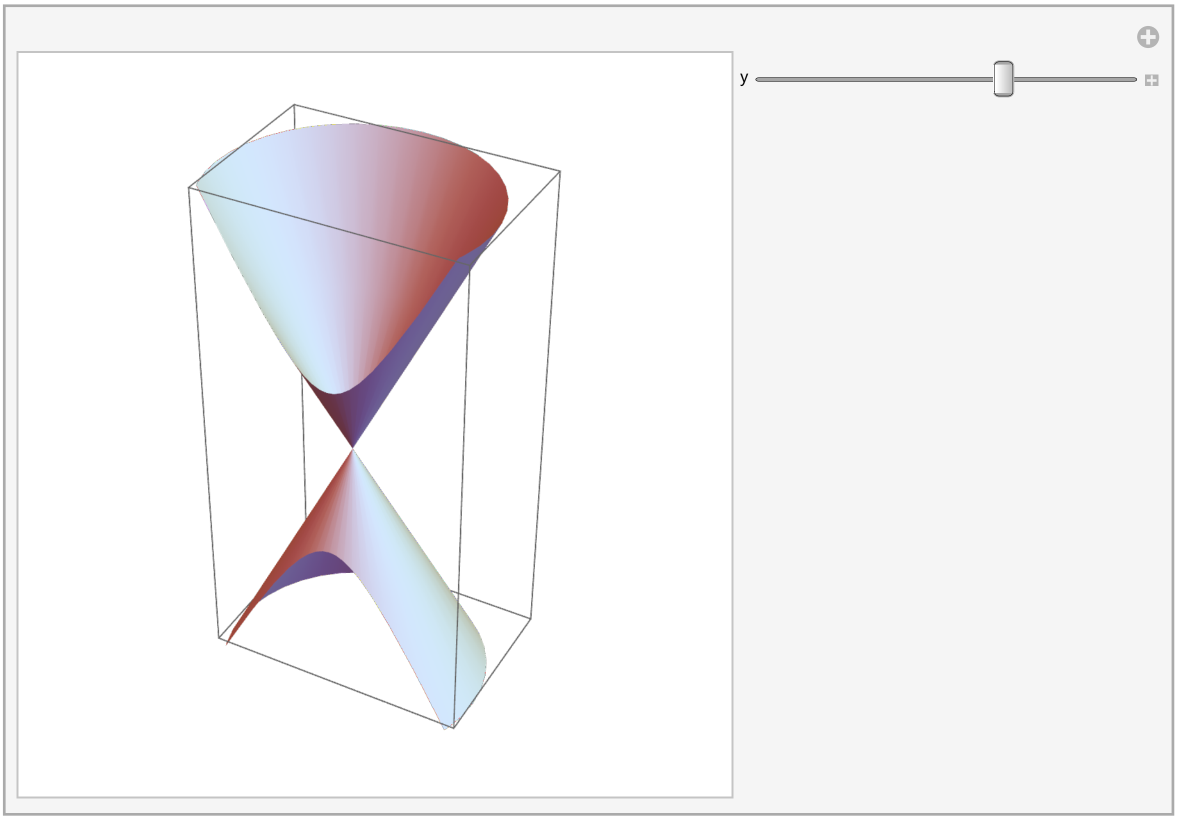
圆锥曲线

垂直截面

In[ ]:=

```
Manipulate[Graphics3D[{EdgeForm[Magenta], Cone[], Cone[{{0, 0, 3}, {0, 0, 1}}]},
交互式操作     三维图形        边的格式  品红色    圆锥    圆锥
  PlotRange → {{-1, 1}, {-y, 1}, {-0.95, 2.95}}], {{y, 0.33}, -1, 1}]
  绘制范围
```

Out[ ]=



不难发现截线是双曲线

在进一步展现其他情况之前，先解决一个问题

根据定点P和法线向量v，计算无限大平面

In[ ]:=

```
p = {px, py, pz}; v = {v1, v2, v3};
p2 = {px + 1, py + 1, zz};
sol = Solve[(p2 - p).v == 0, zz][[1]]
      解方程
```

Out[ ]=

$$\left\{zz \to \frac{-v1 - v2 + pz\, v3}{v3}\right\}$$

In[ ]:=
```
p2 = p2 /. sol
```
Out[ ]=

$$\left\{1+px,\ 1+py,\ \frac{-v1-v2+pz\ v3}{v3}\right\}$$

In[ ]:=
```
p3 = p + (p2 - p) × v
```
Out[ ]=

$$\left\{px+\frac{v1\ v2}{v3}+\frac{v2^2}{v3}+v3,\ py-\frac{v1^2}{v3}-\frac{v1\ v2}{v3}-v3,\ pz-v1+v2\right\}$$

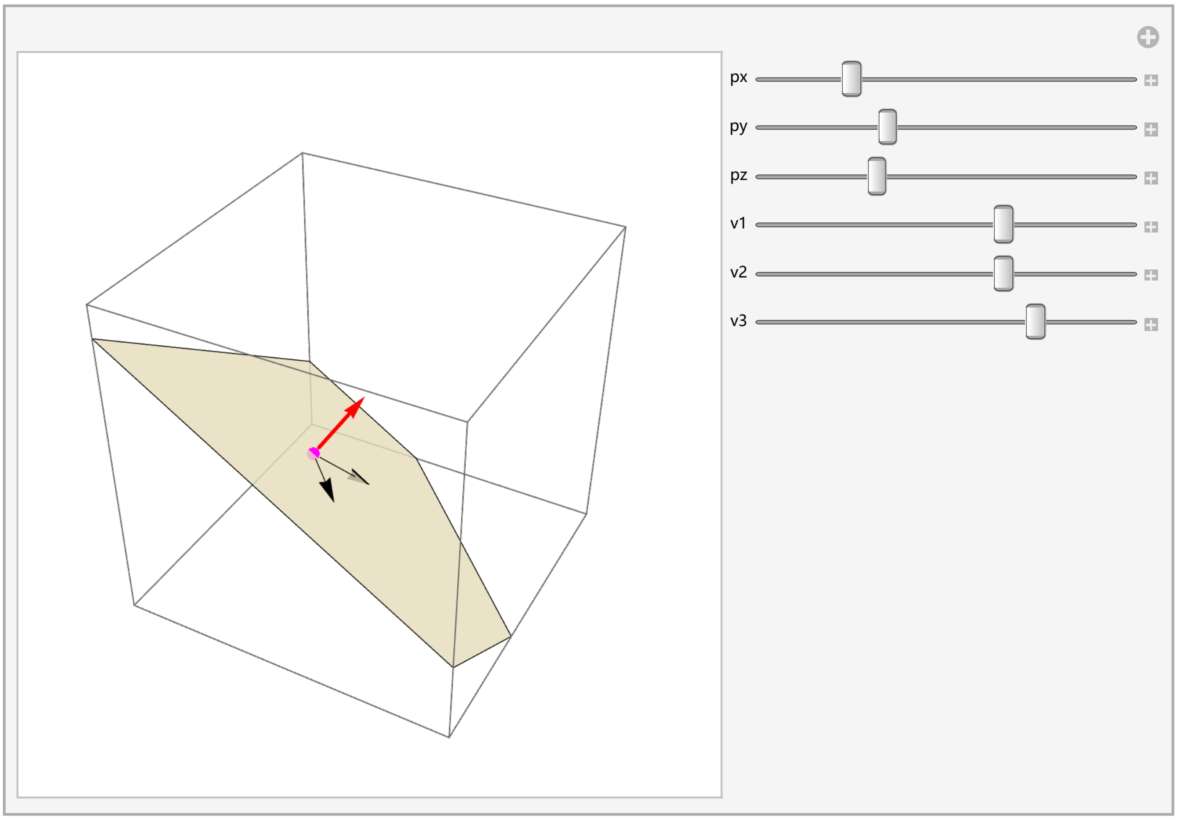
现在我们得到了过p点、法向量为v的平面上的三个点，p，p2，p3

绘制出来看看效果

In[ ]:=

```
DynamicModule[{p, p2, p3, v}, Manipulate[p = {px, py, pz};
动态模块                        交互式操作
  v = {v1, v2, v3};
  p2 = {1 + px, 1 + py, (-v1 - v2 + pz v3)/v3};
  p3 = {px + (v1 v2)/v3 + v2^2/v3 + v3, py - v1^2/v3 - (v1 v2)/v3 - v3, pz - v1 + v2};
  Graphics3D[{{Opacity[0.8], InfinitePlane[{p, p2, p3}]}, {Thick, Red,
  三维图形       不透明度        无限大平面                   粗      红色
    Arrow[{p, p + Normalize@v}, 0.1]}, Arrow[{p, p + Normalize[p2 - p]}, 0.1], Arrow[{p,
    箭头          正规化                 箭头          正规化                    箭头
      p + Normalize[p3 - p]}, 0.1], Magenta, PointSize[Large], Point[p]}, PlotRange → 2],
          正规化                      品红色     点的大小    大      点          绘制范围
  {px, -1, 1}, {py, -1, 1}, {pz, -1, 1}, {v1, -1, 1}, {v2, -1, 1}, {v3, -1, 1}]]
```

Out[ ]=

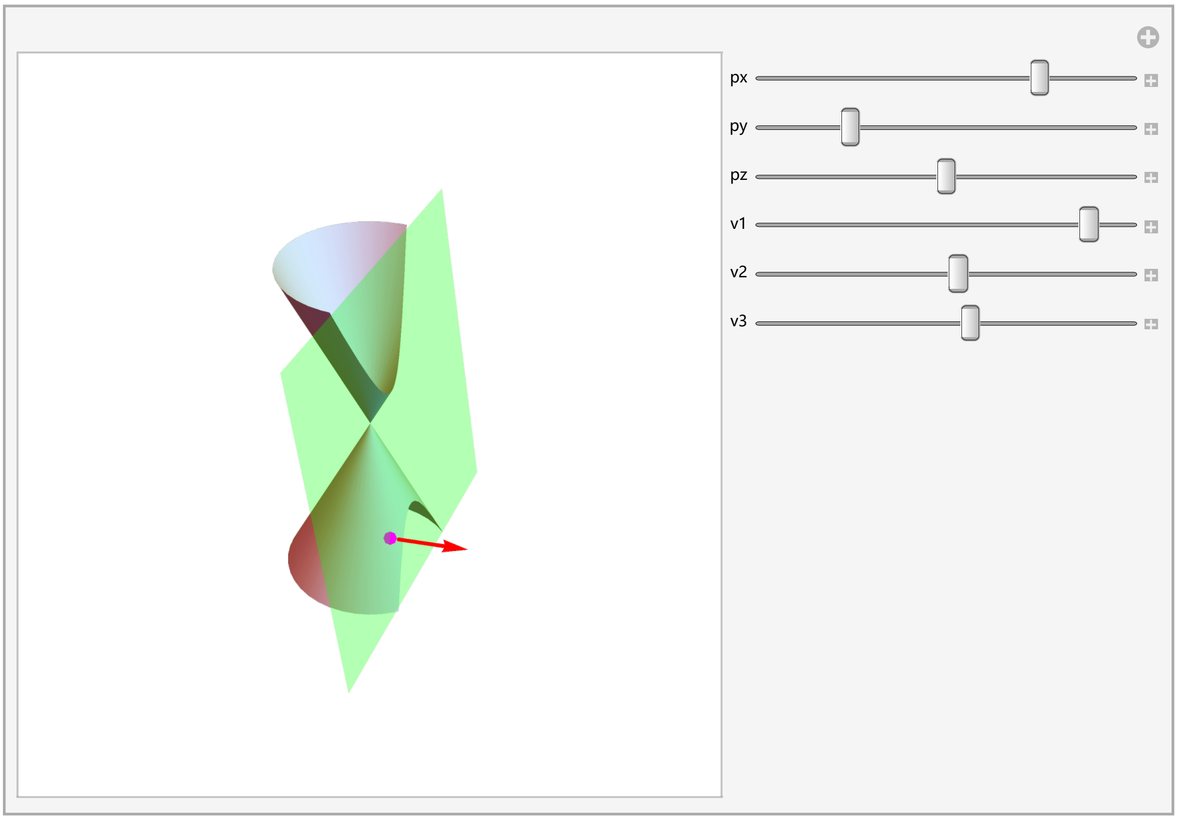


现在，我们让这个平面去截圆锥面

In[ ]:=

```
DynamicModule[{p, p2, p3, v}, Manipulate[p = {px, py, pz};
动态模块                       交互式操作
  v = {v1, v2, v3};
  p2 = {1 + px, 1 + py, (-v1 - v2 + pz v3)/v3};
  p3 = {px + v1 v2/v3 + v2^2/v3 + v3, py - v1^2/v3 - v1 v2/v3 - v3, pz - v1 + v2};
  Graphics3D[{Style[{Cone[], Cone[{{0, 0, 3}, {0, 0, 1}}]}, ClipPlanes →
  三维图形      样式    圆锥     圆锥                            剪切平面
      InfinitePlane[{p, p2, p3}], ClipPlanesStyle → {Directive[Opacity[0.3], Green]}],
      无限大平面                    剪切平面样式        指令       不透明度        绿色
    {Thick, Red, Arrow[{p, p + Normalize@v}, 0.1]}, {Magenta, PointSize[Large],
     粗     红色  箭头         正规化                  品红色    点的大小   大
     Point[p]}}, Boxed → False, PlotRange → {{-2, 2}, {-2, 2}, {-0.9, 2.9}}],
     点           边界框   假     绘制范围
  {{px, 0.5}, -1, 1}, {{py, -0.5}, -1, 1}, {{pz, 0}, -1, 1}, {{v1, 1}, -1, 1},
  {{v2, 0}, -1, 1}, {{v3, 0.05}, -1, 1}]]
```

Out[ ]=



优化

现在我们回头看下p2的值 $\left\{1 + px,\ 1 + py,\ \frac{-v1 - v2 + pz\ v3}{v3}\right\}$

这里v3作为分母是不能为0的。

如果假设v3=0，那么平面就会平行于z轴，这时不一定会过点p2

因此，当v3 = 0 时，我们可以推导出这样的p2和p3

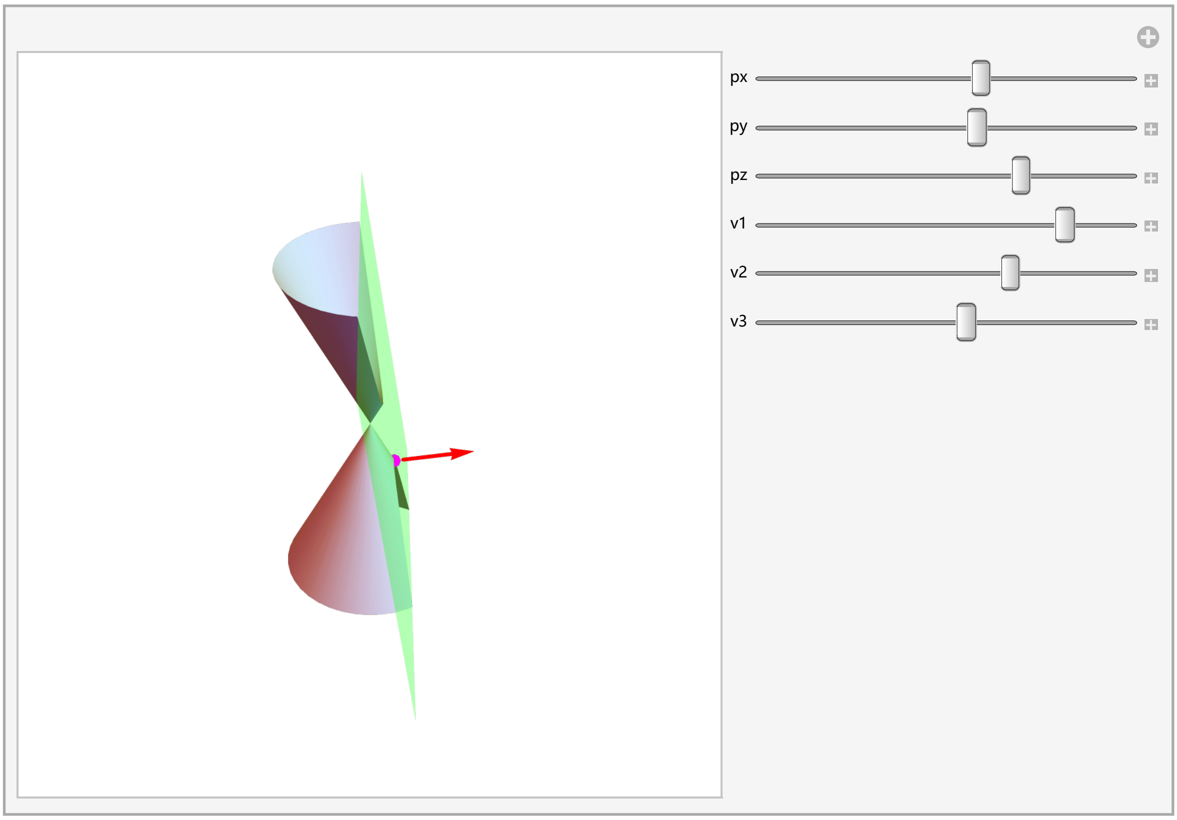
```
p = {px, py, pz}; v = {v1, v2, 0};
p2 = {px, py, pz + 1};
p3 = p + {-v2, v1, 0};
```

稍微修改前面的栗子

In[◦]:=

```
DynamicModule[{p, p2, p3, v}, Manipulate[p = {px, py, pz};
  v = {v1, v2, v3};
  p2 = If[v3 == 0, {px, py, pz + 1}, {1 + px, 1 + py, (-v1 - v2 + pz v3)/v3}];
  p3 =
   If[v3 == 0, p + {-v2, v1, 0}, {px + (v1 v2)/v3 + v2^2/v3 + v3, py - v1^2/v3 - (v1 v2)/v3 - v3, pz - v1 + v2}];
  Graphics3D[{Style[{Cone[], Cone[{{0, 0, 3}, {0, 0, 1}}]}, ClipPlanes ->
      InfinitePlane[{p, p2, p3}], ClipPlanesStyle -> {Directive[Opacity[0.3], Green]}],
    {Thick, Red, Arrow[{p, p + Normalize@v}, 0.1]}, {Magenta, PointSize[Large],
     Point[p]}}, Boxed -> False, PlotRange -> {{-2, 2}, {-2, 2}, {-0.9, 2.9}}],
  {{px, 0.5}, -1, 1}, {{py, -0.5}, -1, 1}, {{pz, 0}, -1, 1}, {{v1, 1}, -1, 1},
  {{v2, 0}, -1, 1}, {{v3, 0.05}, -1, 1}]]
```

Out[◦]=



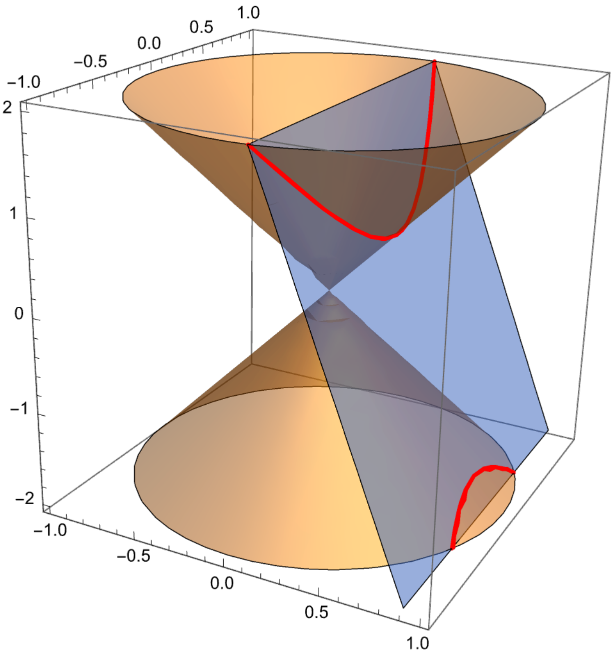
现在，即使v3=0，计算也依然正确

接下来，我们当然要计算得到截线的方程，看起来有些难度啊

先来写出圆锥面和平面的方程

In[ ]:=
```
p = {0.5, 0, 0}; v = {1, 0, 0.2};
g = x^2 + y^2 - (z / 2)^2;
h = (p - {x, y, z}).v;
cp = ContourPlot3D[{g == 0, h == 0}, {x, -1, 1}, {y, -1, 1}, {z, -2, 2},
  MeshFunctions → Function[{x, y, z}, g - h], MeshStyle → {{Red, Thick}},
  Mesh → {{0}}, ContourStyle → Directive[Opacity[0.5], Specularity[White, 30]]]
```
三维等高线 | 网格函数 | 纯函数 | 网格样式 | 红色 | 粗 | 网格 | 等高线样式 | 指令 | 不透明度 | 反射度 | 白色

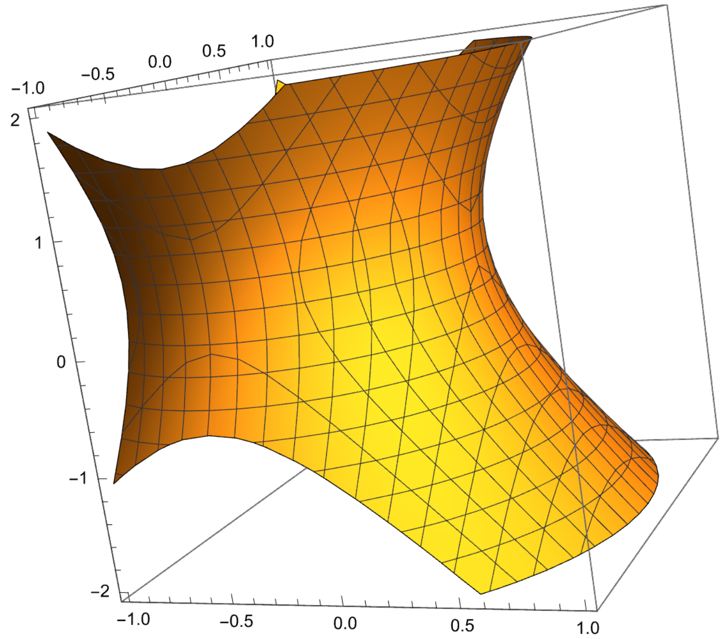
Out[ ]=



计算它们的交线

In[∘]:=
```
ContourPlot3D[x^2 + y^2 - (z / 2)^2 == (p - {x, y, z}).v, {x, -1, 1}, {y, -1, 1}, {z, -2, 2}]
```
三维等高线

Out[∘]=



In[∘]:=
```
Solve[{x^2 + y^2 == (z / 2)^2 && (p - {x, y, z}).v == 0}, {y, z}, Reals]
```
解方程 实数域

Solve: Solve 无法求解具有不精确系数的系统. 答案是通过求解相应的精确系统并且将结果数值化处理得到的.

Out[∘]=

$$\left\{\left\{y \to \boxed{-0.25\sqrt{25. - 100.\,x + 84.\,x^2}\ \text{if}\ x > 0.833333\ ||\ x < 0.357143},\ z \to \boxed{0.5\,(5. - 10.\,x)\ \text{if}\ x > 0.833333\ ||\ x < 0.357143}\right\},\left\{y \to \boxed{0.25\sqrt{25. - 100.\,x + 84.\,x^2}\ \text{if}\ x > 0.833333\ ||\ x < 0.357143},\ z \to \boxed{0.5\,(5. - 10.\,x)\ \text{if}\ x > 0.833333\ ||\ x < 0.357143}\right\}\right\}$$

In[∘]:=
```
p = {px, py, pz}; v = {v1, v2, v3};
SolveValues[(p - {x, y, z}).v == 0, z]
```
由解确定的值

Out[∘]=

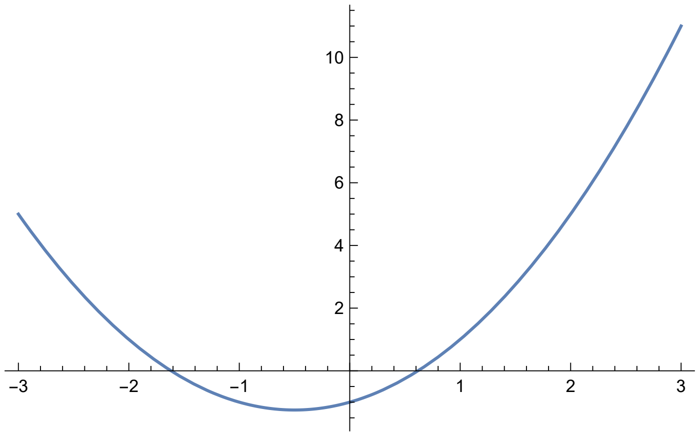
$$\left\{\frac{px\,v1 + py\,v2 + pz\,v3 - v1\,x - v2\,y}{v3}\right\}$$

曲面相交

```
Solve[{x^2 + y^2 + z^2 == 1, z == x^2 + y^2}, {x, y}, Reals]
```
解方程 实数域

In[ ]:= `Plot[z^2 + z - 1, {z, -3, 3}]`
绘图

Out[ ]=



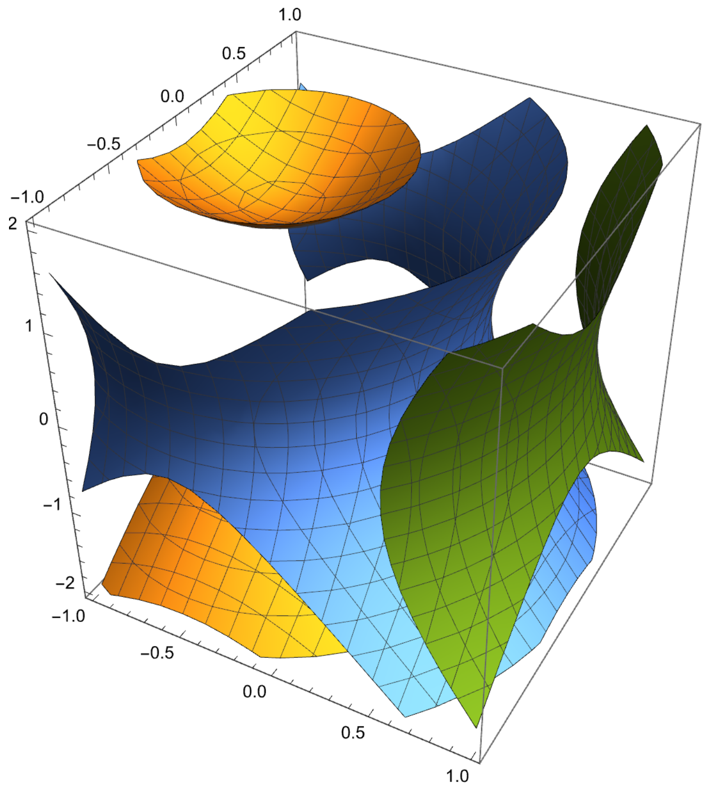
In[ ]:= `x^2 + y^2 - (z / 2)^2 - (p - {x, y, z}).v`

Out[ ]=

$$-0.5 + x + x^2 + y^2 + 0.2\, z - \frac{z^2}{4}$$

In[ ]:= `ContourPlot3D[-0.5` + x + x^2 + y^2 + 0.2` z - z^2/4, {x, -1, 1}, {y, -1, 1}, {z, -2, 2}]`
三维等高线

Out[ ]=



In[ ]:=

```
sol = Solve[x^2 + y^2 + z^2 == 1 && x^3 + x y^2 == z^2, {x, y}, Reals]
```

解方程　　实数域

Out[ ]=

$$\left\{\left\{x \to -\frac{z^2}{-1+z^2} \text{ if } \text{condition},\ y \to -\sqrt{1-z^2-\frac{z^4}{\left(-1+z^2\right)^2}} \text{ if } \text{condition}\right\},\right.$$

$$\left.\left\{x \to -\frac{z^2}{-1+z^2} \text{ if } \text{condition},\ y \to \sqrt{1-z^2-\frac{z^4}{\left(-1+z^2\right)^2}} \text{ if } \text{condition}\right\}\right\}$$

文件操作