Lambda Conf Notes

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***Notes* :**

Over 400 attendees were at this LambaConf. Keynote by Ed Lattimore, who spoke on Fear.

1 Introduction

Category Theory by David Spivak

Set
How to compose Functions
Monad/ Bind Return

Branches of mathematics need to talk to each other

2 Section

Category of Sets
Category of Vect
Category of Hask
Category of Poset
Category of Topological Spaces
Set: bag full of dots
A $\rightarrow_f$ B

Identity function: C $\rightarrow$ C

3 Bijective

People $\rightarrow$ Place
People $\leftarrow$ Place

Initial Set: Set with no elements
The Empty set
For any A, there is a unique set from
$\emptyset \rightarrow$ A

$\forall$ x : X $\exists$! a: A such that f(x) = a

4 Terminal Set

From A to B by
$Hom_{set}(A, B)$ or Set(A, B)
Hom($\emptyset$, A) = 1

5 Universal Properties

Given Sets A and B, what is special about their product
A x B = [(a, b)— a ϵ A, b ϵ b]

6 Coproducts

For any A, B,
A + B is a set such that A+B $\rightarrow$ X

a = [1], b = [1, 2]
a + b
(a, 1), (b, 1), (b, 2)
Isomorphic: way of getting from one to the other

7 Exponentials

Think of Exponentials as Function Objects
(n, n, n) and (m, m)
How many different ways to map from one to the other?
Number of possibilities = 2^3
Hom(A,B) is a set denoted B^A Type A $\rightarrow$ B
Bijection is equivalence relation
Reflective, Transformative, Equivalence
Bijection = Isomorophic in set

Exponential satisfy universal property
For any A,B, C
Hom(A x B, C) $\equiv$ Hom(A, C^B)
currying
$C^{A \times B} \equiv (C^B)^A$

8 General

What is a category?
Def: A category C consists of:
(Ob(C))
Elements of a set are called Objects
For every c, d $\epsilon \leftarrow$ Ob(c), a set
$Hom_C(c, d)$ elements are called morphisms
f ϵ Hom(c, d) write f: c $\rightarrow$ d c $\rightarrow^f$ d
For every c ϵ Ob(c), a choice $id_c \epsilon$ $Hom_c(c, c)$
returns self
For every f $\leftarrow$ Hom(c,d), g ϵ Hom(d,e) an element f; g ϵ Hom(c, e)

satisfying
A. for any f: c $\rightarrow$ d
id_c ; f = f = f (identity)
Unital

B. Associative
Discrete Category
Satisfies all the rules
category each element must map to itself
Any set can be regarded as a discrete category
Any poset (partially ordered set) can be regarded as a category
eg. if I am level 5 clearance I also have clearance for 4, 3, 2 and 1
$\forall$ A ϵ P, a $\leq$ a (reflexive)

(so I have privileges less than or equal to myself)

A poset is a pre-order P $\leq$ such that a $\leq$ b, b $\leq$ a $\rightarrow$ a = b
A poset / preorder can be regarded as category:
it is one where every Hom_c(a, b) has at most one element
$a \rightarrow b$
$a \leq b$
Initial object: map from one thing to another
Everything maps to uniquely -¿ called Terminal object

9 Monoid

Any monoid can be regarded as a category
A monoid consists of a set M
an element e $\leftarrow$ M
a function * M $\times$ M $\rightarrow$ M
such that e * m = m * e for all m
(m * n) * p = m * n + p
eg. (List A, [ ], append)

Path through a graph are called free categories

10 Lunch

Lunch

11 Functors, Natural Transformations Adjunctions

Functor: Maps between categories
Def: Let C and D be categories
A functor F from C to D, write F : C $\rightarrow$ D
A function F : Ob(C) $\rightarrow$ Ob(D)
For every c_1, c_2 ϵ Ob(C) a function
F:$Hom_C(c_1, c_2) \rightarrow Hom_C(F(c_1), F(c_2)$
such that A. for any C ϵ Ob C, F(id_c) = id
B. For any $c_1 \rightarrow c_2 \rightarrow c_3$ F(f;g) = F(f) ; F(g)

monotone : preserves less than
A functor P $\rightarrow$ Q is a monotone map P $\rightarrow$ Q.
P $\leq$ P $\rightarrow$ f(p) $\leq$ f(p)

return : A $\rightarrow$ List A
Let M, N be monoids
A functor M $\rightarrow$ N
id $\rightarrow$ id
F($m_1 \times m_2$) = F(m_1) $\times F(m_2)$
Cat is a category
Ob(Cat) = all categories
Hom_{cat} (C, D) = F: C $\rightarrow$ D functors
A functor F: C $\rightarrow$ D is called faithful if $\forall$ c_1, c_2 if F($Hom_C(c_1, (c_2) \rightarrow F(c_1, c_2)$
is injective ie if f, g : $c_1 \rightarrow c_2$
are such that F(f) = F(g), then f = g

12 Natural Transformations

A natural transformation
Let C, D be cats
F,G:C $\rightarrow$ D be functors
A natural transformation p : F $\rightarrow$ G consists of
for each c $\exists$ Ob(C), morphism P_c : F(c) $\rightarrow$ G(c) in D
such that
for every f : $c_1 \rightarrow c_2$ in C
F(c_1) $\rightarrow_{F(f)} F(c)$
G(c_1) $\rightarrow_{G(f)} G(c_2)$

13 Graph Homomorphisms

Let G, H be graphs
A graph homomorphism
f: G $\rightarrow$ H is
a function G(v) $\rightarrow$ H(v)
a function G(A) $\rightarrow$ H(A)
such that
respecting source and target
database homomorphism
For every two categories C and D
there is a category Fun(C, D) whose objects are the Functors
F : C $\rightarrow$ D
$Hom_{(C,D)}$(F,G) = p: F $\rightarrow$ G
f: $c_1 \rightarrow c_2$ in C

Look at FQL (Topos)
Algebraic Databases 5/26 last session

14 Adjunctions

Adjoints(Adjunctions)
Let C, D be cats,
C $\rightarrow^{L}$ D and D $\rightarrow^{R}$ C functors
we say L is the left adjoint of R
R is right adjoint of L
if there is a natural isomorphism if for all c ϵ Ob(C), d ϵ Ob(D), there is an iso
Hom(L(c), d) $\equiv$ Hom (c, R(d))

Sat 11:30am Adjunctions

currying is an example from adjunctions

Fix a set A
Let Set $\rightarrow^{L_A}$ Set Set $\rightarrow^{R_A}$ Set
For any Set X ϵ Set
$L_A(X) := A \times X$ is a functor
$R_A(Y) := Y^A$
$R_A(L_A(X))$ = (X $\times A)^A$
$L_A(R_A(X)) = X^A \times A$
$Hom_{set}(L_A(X), Y) \equiv Hom_{set}(X, R_A(Y))$
functors X $\times$ A $\rightarrow$ Y $\equiv$ X $\rightarrow$ Y^A

Set $\rightarrow$ Monoid
Free Monoid
X $\rightarrow$ (List(X), [ ], append)
M $\leftarrow$ (M, e, *)
Hom_{Monoid}(Free(X), M) $\equiv$ $Hom_{set}(X, U(M))$
List(X) $\rightarrow$ M
f:X $\rightarrow$ U(M)
Triangle Laws

just: X $\rightarrow$ X + 1
Just

X + 1 $\rightarrow$ X
From Just
X $\rightarrow$ List(X) (unit)
Singleton

List(X), [ ], append) $\rightarrow$ M (counit)

Singleton $\rightarrow$ goes from type to list
Fold goes in other direction

15 Galois connections

If C, D are posets
Then a Galois connection is
a pair of adjoint functors
$\forall$ c ϵ C, d ϵ D
L(c) $\leq$ d $\iff$ c $\leq$ R(d)

P(A) $\rightarrow^(im(f))$ P(B)
P(B) $\leftarrow^{f^*}$ P(A)

16 Monads

Let C be a category.
A monad on C consists of:
1. A Functor M : C $\rightarrow$ C
2. A Natural transformation from $id_C \rightarrow M$
A natural transformation μ : M $\circ$ M $\rightarrow$ M

List: Set $\rightarrow$ Set
$Singleton_X X \rightarrow ListX$
flatten: List [List(X)] $\rightarrow$ List(X)

$[[a, b], [], [b, c]]$

C$\rightarrow_L$ D
D$\leftarrow_R$ C
μ : id_e $\rightarrow$ RL
ϵ : LR $\rightarrow$ id_D
Associated Monad on C:
(M = RL, μ, R ϵ L)
R τ RL $\rightarrow$ RL

Associated Co-monad
N : = (LR, ϵ, L μ R)

Suppose C is a cat and (M, ν, μ) is a monad.

17 Kleisli category

Ob(Kl(C)) = Ob(C)
$Hom_{Kl(C)}(c, d) = Hom_C(c, Md)$
$\eta :$ c $\rightarrow$ Mc
$c \rightarrow Md$
$d \rightarrow Me$
c $\rightarrow$ Md $\rightarrow$ MM_e $\rightarrow_\mu$ Me

18 Eilenberg Moore

Given C, M, η, μ
Ob: M-algebras
an object $c \epsilon C$
a morphism h:$M_c \rightarrow c$
such that get out of monad using join of monad

M : C $\rightarrow$ C (endofunctor)
an M-algebra is
a c ϵ Ob C
a morphism Mc $\rightarrow$ c
A morphism of M-algebras (c, h), ($c^{'}$, $h^{'}$)
consists of a morphism $c \rightarrow^f c^{'}$ such that
Mc $\rightarrow^h$ $c \rightarrow c^{'}$ $M^f \rightarrow Mc^{'} \rightarrow^{h^{'}} c^{'}$
A coalgebra is the choice of an X ϵ Ob(C) taking
h: X $\rightarrow$ MX

catamorphism

Stream(X)
Stream(A) $\rightarrow$ (A $\times$ Stream A) + 1
X $\rightarrow$ A $\times$ X + 1