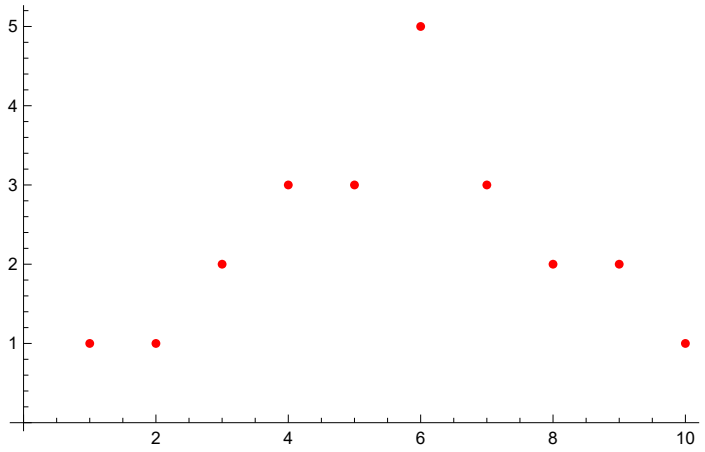
DFT学习

绘制随机的实数列表l

In[ ]:=
```
l = {1, 1, 2, 3, 3, 5, 3, 2, 2, 1};
len = Length[l];
      长度
lp = ListPlot[l, PlotStyle → Red]
     绘制点集      绘图样式      红色
```

Out[ ]=



计算该列表的离散傅里叶变换，并绘图

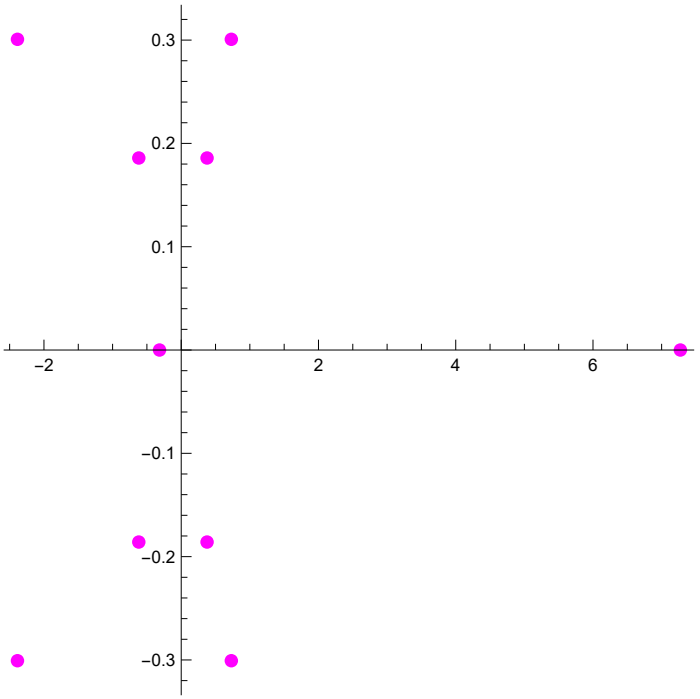
In[ ]:=
```
f = Fourier[l]
    傅立叶
```

Out[ ]=

```
{7.27324 + 0. i, -2.38597 + 0.30075 i, 0.376622 - 0.185874 i,
 -0.618198 - 0.185874 i, 0.730175 + 0.30075 i, -0.316228 + 0. i, 0.730175 - 0.30075 i,
 -0.618198 + 0.185874 i, 0.376622 + 0.185874 i, -2.38597 - 0.30075 i}
```

In[ ]:=
```
ListPlot[(Tooltip[{Re[#1], Im[#1]}] &) /@ f,
 AspectRatio → 1, PlotStyle → {PointSize[Large], Magenta}]
```
绘制点集 提示条 实部 虚部 宽高比 绘图样式 点的大小 大 品红色

Out[ ]=



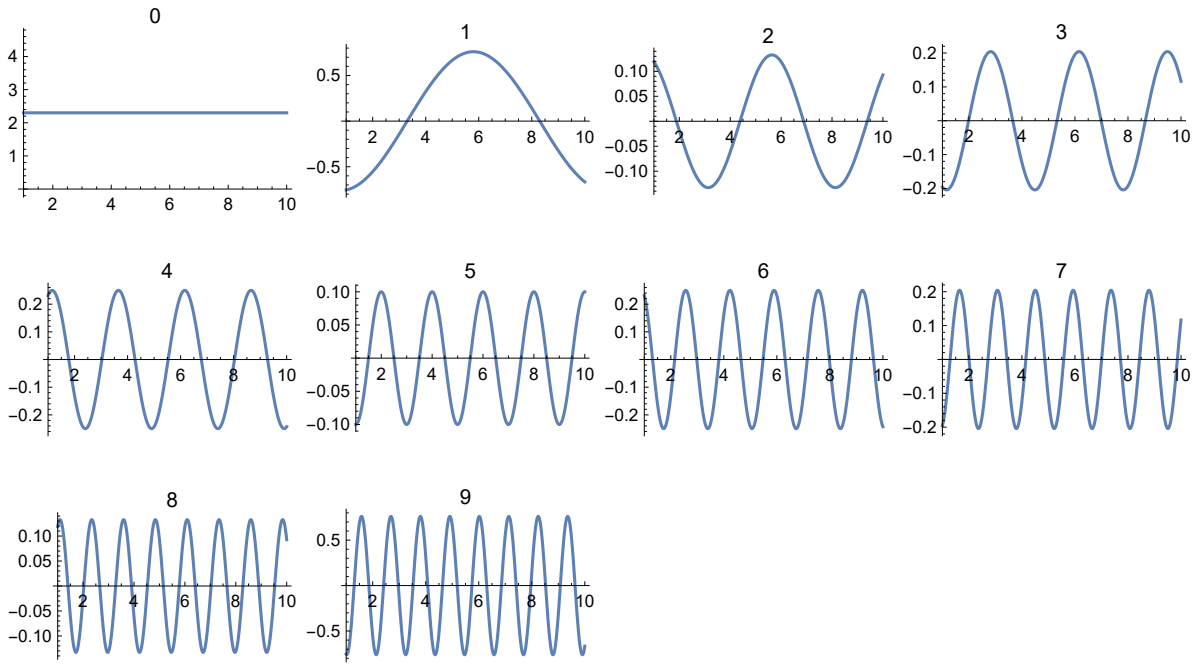
计算每个信号分量，并分别绘图

In[ ]:=
```
funcs = Table[Norm[f[[n]]]/Sqrt[len] Cos[(n - 1) 2 π/len (t - 1) - Arg[f[[n]]]], {n, len}];
```
表格 余弦 辐角

```
Partition[
```
划分

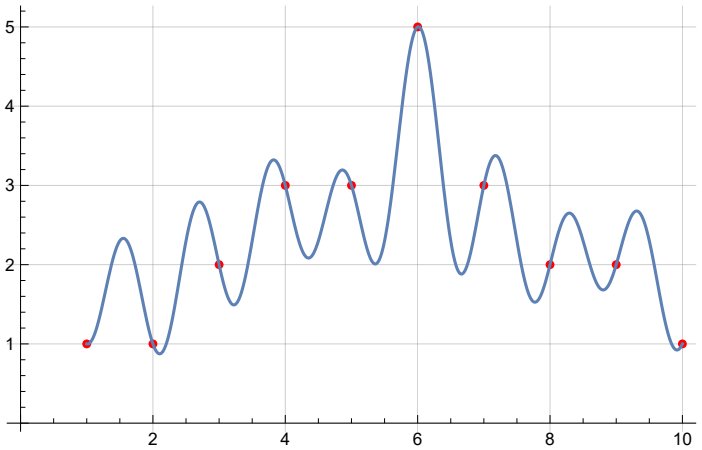
```
  Table[Plot[funcs[[r]], {t, 1, len}, PlotLabel → r - 1], {r, len}], UpTo@4] // Grid
```
表格 绘图 绘图标签 多达 格子

Out[ ]=



In[ ]:=
```
Show[lp, Plot[Total@funcs, {t, 1, len}], GridLines → Automatic]
```
显示 绘图 总计 网格线 自动

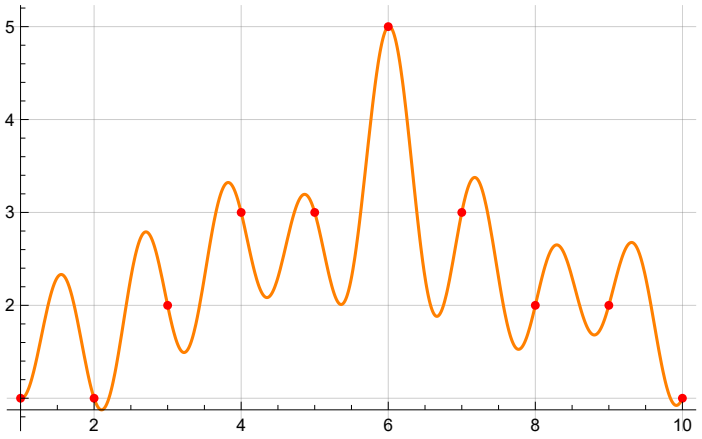
Out[ ]=



将上面过程包装成函数

In[ ]:=
```
draw[l_] := Block[{len = Length[l], f = Fourier[l], func},
                  块          长度          傅立叶
  func = Sum[Norm[f[[r]]] / Sqrt[len] Cos[2 π (r - 1) / len (t - 1) - Arg[f[[r]]]], {r, len}];
             求和                      余弦                        辐角
  Show[Plot[func, {t, 1, len}, PlotStyle → Orange, PlotRange → All],
  显示 绘图                    绘图样式          橙色          绘制范围          全部
   ListPlot[l, PlotStyle → Red], GridLines → Automatic]]
   绘制点集          绘图样式          红色 网格线          自动
draw[l]
```

Out[ ]=

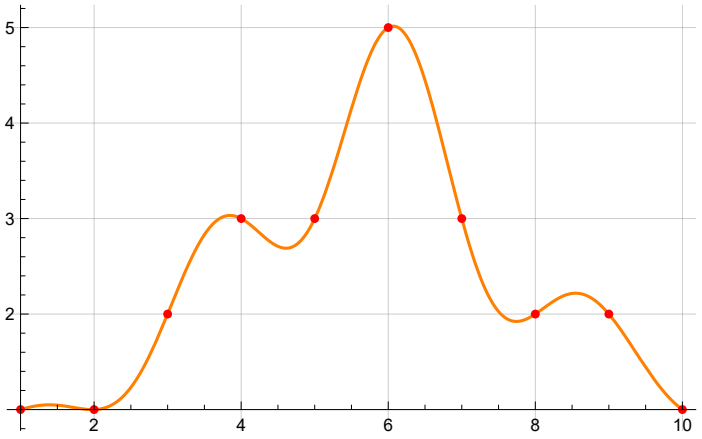


上面的图形问题在于错把有初相的低频基信号当成高频

In[ ]:=
```
draw22[l_] := Block[{len = Length[l], f = Fourier[l], func, k}, k = Ceiling[len / 2];
  func = Sum[c = f[[Mod[n + len, len] + 1]];
    Norm[c]/Sqrt[len] Cos[2 π n/len (t - 1) - Arg[c]], {n, k - len, k - 1}];
  Show[Plot[func, {t, 1, len}, PlotStyle → Orange, PlotRange → All],
   ListPlot[l, PlotStyle → Red], GridLines → Automatic]]
```
块 长度 傅立叶 向上取整 求和 模余 余弦 辐角 显示 绘图 绘图样式 橙色 绘制范围 全部 绘制点集 绘图样式 红色 网格线 自动

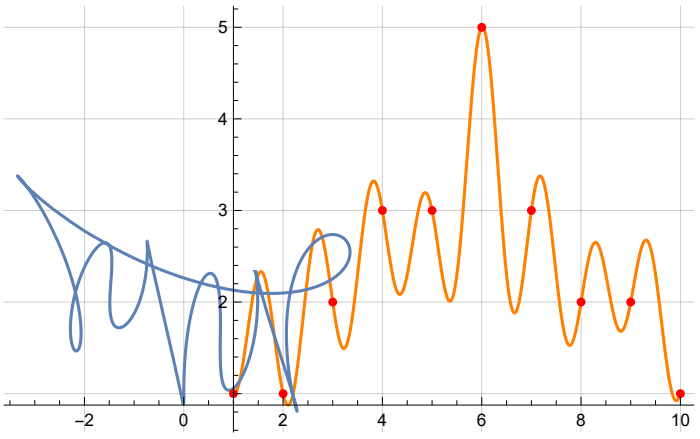
```
draw22[l]
```

Out[ ]=



In[ ]:=
```
draw33[l_] := Block[{len = Length[l], f = Fourier[l], func},
                    块          长度          傅立叶

  func = Sum[Norm[f[[r]]] / Sqrt[len] Cos[2 π (r - 1) / len (t - 1) - Arg[f[[r]]]], {r, len}];
         求和                余弦                        辐角

  m = 8;
  cn = Table[1 / (2 π) Integrate[(t + I func) Exp[-I k t], {t, -π, π}], {k, -m, m}];
       表格            积分                虚数单位 指…  虚数单位

  p = Sum[cn[[n]] Exp[(n - m - 1) I t], {n, 2 m + 1}];
      求和          指数形式          虚数单位
  Show[Plot[func, {t, 1, len}, PlotStyle → Orange],
  显示 绘图                    绘图样式        橙色
    ParametricPlot[ReIm@p, {t, 0, 2 π}],
    绘制参数图          实部虚部列表
    ListPlot[l, PlotStyle → Red], GridLines → Automatic, PlotRange → All]]
    绘制点集        绘图样式        红色 网格线        自动        绘制范围        全部

draw33[l]
```

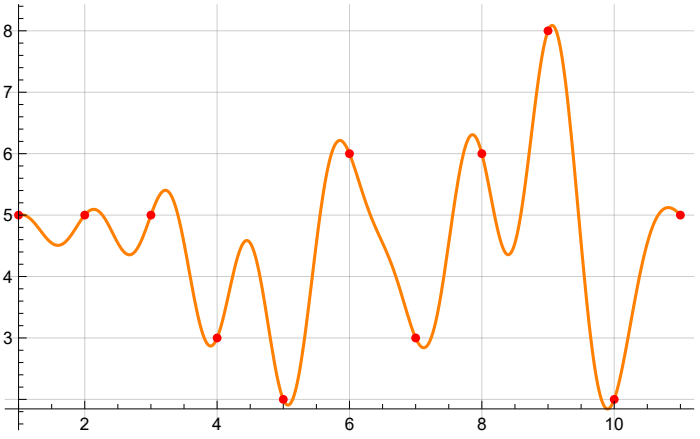
Out[ ]=



使用随机数列对比观察

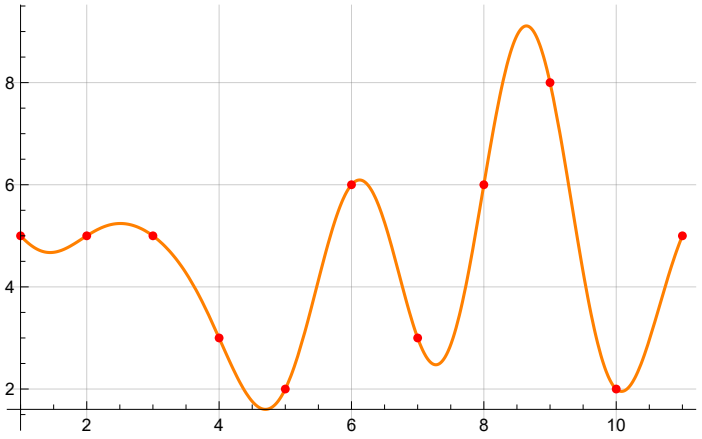
In[ ]:=
```
draw[{5, 5, 5, 3, 2, 6, 3, 6, 8, 2, 5}]
```

Out[ ]=



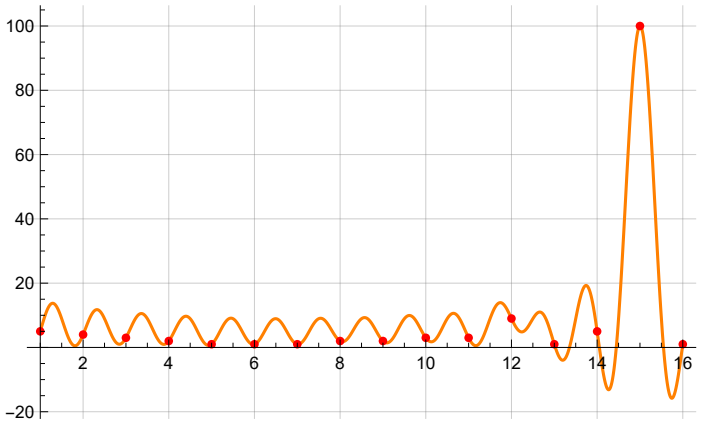
In[ ]:= **draw22[{5, 5, 5, 3, 2, 6, 3, 6, 8, 2, 5}]**

Out[ ]=



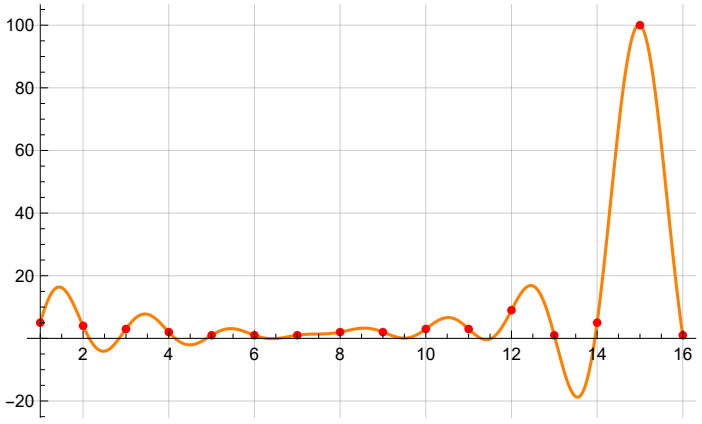
In[ ]:= **draw[{5, 4, 3, 2, 1, 1, 1, 2, 2, 3, 3, 9, 1, 5, 100, 1}]**

Out[ ]=



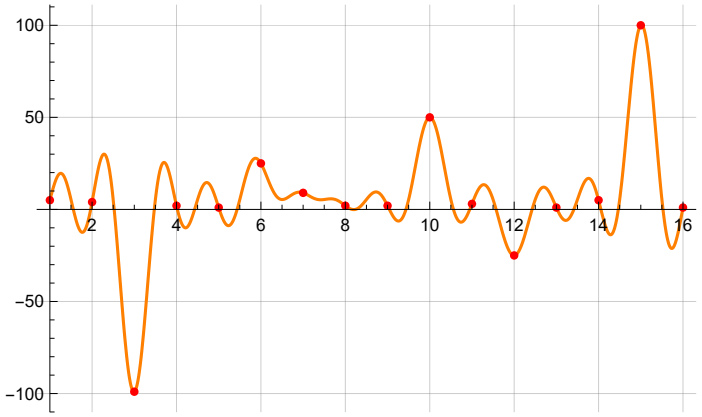
In[ ]:= **draw22[{5, 4, 3, 2, 1, 1, 1, 2, 2, 3, 3, 9, 1, 5, 100, 1}]**

Out[ ]=



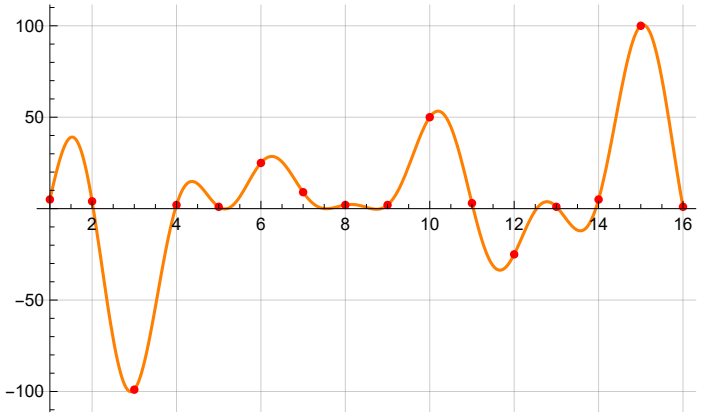
In[ ]:=
```
draw[{5, 4, -99, 2, 1, 25, 9, 2, 2, 50, 3, -25, 1, 5, 100, 1}]
```

Out[ ]=



In[ ]:=
```
draw22[{5, 4, -99, 2, 1, 25, 9, 2, 2, 50, 3, -25, 1, 5, 100, 1}]
```

Out[ ]=



以sin(4πx)信号的k个采样点为例，对比观察

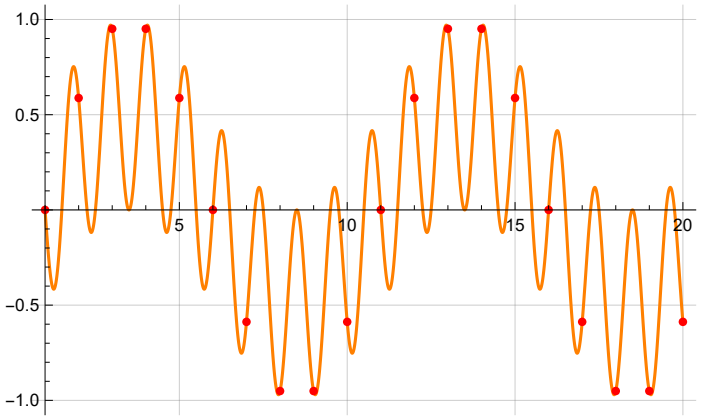
In[ ]:=
```
k = 20;
s = Table[Sin[2 * 2 π (n - 1)/k], {n, k}]
```
表格 正弦

Out[ ]=

$$\left\{0, \sqrt{\frac{5}{8}-\frac{\sqrt{5}}{8}}, \sqrt{\frac{5}{8}+\frac{\sqrt{5}}{8}}, \sqrt{\frac{5}{8}+\frac{\sqrt{5}}{8}}, \sqrt{\frac{5}{8}-\frac{\sqrt{5}}{8}}, 0, -\sqrt{\frac{5}{8}-\frac{\sqrt{5}}{8}},\right.$$
$$-\sqrt{\frac{5}{8}+\frac{\sqrt{5}}{8}}, -\sqrt{\frac{5}{8}+\frac{\sqrt{5}}{8}}, -\sqrt{\frac{5}{8}-\frac{\sqrt{5}}{8}}, 0, \sqrt{\frac{5}{8}-\frac{\sqrt{5}}{8}}, \sqrt{\frac{5}{8}+\frac{\sqrt{5}}{8}}, \sqrt{\frac{5}{8}+\frac{\sqrt{5}}{8}},$$
$$\left.\sqrt{\frac{5}{8}-\frac{\sqrt{5}}{8}}, 0, -\sqrt{\frac{5}{8}-\frac{\sqrt{5}}{8}}, -\sqrt{\frac{5}{8}+\frac{\sqrt{5}}{8}}, -\sqrt{\frac{5}{8}+\frac{\sqrt{5}}{8}}, -\sqrt{\frac{5}{8}-\frac{\sqrt{5}}{8}}\right\}$$

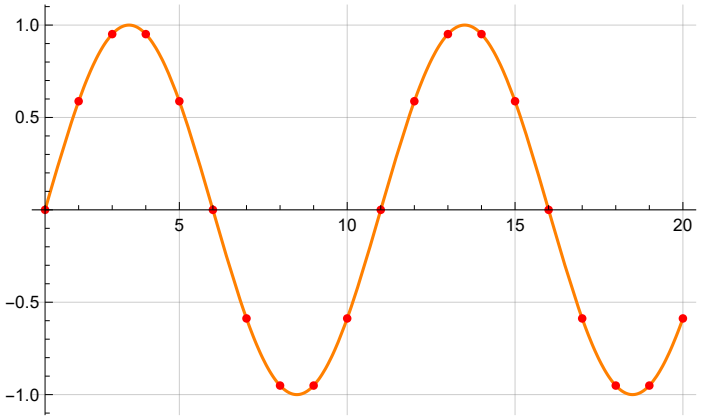
In[ ]:=
```
draw[s]
```

Out[ ]=



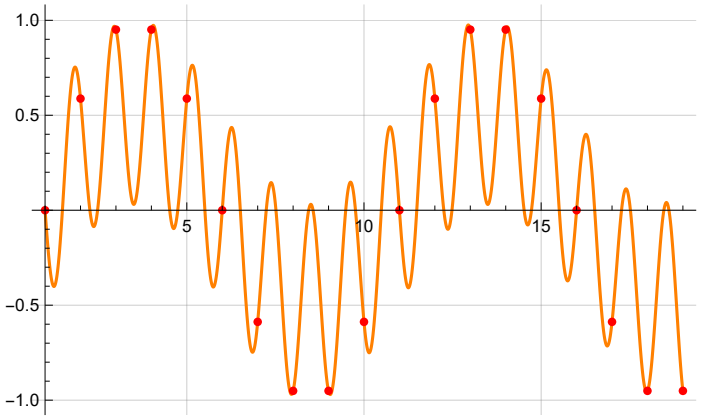
In[ ]:=
```
draw22[s]
```

Out[ ]=



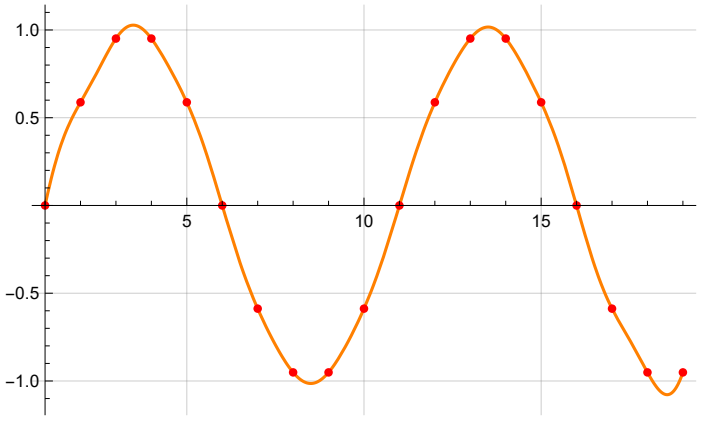
In[ ]:=
```
draw[s[[ ;; -2]]]
```

Out[ ]=



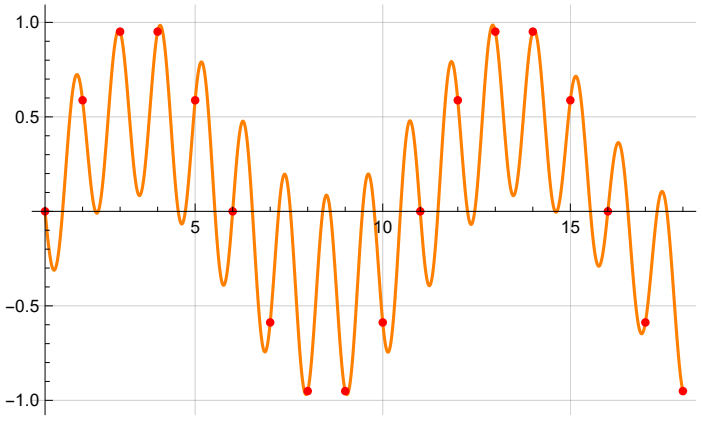
In[ ]:=
```
draw22[s[[ ;; -2]]]
```

Out[ ]=



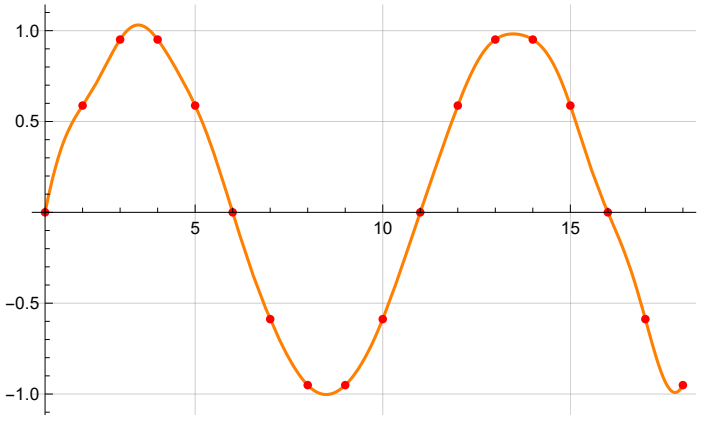
In[ ]:=
```
draw[s[[ ;; -3]]]
```

Out[ ]=



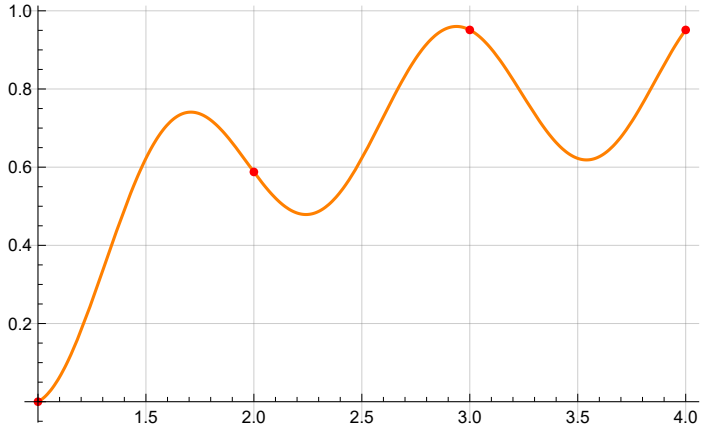
In[ ]:=
```
draw22[s[[ ;; -3]]]
```

Out[ ]=



In[ ]:=
```
draw[s[[ ;; 4]]]
```

Out[ ]=



In[ ]:=
```
draw22[s[[ ;; 4]]]
```

Out[ ]=