Lambda Conf Notes

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***Notes* :**

Over 400 attendees were at this LambaConf. Keynote by Ed Lattimore, who spoke on Fear.

1 Introduction

Category Theory by David Spivak

Set
How to compose Functions
Monad/ Bind Return

Branches of mathematics need to talk to each other

2 Section

Category of Sets
Category of Vect
Category of Hask
Category of Poset
Category of Topological Spaces
Set: bag full of dots
A $\rightarrow_f$ B

Identity function: C $\rightarrow$ C

3 Bijective

People $\rightarrow$ Place
People $\leftarrow$ Place

Initial Set: Set with no elements
The Empty set
For any A, there is a unique sect from
$\emptyset \rightarrow$ A

$\forall$ x : X $\exists!$ a: A such that f(x) = a

4 Terminal Set

From A to B by
$Hom_{set}(A, B)$ or Set(A, B)
Hom($\emptyset$, A) = 1

5 Universal Properties

Given Sets A and B, what is special about their product
A x B = [(a, b)— a ϵ A, b ϵ b]

6 Coproducts

For any A, B,
A + B is a set such that A+B $\rightarrow$ X

a = [1], b = [1, 2]
a + b
(a, 1), (b, 1), (b, 2)
Isomorphic: way of getting from one to the other

7 Exponentials

Think of Exponentials as Function Objects
(n, n, n) and (m, m)
How many different ways to map from one to the other?
Number of possibilities = 2^3
Hom(A,B) is a set denoted B^A Type A $\rightarrow$ B
Bijection is equivalence relation
Reflective, Transformative, Equivalence
Bijection = Isomorophic in set

Exponential satisfy universal property
For any A,B, C
Hom(A x B, C) $\equiv$ Hom(A, C^B)
currying
$C^{A\times B} \equiv (C^B)^A$

8 General

What is a category?
Def: A category C consists of:
(Ob(C))
Elements of a set are called Objects
For every c, d $\epsilon \leftarrow$ Ob(c), a set
$Hom_C(c,d)$ elements are called morphisms
f ϵ Hom(c, d) write f: c $\rightarrow$ d c $\rightarrow^f$ d
For every c ϵ Ob(c), a choice $id_c \epsilon$ $Hom_c(c,c)$
returns self
For every f $\leftarrow$ Hom(c,d), g ϵ Hom(d,e) an element f; g ϵ Hom(c, e)

satisfying
A. for any f: c $\rightarrow$ d
id_c ; f = f = f (identity)
Unital

B. Associative
Discrete Category
Satisfies all the rules
category each element must map to itself
Any set can be regarded as a discrete category
Any poset (partially ordered set) can be regarded as a category
eg. if I am level 5 clearance I also have clearance for 4, 3, 2 and 1
$\forall$ A ϵ P, a $\leq$ a (reflexive)

(so I have privileges less than or equal to myself)

A poset is a pre-order P $\leq$ such that a $\leq$ b, b $\leq$ a $\rightarrow$ a = b
A poset / preorder can be regarded as category:
it is one where every Hom_c(a, b) has at most one element
$a \rightarrow b$
$a \leq b$
Initial object: map from one thing to another
Everything maps to uniquely -¿ called Terminal object

9 Monoid

Any monoid can be regarded as a category
A monoid consists of a set M
an element e $\leftarrow$ M
a function * M $\times$ M $\rightarrow$ M
such that e * m = m * e for all m
(m * n) * p = m * n + p
eg. (List A, [ ], append)

Path through a graph are called free categories

10 Lunch