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A modification of the test of Shapiro and Wilk for normality

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SUMMARY *The* W *statistic of Shapiro and Wilk provides the best omnibus test of normality, but its application is limited up to* n = *50. This study modifies* W, *such that it can be extended for all sample sizes. The critical values of* $\tilde{W}$, *i.e. the modification of* W, *is given for* n *up to 5000. The empirical moments show that the null distribution of* $\tilde{W}$ *is skewed to the left and is consistant for all sample sizes. Empirical powers of* $\tilde{W}$ *are also comparable with those of* W.

1 Introduction

The recent interest in goodness-of-fit (GOF) tests for normality has arisen from the exciting work of Shapiro and Wilk (1965). The Shapiro–Wilk W statistic is the square of the ratio of the best linear estimator of the scale parameter to the sample standard deviation. The numerator of this ratio is actually a linear combination of sample order statistics. Shapiro and Wilk provided a table for the coefficients of order statistics, which enables us to compute the values of the test statistic W; they also gave a second table for the empirical percentage points of W for $n = 3(1)50$. The statistic test W' of Shapiro and Francia (1972) extends W, replacing the variance–covariance of the order statistics by the identity matrix, which makes the coefficients proportional to the expected values of normal order statistics. Weisberg and Bingham (1975) gave an approximation to the Shapiro–Francia test statistic using Blom's (1958) approximation to the expected values of normal order statistics. D'Agostino (1971) modified the W test by replacing the coefficients by the simple linear function

$$c_i = i - \frac{1}{2}(n + 1)$$

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Probability points are also available for this test for samples of size up to 1000. Royston (1982a) gave an approximate normalizing transformation for W as $y = (1 - W)^{\lambda}$ for some choice of λ. Later, Royston (1982b, c) provided algorithms in FORTRAN 66 for computing W and its critical values for n up to 2000.

A serious drawback of the Shapiro–Wilk W test statistic is that its applicability is limited to sample sizes only up to $n = 50$. The Shapiro–Francia extension W' provides a test for up to $n = 99$. Both these tests require two tables, i.e. one table for the coefficients and one table for the critical values. Moreover, the extension by Shapiro and Francia (1972) assumes a zero-correlation among the ordered observations, which is not true. The proposed modification of the W test statistic, i.e. $\tilde{W}$, will extend the test for all sample sizes, and the computation of $\tilde{W}$ will be much simpler than computing Royston's (1982a) approximation.

2 The Shapiro–Wilk W test and its extensions

Let $X_1 \leqslant X_2 \leqslant \ldots \leqslant X_n$ denote an ordered random sample of size n from a standard normal distribution. Also, let $\mathbf{m}' = (m_1, m_2, \ldots, m_n)$ be the vector of expected values of standard normal order statistics, and let $\mathbf{V} = (v_{ij})$ be the corresponding $n \times n$ covariance matrix, so that

$$E(X_i) = m_i \quad \text{and} \quad \operatorname{cov}(X_i, X_j) = v_{ij}, \qquad i, j = 1, 2, \ldots, n$$

Let $\mathbf{Y}' = (Y_1, Y_2, \ldots, Y_n)$ denote a vector of ordered random sample from an arbitrary population. If the $\{Y_i\}$ terms are ordered observations in a sample from a normal distribution with mean μ and variance σ^2, then Y_i may be expressed as

$$Y_i = \mu + \sigma X_i, \qquad i = 1, 2, \ldots, n$$

The best linear unbiased estimates of μ and σ for a symmetric distribution, which follow from the generalized least-squares theorem (Aitken, 1935), are given by

$$\hat{\mu} = \frac{1}{n} \sum_{i=1}^{n} Y_i = \bar{Y}, \qquad \hat{\sigma} = \frac{\mathbf{m} \mathbf{V}^{-1} \mathbf{Y}}{\mathbf{m} \mathbf{V}^{-1} \mathbf{m}}$$

The W test statistic (Shapiro & Wilk, 1965) for normality is then defined by

$$W = \frac{\left(\sum_{i=1}^{n} a_i Y_i \right)}{\sum_{i=1}^{n} (Y_i \bar{Y})^2} = \frac{(\mathbf{a}\,\mathbf{Y})}{S^2} \tag{1}$$

where

$$\mathbf{a}' = (a_1, a_2, \ldots, a_n) = \mathbf{m} \mathbf{V}^{-1} (\mathbf{m} \mathbf{V}^{-1} \mathbf{V}^{-1} \mathbf{m})^{-1/2} \tag{2}$$

and

$$S^2 = \sum_{i=1}^{n} (Y_i - \bar{Y})^2 \tag{3}$$

is an unbiased estimate of $(n - 1)\sigma^2$.

To calculate W, the vector $\mathbf{a}$ is needed, which in turn requires the values of the vector $\mathbf{m}$ and $\mathbf{V}^{-1}$ to be known. The exact values of $\mathbf{m}$ are given for sample sizes $n = 2(1)100(25)300(50)400$ by Harter (1961), but the exact values of $\mathbf{V}$ were available before 1965 only for sample sizes up to $n = 20$ (see Sarhan & Greenberg, 1962). Shapiro and Wilk (1965) used the following approximation for $n = 21(1)50$:

$$\hat{a}_1^2 = \hat{a}_n^2 = \Gamma\left[\frac{1}{2}(n+1)\right] \Big/ \left[2^{1/2}\Gamma\left(\frac{n}{2}+1\right)\right] \tag{4}$$

and

$$\hat{a}_i^\star = 2m_i, \qquad i = 2, 3, \ldots, (n-1) \tag{5}$$

where

$$\mathbf{a}^{\star\prime} = \mathbf{m}'\mathbf{V}^{-1}$$

The coefficient vector $\mathbf{a}$ was computed using the exact values of $\mathbf{m}$ and $\mathbf{V}$ for n up to 20, and with the approximations of equations (4) and (5) for $n = 21(1)50$.

Shapiro and Francia (1972) suggested replacing the covariance matrix $\mathbf{V}$ by the identity matrix $\mathbf{I}$, because for large samples, the observations $\{Y_i\}$ may be treated as if they are independent (see Gupta, 1952). With this approach, the vector $\mathbf{a}$ in equation (2) can be replaced by

$$\mathbf{b}' = \frac{\mathbf{m}'}{(\mathbf{m}'\mathbf{m})^{1/2}} \tag{6}$$

and the W statistic of equation (2) can be replaced by

$$W' = \frac{\left(\sum_{i=1}^{n} b_i Y_i\right)^2}{\sum_{i=1}^{n}(Y_i - \bar{Y})^2} \tag{7}$$

Recall that $\mathbf{m}$ is the vector of the expected values of the n ordered statistics from the standard normal distribution.

Shapiro and Francia (1972) provided critical values of $\mathbf{W}'$ for $n = 35, 50, 51(2)99$. These critical values were calculated using a simulation study based on 1000 samples. Pearson *et al.* (1977) re-evaluated the percentage points of $\mathbf{W}'$ for $n = 99, 100$ and 125, based on 50 000 simulations and found that Shapiro–Francia values in the lower tail were higher than they should be. Hence, the actual levels of significance are greater than those indicated by the Shapiro–Francia table.

Another asymptotic extension was suggested by Weisberg and Bingham (1975), who replaced the vector $\mathbf{b}'$ in equation (6) with

$$\tilde{\mathbf{b}}' = \frac{\tilde{\mathbf{m}}'}{(\tilde{\mathbf{m}}'\tilde{\mathbf{m}})^{1/2}} \tag{8}$$

where the elements of the vector $\tilde{\mathbf{m}}$ are

$$\tilde{m} = \Phi^{-1}\left(\frac{i-\frac{3}{8}}{n+\frac{1}{4}}\right) \qquad i = 1, 2, \ldots, n \tag{9}$$

and Φ^{-1} denotes the inverse of the standard normal cumulative distribution function. This approximation ($\tilde{m}_i$) to m_i was suggested by Blom (1958). Then, the resulting statistic W'' is given by

$$W'' = \frac{\left(\sum_{i=1}^{n} \tilde{b}_i Y_i\right)^2}{\sum_{i=1}^{n} (Y_i - \bar{Y})^2} \tag{10}$$

Weisberg and Bingham (1975) made a comparison for $n = 5, 20, 35$ of the null distribution of W'' with that of W', and found that both are very close. They also suggested using W'' with the critical values of W'. The only advantage of this procedure is that it removes the need to have tables of weights for computation of the test statistics.

3 Proposed modification of the W test statistic

To compute W as defined in equation (1), the vector **a** is needed, where **a** is defined as in equation (2). The elements of the vector **a** cannot be computed unless we know both the vector of means **m** and the covariance matrix **V**. Because the exact values of the elements of **m** and **V** are only known for certain sample sizes, it is difficult to compute W for arbitrary sample sizes. This study proposes a modification of the Shapiro–Wilk W test for normality, which can be applied for any sample size.

The proposed modification uses the approximation for the expected values of order statistics given by Blom (1958) and the approximations for the elements of the variance–covariance matrix given by Blom (1958) and Mosteller (1946). These approximations are

$$E(X_i) = m_i \approx F^{-1}(p_i), \qquad i = 1, 2, \ldots, n \tag{11}$$

and

$$\operatorname{cov}(X_i, X_j) = v_{ij} \approx \frac{p_i p_j}{(n+2) f(m_i) f(m_j)}, \qquad i, j = 1, 2, \ldots, n \tag{12}$$

where

$$p_i = \frac{i}{n+1} \tag{13}$$

Under the null hypothesis, equations (11) and (12) become

$$m_i \approx \Phi^{-1}(p_i) \tag{14}$$

and

$$v_{ij} \approx \frac{p_i p_j}{(n+2)\phi(m_i)(m_j)} \tag{15}$$

where Φ^{-1} is the inverse of the standard normal cumulative distribution function and ϕ is the normal density function.

It is more suitable to use equation (14) than equation (9), the simple expression of equation (14) provides a better overall approximation to the expected values of normal order statistics. The approximation of equation (9) works better for small sample sizes in the normal case (Blom, 1958).

We know (see Hammersley & Morton, 1954; Plackett, 1958) that

$$[(n+2)\mathbf{V}]^{-1} = ((v^{ij}))$$

where

$$\begin{aligned} v^{ii} &= f^2(m_i)\,[(p_{i+1} - p_i)^{-1} + (p_i - p_{i-1})^{-1} \\ &= 2(n+1)f^2(m_i) \\ v^{i,i+1} = v^{i+1,i} &= -f(m_i)f(m_{i+1})(p_{i+1} - p_i)^{-1} \\ &= -(n+1)f(m_i)f(m_{i+1}) \end{aligned}$$

and

$$v^{i,j} = v^{j,i} = 0, \qquad \text{for } j = i+2, i+3, \ldots, n, i = 1, 2, \ldots, n$$

Now, when the null hypothesis is true, we have

$$\mathbf{V}^{-1} = (n+1)(n+2)$$
$$\times \begin{pmatrix} 2\phi^2(m_1) & -\phi(m_1)\phi(m_2) & 0 & 0 & \ldots & 0 \\ -\phi(m_1)\phi(m_2) & 2\phi^2(m_2) & -\phi(m_2)\phi(m_3) & 0 & \ldots & 0 \\ 0 & -\phi(m_2)\phi(m_3) & 2\phi^2(m_3) & -\phi(m_3)\phi(m_4) & \ldots & 0 \\ \vdots & & & & & \\ 0 & 0 & 0 & 0 & \ldots & 2\phi^2(m_n) \end{pmatrix} \tag{16}$$

Equations (14) and (16) provide us with a uniform computing technique for the vector **a** for all sample sizes. Because the computation of the value of the test statistic W depends only on the vector **a**, the approximation suggested above for $\mathbf{m}'$ and $\mathbf{V}^{-1}$ will let us derive the proposed test statistic $\tilde{W}$ as an approximation to the Shapiro–Wilk W test statistic for normality. The proposed $\tilde{W}$ statistic will provide a GOF test for normality for all sample sizes, without the assumption of zero-correlation among the ordered observations assumed in Shapiro and Francia (1972).

4 Computation of the coefficient vector ã

As we know, the value of the test statistic W depends on the elements of **a**, which in turn requires the values of the vector **m** of the expected values of the order statistics and the values of the inverse of the variance–covariance matrix $\mathbf{V}^{-1}$ to be known. The exact values of the elements of the variance–covariance matrix are available for sample sizes up to $n = 50$ (see Tietjen *et al.*, 1977). Hence, for n greater than 50, we have to apply some kind of an approximation to the elements of **a**. The proposed approximation (see equations (14) and (16)) will provide a computational technique for evaluating numerically the values of the elements of **ã** for every sample size.

Equation (2) can be written as

$$\mathbf{a}' = \frac{\mathbf{m}'\mathbf{V}^{-1}}{C} = \frac{\mathbf{a}^{\star\prime}}{(\mathbf{a}^{\star\prime}\mathbf{a}^{\star})^{1/2}} \tag{17}$$

where $C = (\mathbf{m}'\mathbf{V}^{-1}\mathbf{m})^{1/2}$ and $\mathbf{a}^{\star\prime} = \mathbf{m}'\mathbf{V}^{-1}$. Equation (17) shows that **a** depends on the values of $\mathbf{a}^{\star\prime} = \mathbf{m}\,\mathbf{V}^{-1}$. The elements of the vector $\mathbf{a}^\star$ can be written as

$$\mathbf{a}_i^\star = \sum_{j=1}^{n} m_j v^{ij}, \qquad i = 1, 2, \ldots, n \tag{18}$$

Using equation (16) in equation (18), we obtain

$$\tilde{a}_i^\star = -(n+1)(n+2)\phi(m_i)[m_{i-1}\phi(m_{i-1}) - 2m_i\phi(m_i) + m_{i+1}\phi(m_{i+1})], \tag{19}$$
$$i = 1, 2, \ldots, n$$

where we assume that

$$m_0\phi(m_0) = m_{n+1}\phi(m_{n+1}) = 0 \tag{20}$$

Equation (17) becomes

$$\tilde{\mathbf{a}}' = \frac{\mathbf{m}\,\mathbf{V}^{-1}}{C} = \frac{\tilde{\mathbf{a}}^{\star\prime}}{C} \tag{21}$$

Then

$$\tilde{W} = (\tilde{\mathbf{a}}'Y)^2/S^2 \tag{22}$$

The values of m_i and $\phi(m_i)$ are available for all i, so equation (19) is computable. Most scientific calculators or any computer package can provide the values of m_i and $\phi(m_i)$.

Table 1 compares the values of $\tilde{a}_i^\star$ for sample sizes $n = 10, 20, 30$ with the exact values of $\mathbf{m}\,\mathbf{V}^{-1}$. The values of $\tilde{a}_i^\star$ suggest that, for n greater than 15, the approximations $\tilde{a}_i^\star$ are within 2% of the corresponding exact values, and most of the errors are less than 1%—except for $\tilde{a}_1^\star$. The difference between the exact and approximate values decreases significantly as n increases for $i > 1$. These results encourage the use of the proposed approximation. The approximation of equation (19) leads $\tilde{a}_1^\star$ to have a discrepancy of close to 20% from the exact values for all sample sizes. However, this large difference in $\tilde{a}_1^\star$ will not greatly affect the computation of the value of the test statistic, because the proposed test statistic $\tilde{W}$ will depend on $\tilde{\mathbf{a}}$. However, the Shapiro–Wilk W statistic depends on $\mathbf{a}$. Moreover, keeping $\tilde{a}_1^\star$ as it is makes the computation of equation (19) easier and consistent for all sample sizes. Rahman (1992) provides the coefficients $\{\tilde{a}_{n-i+1}\}$ for $\tilde{W}$ for $n = 2(1)100$. For the sake of brevity, they are not reproduced here.

5 The empirical distribution of $\tilde{W}$

The first and second moments of normal order statistics are known (see Shapiro & Wilk, 1965, Lemma 4); hence, using these, we can evaluate only the half and first moments of W (or $\tilde{W}$). In this circumstance, it is appropriate to employ an empirical sampling study to obtain an approximation for the distribution of $\tilde{W}$ under the null hypothesis.

To find an approximation to the null distribution of $\tilde{W}$, normal random samples were generated using SAS (1990). With 20 000 simulations, the different values of $\tilde{W}$ were computed for $n = 3(1)100(10)200(50)1000(500)5000$, and the empirical percentage points were determined for each value of n. The 1%, 2%, 5%, 10%, 50%, 90%, 95%, 98% and 99% points of the empirical distribution of $\tilde{W}$ are given in Table 2. Small values of $\tilde{W}$ indicate non-normality.

Using the same simulations, the sample mean and standard deviation, and the

TABLE 1. Comparison of the values of $\hat{\mathbf{a}}^{\star\prime} = \mathbf{m}\,\mathbf{V}^{-1}$

	$n = 10$			$n = 20$			$n = 30$		
i	Exact	Approx.	Error	Exact	Approx.	Error	Exact	Approx.	Error
1	− 3.5492	− 4.2545	− 0.198	− 4.2009	− 5.0087	− 0.192	− 4.5472	− 5.4314	− 0.194
2	− 2.0335	− 2.1060	− 0.035	− 2.8495	− 2.8932	− 0.015	− 3.2655	− 3.2983	− 0.010
3	− 1.3225	− 1.3600	− 0.028	− 2.2778	− 2.2915	− 0.006	− 2.7448	− 2.7455	− 0.000
4	− 0.7546	− 0.7772	− 0.029	− 1.8487	− 1.8624	− 0.007	− 2.3663	− 2.3657	+ 0.000
5	− 0.2605	− 0.2536	+ 0.026	− 1.4970	− 1.5078	− 0.007	− 2.0581	− 2.0601	− 0.000
6	—	—	—	− 1.1818	− 1.1949	− 0.011	− 1.7924	− 1.7970	− 0.002
7	—	—	—	− 0.9007	− 0.9082	− 0.008	− 1.5574	− 1.5617	− 0.002
8	—	—	—	− 0.6333	− 0.6383	− 0.007	− 1.3424	− 1.3460	− 0.002
9	—	—	—	− 0.3724	− 0.3791	− 0.017	− 1.1333	− 1.1444	− 0.009
10	—	—	—	− 0.1208	− 0.1257	− 0.040	− 0.9505	− 0.9535	− 0.003
11	—	—	—	—	—	—	− 0.7675	− 0.7707	− 0.004
12	—	—	—	—	—	—	− 0.5938	− 0.5938	+ 0.000
13	—	—	—	—	—	—	− 0.4062	− 0.4212	− 0.036
14	—	—	—	—	—	—	− 0.2740	− 0.2516	+ 0.081
15	—	—	—	—	—	—	− 0.0804	− 0.0837	− 0.041

TABLE 2. Empirical percentage points of $\tilde{W}$

	Level								
n	0.01	0.02	0.05	0.10	0.50	0.90	0.95	0.98	0.99
3	0.754	0.758	0.771	0.793	0.933	0.997	0.999	0.999	1.000
4	0.703	0.722	0.760	0.795	0.911	0.983	0.992	0.996	0.998
5	0.702	0.728	0.770	0.803	0.906	0.975	0.984	0.991	0.994
6	0.722	0.748	0.784	0.816	0.909	0.971	0.980	0.988	0.991
7	0.737	0.763	0.799	0.829	0.911	0.968	0.978	0.985	0.989
8	0.755	0.775	0.809	0.836	0.915	0.967	0.976	0.984	0.987
9	0.768	0.790	0.819	0.843	0.917	0.967	0.975	0.983	0.987
10	0.779	0.800	0.828	0.852	0.920	0.967	0.975	0.982	0.986
11	0.794	0.814	0.839	0.859	0.923	0.966	0.975	0.982	0.986
12	0.805	0.823	0.846	0.866	0.926	0.967	0.975	0.982	0.986
13	0.814	0.830	0.852	0.871	0.928	0.968	0.975	0.982	0.986
14	0.817	0.834	0.856	0.875	0.930	0.968	0.975	0.982	0.985
15	0.827	0.842	0.863	0.881	0.932	0.968	0.975	0.982	0.985
16	0.834	0.848	0.868	0.886	0.935	0.969	0.976	0.982	0.985
17	0.839	0.853	0.872	0.889	0.936	0.969	0.976	0.982	0.985
18	0.842	0.857	0.878	0.893	0.938	0.969	0.976	0.982	0.985
19	0.848	0.862	0.881	0.896	0.940	0.970	0.977	0.982	0.985
20	0.853	0.867	0.884	0.899	0.941	0.971	0.977	0.983	0.986
21	0.857	0.870	0.888	0.902	0.943	0.971	0.977	0.983	0.986
22	0.863	0.875	0.891	0.904	0.944	0.971	0.977	0.983	0.986
23	0.865	0.877	0.893	0.907	0.945	0.972	0.978	0.983	0.986
24	0.869	0.880	0.895	0.909	0.946	0.972	0.978	0.983	0.986
25	0.873	0.884	0.899	0.911	0.947	0.973	0.978	0.983	0.986
26	0.875	0.887	0.902	0.914	0.949	0.974	0.979	0.984	0.986
27	0.880	0.890	0.904	0.916	0.950	0.974	0.979	0.984	0.986
28	0.883	0.892	0.906	0.917	0.950	0.974	0.979	0.984	0.986
29	0.885	0.896	0.909	0.920	0.952	0.974	0.980	0.984	0.987
30	0.887	0.897	0.911	0.921	0.952	0.975	0.980	0.984	0.987
31	0.890	0.900	0.913	0.923	0.953	0.975	0.980	0.984	0.987
32	0.891	0.901	0.913	0.924	0.954	0.976	0.980	0.985	0.987
33	0.894	0.904	0.916	0.926	0.955	0.976	0.980	0.985	0.987
34	0.897	0.906	0.917	0.927	0.956	0.976	0.981	0.985	0.987
35	0.899	0.907	0.919	0.928	0.956	0.976	0.981	0.985	0.988
36	0.901	0.909	0.921	0.930	0.957	0.977	0.981	0.985	0.988

TABLE 2.—(*Continued*)

	Level								
n	0.01	0.02	0.05	0.10	0.50	0.90	0.95	0.98	0.99
37	0.902	0.910	0.922	0.931	0.958	0.977	0.981	0.985	0.988
38	0.905	0.913	0.924	0.933	0.958	0.977	0.982	0.986	0.988
39	0.906	0.913	0.925	0.934	0.959	0.977	0.982	0.986	0.988
40	0.908	0.916	0.925	0.934	0.960	0.978	0.982	0.986	0.988
41	0.909	0.916	0.927	0.935	0.960	0.978	0.982	0.986	0.988
42	0.912	0.918	0.928	0.936	0.961	0.978	0.982	0.986	0.988
43	0.913	0.920	0.930	0.938	0.961	0.979	0.982	0.986	0.988
44	0.914	0.921	0.931	0.939	0.962	0.979	0.982	0.986	0.988
45	0.915	0.923	0.932	0.939	0.962	0.979	0.983	0.986	0.988
46	0.917	0.923	0.933	0.940	0.963	0.979	0.983	0.987	0.988
47	0.918	0.924	0.934	0.942	0.963	0.979	0.983	0.987	0.989
48	0.919	0.926	0.934	0.942	0.964	0.980	0.983	0.987	0.989
49	0.921	0.927	0.936	0.943	0.964	0.980	0.983	0.987	0.989
50	0.921	0.928	0.937	0.944	0.965	0.980	0.984	0.987	0.989
51	0.922	0.928	0.937	0.944	0.965	0.980	0.984	0.987	0.989
52	0.923	0.930	0.938	0.945	0.966	0.981	0.984	0.987	0.989
53	0.925	0.930	0.939	0.946	0.966	0.981	0.984	0.987	0.989
54	0.925	0.932	0.940	0.947	0.966	0.981	0.984	0.987	0.989
55	0.927	0.933	0.941	0.947	0.967	0.981	0.984	0.988	0.989
56	0.928	0.934	0.942	0.948	0.967	0.981	0.985	0.988	0.989
57	0.928	0.934	0.942	0.949	0.967	0.982	0.985	0.988	0.990
58	0.929	0.935	0.942	0.949	0.968	0.982	0.985	0.988	0.990
59	0.930	0.936	0.943	0.950	0.968	0.982	0.985	0.988	0.990
60	0.931	0.936	0.944	0.950	0.968	0.982	0.985	0.988	0.990
61	0.932	0.937	0.945	0.951	0.969	0.982	0.985	0.988	0.990
62	0.933	0.938	0.946	0.952	0.969	0.982	0.985	0.988	0.990
63	0.934	0.939	0.946	0.952	0.969	0.982	0.985	0.988	0.990
64	0.934	0.939	0.946	0.952	0.970	0.983	0.986	0.989	0.990
65	0.935	0.941	0.947	0.953	0.970	0.983	0.986	0.989	0.990
66	0.936	0.941	0.947	0.953	0.970	0.983	0.986	0.989	0.990
67	0.937	0.941	0.948	0.954	0.971	0.983	0.986	0.989	0.990
68	0.937	0.942	0.949	0.954	0.971	0.983	0.986	0.989	0.990
69	0.937	0.942	0.949	0.955	0.971	0.983	0.986	0.989	0.990
70	0.939	0.944	0.950	0.955	0.971	0.983	0.986	0.989	0.990

TABLE 2.—(*Continued*)

	Level								
n	0.01	0.02	0.05	0.10	0.50	0.90	0.95	0.98	0.99
71	0.940	0.944	0.950	0.956	0.972	0.984	0.986	0.989	0.991
72	0.939	0.944	0.951	0.956	0.972	0.984	0.986	0.989	0.991
73	0.940	0.945	0.951	0.956	0.972	0.984	0.986	0.989	0.991
74	0.940	0.945	0.952	0.957	0.972	0.984	0.987	0.989	0.991
75	0.940	0.945	0.952	0.957	0.973	0.984	0.987	0.989	0.991
16	0.942	0.946	0.953	0.958	0.973	0.984	0.987	0.989	0.991
77	0.942	0.948	0.953	0.958	0.973	0.984	0.987	0.990	0.991
78	0.943	0.947	0.954	0.959	0.973	0.984	0.987	0.990	0.991
79	0.944	0.948	0.954	0.959	0.974	0.984	0.987	0.990	0.991
80	0.944	0.948	0.954	0.959	0.974	0.985	0.987	0.990	0.991
81	0.945	0.949	0.955	0.960	0.974	0.985	0.987	0.990	0.991
82	0.945	0.950	0.955	0.960	0.974	0.985	0.987	0.990	0.991
83	0.946	0.950	0.956	0.960	0.974	0.985	0.987	0.990	0.991
84	0.946	0.950	0.956	0.961	0.975	0.985	0.987	0.990	0.991
85	0.947	0.951	0.957	0.961	0.975	0.985	0.988	0.990	0.991
86	0.947	0.952	0.957	0.962	0.975	0.985	0.988	0.990	0.991
87	0.947	0.952	0.957	0.962	0.975	0.985	0.988	0.990	0.991
88	0.948	0.953	0.958	0.962	0.975	0.985	0.988	0.990	0.991
89	0.948	0.952	0.958	0.962	0.976	0.986	0.988	0.990	0.992
90	0.948	0.953	0.958	0.963	0.976	0.986	0.988	0.990	0.992
91	0.949	0.954	0.959	0.963	0.976	0.986	0.988	0.990	0.992
92	0.950	0.953	0.959	0.963	0.976	0.986	0.988	0.990	0.992
93	0.951	0.954	0.959	0.963	0.976	0.986	0.988	0.990	0.992
94	0.951	0.955	0.960	0.964	0.976	0.986	0.988	0.991	0.992
95	0.951	0.955	0.960	0.964	0.977	0.986	0.988	0.990	0.992
96	0.951	0.955	0.960	0.965	0.977	0.986	0.988	0.991	0.992
97	0.951	0.955	0.961	0.965	0.977	0.986	0.988	0.991	0.992
98	0.952	0.956	0.961	0.965	0.977	0.986	0.988	0.991	0.992
99	0.952	0.956	0.961	0.965	0.977	0.986	0.989	0.991	0.992
100	0.953	0.956	0.961	0.965	0.977	0.987	0.989	0.991	0.992
110	0.957	0.960	0.964	0.968	0.979	0.987	0.989	0.991	0.992
120	0.959	0.962	0.966	0.970	0.980	0.988	0.990	0.992	0.993

TABLE 2.—(*Continued*)

	Level								
n	0.01	0.02	0.05	0.10	0.50	0.90	0.95	0.98	0.99
130	0.962	0.964	0.968	0.972	0.981	0.988	0.990	0.992	0.993
140	0.964	0.966	0.970	0.973	0.982	0.989	0.991	0.992	0.993
150	0.966	0.968	0.972	0.975	0.983	0.989	0.991	0.993	0.993
160	0.968	0.970	0.973	0.976	0.984	0.990	0.991	0.993	0.994
170	0.969	0.971	0.974	0.977	0.984	0.990	0.992	0.993	0.994
180	0.970	0.972	0.975	0.978	0.985	0.991	0.992	0.993	0.994
190	0.971	0.973	0.976	0.979	0.986	0.991	0.992	0.994	0.994
200	0.973	0.975	0.977	0.979	0.986	0.991	0.993	0.994	0.995
250	0.977	0.979	0.981	0.983	0.988	0.992	0.993	0.994	0.995
300	0.981	0.982	0.984	0.985	0.990	0.993	0.994	0.995	0.996
350	0.983	0.984	0.985	0.987	0.991	0.994	0.995	0.996	0.996
400	0.984	0.985	0.987	0.988	0.992	0.994	0.995	0.996	0.996
450	0.986	0.987	0.988	0.989	0.992	0.995	0.996	0.996	0.997
500	0.987	0.988	0.989	0.990	0.993	0.995	0.996	0.996	0.997
550	0.988	0.989	0.990	0.991	0.993	0.995	0.996	0.997	0.997
600	0.989	0.990	0.991	0.991	0.994	0.996	0.996	0.997	0.997
650	0.990	0.990	0.991	0.992	0.994	0.996	0.996	0.997	0.997
700	0.990	0.991	0.992	0.992	0.994	0.996	0.997	0.997	0.997
750	0.991	0.991	0.992	0.993	0.995	0.996	0.997	0.997	0.997
800	0.991	0.992	0.992	0.993	0.995	0.997	0.997	0.997	0.998
850	0.992	0.992	0.993	0.993	0.995	0.997	0.997	0.997	0.998
900	0.992	0.993	0.993	0.994	0.995	0.997	0.997	0.997	0.998
950	0.992	0.993	0.993	0.994	0.996	0.997	0.997	0.998	0.998
1000	0.993	0.993	0.994	0.994	0.996	0.997	0.997	0.998	0.998
1500	0.995	0.995	0.995	0.996	0.997	0.998	0.998	0.998	0.998
2000	0.996	0.996	0.996	0.997	0.997	0.998	0.998	0.998	0.999
2500	0.996	0.997	0.997	0.997	0.998	0.998	0.998	0.999	0.999
3000	0.997	0.997	0.997	0.997	0.998	0.998	0.999	0.999	0.999
3500	0.997	0.997	0.998	0.998	0.998	0.999	0.999	0.999	0.999
4000	0.998	0.998	0.998	0.998	0.998	0.999	0.999	0.999	0.999
4500	0.998	0.998	0.998	0.998	0.998	0.999	0.999	0.999	0.999
5000	0.998	0.998	0.998	0.998	0.999	0.999	0.999	0.999	0.999

TABLE 3. Some empirical moments of W

	$\hat{\mu}_1$			$(\hat{\mu}_2)^{1/2}$			$(\beta_1)^{1/2}$			β_2		
n	S–W	Royston	This study	S–W	Royston	This study	S–W	Royston	This study	S–W	Royston	This study
7	0.9120	0.9113	0.9037	0.0547	0.0542	0.0554	−1.32	−1.23	−0.90	6.41	5.44	4.03
8	0.9175	0.9174	0.9074	0.0497	0.0499	0.0522	−1.38	−1.27	−0.92	7.11	5.44	4.04
9	0.9215	0.9227	0.9100	0.0479	0.0466	0.0486	−1.60	−1.48	−0.89	8.45	6.49	4.02
10	0.9260	0.9277	0.9138	0.0444	0.0432	0.0458	−1.67	−1.47	−0.94	9.28	6.45	4.39
15	0.9422	0.9423	0.9281	0.0320	0.0323	0.0346	−1.89	−1.52	−0.89	16.74	6.47	4.39
20	0.9527	0.9523	0.9375	0.0255	0.0253	0.0288	−2.28	−1.55	−0.86	32.59	7.76	4.20
30	0.9626	0.9631	0.9500	0.0185	0.0183	0.0214	−2.73	−1.46	−0.83	71.77	6.73	4.30
40	0.9682	0.9684	0.9579	0.0151	0.0147	0.0173	−3.17	−1.34	−0.78	136.48	6.17	4.02
50	0.9714	0.9718	0.9635	0.0124	0.0126	0.0144	−3.32	−1.20	−0.72	212.43	6.05	3.76
60		0.9738	0.9674		0.0110	0.0126		−1.04	−0.72		5.08	3.73
70		0.9755	0.9706		0.0099	0.0109		−1.02	−0.69		5.64	3.74
80		0.9768	0.9731		0.0090	0.0100		−0.92	−0.75		5.02	3.99
90		0.9778	0.9752		0.0083	0.0092		−0.84	−0.67		4.17	3.69
100		0.9786	0.9769		0.0078	0.0084		−0.73	−0.66		3.86	3.77
150		0.9810	0.9828		0.0063	0.0060		−0.56	−0.63		3.69	3.65
200		0.9821	0.9861		0.0054	0.0046		−0.44	−0.61		3.60	3.81
300		0.9835	0.9898		0.0044	0.0033		−0.25	−0.53		3.18	3.42
400		0.9844	0.9918		0.0037	0.0025		−0.16	−0.54		3.17	3.63
500		0.9850	0.9932		0.0033	0.0020		−0.10	−0.50		3.00	3.49
750		0.9860	0.9950		0.0026	0.0014		−0.02	−0.45		3.06	3.41
1000		0.9866	0.9961		0.0022	0.0010		0.05	−0.48		3.16	3.52
1500		0.9876	0.9972		0.0018	0.0007		0.18	−0.50		3.09	3.66
2000		0.9884	0.9978		0.0015	0.0005		0.24	−0.43		3.16	3.54
2500			0.9982			0.0004			−0.38			3.39
3000			0.9984			0.0004			−0.41			3.43
3500			0.9986			0.0003			−0.38			3.36
4000			0.9988			0.0003			−0.33			3.22
4500			0.9989			0.0002			−0.37			3.53
5000			0.9990			0.0002			−0.35			3.45

Note: S–W, Shapiro–Wilk.

third and fourth standardized moments of $\tilde{W}$ were computed for $n = 3(1)50$. Table 3 provides a comparison of these moments of $\tilde{W}$ with those of Shapiro and Wilk and those of Royston. The results show that the mean and the standard deviation of the distribution of $\tilde{W}$ are close to those of W and those of the approximation given by Royston, and the differences are less than 1% for all sample sizes. These results indicate that $\tilde{W}$ can be considered to be equivalent to W. Table 3 also provides a comparison of third and fourth standardized moments of W and $\tilde{W}$. Here, the moments indicate disagreement in the skewness and peakness. The Shapiro–Wilk W statistic seems to be more skewed and steeper as the sample size n increases; for Royston's W statistic, the skewness and kurtosis both reduce in magnitude as n increases to about 750, and they then diverge. Royston (1982a) claimed that the values of $(\hat{\beta})^{1/2}$ and $\hat{\beta}_2$ given by Shapiro and Wilk are incorrect, because they are not consistant with the empirical cumulative distribution of W (see Royston, 1982a, Figure 1; Shapiro & Wilk, 1965, Figure 4). However, $\tilde{W}$ remains almost the same with respect to skewness and peakness for all sample sizes. The null distributions of W and $\tilde{W}$ are both skewed to the left. These findings confirm that the distribution of $\tilde{W}$ under the null hypothesis is consistent, i.e. the shape of the null distribution remains the same for all sample sizes.

6 Power considerations

Shapiro *et al.* (1968) evaluated the W statistic relative to other competitors for normality tests and found that, for many alternative distributions, the Shapiro–Wilk W test statistic is more powerful than are $b_1^{1/2}$, b_2, Kolmogorov–Smirnov, Cramér–von Mises, weighted Cramer–von Mises, Durbin, χ^2 and David's U tests. Pearson *et al.* (1977) studied the power comparisons of the tests for normality and concluded that the W test statistic is the most powerful for skewed alternatives and that it is also more powerful for symmetric distributions with non-normal kurtosis ($\beta_2 \neq 3$). Shapiro and Francia (1972) surmise that their W' test statistic is equivalent to the W test statistics in terms of power. The objective of this section is to compare the powers of $\tilde{W}$ with those of W and W'.

Empirical power studies have been carried out by generating 5000 samples of sizes 10, 20, 35, 50, 75 and 99. The alternative distributions used in this comparison are uniform, logistic, log-normal χ^2 with one, two, four and 10 degrees of freedom, and beta (2, 1). The empirical power is defined here by the proportion of times that the null hypothesis is rejected when the sample comes from the above alternative distributions. Recall here that the W test for normality is a one-tailed test and we reject the null hypothesis if the value of W statistic is less than the critical value.

Therefore, the values of the W statistic were computed using appropriate coefficients $(a_1, \ldots, a_n)$ (see equation (2)). We used linear coefficients from Table 5 of Shapiro and Wilk (1965) to compute W. The vector $\mathbf{b}$ (equation (6)) was computed using expected normal order statistics obtained from Harter (1961) to compute W'. $\tilde{W}$ values were computed using linear coefficients from Table 4.2 of Rahman (1992).

Table 4 shows the empirical powers of the tests for the selected alternative distributions when the level of significance is $\alpha = 0.01$, 0.05 and 0.10. The powers indicate that $\tilde{W}$ is as good or better than W or W' for all the selected sample sizes for the skewed alternatives. It seems that W' performs better than W or $\tilde{W}$ if the

TABLE 4. Empirical power of 1%, 5% and 10% tests of selected alternative distributions

Alternative	n= 10		n= 20		n= 35			n= 50			n= 75		n= 99	
Distributions	W	$\tilde{W}$	W	$\tilde{W}$	W	W'	$\tilde{W}$	W	W'	$\tilde{W}$	W'	$\tilde{W}$	W'	$\tilde{W}$
$\alpha = 0.01$														
Uniform	0.01	0.03	0.03	0.12	0.18	0.06	0.43	0.53	0.13	0.73	0.51	0.95	0.84	0.99
Logistic	0.02	0.02	0.05	0.02	0.05	0.11	0.03	0.06	0.11	0.04	0.17	0.03	0.23	0.04
Log-normal	0.39	0.42	0.83	0.83	0.98	0.98	0.98	0.99	0.99	0.99	1.00	1.00	1.00	1.00
(1)	0.47	0.53	0.92	0.93	0.99	0.99	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00
(2)	0.22	0.25	0.63	0.64	0.92	0.92	0.94	0.99	0.98	0.99	1.00	1.00	1.00	1.00
(4)	0.09	0.09	0.30	0.27	0.61	0.63	0.62	0.83	0.78	0.83	0.96	0.97	0.99	0.99
(10)	0.03	0.03	0.10	0.08	0.20	0.25	0.19	0.33	0.34	0.32	0.56	0.51	0.75	0.69
Cauchy	0.46	0.43	0.80	0.72	0.94	0.97	0.91	0.98	0.99	0.97	1.00	0.99	1.00	0.99
Laplace	0.01	0.03	0.05	0.14	0.23	0.10	0.46	0.59	0.19	0.76	0.60	0.96	0.89	0.99
Poisson (1)	0.36	0.43	0.89	0.94	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Beta (2, 1)	0.10	0.18	0.39	0.56	0.84	0.73	0.93	0.98	0.91	0.99	0.99	1.00	1.00	1.00
Bin. (4, 0.5)	0.15	0.19	0.40	0.46	0.92	0.84	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
$\alpha = 0.05$														
Uniform	0.07	0.15	0.21	0.37	0.55	0.31	0.75	0.86	0.46	0.94	0.87	0.99	0.97	1.00
Logistic	0.07	0.07	0.12	0.08	0.13	0.23	0.07	0.12	0.23	0.08	0.33	0.08	0.40	0.09
Log-normal	0.59	0.61	0.93	0.93	0.99	0.99	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00
(1)	0.72	0.75	0.98	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
(2)	0.43	0.46	0.83	0.83	0.98	0.98	0.98	1.00	0.99	1.00	1.00	1.00	1.00	1.00
(4)	0.22	0.23	0.52	0.49	0.82	0.82	0.81	0.94	0.92	0.94	0.99	0.99	1.00	0.99
(10)	0.10	0.11	0.24	0.21	0.40	0.44	0.37	0.57	0.54	0.55	0.77	0.73	0.89	0.86
Cauchy	0.59	0.53	0.87	0.79	0.97	0.98	0.94	0.99	0.99	0.98	1.00	0.99	1.00	1.00
Laplace	0.09	0.16	0.24	0.39	0.60	0.39	0.77	0.88	0.55	0.94	0.90	0.99	0.98	1.00
Poisson (1)	0.75	0.75	0.99	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Beta (2, 1)	0.32	0.43	0.73	0.82	0.97	0.92	0.99	0.99	0.98	0.99	1.00	1.00	1.00	1.00
Bin. (4, 0.5)	0.47	0.46	0.72	0.66	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

TABLE 4.—*(Continued)*

Alternative	$n=10$		$n=20$		$n=35$			$n=50$			$n=75$		$n=99$	
Distributions	W	$\tilde{W}$	W	$\tilde{W}$	W	W'	$\tilde{W}$	W	W'	$\tilde{W}$	W'	$\tilde{W}$	W'	$\tilde{W}$
$\alpha=0.10$														
Uniform	0.16	0.27	0.36	0.54	0.74	0.48	0.86	0.95	0.71	0.97	0.93	0.99	0.99	1.00
Logistic	0.13	0.12	0.18	0.13	0.19	0.30	0.12	0.19	0.35	0.13	0.40	0.13	0.50	0.14
Log-norma1	0.69	0.71	0.96	0.95	0.99	0.99	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00
(1)	0.81	0.84	0.99	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
(2)	0.56	0.60	0.90	0.90	0.99	0.99	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00
(4)	0.32	0.35	0.64	0.62	0.89	0.88	0.88	0.97	0.96	0.97	0.99	0.99	1.00	1.00
(10)	0.18	0.20	0.34	0.30	0.53	0.54	0.49	0.69	0.67	0.66	0.83	0.82	0.93	0.92
Cauchy	0.65	0.59	0.90	0.82	0.98	0.99	0.95	0.99	0.99	0.98	1.00	0.99	1.00	1.00
Laplace	0.18	0.29	0.40	0.56	0.77	0.56	0.87	0.95	0.76	0.98	0.95	0.99	0.99	1.00
Poisson (1)	0.83	0.84	1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Beta (2, 1)	0.47	0.59	0.84	0.91	0.99	0.97	0.99	1.00	0.99	1.00	1.00	1.00	1.00	1.00
Bin. (4, 0.5)	0.56	0.57	0.91	0.80	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Note: W, Shapiro–Wilk test statistic; W', Shapiro–Francias test statistic; $\tilde{W}$, proposed modified test statistic.

alternative distribution is symmetric but leptokurtic ($\beta_2 > 3$), as in the case of the logistic alternative, where all the tests have low power.

Hence, we recommend that the modified version ($\tilde{W}$) of the W test statistic can be applied for sample sizes as small as 10. While making the comparison of powers via simulation, the effective levels of significance will not be the same as the nominal levels of significance. However, on the basis of empirical study for normal samples, we find that the effective levels and nominal levels of significance are quite similar. It should be noted that this is not relevant when comparing the performance of $\tilde{W}$ relative to W, because both tests will be operating at the same level of significance.

7 Computation of $\tilde{W}$ using SAS

The computation of the test statistic $\tilde{W}$ (or W) is not that simple, because it requires the use of a table of linear coefficients (see Rahman, 1992, Table 1) to compute the value of the numerator of equation (2). This becomes more difficult, especially for large samples. To ease the difficulty in computing for $\tilde{W}$, we provide a simple computer program written in SAS using the procedure IML. The SAS program given in the appendix computes the value of the test statistic $\tilde{W}$ for all sample sizes.

REFERENCES

AITKEN, A. C. (1935) On least squares and linear combination of observations, *Proceedings of the Royal Society, Edinburgh*, 55, pp. 42–48.

BLOM, G. (1958) *Statistical Estimates and Transformed Beta-Variables* (New York, Wiley).

D'AGOSTINO, R. B. (1971) An omnibus test of normality for moderate and large sample sizes, *Biometrika*, 58, pp. 341–348.

GUPTA, A. K. (1952) Estimation of the mean and standard deviation of a normal population from a censored sample, *Biometrika*, 39, pp. 266–273.

HAMMERSLEY, J. M. & MORTON, K. W. (1954) The estimation of location and scale parameters from group data, *Biometrika*, 41, pp. 296–301.

HARTER, H. L. (1961) Expected values of normal order statistics, *Biometrika*, 48, pp. 151–165.

MOSTELLER, F. (1946) On some useful 'inefficient' statistics, *Annals of Mathematical Statistics*, 17, pp. 377–408.

PEARSON, E. S., D'AGOSTINO, R. B. & BOWMAN, K. O. (1977) Tests for departure from normality: comparison of powers, *Biometrika*, 64, pp. 231–246.

PLACKETT, R. L. (1958) Linear estimation from censored data, *Annals of Mathematical Statistics*, 29, pp. 131–142.

RAHMAN, M. M. (1992) Some contributions to goodness of fit tests and nonparametric estimation of variance components, *PhD Dissertation* (Lexington, KY, University of Kentucky).

ROYSTON, J. P. (1982a) An extension of Shapiro and Wilk's W test for normality to large samples, *Applied Statistics*, 31, pp. 115–124.

ROYSTON, J. P. (1982b) Algorithm As 177, expected normal order statistics (exact and approximate), *Applied Statistics*, 31, pp. 161–165.

ROYSTON, J. P. (1982c) Algorithm as 181, the W test for normality, *Applied Statistics*, 31, pp. 176–180.

SARHAN, A. E. & GREENBERG, B. G. (1962) *Contributions to Order Statistics* (New York, Wiley).

SAS (1990) *SAS Language: Reference Version 6*, 1st Edn (Cary, NC, SAS Institute Inc.).

SHAPIRO, S. S. & FRANCIA, R. S. (1972) An analysis of variance test for normality, *Journal of the American Statistical Association*, 67, pp. 215–216.

SHAPIRO, S. S. & WILK, M. B. (1965) An analysis of variance test for normality (complete sample), *Biometrika*, 52, pp. 591–611.

SHAPIRO, S. S., WILK, M. B. & CHEN, H. J. (1968) A comparative study of various tests for normality, *Journal of the American Statistical Association*, 63, pp. 1343–1372.

TIETJEN, G. L., KAHANER, D. K. & BECKMAN, R. J. (1977) Variance and covariances of the normal order statistics for sample sizes 2 to 50. *In: Selected Tables in Mathematical Statistics* (Providence, RI, American Mathematical Society).

WEISBURG, S. & BINHAM, C. (1975) An approximate analysis of variance test for non-normality suitable for machine computation, *Technometrics*, 17, pp. 133–134.

Appendix: Program for computing $\tilde{W}$

```
proc iml;
n= _ _ _ _ _;
/*Enter sample size*/
start one;
x= {_ _, _ _, _ _, _ _, _ _};
/*Enter the sample observations as a column vector*/
p= 1/(n+ 1);
do i= 2 to n by 1;
pi= i/(n+ 1);
p= p//pi;
end;
m= probit(p);
f= (1/sqrt(2*3.1415926))*(exp(- 0.5*m##2));
q= {0};
mf= m#f;
mf1= mf(|2:nrow(mf),|)//q;
mf2= q//mf(|1:nrow(mf) - 1,|);
mf3= f#(2*mf- mf1 - mf2);
a= mf3/sqrt(mf3`*mf3);
/*Computes the linear coefficients*/
finish;
run one;
y= x; y(|rank(y)|)= x; x= y;
/*y is the ordered observations*/
b= a`*y;
bsqr= b**2;
/*bsqr is the numerator of (2)*/
ymean= x(|:|);
ssqr= ((x- ymean)#(x- ymean))(|+ ,|);
/*ssqr is the denominator of (2)*/
w= bsqr/ssqr;
/*w is the value of the test statistic*/
print w;
```