Limita funkce

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Připomenutí pojmů

Demonstrujme různé případy funkcí a jejich limit. Nejprve ale připomeňme samotnou definici:

Nechť f je reálná funkce reálné proměnné definovaná na okolí bodu $a \in \overline{\mathbb{R}}$ s možnou vyjímkou bodu a samotného. Řekneme, že f má limitu v bodě a rovnu $c \in \overline{\mathbb{R}}$ právě když pro každé okolí H_c bodu c existuje okolí H_a bodu a takové, že pokud $x \in H_a \setminus a$ platí $f(x) \in H_c$.

Připomeňme také definici okolí bodů z rozšířené reálné osy $\bar{\mathbb{R}}$. Pokud $a \in \mathbb{R}$, pak velikost okolí je udána kladným reálným parametrem ϵ,

$$H_a(\epsilon) = (a - \epsilon, a + \epsilon).$$

Pokud $a = +\infty$, pak pro $\alpha \in \mathbb{R}$ klademe

$$H_{+\infty}(\alpha) = (\alpha, +\infty).$$

Podobně v případě $a = -\infty$ máme pro $\alpha \in \mathbb{R}$

$$H_{-\infty}(\alpha) = (-\infty, -\alpha).$$

$a, c, \in \mathbb{R}$

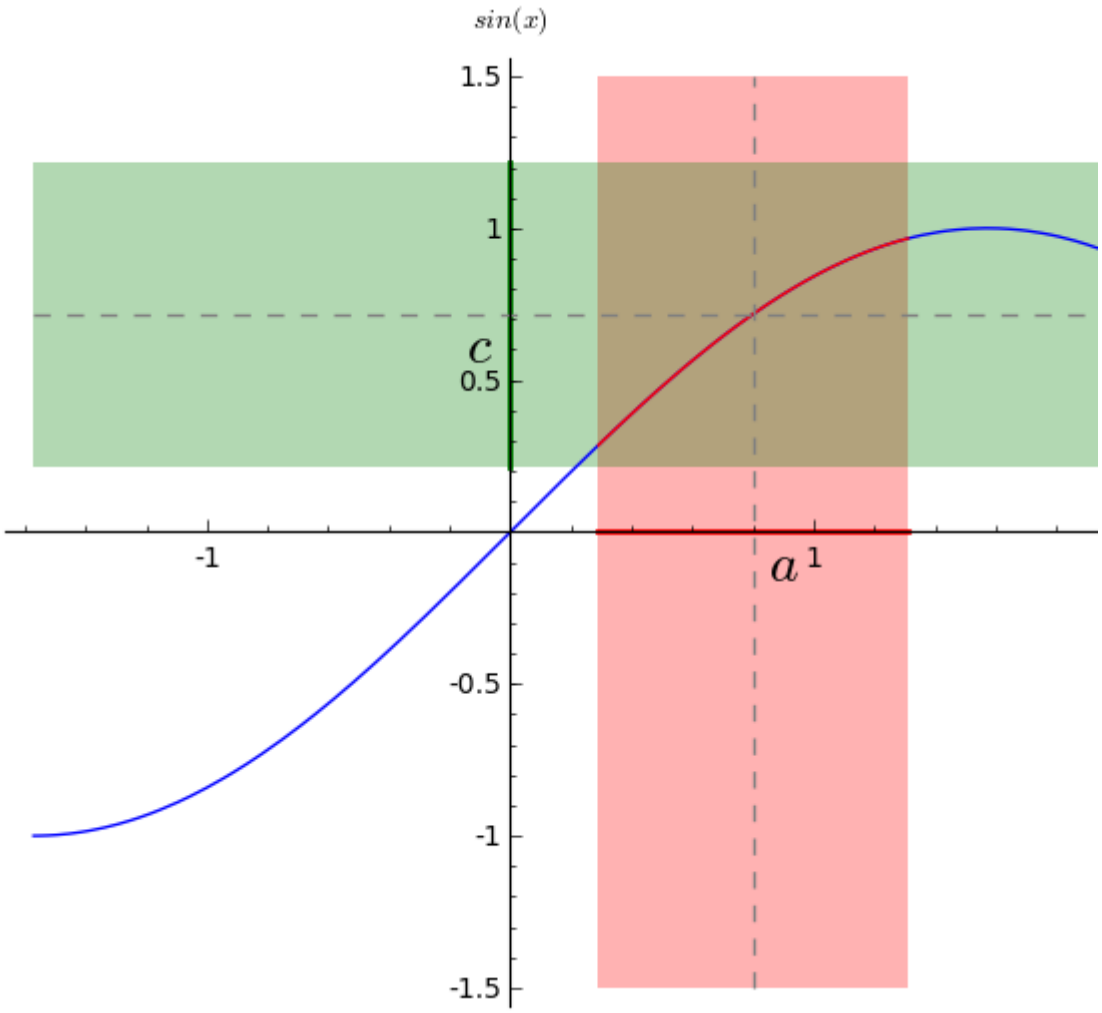
$\sin(x) \to \sin(1)$, když $x \to 1$.

```
a = .8
c = sin(a)
xm = -pi/2
xM = pi
ym = -1.5
yM = 1.5
graph_params = dict(xmin = xm,    xmax = xM,
   ymin = ym, ymax = yM,
```

```
   aspect_ratio=1,
   axes = True)
def sine(delta = 1, epsilon = .5, instant_show = True,
show_pi=True ):
    graph = Graphics()
    sine = plot( sin(x) , (x, -pi/2, pi), color="blue",
axes_labels=['$x$','$sin(x)$'])
    redsine = plot( sin(x) , (x, a - delta, a + delta),
color="red" )
    graph += sine + redsine
    graph += text( '$a$', (a+.1,-.1), color = "black"
,fontsize=20 )
    graph += text( '$c$', (-.1,c-.1), color = "black"
,fontsize=20 )
    graph += line( [(a - delta,0),(a + delta,0)],
linestyle='-', thickness = 2, color="red" ) #redline
    graph += line( [(0,c - epsilon),(0,c + epsilon)],
linestyle='-', thickness = 2, color="green" ) #green line
    graph += line( [(a,ym),(a,yM)], linestyle='--',
color="grey" )
    graph += line( [(xm,c),(xM,c)], linestyle='--',
color="grey" )
    graph += polygon([(a - delta,ym), (a - delta, yM), (a +
delta,yM),(a + delta, ym)], color = "red", alpha = .3)
    graph += polygon([(xm,c - epsilon), (xM,c - epsilon),
(xM,c + epsilon),(xm,c + epsilon)], color = "green", alpha =
.3)
    # LABEL on sine
    if instant_show:
        show (graph,  **graph_params)
    return graph
######################
# make Interaction
######################
@interact
def _( epsilon= slider(srange(.05,.501,.01),default = .5,
    label="$\epsilon$ :    ", display_value=True), delta=
slider(srange(.05,1.001,.01),default = 1.,
    label="$\delta$ :    ", display_value=True) ):
    sine(delta,epsilon)
```

ϵ : 0.500000000000000

δ : 1.00000000000000



Tuto limitu si můžeme nechat ověřit:

```
show(limit(sin(x),x=1))
show(n(sin(1)))
```

$$\sin(1)$$

$$0.841470984807897$$

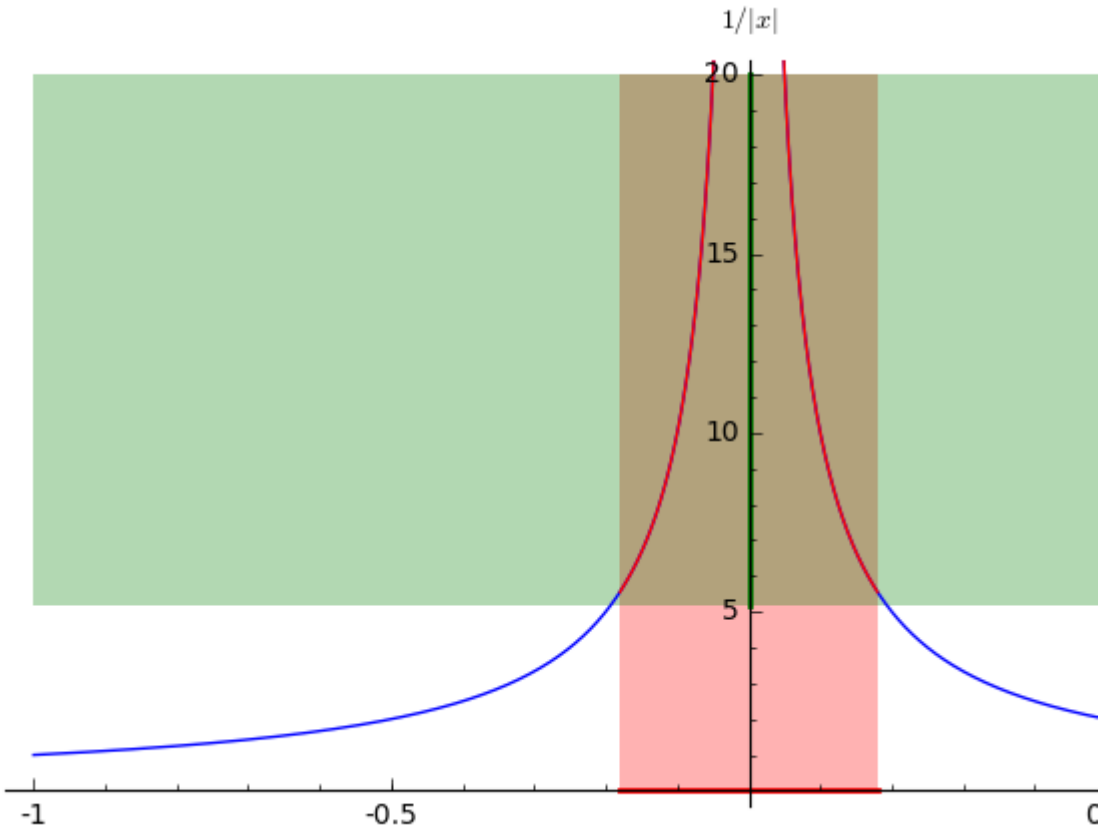
$$a \in \mathbb{R}, c = +\infty$$

$\frac{1}{|x|} \to +\infty$ když $x \to 0$.

```
a = 0
xm = -1
xM = 1
ym = 0
yM = 20
graph_params = dict(xmin = xm,    xmax = xM,
   ymin = ym, ymax = yM,
   aspect_ratio=.05,
   axes = True)
def sine(alpha = 5,delta = .8, instant_show = True,
show_pi=True ):
    graph = Graphics()
    f = plot( 1 / abs(x) , (x, xm, xM), color="blue",
axes_labels=['$x$','$1/|x|$'])
    redf = plot( 1/ abs(x) , (x, a - delta, a + delta),
color="red" )
    graph += f + redf
    graph += line( [(a - delta,0),(a + delta,0)],
linestyle='-', thickness = 2, color="red" ) #redline
    graph += line( [(0,alpha),(0,yM)], linestyle='-',
thickness = 2, color="green" ) #green line
    graph += polygon([(a - delta,ym), (a - delta, yM), (a +
delta,yM),(a + delta, ym)], color = "red", alpha = .3)
    graph += polygon([(xm,alpha), (xM,alpha), (xM,yM),
(xm,yM)], color = "green", alpha = .3)
    # LABEL on sine
    if instant_show:
        show (graph,  **graph_params)
    return graph
######################
# make Interaction
######################
@interact
def _( delta= slider(srange(.05,.801,.01),default = .8,
    label="$\delta$ :    ", display_value=True), alpha=
slider(srange(5,19.001,.01),default = 1.,
    label="$\\alpha$ :    ", display_value=True) ):
    sine(alpha,delta)
```

δ : 0.800000000000000

α : 5.00000000000000



Ověření:

```
show(limit(1/abs(x),x=0))
```

$$+\infty$$

Upozorňujeme, že sage počítá v komplexním oboru, takže z pohledu reálné analýzy vrací nesprávně tuto limitu:

```
show(limit(1/x,x=0))
```

$$\infty$$

Toto je pravda, pokud se na funkci 1/x díváme jakožto na funkci $\mathbb{C} \to \mathbb{C}$. V komplexní rovině je totiž okolí bodu nula dáno jako disk se středem v nule a poloměrem ϵ

$$H_0(\epsilon) = \{z \in \mathbb{C} : |z| < \epsilon\}$$

a okolí bodu ∞(v $\mathbb{C}$ nerozlišujeme $+\infty$ a $-\infty$, máme pouze jedno) je

doplňek disku o daném poloměru $c > 0$,

$H_\infty(c) = \{z \in \mathbb{C} : |z| > c\}$.

$a = +\infty, c \in \mathbb{R}$

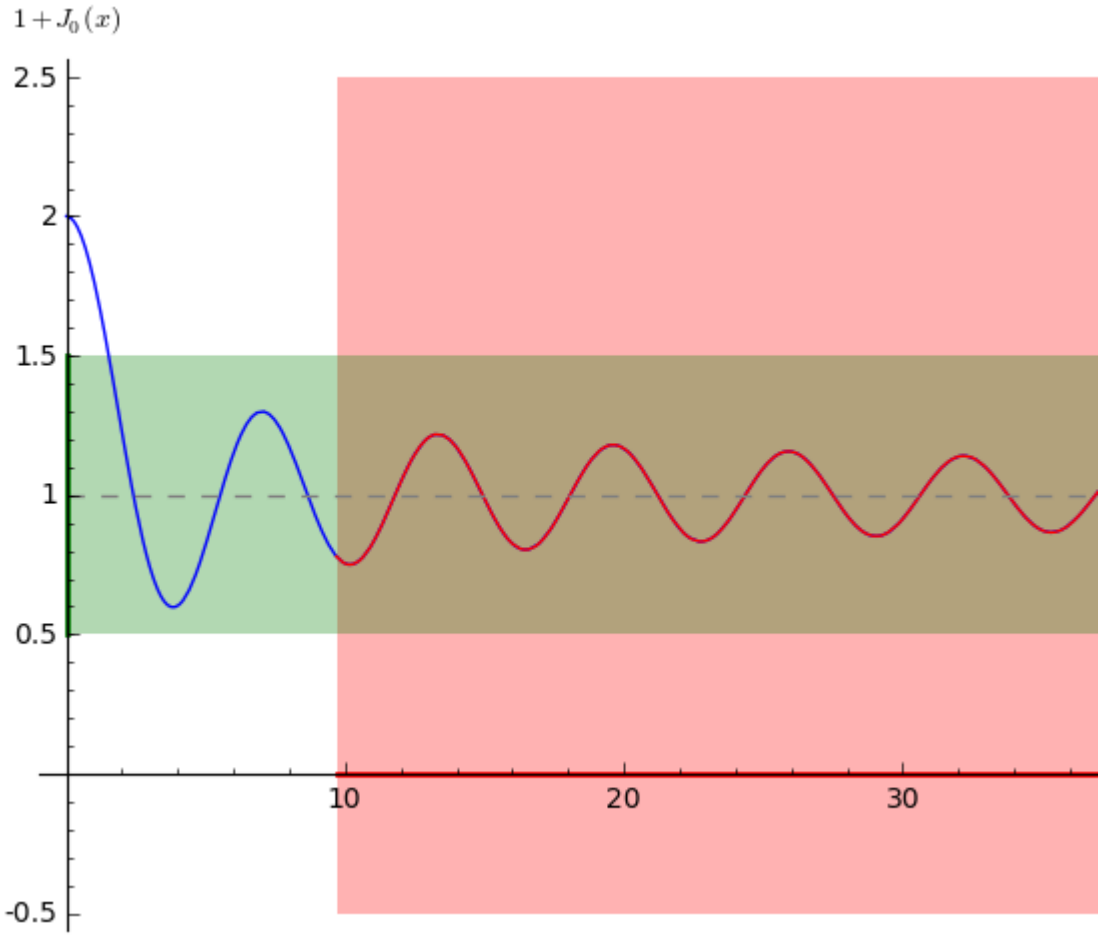
$1 + J_0(x) \to 1$, když $x \to +\infty$

```
a = 0
c = 1
xm = 0
xM = 50
ym = -.5
yM = 2.5
graph_params = dict(xmin = xm,    xmax = xM,
   ymin = ym, ymax = yM,
   aspect_ratio=10,
   axes = True)
def sine(alpha = 5,epsilon= .8, instant_show = True,
show_pi=True ):
    graph = Graphics()
    f = plot( bessel_J(0,x) + 1 , (x, xm, xM), color="blue",
axes_labels=['$x$','$1+J_0(x)$'])
    redf = plot( bessel_J(0,x) + 1  , (x, alpha, xM),
color="red" )
    graph += f + redf
    graph += line( [(xm,c),(xM,c)], linestyle='--',
color="grey" )
    graph += line( [(alpha,0),(xM,0)], linestyle='-',
thickness = 2, color="red" ) #redline
    graph += line( [(0,c - epsilon),(0,c + epsilon)],
linestyle='-', thickness = 2, color="green" ) #green line
    graph += polygon([(alpha,ym), (alpha, yM), (xM,yM),(xM,
ym)], color = "red", alpha = .3)
    graph += polygon([(xm,c - epsilon), (xM,c - epsilon),
(xM,c + epsilon),(xm,c + epsilon)], color = "green", alpha =
.3)
    # LABEL on sine
    if instant_show:
        show (graph,  **graph_params)
    return graph
######################
# make Interaction
######################
@interact
def _( alpha= slider(srange(.1,40.001,.01),default = .1,
```

```
    label="$\\alpha$ :    ", display_value=True), epsilon=
slider(srange(.05,.901,.01),default = .5,
    label="$\epsilon$ :    ", display_value=True) ):
    sine(alpha,epsilon)
```

α : 0.100000000000000

ϵ : 0.500000000000000



Ověření:

```
show(limit(1+bessel_J(0,x),x=infinity))
```

$$1$$