

20220704求解PDE热传导方程

5、一维热传导方程: $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, $x \in (0,1), t \in (0,5)$

初始条件: $u(x,0) = x, x \in (0,0.5)$, $u(x,0) = 1-x, x \in (0.5,1)$

边界条件: $u(0,t) = u(1,t) = 0$

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In[*]:= pde = D[u[x, t], t] == D[u[x, t], {x, 2}] - D[u[x, t], x];
          |偏导          |偏导          |偏导
ic = u[x, 0] == Piecewise[{{x, 0 ≤ x ≤ 0.5}, {1 - x, 0.5 ≤ x ≤ 1}}];
          |分段函数
bcs = u[0, t] == 0 && u[1, t] == 0;
DSolveValue[pde && ic && bcs, u[x, t], {x, t} ∈ Rectangle[{0, 0}, {1, 5}]]
          |微分方程解          |矩形
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Out[*]=

$$e^{\frac{1}{2} \left(-\frac{t}{2} + x \right)} \sum_{K[1]=1}^{\infty} \left(4.85225 e^{-\pi^2 t K[1]^2} \right. \\ \left. \left(20.7184 K[1] - 32.2711 \cos[1.5708 K[1]] K[1] + 12.5664 \cos[3.14159 K[1]] K[1] - \right. \right. \\ \left. \left. 2.56805 \sin[1.5708 K[1]] + 101.383 K[1]^2 \sin[1.5708 K[1]] + \sin[3.14159 K[1]] - \right. \right. \\ \left. \left. 39.4784 K[1]^2 \sin[3.14159 K[1]] \right) \sin[\pi x K[1]] \right) / \left(1. + 39.4784 K[1]^2 \right)^2$$

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In[*]:= pde = D[u[x, t], t] == D[u[x, t], {x, 2}] - D[u[x, t], x];
ic = u[x, 0] == { x      0 ≤ x ≤ 0.5
                  1 - x  0.5 ≤ x ≤ 1 };
bcs = u[0, t] == 0 && u[1, t] == 0;
DSolveValue[{pde, ic, bcs}, u[x, t], {x, t} ∈ Rectangle[{0, 0}, {1, 5}]]
          |微分方程解          |矩形
```

Out[*]=

$$e^{\frac{1}{2} \left(-\frac{t}{2} + x \right)} \sum_{K[1]=1}^{\infty} \frac{1}{\left(1. + 39.4784 K[1]^2 \right)^2} 4.85225 e^{-\pi^2 t K[1]^2} \\ \left(20.7184 K[1] - 32.2711 \cos[1.5708 K[1]] K[1] + 12.5664 \cos[3.14159 K[1]] K[1] - \right. \\ \left. 2.56805 \sin[1.5708 K[1]] + 101.383 K[1]^2 \sin[1.5708 K[1]] + \right. \\ \left. \sin[3.14159 K[1]] - 39.4784 K[1]^2 \sin[3.14159 K[1]] \right) \sin[\pi x K[1]]$$