
40	1	33	-1.89687 05761 07732	40	3	18	0.25564 44121 36302
40	1	34	-2.10483 67795 00855	40	3	19	0.15940 09685 24552
40	1	35	-2.33445 12802 20649	40	3	20	0.06379 50519 31725
40	1	36	-2.59405 34311 96260	40	3	21	-0.03155 91033 22378
40	1	37	-2.89776 64851 57515	40	3	22	-0.12704 02489 58392
40	1	38	-3.27274 50376 68744	40	3	23	-0.22303 07355 14647
40	1	39	-3.78336 39763 11029	40	3	24	-0.31992 73923 92313
40	1	40	-4.66487 19458 07889	40	3	25	-0.41815 34938 45222
40	2	2	3.19836 41698 12639	40	3	26	-0.51817 27877 30022
40	2	3	2.74398 83557 07775	40	3	27	-0.62050 68800 22143
40	2	4	2.42033 63029 61347	40	3	28	-0.72575 78256 38663
40	2	5	2.16239 75171 68666	40	3	29	-0.83463 87296 84667
40	2	6	1.94411 93859 85393	40	3	30	-0.94801 68284 90893
40	2	7	1.75236 96991 85027	40	3	31	-1.06697 64943 90930
40	2	8	1.57955 90041 05589	40	3	32	-1.19291 51682 09321
40	2	9	1.42087 87131 45322	40	3	33	-1.32769 62635 87259
40	2	10	1.27307 15580 88166	40	3	34	-1.47390 62041 54195
40	2	11	1.13381 45324 65041	40	3	35	-1.63531 61277 13056
40	2	12	1.00138 05388 37780	40	3	36	-1.81778 41275 21304
40	2	13	0.87443 96661 46947	40	3	37	-2.03123 07126 91475
40	2	14	0.75193 58155 14183	40	3	38	-2.29472 90414 54010
40	2	15	0.63300 64476 54956	40	4	4	1.88188 11693 70743
40	2	16	0.51692 81945 16665	40	4	5	1.67936 40006 55909
40	2	17	0.40307 85681 44489	40	4	6	1.50880 61612 38797
40	2	18	0.29090 79643 96926	40	4	7	1.35947 40838 72592
40	2	19	0.17991 83545 48914	40	4	8	1.22521 86153 16449
40	2	20	0.06964 63152 44837	40	4	9	1.10216 97862 39472
40	2	21	-0.04035 12195 02696	40	4	10	0.98772 01603 61489
40	2	22	-0.15050 96779 44146	40	4	11	0.88001 84092 95202

(continued)

TABLE 2. PRODUCT MOMENTS OF NORMAL ORDER STATISTICS (cont'd)

n	i	j	$E[X_{i:n} X_{j:n}]$
40	4	12	0.77769 33131 88807
40	4	13	0.67969 24897 57576
40	4	14	0.58518 27140 87691
40	4	15	0.49348 53331 80189
40	4	16	0.40403 26534 12899
40	4	17	0.31633 73420 87870
40	4	18	0.22997 01332 69686
40	4	19	0.14454 29213 07800
40	4	20	0.05969 53499 11974
40	4	21	-0.02491 63986 77408
40	4	22	-0.10962 95654 65808
40	4	23	-0.19478 42948 04999
40	4	24	-0.28073 33320 60631
40	4	25	-0.36785 26695 09321
40	4	26	-0.45655 40094 01201
40	4	27	-0.54730 01932 78821
40	4	28	-0.64062 52351 45607
40	4	29	-0.73716 14542 77740
40	4	30	-0.83767 76499 44857
40	4	31	-0.94313 49286 06416
40	4	32	-1.05477 17305 37196
40	4	33	-1.17423 92283 67394
40	4	34	-1.30382 91662 17385
40	4	35	-1.44688 27562 96699
40	4	36	-1.60859 01259 70019
40	4	37	-1.79774 03020 73343
40	5	5	1.51453 82604 29449
40	5	6	1.36023 30122 77255
40	5	7	1.22536 35686 60535
40	5	8	1.10426 46233 04649
40	5	9	0.99338 23081 40295
40	5	10	0.89032 84683 81682
40	5	11	0.79341 12023 63128
40	5	12	0.70137 97454 75449
40	5	13	0.61327 57674 22693
40	5	14	0.52834 16570 09889
40	5	15	0.44596 12113 79402
40	5	16	0.36561 96564 93082
40	5	17	0.28687 56457 41335
40	5	18	0.20934 08934 57195
40	5	19	0.13266 47587 07442
40	5	20	0.05652 20411 88139
40	5	21	-0.01939 71980 27229
40	5	22	-0.09539 66598 98676
40	5	23	-0.17178 23735 53926
40	5	24	-0.24887 14389 78003
40	5	25	-0.32700 16057 63204
40	5	26	-0.40654 24704 22342
40	5	27	-0.48790 93205 86471
40	5	28	-0.57158 11087 13777
40	5	29	-0.65812 47517 52552
40	5	30	-0.74822 93714 54447
40	5	31	-0.84275 62916 36495
40	5	32	-0.94281 51625 83257
40	5	33	-1.04988 54253 96282
40	5	34	-1.16602 00753 40016
40	5	35	-1.29421 23514 98580
40	5	36	-1.43911 14072 13766
40	6	6	1.23359 99413 01782
40	6	7	1.11105 05830 80004
40	6	8	1.00116 44024 17916
40	6	9	0.90065 48984 04491
40	6	10	0.80731 95981 74764
40	6	11	0.71960 17243 22614
40	6	12	0.63635 26873 59036
40	6	13	0.55669 39920 54596
40	6	14	0.47993 22520 75205
40	6	15	0.40550 43397 83820
40	6	16	0.33294 04814 47102
40	6	17	0.26183 84555 05271
40	6	18	0.19184 48600 70862
40	6	19	0.12264 09609 89532
40	6	20	0.05393 15124 68308
40	6	21	-0.01456 45453 83674
40	6	22	-0.08312 23115 76450
40	6	23	-0.15201 87119 57498
40	6	24	-0.22154 04277 74351
40	6	25	-0.29199 25642 65399
40	6	26	-0.36370 87657 62230
40	6	27	-0.43706 37152 00671
40	6	28	-0.51248 93164 06628
40	6	29	-0.59049 66553 34160
40	6	30	-0.67170 67358 28667
40	6	31	-0.75689 56381 88256
40	6	32	-0.84706 30644 13305
40	6	33	-0.94354 14768 32203
40	6	34	-1.04818 02034 99745
40	6	35	-1.16367 47805 90113
40	7	7	1.01022 22339 77034
40	7	8	0.91022 49700 43416
40	7	9	0.81886 68629 36834
40	7	10	0.73410 81706 61187
40	7	11	0.65451 06445 29868
40	7	12	0.57901 53906 91513
40	7	13	0.50681 40460 55206
40	7	14	0.43726 96784 61019
40	7	15	0.36986 58406 48812
40	7	16	0.30417 23587 23936
40	7	17	0.23982 14586 27782
40	7	18	0.17649 04670 64127
40	7	19	0.11388 87695 52588
40	7	20	0.05174 75315 40345
40	7	21	-0.01018 88328 32837
40	7	22	-0.07217 01827 83559
40	7	23	-0.13444 77525 24214
40	7	24	-0.19728 13617 13428
40	7	25	-0.26094 72799 40195
40	7	26	-0.32574 73898 84014
40	7	27	-0.39202 04782 95239
40	7	28	-0.46015 69097 01660
40	7	29	-0.53061 83196 28177
40	7	30	-0.60396 56392 14168
40	7	31	-0.68089 94689 07886
40	7	32	-0.76232 22446 06998
40	7	33	-0.84943 67998 02051
40	7	34	-0.94391 18098 70390
40	8	8	0.82808 90081 42881
40	8	9	0.74499 84294 47890
40	8	10	0.66799 01879 61319
40	8	11	0.59573 25039 92833
40	8	12	0.52724 71146 52236
40	8	13	0.46178 87440 64071

(continued)

TABLE 2. PRODUCT MOMENTS OF NORMAL ORDER STATISTICS (cont'd)

n	i	j	$E[X_{i:n} X_{j:n}]$
40	8	14	0.39877 12745 43797
40	8	15	0.33772 02999 07113
40	8	16	0.27824 13297 50303
40	8	17	0.21999 76467 65030
40	8	18	0.16269 42953 49823
40	8	19	0.10606 60376 03816
40	8	20	0.04986 78847 34651
40	8	21	-0.00613 27395 25100
40	8	22	-0.06216 28871 49006
40	8	23	-0.11845 05682 72261
40	8	24	-0.17523 13127 79971
40	8	25	-0.23275 53090 44570
40	8	26	-0.29129 56986 94847
40	8	27	-0.35115 88105 76776
40	8	28	-0.41269 73321 43936
40	8	29	-0.47632 83313 23740
40	8	30	-0.54255 80339 05239
40	8	31	-0.61201 90652 52388
40	8	32	-0.68552 56252 05373
40	8	33	-0.76416 29593 56332
40	9	9	0.67710 90967 73114
40	9	10	0.60722 81473 64460
40	9	11	0.54172 17415 42327
40	9	12	0.47968 53724 62904
40	9	13	0.42043 15114 82943
40	9	14	0.36342 05829 50776
40	9	15	0.30821 67091 13770
40	9	16	0.25445 81194 14675
40	9	17	0.20183 65887 85745
40	9	18	0.15008 26033 43283
40	9	19	0.09895 42293 47974
40	9	20	0.04822 83865 74198
40	9	21	-0.00230 63515 84803
40	9	22	-0.05285 60648 22334
40	9	23	-0.10362 73996 70994
40	9	24	-0.15483 35348 93162
40	9	25	-0.20670 06506 78947
40	9	26	-0.25947 54351 49540
40	9	27	-0.31343 42915 28741
40	9	28	-0.36889 53263 28707
40	9	29	-0.42623 42562 21551
40	9	30	-0.48590 74529 38814
40	9	31	-0.54848 42050 43050
40	9	32	-0.61469 80260 05749
40	10	10	0.55063 33960 12163
40	10	11	0.49142 11216 75270
40	10	12	0.43539 82918 51303
40	10	13	0.38193 09489 17420
40	10	14	0.33052 27203 98388
40	10	15	0.28077 35138 81571
40	10	16	0.23235 19659 57307
40	10	17	0.18497 63440 09532
40	10	18	0.13840 08023 60990
40	10	19	0.09240 50977 05218
40	10	20	0.04678 65510 22528
40	10	21	0.00135 34391 25412
40	10	22	-0.04408 07712 83243
40	10	23	-0.08970 28080 17501
40	10	24	-0.13570 50029 26137
40	10	25	-0.18229 11392 96715
40	10	26	-0.22968 32090 32920
40	10	27	-0.27812 97256 85770
40	10	28	-0.32791 63693 38011
40	10	29	-0.37938 06137 17612
40	10	30	-0.43293 15728 79183
40	10	31	-0.48908 04130 38666
40	11	11	0.44407 17323 54168
40	11	12	0.39371 70833 97361
40	11	13	0.34570 48875 86760
40	11	14	0.29957 93053 67237
40	11	15	0.25497 38379 35630
40	11	16	0.21158 56829 12637
40	11	17	0.16915 80052 95846
40	11	18	0.12746 72145 66632
40	11	19	0.08631 34733 22633
40	11	20	0.04551 33050 09874
40	11	21	0.00489 35407 70934
40	11	22	-0.03571 39382 27969
40	11	23	-0.07647 70870 99771
40	11	24	-0.11756 85836 51300
40	11	25	-0.15917 10869 24760
40	11	26	-0.20148 33022 79478
40	11	27	-0.24472 73865 59922
40	11	28	-0.28915 85580 98195
40	11	29	-0.33507 78358 39503
40	11	30	-0.38285 04492 70340
40	12	12	0.35414 28525 56844
40	12	13	0.31131 92147 60940
40	12	14	0.27021 88995 69728
40	12	15	0.23050 72204 65549
40	12	16	0.19190 84374 79149
40	12	17	0.15418 93201 51437
40	12	18	0.11714 73888 44651
40	12	19	0.08060 21731 32165
40	12	20	0.04438 84305 63833
40	12	21	0.00835 06194 64525
40	12	22	-0.02766 18767 99974
40	12	23	-0.06379 91633 39358
40	12	24	-0.10021 52227 28234
40	12	25	-0.13707 26161 33346
40	12	26	-0.17454 78769 10240
40	12	27	-0.21283 81054 08783
40	12	28	-0.25216 94053 73896
40	12	29	-0.29280 83671 11119
40	13	13	0.27843 96790 39769
40	13	14	0.24215 57165 81491
40	13	15	0.20713 50381 59772
40	13	16	0.17312 77347 56644
40	13	17	0.13992 31195 65184
40	13	18	0.10733 88864 99368
40	13	19	0.07521 30824 33258
40	13	20	0.04339 79083 17299
40	13	21	0.01175 46937 89540
40	13	22	-0.01985 04182 03716
40	13	23	-0.05155 04327 76092
40	13	24	-0.08348 14161 92806
40	13	25	-0.11578 66609 94351
40	13	26	-0.14862 14454 22774
40	13	27	-0.18215 88128 65970
40	13	28	-0.21659 70091 36497
40	14	14	0.21516 37947 80206
40	14	15	0.18466 88857 97074

(continued)

TABLE 2. PRODUCT MOMENTS OF NORMAL ORDER STATISTICS (cont'd)

n	i	j	$E[X_{i:n} X_{j:n}]$
40	14	16	0.15509 19115 01356
40	14	17	0.12624 38514 58085
40	14	18	0.09796 17996 26537
40	14	19	0.07010 16204 72791
40	14	20	0.04253 23085 64695
40	14	21	0.01513 14449 77579
40	14	22	-0.01221 86755 60075
40	14	23	-0.03963 45708 89098
40	14	24	-0.06723 50258 42870
40	14	25	-0.09514 47416 93127
40	14	26	-0.12349 84777 25951
40	14	27	-0.15244 60661 92645
40	15	15	0.16295 64674 98878
40	15	16	0.13767 87232 13462
40	15	17	0.11305 88157 20848
40	15	18	0.08895 25984 39210
40	15	19	0.06523 31755 55841
40	15	20	0.04178 58001 11093
40	15	21	0.01850 37955 12943
40	15	22	-0.00471 48895 74065
40	15	23	-0.02797 07863 12610
40	15	24	-0.05136 59157 84988
40	15	25	-0.07500 69377 56103
40	15	26	-0.09900 86955 93615
40	16	16	0.12078 76369 26618
40	16	17	0.10029 22619 76921
40	16	18	0.08026 00099 68926
40	16	19	0.06058 07482 28949
40	16	20	0.04115 55485 88758
40	16	21	0.02189 30600 79700
40	16	22	0.00270 64793 10100
40	16	23	-0.01648 91450 05558
40	16	24	-0.03577 94894 33872
40	16	25	-0.05525 36167 64199
40	17	17	0.08788 13046 97530
40	17	18	0.07184 21920 24250
40	17	19	0.05612 33727 39935
40	17	20	0.04064 13790 74956
40	17	21	0.02531 98478 13454
40	17	22	0.01008 68960 48114
40	17	23	-0.00512 71740 89806
40	17	24	-0.02039 20295 27205
40	18	18	0.06366 47683 97020
40	18	19	0.05184 50661 40160
40	18	20	0.04024 56347 57541
40	18	21	0.02880 48297 05505
40	18	22	0.01746 54297 92028
40	18	23	0.00617 27345 37041
40	19	19	0.04773 41570 54099
40	19	20	0.03997 31976 89912
40	19	21	0.03236 94510 62318
40	19	22	0.02488 01604 00802
40	20	20	0.03983 16610 30628
40	20	21	0.03603 66528 12149

n	i	j	$E[X_{i:n} X_{j:n}]$
50	1	1	5.27404 46004
50	1	2	4.27404 46004
50	1	3	3.73167 39024
50	1	4	3.34457 24216
50	1	5	3.03657 00818
50	1	6	2.77677 53769
50	1	7	2.54950 88504
50	1	8	2.34567 37251
50	1	9	2.15949 74406
50	1	10	1.98707 20666
50	1	11	1.82561 85190
50	1	12	1.67308 18768
50	1	13	1.52789 33297
50	1	14	1.38882 24266
50	1	15	1.25488 12445
50	1	16	1.12525 98889
50	1	17	0.99928 16753
50	1	18	0.87637 10958
50	1	19	0.75603 03292
50	1	20	0.63782 15936
50	1	21	0.52135 35690
50	1	22	0.40627 06936
50	1	23	0.29224 45016
50	1	24	0.17896 64067
50	1	25	0.06614 14890
50	1	26	-0.04651 70559
50	1	27	-0.15929 30867
50	1	28	-0.27247 25498
50	1	29	-0.38634 86091
50	1	30	-0.50122 71174
50	1	31	-0.61743 26922
50	1	32	-0.73531 56980
50	1	33	-0.85526 09408
50	1	34	-0.97769 68830
50	1	35	-1.10311 07419
50	1	36	-1.23206 47533
50	1	37	-1.36520 94486
50	1	38	-1.50334 64878
50	1	39	-1.64740 46745
50	1	40	-1.79859 46408
50	1	41	-1.95840 45054
50	1	42	-2.12879 56713
50	1	43	-2.31241 89365
50	1	44	-2.51294 35842
50	1	45	-2.73578 19546
50	1	46	-2.98934 41682
50	1	47	-3.28792 65094
50	1	48	-3.65907 18588
50	1	49	-4.16822 43781
50	1	50	-5.05526 47583
50	2	2	3.55703 04751
50	2	3	3.09940 11731
50	2	4	2.77508 38525
50	2	5	2.51802 54006
50	2	6	2.30172 19941
50	2	7	2.11281 25394
50	2	8	1.94358 22569
50	2	9	1.78915 29629
50	2	10	1.64623 13440
50	2	11	1.51248 06119
50	2	12	1.38617 60751

n	i	j	$E[X_{i:n} X_{j:n}]$
50	2	13	1.26600 30903
50	2	14	1.15093 19488
50	2	15	1.04013 68884
50	2	16	0.93294 16779
50	2	17	0.82878 18635
50	2	18	0.72717 78226
50	2	19	0.62771 50291
50	2	20	0.53002 92445
50	2	21	0.43379 51392
50	2	22	0.33871 73319
50	2	23	0.24452 31472
50	2	24	0.15095 65898
50	2	25	0.05777 31638
50	2	26	-0.03526 47489
50	2	27	-0.12839 21866
50	2	28	-0.22184 57860
50	2	29	-0.31586 80427
50	2	30	-0.41071 17753
50	2	31	-0.50664 53256
50	2	32	-0.60395 88806
50	2	33	-0.70296 53786
50	2	34	-0.80403 32503
50	2	35	-0.90754 28992
50	2	36	-1.01395 17969
50	2	37	-1.12391 61558
50	2	38	-1.23776 87126
50	2	39	-1.35682 88460
50	2	40	-1.48145 34528
50	2	41	-1.61337 17985
50	2	42	-1.75400 46749
50	2	43	-1.90544 48216
50	2	44	-2.07095 41811
50	2	45	-2.25477 58290
50	2	46	-2.46398 44485
50	2	47	-2.71031 13705
50	2	48	-3.01649 50935
50	2	49	-3.43651 23891
50	3	3	2.73790 52002
50	3	4	2.44932 40016
50	3	5	2.22131 85087
50	3	6	2.02984 85571
50	3	7	1.86285 96838
50	3	8	1.71341 80541
50	3	9	1.57715 18754
50	3	10	1.45111 63598
50	3	11	1.33322 58375
50	3	12	1.22194 33004
50	3	13	1.11609 86848
50	3	14	1.01477 65576
50	3	15	0.91724 35302
50	3	16	0.82289 95576
50	3	17	0.73124 41910
50	3	18	0.64185 25188
50	3	19	0.55435 75643
50	3	20	0.46843 70913
50	3	21	0.38380 34751
50	3	22	0.30019 57337
50	3	23	0.21737 30948
50	3	24	0.13510 96481
50	3	25	0.05318 97540
50	3	26	-0.02859 60479

(continued)

TABLE 2. PRODUCT MOMENTS OF NORMAL ORDER STATISTICS (cont'd)

n	i	j	$E[X_{i:n} X_{j:n}]$
50	3	27	-0.11045 48391
50	3	28	-0.19259 50105
50	3	29	-0.27522 99383
50	3	30	-0.35858 26752
50	3	31	-0.44288 80167
50	3	32	-0.52839 54360
50	3	33	-0.61542 86601
50	3	34	-0.70414 60446
50	3	35	-0.79518 63625
50	3	36	-0.88879 24265
50	3	37	-0.98473 04419
50	3	38	-1.08618 66272
50	3	39	-1.18884 75897
50	3	40	-1.30008 67843
50	3	41	-1.41498 98387
50	3	42	-1.53862 75515
50	3	43	-1.67204 98147
50	3	44	-1.81697 26607
50	3	45	-1.97871 78393
50	3	46	-2.16234 48864
50	3	47	-2.37872 57181
50	3	48	-2.64762 56933
50	4	4	2.21250 48612
50	4	5	2.00557 11458
50	4	6	1.83212 96287
50	4	7	1.68106 50092
50	4	8	1.54600 58728
50	4	9	1.42294 56584
50	4	10	1.30919 12420
50	4	11	1.20283 84526
50	4	12	1.10248 60385
50	4	13	1.00706 85844
50	4	14	0.91575 34191
50	4	15	0.82787 40886
50	4	16	0.74288 57784
50	4	17	0.66033 44594
50	4	18	0.57983 49146
50	4	19	0.50105 46795
50	4	20	0.42370 20144
50	4	21	0.34751 66800
50	4	22	0.27226 26876
50	4	23	0.19772 24514
50	4	24	0.12369 19332
50	4	25	0.04997 64782
50	4	26	-0.02361 28915
50	4	27	-0.09726 29107
50	4	28	-0.17116 14163
50	4	29	-0.24550 11944
50	4	30	-0.32047 87237
50	4	31	-0.39631 75577
50	4	32	-0.47326 42673
50	4	33	-0.55131 46607
50	4	34	-0.63175 77080
50	4	35	-0.71269 21536
50	4	36	-0.79690 36120
50	4	37	-0.88638 85420
50	4	38	-0.96930 11499
50	4	39	-1.07421 70997
50	4	40	-1.16124 33490
50	4	41	-1.27364 90541
50	4	42	-1.38126 97957
50	4	43	-1.50077 79837
50	4	44	-1.63287 65345
50	4	45	-1.77685 94891
50	4	46	-1.94264 18613
50	4	47	-2.13694 08008
50	5	5	1.83239 07235
50	5	6	1.67340 03036
50	5	7	1.53510 75063
50	5	8	1.41158 85786
50	5	9	1.29912 80384
50	5	10	1.19523 35324
50	5	11	1.09814 58028
50	5	12	1.00657 20643
50	5	13	0.91953 05615
50	5	14	0.83625 48205
50	5	15	0.75613 19491
50	5	16	0.67866 13472
50	5	17	0.60342 61605
50	5	18	0.53007 29728
50	5	19	0.45829 69770
50	5	20	0.38783 08809
50	5	21	0.31843 64053
50	5	22	0.24989 76091
50	5	23	0.18201 55102
50	5	24	0.11460 36245
50	5	25	0.04748 41431
50	5	26	-0.01951 54648
50	5	27	-0.08656 56199
50	5	28	-0.15383 75970
50	5	29	-0.22150 46473
50	5	30	-0.28976 88251
50	5	31	-0.35875 60682
50	5	32	-0.42867 55090
50	5	33	-0.50072 69312
50	5	34	-0.57062 79326
50	5	35	-0.65014 35234
50	5	36	-0.72277 93401
50	5	37	-0.79576 23459
50	5	38	-0.90288 08431
50	5	39	-0.94326 82465
50	5	40	-1.08607 33583
50	5	41	-1.13669 35280
50	5	42	-1.26274 00913
50	5	43	-1.36394 52391
50	5	44	-1.48148 15787
50	5	45	-1.61665 71332
50	5	46	-1.76497 54612
50	6	6	1.53887 09129
50	6	7	1.41139 31842
50	6	8	1.29765 02904
50	6	9	1.19417 17852
50	6	10	1.09863 44837
50	6	11	1.00940 13240
50	6	12	0.92527 10401
50	6	13	0.84533 25049
50	6	14	0.76887 51558
50	6	15	0.69533 13595
50	6	16	0.62423 78902
50	6	17	0.55520 93291
50	6	18	0.48791 91590
50	6	19	0.42208 59755
50	6	20	0.35746 31781
50	6	21	0.29383 10800
50	6	22	0.23099 07178
50	6	23	0.16875 88685
50	6	24	0.10696 39215
50	6	25	0.04544 23455
50	6	26	-0.01596 44474
50	6	27	-0.07741 29313
50	6	28	-0.13906 06906
50	6	29	-0.20107 53861
50	6	30	-0.26355 26279
50	6	31	-0.32696 44021
50	6	32	-0.39118 55258
50	6	33	-0.45396 25973
50	6	34	-0.53020 29397
50	6	35	-0.57767 81485
50	6	36	-0.67215 34793
50	6	37	-0.74300 56697
50	6	38	-0.76178 89925
50	6	39	-0.96789 58469
50	6	40	-0.87839 89636
50	6	41	-1.12631 10916
50	6	42	-1.11030 35783
50	6	43	-1.25980 76531
50	6	44	-1.35685 08406
50	6	45	-1.47534 23090
50	7	7	1.30287 19130
50	7	8	1.19769 99086
50	7	9	1.10209 84925
50	7	10	1.01389 20324
50	7	11	0.93155 01245
50	7	12	0.85395 14339
50	7	13	0.78024 65494
50	7	14	0.70977 37876
50	7	15	0.64200 50974
50	7	16	0.57650 99518
50	7	17	0.51293 04436
50	7	18	0.45096 36054
50	7	19	0.39034 85267
50	7	20	0.33085 67332
50	7	21	0.27228 48288
50	7	22	0.21444 87303
50	7	23	0.15717 90307
50	7	24	0.10031 71618
50	7	25	0.04371 21155
50	7	26	-0.01278 24670
50	7	27	-0.06931 11938
50	7	28	-0.12601 79764
50	7	29	-0.18303 46047
50	7	30	-0.24072 11913
50	7	31	-0.29827 33310
50	7	32	-0.35768 15920
50	7	33	-0.42277 32925
50	7	34	-0.45902 44559
50	7	35	-0.58369 78204
50	7	36	-0.55560 05801
50	7	37	-0.68677 84873
50	7	38	-0.81557 20758
50	7	39	-0.63995 33230
50	7	40	-1.11281 33852
50	7	41	-0.77090 81450

(continued)

TABLE 2. PRODUCT MOMENTS OF NORMAL ORDER STATISTICS (cont'd)

n	i	j	$E[X_{i:n}\ X_{j:n}]$
50	7	42	-1.17895 06345
50	7	43	-1.09091 59255
50	7	44	-1.25949 73792
50	8	8	1.10791 72780
50	8	9	1.01938 97110
50	8	10	0.93776 80656
50	8	11	0.86161 72773
50	8	12	0.78988 75016
50	8	13	0.72178 45544
50	8	14	0.65669 05113
50	8	15	0.59411 27707
50	8	16	0.53365 01014
50	8	17	0.47496 92527
50	8	18	0.41778 83695
50	8	19	0.36186 49196
50	8	20	0.30698 66878
50	8	21	0.25296 48953
50	8	22	0.19962 88139
50	8	23	0.14682 14391
50	8	24	0.09439 59146
50	8	25	0.04221 24794
50	8	26	-0.00986 42301
50	8	27	-0.06196 71682
50	8	28	-0.11423 37105
50	8	29	-0.16683 12200
50	8	30	-0.21943 72636
50	8	31	-0.27468 33251
50	8	32	-0.32647 40449
50	8	33	-0.37516 31505
50	8	34	-0.47992 49100
50	8	35	-0.39748 26705
50	8	36	-0.70537 43311
50	8	37	-0.50061 08182
50	8	38	-0.64697 06980
50	8	39	-0.99621 23712
50	8	40	-0.41773 62337
50	8	41	-1.29483 42750
50	8	42	-0.67871 75644
50	8	43	-1.20419 08699
50	9	9	0.94379 01167
50	9	10	0.86818 69545
50	9	11	0.79769 58672
50	9	12	0.73133 20756
50	9	13	0.66835 16835
50	9	14	0.60817 66296
50	9	15	0.55034 66188
50	9	16	0.49448 71270
50	9	17	0.44028 73892
50	9	18	0.38748 48096
50	9	19	0.33585 36260
50	9	20	0.28519 64639
50	9	21	0.23533 78902
50	9	22	0.18611 93727
50	9	23	0.13739 52367
50	9	24	0.08902 93264
50	9	25	0.04089 21602
50	9	26	-0.00714 15956
50	9	27	-0.05519 64272
50	9	28	-0.10338 80468
50	9	29	-0.15180 22383
50	9	30	-0.20140 89467
50	9	31	-0.24728 64491
50	9	32	-0.30518 67351
50	9	33	-0.35778 22868
50	9	34	-0.34156 46823
50	9	35	-0.63688 35274
50	9	36	-0.19292 64721
50	9	37	-0.91899 74448
50	9	38	-0.45303 58739
50	9	39	-0.51524 96672
50	9	40	-1.24057 57474
50	9	41	-0.21955 93862
50	9	42	-1.38210 42238
50	10	10	0.80373 31928
50	10	11	0.73848 54571
50	10	12	0.67709 36794
50	10	13	0.61886 03692
50	10	14	0.56324 42659
50	10	15	0.50981 49108
50	10	16	0.45822 24540
50	10	17	0.40817 69112
50	10	18	0.35943 34908
50	10	19	0.31178 19391
50	10	20	0.26503 86088
50	10	21	0.21904 04146
50	10	22	0.17364 01121
50	10	23	0.12870 25143
50	10	24	0.08410 13743
50	10	25	0.03971 67305
50	10	26	-0.00456 74175
50	10	27	-0.04886 36724
50	10	28	-0.09330 23083
50	10	29	-0.13803 29742
50	10	30	-0.18196 75038
50	10	31	-0.23354 82937
50	10	32	-0.26298 49913
50	10	33	-0.32560 24683
50	10	34	-0.43557 66506
50	10	35	-0.16975 42989
50	10	36	-0.98236 36357
50	10	37	0.17218 26248
50	10	38	-1.16437 01077
50	10	39	-0.45293 36999
50	10	40	-0.37418 30000
50	10	41	-1.36182 24857
50	11	11	0.68305 62899
50	11	12	0.62632 04781
50	11	13	0.57253 30908
50	11	14	0.52118 72017
50	11	15	0.47188 01991
50	11	16	0.42428 52706
50	11	17	0.37813 18405
50	11	18	0.33319 17513
50	11	19	0.28926 92420
50	11	20	0.24619 34982
50	11	21	0.20381 29807
50	11	22	0.16199 10023
50	11	23	0.12060 21884
50	11	24	0.07952 95642
50	11	25	0.03866 20794
50	11	26	-0.00210 75992
50	11	27	-0.04288 72365
50	11	28	-0.08375 76413
50	11	29	-0.12483 16817
50	11	30	-0.16760 27072
50	11	31	-0.20114 25369
50	11	32	-0.27099 82696
50	11	33	-0.26880 48304
50	11	34	-0.30023 74456
50	11	35	-0.63590 46465
50	11	36	0.19959 74478
50	11	37	-1.47500 92477
50	11	38	0.55499 59249
50	11	39	-1.32552 61514
50	11	40	-0.46633 04148
50	12	12	0.57838 02155
50	12	13	0.52879 27762
50	12	14	0.48148 13162
50	12	15	0.43606 94383
50	12	16	0.39225 20242
50	12	17	0.34977 67815
50	12	18	0.30843 12275
50	12	19	0.26803 32619
50	12	20	0.22842 41700
50	12	21	0.18946 33021
50	12	22	0.15102 39308
50	12	23	0.11298 99409
50	12	24	0.07525 31094
50	12	25	0.03771 07992
50	12	26	0.00026 39651
50	12	27	-0.03718 22722
50	12	28	-0.07475 79740
50	12	29	-0.11249 47871
50	12	30	-0.14928 86926
50	12	31	-0.19707 97542
50	12	32	-0.20078 70908
50	12	33	-0.31766 64861
50	12	34	-0.28781 51786
50	12	35	-0.18254 34917
50	12	36	-0.96321 54176
50	12	37	0.63663 17448
50	12	38	-1.85903 27263
50	12	39	0.72110 78200
50	13	13	0.48719 78019
50	13	14	0.44372 55405
50	13	15	0.40202 06803
50	13	16	0.36179 86789
50	13	17	0.32282 45903
50	13	18	0.28490 08076
50	13	19	0.24785 81999
50	13	20	0.21154 95427
50	13	21	0.17584 45303
50	13	22	0.14062 58990
50	13	23	0.10578 63337
50	13	24	0.07122 59318
50	13	25	0.03685 00566
50	13	26	0.00256 74169
50	13	27	-0.03171 39606
50	13	28	-0.06604 82012
50	13	29	-0.10065 68576
50	13	30	-0.13636 22028
50	13	31	-0.16319 52526
50	13	32	-0.23505 48480

(continued)

TABLE 2. PRODUCT MOMENTS OF NORMAL ORDER STATISTICS (cont'd)

n	i	j	$E[X_{i:n} X_{j:n}]$
50	13	33	-0.17603 88260
50	13	34	-0.36068 14939
50	13	35	-0.34316 66827
50	13	36	-0.00762 52798
50	13	37	-1.24023 82994
50	13	38	0.83876 84853
50	14	14	0.40760 49699
50	14	15	0.36944 97754
50	14	16	0.33267 08924
50	14	17	0.29704 99320
50	14	18	0.26240 34915
50	14	19	0.22857 48271
50	14	20	0.19542 76918
50	14	21	0.16284 16661
50	14	22	0.13070 85329
50	14	23	0.09892 93923
50	14	24	0.06741 23000
50	14	25	0.03607 02735
50	14	26	0.00481 95808
50	14	27	-0.02642 00202
50	14	28	-0.05776 26384
50	14	29	-0.08912 82578
50	14	30	-0.12043 50659
50	14	31	-0.15834 10947
50	14	32	-0.16130 92337
50	14	33	-0.28428 19615
50	14	34	-0.13897 93867
50	14	35	-0.38260 86889
50	14	36	-0.40328 19433
50	14	37	0.08072 39160
50	15	15	0.33812 86636
50	15	16	0.30466 45742
50	15	17	0.27227 20458
50	15	18	0.24078 13086
50	15	19	0.21004 75119
50	15	20	0.17994 49521
50	15	21	0.15036 27068
50	15	22	0.12120 12530
50	15	23	0.09236 97798
50	15	24	0.06378 39952
50	15	25	0.03536 42784
50	15	26	0.00703 40408
50	15	27	-0.02128 29383
50	15	28	-0.04963 72991
50	15	29	-0.07825 90678
50	15	30	-0.10700 12586
50	15	31	-0.13297 68212
50	15	32	-0.18119 54493
50	15	33	-0.14524 89780
50	15	34	-0.33046 18020
50	15	35	-0.11283 84187
50	15	36	-0.38691 60080
50	16	16	0.27761 27511
50	16	17	0.24834 31005
50	16	18	0.21990 51303
50	16	19	0.19216 54605
50	16	20	0.16500 85597
50	16	21	0.13833 26717
50	16	22	0.11204 66664
50	16	23	0.08606 75449
50	16	24	0.06031 84068
50	16	25	0.03472 67445
50	16	26	0.00922 29768
50	16	27	-0.01626 02504
50	16	28	-0.04180 56436
50	16	29	-0.06736 79707
50	16	30	-0.09337 29970
50	16	31	-0.12045 80453
50	16	32	-0.13791 55778
50	16	33	-0.20171 42235
50	16	34	-0.12931 10547
50	16	35	-0.34973 73901
50	17	17	0.22514 00865
50	17	18	0.19966 75769
50	17	19	0.17483 66181
50	17	20	0.15054 15328
50	17	21	0.12668 94468
50	17	22	0.10319 73661
50	17	23	0.07998 98662
50	17	24	0.05699 72163
50	17	25	0.03415 38101
50	17	26	0.01139 67959
50	17	27	-0.01133 54226
50	17	28	-0.03409 61797
50	17	29	-0.05701 39823
50	17	30	-0.07986 19326
50	17	31	-0.10311 90133
50	17	32	-0.12952 88618
50	17	33	-0.13824 99781
50	17	34	-0.21040 30923
50	18	18	0.17997 79468
50	18	19	0.15798 33119
50	18	20	0.13647 90151
50	18	21	0.11538 08206
50	18	22	0.09461 36497
50	18	23	0.07410 94492
50	18	24	0.05380 54684
50	18	25	0.03364 28212
50	18	26	0.01356 52528
50	18	27	-0.00648 18841
50	18	28	-0.02655 59350
50	18	29	-0.04668 07310
50	18	30	-0.06707 85861
50	18	31	-0.08732 58914
50	18	32	-0.10775 64240
50	18	33	-0.13287 26861
50	19	19	0.14153 90642
50	19	20	0.12276 56046
50	19	21	0.10436 23202
50	19	22	0.08626 18584
50	19	23	0.06840 33641
50	19	24	0.05073 09037
50	19	25	0.03319 21577
50	19	26	0.01573 72978
50	19	27	-0.00168 20309
50	19	28	-0.01911 26295
50	19	29	-0.03661 37637
50	19	30	-0.05416 50568
50	19	31	-0.07208 85024
50	19	32	-0.08982 01516
50	20	20	0.10935 33621
50	20	21	0.09359 56027
50	20	22	0.07811 31467
50	20	23	0.06285 21888
50	20	24	0.04776 34750
50	20	25	0.03280 11215
50	20	26	0.01792 16091
50	20	27	0.00308 28944
50	20	28	-0.01175 66388
50	20	29	-0.02663 55147
50	20	30	-0.04161 77437
50	20	31	-0.05664 81931
50	21	21	0.08304 72370
50	21	22	0.07014 25547
50	21	23	0.05743 93715
50	21	24	0.04489 49963
50	21	25	0.03246 98708
50	21	26	0.02012 66764
50	21	27	0.00782 95522
50	21	28	-0.00445 65889
50	21	29	-0.01676 73769
50	21	30	-0.02913 38253
50	22	22	0.06232 83035
50	22	23	0.05215 07519
50	22	24	0.04211 88900
50	22	25	0.03219 93936
50	22	26	0.02236 10577
50	22	27	0.01257 41190
50	22	28	0.00280 96447
50	22	29	-0.00696 09766
50	23	23	0.04697 42040
50	23	24	0.03943 00123
50	23	25	0.03199 15139
50	23	26	0.02463 35467
50	23	27	0.01733 23778
50	23	28	0.01006 51961
50	24	24	0.03682 45403
50	24	25	0.03184 89309
50	24	26	0.02695 33689
50	24	27	0.02212 00983
50	25	25	0.03177 52896
50	25	26	0.02933 03801

2. COMPUTATIONAL TECHNIQUE

2.1 Product moments

The general product moment of the ith and jth smallest order statistics in a sample of size n from a normal parent distribution is given by

$$E[X_{i|n}X_{j|n}] = K_{ijn} \int_{-\infty}^{\infty} \int_{-\infty}^{y} xy f(x) f(y) [F(x)]^{i-1} [1-F(y)]^{n-j} [F(y)-F(x)]^{j-i-1} dx dy \tag{1}$$

where

$$K_{ijn} = \frac{n!}{(i-1)!\,(n-j)!\,(j-i-1)!} ,$$

$$f(t) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t^2} , \text{ and}$$

$$F(u) = \int_{-\infty}^{u} f(t)\, dt .$$

Given the product moments, variances and covariances can be computed simply by subtracting the product of the corresponding expected values as obtained to high precision (i.e., more than 25 d.p.) by Parrish (1991). Thus, to obtain covariances the task is to evaluate numerically the integral in Eq.(1) to the desired precision for various values of i, j, and n.

Godwin (1949) presented a method for the evaluation of this integral. That technique was employed here in conjunction with the use of Gauss-Legendre quadrature for numerical evaluation of associated single integrals with finite limits. Godwin developed the expression

$$E[X_{i|n}X_{j|n}] = K_{ijn} \sum_{r=0}^{j-i-1} \sum_{s=0}^{j-i-1-r} \frac{(-1)^{r+s}(j-i-1)!}{r!\,s!\,(j-i-1-r-s)!} \gamma_{i+r,\,n-j+s+1} \tag{2}$$

which is applicable for order statistics from the normal distribution. He defined the function

$$\gamma_{i,j} = \frac{1}{ij}(\alpha_{i,j} + i\,\beta_{i-1,j} - \psi_{i,j}) \tag{3}$$

where

$$\alpha_{i,j} = \int_{-\infty}^{\infty} x\,[F(x)]^i\,[1-F(x)]^j\,dx, \tag{4}$$

$$\beta_{i,j} = \int_{-\infty}^{\infty} x^2 f(x)\,[F(x)]^i\,[1-F(x)]^j\,dx, \text{ and} \tag{5}$$

$$\Psi_{i,j} = \int_{-\infty}^{\infty} [F(x)]^i \int_{x}^{\infty} [1-F(y)]^j\,dy\,dx = \int_{-\infty}^{\infty} [F(x)]^i \int_{-\infty}^{-x} [F(y)]^j\,dy\,dx. \tag{6}$$

The following symmetry relationships hold:

$$\alpha_{i,j} = -\alpha_{j,i}, \quad \beta_{i,j} = \beta_{j,i}, \quad \psi_{i,j} = \psi_{j,i}. \tag{7}$$

As was done in Parrish (1991), the infinite-limit integrals in Eqs.(4)-(6) were truncated for the purpose of numerical evaluation. In order to permit the use of a Gauss-Legendre integration method, the infinite limits were replaced by finite limits corresponding to 12.2 standard deviations above and below the mean. These limits were chosen as large as possible, constrained only by the limitations of the computing equipment and software in regard to computational use of Gaussian integration points within this range. For expected value computations, Parrish observed that the loss of tail area due to such truncation was negligible, being less than the precision required. It was considered, therefore, to be a reasonable choice also for the covariance computations. The errors introduced as a result of the truncated integration limits are shown in Appendix A to be negligible in all cases.

With the infinite limits replaced by finite limits, selected integrals of Eqs.(4)-(5) were evaluated to high precision using 3072-point Gauss-Legendre quadrature (see Stroud and Secrest (1969), Davis and Rabinowitz (1984), Lether (1978), and Parrish (1991)). Thus, the numerical approximation of the α_{ij} values, Eq.(4), are given by the summation expression

$$\sum_{k=1}^{N} w_k x_k\,[F(x_k)]^i\,[1-F(x_k)]^j \tag{8}$$

where the w_k values represent appropriate weights and the x_k values represent appropriate integration points. Standard Gauss-Legendre points and weights were generated for the interval (-1,1) and then linearly transformed for application to the interval (-12.2, 12.2) (*cf.* Stroud and

Secrest, 1969). The points and weights were calculated using a routine developed by Lether (personal communication) which produces values to full machine precision. This type of numerical integration produces values for the summation that converge as the number of points N increases. Similarly, β_{ij} values, Eq.(5), were computed as

$$\sum_{k=1}^{N} w_k x_k^2 f(x_k) [F(x_k)]^i [1 - F(x_k)]^j \,. \tag{9}$$

The evaluation of the double integral expression for ψ_{ij} in Eq.(6) requires computation of the double summation

$$\sum_{k=1}^{N} \sum_{m=1}^{N} w_{km} [F(x_k)]^i [F(x_m)]^j \tag{10}$$

where w_{km} values represent numerical integration weights. Lether (1976) presented a general cubature method for integration over a two-dimensional triangle, and he also provided a computer code for its implementation. This code was adapted for use in the present application in order to compute the weights and intermediate functional values appearing in Eq.(10), thus providing a basis for the numerical evaluations of ψ_{ij}.

Computation of the values ψ_{ij} required the numerical evaluation of many double integrals that were computed using 512 Gaussian points on both the inner and the outer integrals. The computational expense was highest for this phase of the work. Generally, the expense of cpu time and storage resources increases by a factor of four for each doubling of the number of integration points. (Approximately 100 hours of cpu time on a DEC VAX 11/785 computer system with floating point hardware were required for the computations of ψ values using 512 points.) All computations were carried out using 128-bit floating-point variables with 112-bit mantissa, providing approximately 33 significant digits of precision.

2.2 Variances

The computation of variances of normal order statistics was based on a single integral representation of $E[X_{i|n}^2]$ as given in Parrish (1991). The precision for the variances is on the order of 29 d.p. for all values of n up to 50.

2.3 Table entries

Table 1 contains values, given to 25-decimal-digit precision, for variances and covariances of normal order statistics for sample sizes ranging from 2 to 20. Table 2 contains product moments for a sample size of 20 to 25 d.p., for a sample size of 30 to 20 d.p., for a sample size of 40 to 15 d.p,, and for a sample size of 50 to 10 d.p. Covariances and product moments that are not included in the tables can be obtained using the identities

$$Cov(X_{i|n}, X_{j|n}) = Cov(X_{j|n}, X_{i|n}) = Cov(X_{n-i+1|n}, X_{n-j+1|n}), \text{ and}$$

$$E(X_{i|n}\, X_{j|n}) = E(X_{j|n}\, X_{i|n}) = E(X_{n-i+1|n}\, X_{n-j+1|n}) \;.$$

The following recursion relation (Teichroew, 1956) can be applied to obtain values corresponding to sample sizes not included in Table 2.

$$E[X_{i|n} X_{j|n}] = \left(\frac{j-i}{n+1}\right) E[X_{i|n+1} X_{j+1|n+1}] + \left(\frac{i}{n+1}\right) E[X_{i+1|n+1} X_{j+1|n+1}] + \left(\frac{n-j+1}{n+1}\right) E[X_{i|n+1} X_{j|n+1}] \tag{11}$$

The maximum attainable precision for product moments appeared mainly to be a function of the relative magnitudes of the terms occurring in the summation of Eq.(2) in conjunction with the inherent limited precision of floating-point storage of these values (approximately 33 significant digits). For large values of n, where the precision was relatively low, the summation contained terms that approached magnitudes of 10^{20}, thereby contributing to the loss of precision in the low-order decimal values. The number of Gaussian points used for the evaluation of $\psi_{i,j}$ values, Eq.(6), was fixed at 512 on each integral. Comparison with preliminary results using 256 points revealed only marginal improvement in the precision of the covariances.

3. CHECKS AND COMPARISONS

3.1 Relations for intermediate quantities

The following relations hold for the quantities given in Eqs.(4)-(5):

$$\alpha_{i,j} = \alpha_{i,j+1} + \alpha_{i+1,j},$$

$$\beta_{i,i} = \left(\frac{i}{4i+2}\right)\beta_{i-1,i-1} - \left(\frac{2}{2i+1}\right)\alpha_{i+1,i}, \text{ and}$$

$$\beta_{i,j} = \beta_{i,j+1} + \beta_{i+1,j}\,.$$

Given that $\beta_{0,0} = 1$ for the standard normal distribution, these relations can be used to check the values obtained via numerical quadrature. These identities are satisfied in all cases.

3.2 Exact values

Jones (1948) and Godwin (1949) gave exact mathematical expressions for product moments of normal order statistics in samples of size six and less. Values computed by Eq.(3) agree with exact values to at least 30 d.p. for $n \leq 6$.

3.3 Other tables

The computed product moments agree completely with the 8-place values of Yamauti (1972). In comparison to the table given by Tietjen *et al.* (1977), however, there are several instances where covariance values differ, some as early as the fourth decimal digit for the larger sample sizes.

3.4 Summations of product moments

Teichroew (1956) noted that (with corrected upper limit on the summation)

$$\sum_{j=1}^{n} E[X_{i|n} X_{j|n}] = 1 , \tag{12}$$

for $i=1, \dots, n$-1. As the precision and accuracy of the calculated product moments improve, the summation more nearly will approach unity. The results of evaluating this summation for each sample size show agreement as follows: 31 d.p. at $n=2$, 29 at $n=10$, 25 at $n=20$, 20 at $n=30$, 15 at $n=40$, 10 d.p. at $n=50$. For example, with $n = 50$ and $i = 12$, the summation in Eq.(12) evaluates to 1.0000000000342; by contrast, the Tietjen *et al.* (1977) tabled values produce 0.9999932112. Although this relationship is not a sufficient condition for the covariance values to be as accurate as indicated, it is a necessary condition.

3.5 Summations of expected squared values

Teichroew (1956) also noted that

$$\sum_{i=1}^{n} E[X_{i|n}^2] = n .$$

This summation was calculated for each value of n. The maximum deviation observed was on the order of 10^{-30}. Thus, given accurate expected values, the variance computations are considered to be accurate beyond the precision reported in Table 1, and similarly for Table 2.

3.6 Recurrence for product moments

The recurrence relation among product moments (Eq.11) was applied for each $n=2(1)49$ and the result was compared against the corresponding computed value. Differences between the recurrence values and the computed values were on the order of 10^{-28} at $n=10$, 10^{-24} at $n=20$, 10^{-19} at $n=30$, 10^{-14} at $n=40$, and 10^{-9} at $n=49$. These results are consistent with the indications of maximum significance levels attainable when considering the magnitudes of the terms appearing in Eq.(2).

3.7 Variance of the sample range

The values in Tables 1 and 2 can be used to evaluate the variance of the range W for a sample of size n for $n \leq 50$. Of course, the range may be written as the difference between the nth order statistic and the first order statistic, so that the variance is

$$Var(W) = Var(X_{n|n}) + Var(X_{1|n}) - 2Cov(X_{1|n}, X_{n|n}) = 2[Var(X_{1|n}) - Cov(X_{1|n}, X_{n|n})].$$

The moments of W were computed by Harter (1969a, Table A8) and were presented in a 10-decimal-place table. Variances of W computed using the values in Tables 1 and 2 agree with the results of Harter to 10 d.p. for all n except for $n = 3$ where there is a difference of one digit in the tenth place.

3.8 Variances of quasi-ranges

The rth quasi-range W_r may be defined as

$$W_r = X_{n-r+1|n} - X_{r|n},$$

for $r \leq [n/2]$. The values in Tables 1 and 2 can be used to evaluate the variance of W_r for samples of size $n \leq 50$. The variance of W_r is

$$Var(W_r) = Var(X_{n-r+1|n}) + Var(X_{r|n}) - 2Cov(X_{r|n}, X_{n-r+1|n})$$

$$= 2[Var(X_{r|n}) - Cov(X_{r|n}, X_{n-r+1|n})].$$

The variances of W_r were given by Harter (1969b, Table A2) to five decimal-places for $n \leq 100$ and $r \leq 9$. Variances of W_r computed using the values in Tables 1 and 2 agree completely with the 5-decimal-place results of Harter. In comparison to quasi-range values computed using the Tietjen *et al.* (1977) table, differences occur in the fourth decimal place for larger n values.

APPENDIX A.
BOUNDS ON ERRORS RESULTING FROM THE USE OF TRUNCATED INTEGRATION LIMITS

If the infinite limits of Eq.(1) are replaced by finite constants, the resulting integral may be considered as an approximation to the true value. The amount of error introduced by this truncation depends upon the magnitude of the finite constants used, but if these values are suitably chosen, the error can be made quite small. Upper bounds on the total magnitude of the error can be derived mathematically as follows.

Letting $A > 0$, the product moments of Eq.(1) can be approximated by the finite integral

$$K_{ijn} \int_{-A}^{A} y f(y) [1 - F(y)]^{n-j} \int_{-A}^{y} x f(x) [F(x)]^{i-1} [F(y) - F(x)]^{j-i-1} dx\, dy \qquad \text{(A.1)}$$

where

$$K_{ijn} = \frac{n!}{(i-1)!\,(n-j)!\,(j-i-1)!}\,.$$

The factors in the integrand have been rearranged to isolate the inner integral. In comparison to the integral in Eq.(1), there are several regions in the x-y plane that collectively define the domain that has been eliminated. These regions are identified below using the notation: $a < x < b, c < y < d$.

Region	a	b	c	d
R1	$-\infty$	y	$-\infty$	$-A$
R2	$-\infty$	$-A$	$-A$	0
R3	$-\infty$	$-A$	0	A
R4	$-\infty$	$-A$	A	∞
R5	$-A$	0	A	∞
R6	0	A	A	∞
R7	A	y	A	∞

By placing an upper bound on the absolute value of the integral for each of these regions and summing, an overall upper bound for the truncation error can be obtained.

The infinite domain of integration for Eq.(1) covers that half-plane defined by $x < y$; thus, $F(y) > F(x)$ for all x and y values. Also, for any values $a < x < b$, $F(a) < F(x) < F(b)$.

Hence, the absolute value of the inner integral in Eq.(A.1) can be immediately bounded as follows. Let $g(y)$ denote the inner integral taken over any of the excluded regions, then since

$$\int_a^b x f(x)\, dx = f(a) - f(b),$$

it follows that

$$|g(y)| = \left| \int_a^b x f(x) [F(x)]^{i-1} [F(y) - F(x)]^{j-i-1} dx \right|$$

$$\le [F(b)]^{i-1} [1 - F(a)]^{j-i-1} |f(a) - f(b)| = U(a,b), \text{ say}.$$

For each of the regions $R2$ through $R6$, c and d are either both nonnegative or both nonpositive, with $c < y < d$. Thus, the double integral derived from Eq.(A.1), taken over any of these regions, has the following property.

$$\left| \int_c^d y f(y) [1 - F(y)]^{n-j} g(y)\, dy \right|$$

$$\le \left| \int_c^d y f(y) [1 - F(y)]^{n-j} |g(y)|\, dy \right|$$

$$\le U(a,b) \left| \int_c^d y f(y) [1 - F(y)]^{n-j} dy \right|.$$

Since $[1 - F(y)] \le [1 - F(c)]$, this last quantity does not exceed

$$U(a,b) [1 - F(c)]^{n-j} |f(c) - f(d)|$$

$$= [1 - F(a)]^{j-i-1} [F(b)]^{i-1} |f(a) - f(b)| [1 - F(c)]^{n-j} |f(c) - f(d)|$$

$$= U(a,b,c,d), \text{ say}.$$

For regions $R2$ through $R6$, this produces the following bounds.

Region	*Bound* $= K_{ijn}\ U(a,b,c,d)$
$R2$	$K_{ijn}\ [F(-A)]^{i-1}\ f(-A)\ f(0)$
$R3$	$K_{ijn}\ [F(-A)]^{i-1}\ f(-A)\ (0.5)^{n-j}\ f(0)$
$R4$	$K_{ijn}\ [F(-A)]^{i-1}\ f(-A)\ [1-F(A)]^{n-j}\ f(A)$
$R5$	$K_{ijn}\ (0.5)^{i-1}\ f(0)\ [1-F(A)]^{n-j}\ f(A)$
$R6$	$K_{ijn}\ (0.5)^{j-i-1}\ f(0)\ [1-F(A)]^{n-j}\ f(A)$

Numerically, using $A = 12.2, f(A) = f(-A) = 1.0 \times 10^{-33}, F(-A) = [1 - F(A)] = 1.5 \times 10^{-34}$. For calculating these bounds, $[1 - F(-A)]$ and $F(A)$ are taken as unity, and $|f(-A)-f(0)|$ and $|f(0)-f(A)|$ are taken as $f(0) = (2\pi)^{-1/2}$.

Regions *R1* and *R7* can be treated separately. For *R1*, the limits on the inner integral are $-\infty$ to y. This integral can be transformed using $u = -x$ to produce

$$\int_{-y}^{\infty} u f(u)\,[1 - F(u)]^{i-1}\,\{F(y) - [1 - F(u)]\}^{j-i-1}\,du .$$

Since $F(y) \leq 1$, the absolute value of this integral does not exceed

$$\int_{-y}^{\infty} u f(u)\,[1 - F(u)]^{i-1}\,[F(u)]^{j-i-1}\,du$$

$$\leq [F(\infty)]^{j-i-1}\,[1 - F(-y)]^{i-1}\,|f(-y) - f(\infty)|$$

$$= [F(y)]^{i-1} f(y)$$

$$\leq [F(y)]^{i-1} f(-A),$$

since the range of y is $-\infty$ to $-A$ for this region. Thus, an upper bound on the absolute value of the double integral over region *R1* is

$$\left| \int_{-\infty}^{-A} y f(y)\,[1 - F(y)]^{n-j}\,F(y)^{i-1}\,dy \right| \times f(-A)$$

$$= \int_{A}^{\infty} u f(u)\,[F(u)]^{n-j}\,[1 - F(u)]^{i-1}\,du \times f(A)$$

$$\leq [F(\infty)]^{n-j}\,[1 - F(A)]^{i-1}\,|f(A) - f(\infty)|\,f(A)$$

$$= [f(A)]^2\,[1 - F(A)]^{i-1} .$$

For region R7, the inner integral has limits A to y which produces an upper bound of

$$[F(y)]^{i-1}[1-F(A)]^{j-i-1}|f(A)-f(y)|.$$

Thus, an upper bound on the double integral can be written as

$$[1-F(A)]^{j-i-1}\int_A^{\infty} yf(y)[1-F(y)]^{n-j}[F(y)]^{i-1}[f(A)-f(y)]\,dy$$

$$= [1-F(A)]^{j-i-1}\{f(A)\int_A^{\infty} yf(y)[1-F(y)]^{n-j}[F(y)]^{i-1}dy - \int_A^{\infty} y[f(y)]^2[1-F(y)]^{n-j}[F(y)]^{i-1}dy\}$$

$$\leq [1-F(A)]^{j-i-1}f(A)[F(\infty)]^{i-1}[1-F(A)]^{n-j}[f(A)-f(\infty)]$$

$$= [f(A)]^2[1-F(A)]^{n-i-1}.$$

Hence, the remaining two regions can be bounded as follows.

Region	*Bound*
R1	$K_{ijn}\ [f(A)]^2\ [1-F(A)]^{i-1}$
R7	$K_{ijn}\ [f(A)]^2\ [1-F(A)]^{n-i-1}$

For each n from 2 to 50, numerical calculations were made to determine the maximum of the sum of the regional upper bounds over all i and j values. The results indicated values of order 10^{-30} at $n=10$, 10^{-26} at $n=20$, 10^{-23} at $n=30$, 10^{-20} at $n=40$, and 10^{-17} at $n=50$. Furthermore, these bounds are considered conservative.

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TABLE AVAILABILITY

Complete tables of product moments, variances, and covariances are available for sample sizes up to 50. Current distribution information may be obtained by contacting the author in writing.

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