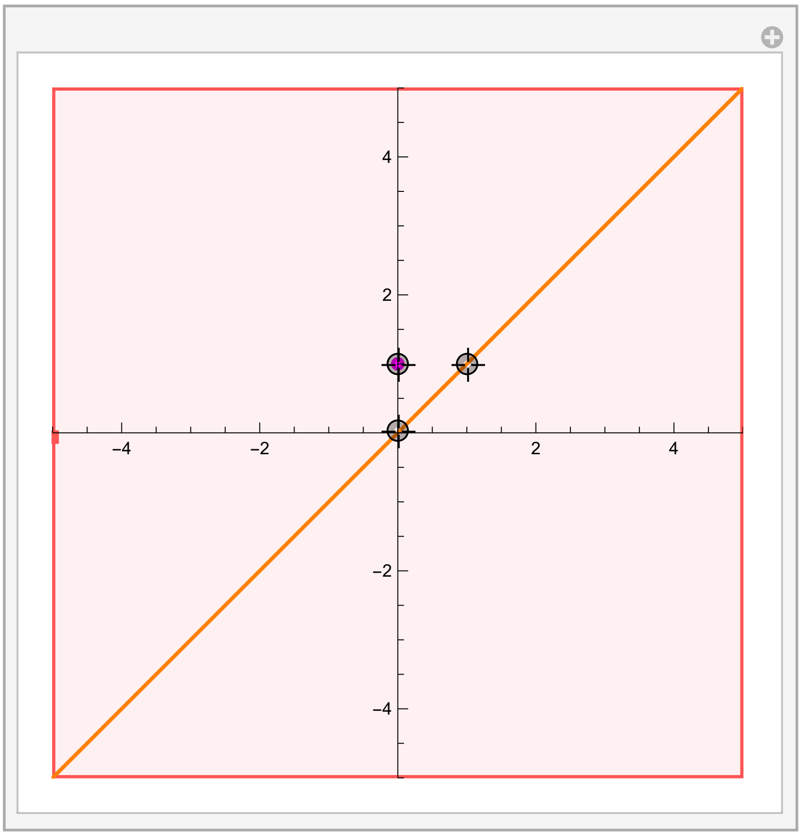
利用矩阵计算点关于直线的对称点

直线l由a，b两点确定，求点p关于l的对称点ps

方式一：利用相似变换，求对称变换矩阵
https://www.bilibili.com/video/BV1zu411673J

In[ ]:=
```
base[a_, b_] := {a - b, Reverse[a - b] {1, -1}}
                        反向排序
m1[a_, b_] := m.{{1, 0}, {0, -1}}.Inverse[m] /. m → Transpose[base[a, b]]
                                   逆              转置
(*可视化*)
visual[ps_] := Manipulate[
               交互式操作
  Graphics[{Thick, Orange, InfiniteLine[{a, b}], PointSize[Large], Magenta, Point[p],
  图形      粗     橙色     无限长直线              点的大小   大       品红色    点
    Point[ps[a, b, p]], Green, Arrow[{{0, 0}, (*2Normalize@*)#} & /@ base[a, b]]},
    点                  绿色   箭头               正规化
   Axes → True, PlotRange → 5], {{p, {0, 1}}, Locator},
   坐标轴  真    绘制范围                         定位器
  {{a, {0, 0}}, Locator}, {{b, {1, 1}}, Locator}]
                定位器                 定位器
visual[m1[#1, #2].#3 &]
```

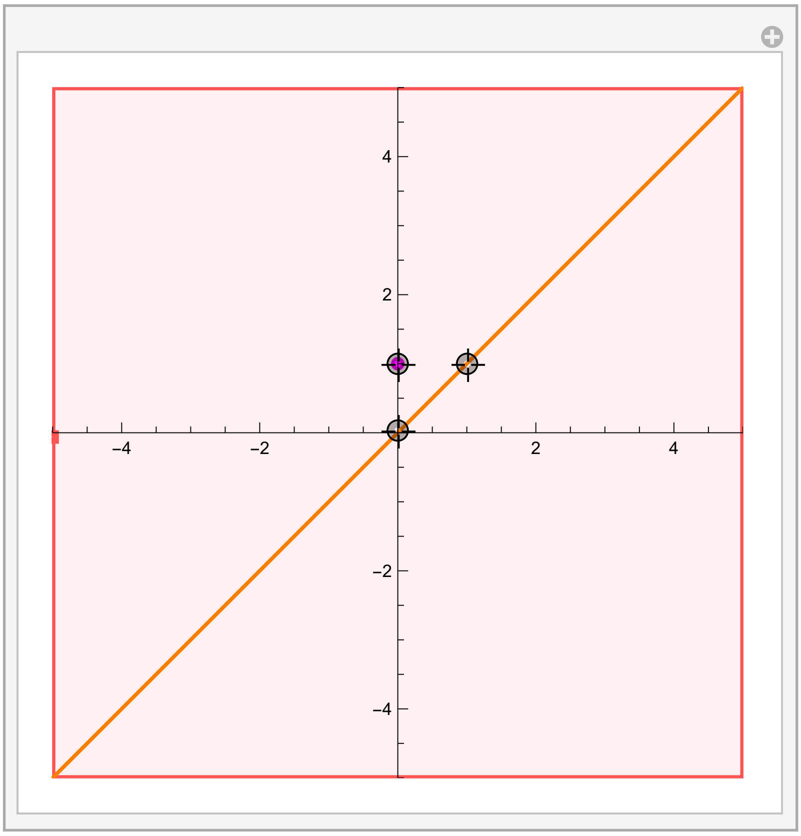
Out[ ]=



方式二：利用辐角
https://www.zhihu.com/question/63108236

In[ ]:=
```
m2[a_, b_] :=
 {{Cos[2 θ], Sin[2 θ]}, {Sin[2 θ], -Cos[2 θ]}} /. {θ → Arg[Complex @@ (a - b)]}
   余弦        正弦         正弦        余弦                辐角  复数
visual[m2[#1, #2].#3 &]
```

Out[ ]=



加上平移

In[ ]:=
```
Solve[{x, y}.({x, y} - #) == 0 & /@ {{a, b}, {c, d}}, {x, y}]
解方程
```

Out[ ]=

$$\left\{\{x\to 0,\ y\to 0\},\ \left\{x\to \frac{b^2 c-a b d-b c d+a d^2}{a^2+b^2-2 a c+c^2-2 b d+d^2},\ y\to \frac{-a b c+b c^2+a^2 d-a c d}{a^2+b^2-2 a c+c^2-2 b d+d^2}\right\}\right\}$$

In[ ]:=
```
Solve[{x, y}.{x - a, y - b} == 0 && {x, y}.{x - c, y - d} == 0, {x, y}]
解方程
```

Out[ ]=

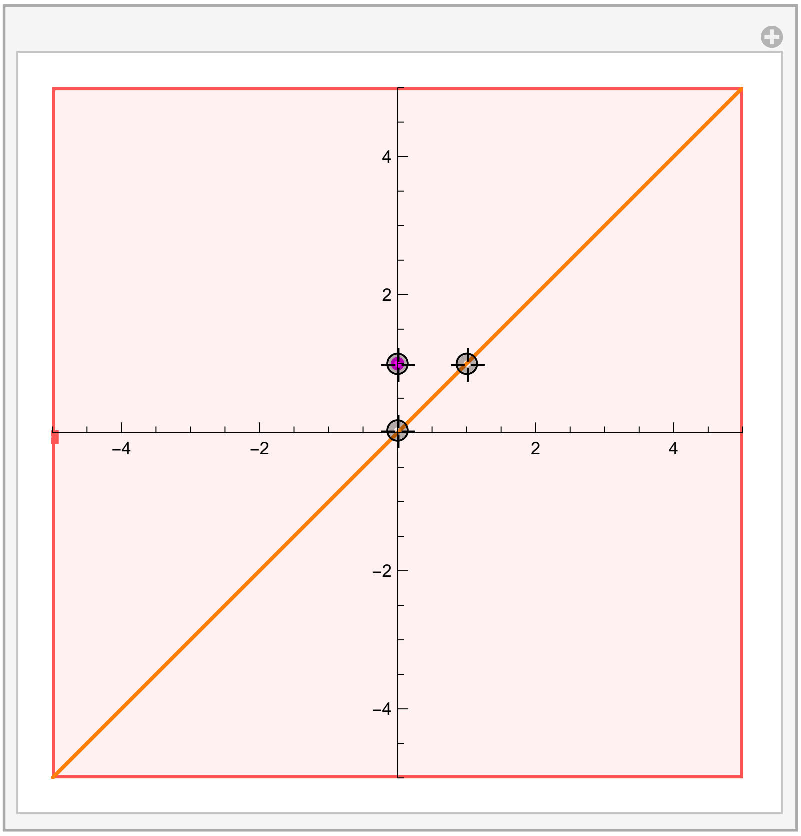
$$\left\{\{x\to 0,\ y\to 0\},\ \left\{x\to \frac{b^2 c-a b d-b c d+a d^2}{a^2+b^2-2 a c+c^2-2 b d+d^2},\ y\to \frac{-a b c+b c^2+a^2 d-a c d}{a^2+b^2-2 a c+c^2-2 b d+d^2}\right\}\right\}$$

In[ ]:=
```
{x, y} /. %[[2]]
```
Out[ ]=
$$\left\{\frac{b^2 c - a b d - b c d + a d^2}{a^2 + b^2 - 2 a c + c^2 - 2 b d + d^2}, \frac{-a b c + b c^2 + a^2 d - a c d}{a^2 + b^2 - 2 a c + c^2 - 2 b d + d^2}\right\}$$

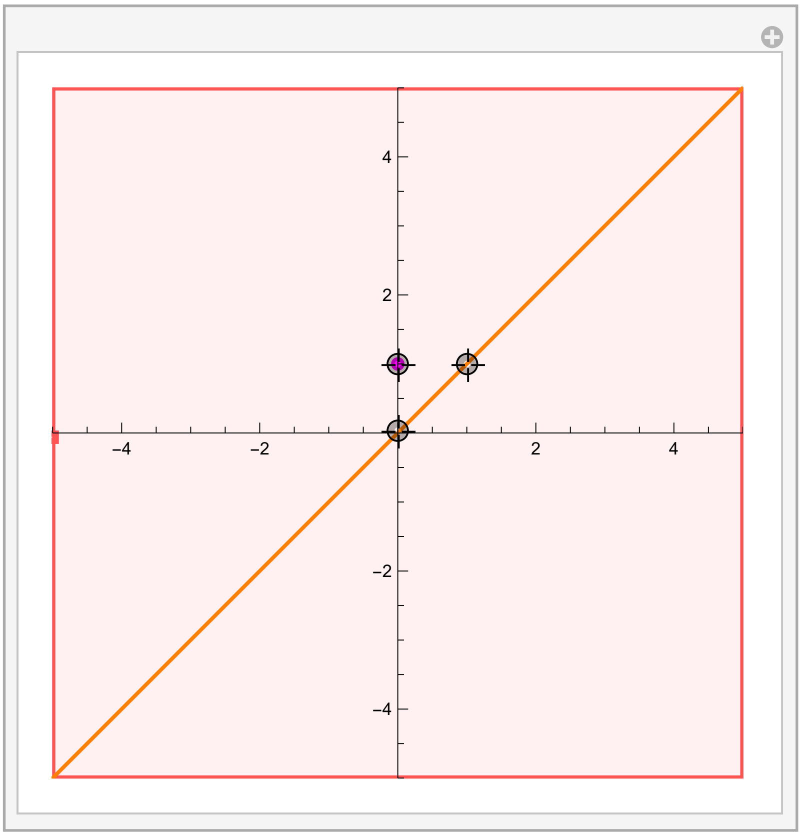
In[ ]:=
```
span[p1_, p2_] := Block[{a, b, c, d}, {a, b} = p1;
  {c, d} = p2;
  {(b^2 c - a b d - b c d + a d^2)/(a^2 + b^2 - 2 a c + c^2 - 2 b d + d^2), (-a b c + b c^2 + a^2 d - a c d)/(a^2 + b^2 - 2 a c + c^2 - 2 b d + d^2)}]
span[{1, 0}, {0, 1}]
```
Out[ ]=
$$\left\{\frac{1}{2}, \frac{1}{2}\right\}$$

In[ ]:=
```
span2[p1_, p2_] := {(b^2 c - a b d - b c d + a d^2)/(a^2 + b^2 - 2 a c + c^2 - 2 b d + d^2), (-a b c + b c^2 + a^2 d - a c d)/(a^2 + b^2 - 2 a c + c^2 - 2 b d + d^2)} /.
  {a → p1[[1]], b → p1[[2]], c → p2[[1]], d → p2[[2]]}
span2[{1, 0}, {0, 1}]
```
Out[ ]=
$$\left\{\frac{1}{2}, \frac{1}{2}\right\}$$

In[ ]:=
```
visual[m1[#1, #2].#3 + 2 span[#1, #2] &]
```
Out[ ]=



In[ ]:=
```
visual[m2[#1, #2].#3 + 2 span[#1, #2] &]
```
Out[ ]=



整理成一个函数

In[ ]:=
```
m1[#1, #2].#3 + 2 span[#1, #2] &[{a, b}, {c, d}, {x, y}]
```
Out[ ]=

$$\left\{\frac{2\left(b^2 c - a b d - b c d + a d^2\right)}{a^2 + b^2 - 2 a c + c^2 - 2 b d + d^2} + \left(\frac{(a-c)(-a+c)}{-a^2 - b^2 + 2 a c - c^2 + 2 b d - d^2} + \frac{(-b+d)^2}{-a^2 - b^2 + 2 a c - c^2 + 2 b d - d^2}\right) x + \frac{2 (a-c)(-b+d) y}{-a^2 - b^2 + 2 a c - c^2 + 2 b d - d^2}, \frac{2\left(-a b c + b c^2 + a^2 d - a c d\right)}{a^2 + b^2 - 2 a c + c^2 - 2 b d + d^2} + \left(\frac{(-a+c)(b-d)}{-a^2 - b^2 + 2 a c - c^2 + 2 b d - d^2} + \frac{(a-c)(-b+d)}{-a^2 - b^2 + 2 a c - c^2 + 2 b d - d^2}\right) x + \left(\frac{(a-c)^2}{-a^2 - b^2 + 2 a c - c^2 + 2 b d - d^2} + \frac{(b-d)(-b+d)}{-a^2 - b^2 + 2 a c - c^2 + 2 b d - d^2}\right) y\right\}$$

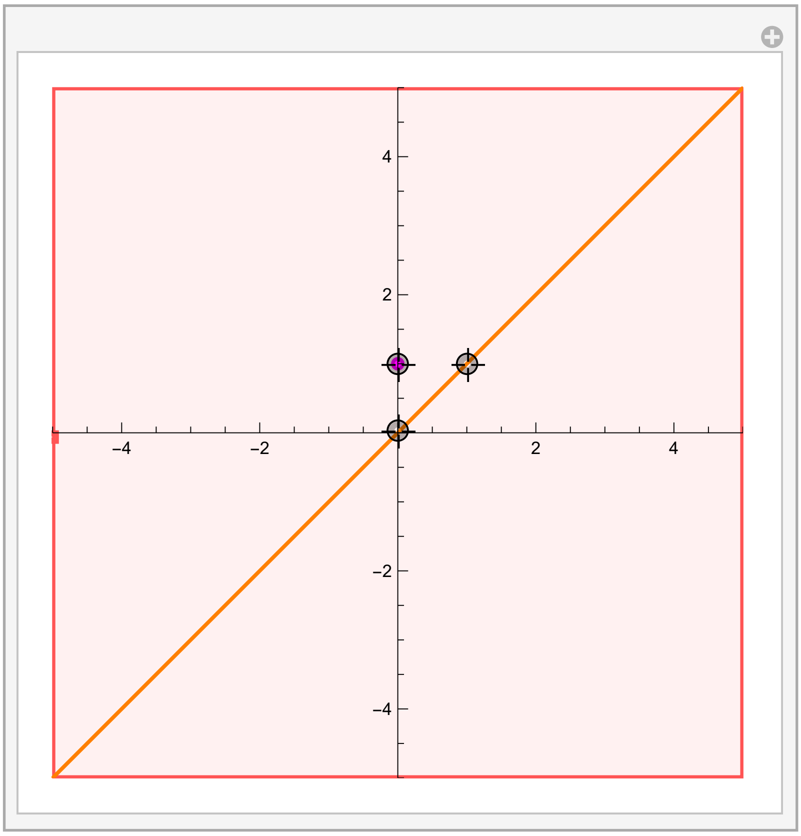
In[ ]:=
```
m1[#1, #2].#3 + 2 span[#1, #2] &[{a, b}, {c, d}, {x, y}] // FullSimplify
```
完全简化

Out[ ]=

$$\left\{\frac{-2\,b\,d\,(a+c-x)+b^2\,(2\,c-x)+a^2\,x+c^2\,x-d^2\,x+2\,b\,(a-c)\,y+2\,c\,d\,y+2\,a\,\left(d^2-c\,x-d\,y\right)}{(a-c)^2+(b-d)^2},\right.$$
$$\left.\frac{2\,(a-c)\,(d\,(a-x)+b\,(-c+x))-(a+b-c-d)\,(a-b-c+d)\,y}{(a-c)^2+(b-d)^2}\right\}$$

In[ ]:=
```
symmetry[pa_, pb_, p_] := Block[{a, b, c, d, x, y}, {a, b} = pa;
  {c, d} = pb;
  {x, y} = p;
  {(-2 b d (a + c - x) + b^2 (2 c - x) + a^2 x + c^2 x - d^2 x + 2 b (a - c) y + 2 c d y + 2 a (d^2 - c x - d y)) /
    ((a - c)^2 + (b - d)^2), (2 (a - c) (d (a - x) + b (-c + x)) - (a + b - c - d) (a - b - c + d) y)/((a - c)^2 + (b - d)^2)}]
visual[symmetry]
```
块

Out[ ]=



In[ ]:=

In[ ]:=
```
m2[#1, #2].#3 + 2 span[#1, #2] &[{a, b}, {c, d}, {x, y}]
```

Out[ ]=

$$\left\{\frac{2\left(b^2 c - a b d - b c d + a d^2\right)}{a^2 + b^2 - 2 a c + c^2 - 2 b d + d^2} + x \operatorname{Cos}[2 \operatorname{Arg}[\operatorname{Complex}[a - c, b - d]]] + y \operatorname{Sin}[2 \operatorname{Arg}[\operatorname{Complex}[a - c, b - d]]], \frac{2\left(-a b c + b c^2 + a^2 d - a c d\right)}{a^2 + b^2 - 2 a c + c^2 - 2 b d + d^2} - y \operatorname{Cos}[2 \operatorname{Arg}[\operatorname{Complex}[a - c, b - d]]] + x \operatorname{Sin}[2 \operatorname{Arg}[\operatorname{Complex}[a - c, b - d]]]\right\}$$

In[ ]:=
```
m2[#1, #2].#3 + 2 span[#1, #2] &[{a, b}, {c, d}, {x, y}] // FullSimplify
```

完全简化

Out[ ]=

$$\left\{\frac{2 (b - d) (b c - a d)}{(a - c)^2 + (b - d)^2} + x \operatorname{Cos}[2 \operatorname{Arg}[\operatorname{Complex}[a - c, b - d]]] + y \operatorname{Sin}[2 \operatorname{Arg}[\operatorname{Complex}[a - c, b - d]]], \frac{2 (a - c) (-b c + a d)}{(a - c)^2 + (b - d)^2} - y \operatorname{Cos}[2 \operatorname{Arg}[\operatorname{Complex}[a - c, b - d]]] + x \operatorname{Sin}[2 \operatorname{Arg}[\operatorname{Complex}[a - c, b - d]]]\right\}$$

In[ ]:=

```
symmetry2[pa_, pb_, p_] := Block[{a, b, c, d, x, y}, {a, b} = pa;
                             块
  {c, d} = pb;
  {x, y} = p;
  {(2 (b - d) (b c - a d))/((a - c)^2 + (b - d)^2) + x Cos[2 Arg[Complex[a - c, b - d]]] +
                                                     余弦   辐角 复数
    y Sin[2 Arg[Complex[a - c, b - d]]], (2 (a - c) (-b c + a d))/((a - c)^2 + (b - d)^2) -
      正弦   辐角 复数
    y Cos[2 Arg[Complex[a - c, b - d]]] + x Sin[2 Arg[Complex[a - c, b - d]]]}]
      余弦   辐角 复数                        正弦   辐角 复数
visual[symmetry]
```

Out[ ]=