CS221 Fall 2018 Homework 7

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Collaborators:

By turning in this assignment, I agree by the Stanford honor code and declare that all of this is my own work.

Problem 1

(a) We compute $\mathbb{P}(C_2 = 1 \mid D_2 = 0)$. We note that by the factor graph, we have the following:

$$\begin{aligned}
\mathbb{P}(C_2 = c_2 \mid D_2 = 0) &\propto p(D_2 = 0 \mid C_2 = c_2) \sum_{c_1 \in \{0,1\}} p(C_2 = c_2 \mid C_1 = c_1) p(C_1 = c_1) \\
&\propto p(D_2 = 0 \mid C_2 = c_2) \sum_{c_1 \in \{0,1\}} p(C_2 = c_2 \mid C_1 = c_1) \\
&\qquad (p(C_1 = c_1) = 0.5, \text{ which we can drop since it's just a proportionality constant}) \\
&\propto p(D_2 = 0 \mid C_2 = c_2) \\
&\qquad (\forall c_2, \textstyle\sum_{c_1} p(c_2 \mid c_1) = 1 \text{ is a valid probability distribution}) \\
&\propto p(D_2 = 0 \mid C_2 = c_2)
\end{aligned}$$

Note that $p(d_2 \mid c_2)$ is a valid probability distribution, so the proportionality constant is 1. Then we have:

$$\mathbb{P}(C_2 = 0 \mid D_2 = 0) = p(D_2 = 0 \mid C_2 = 0) = 1 - \eta$$
$$\mathbb{P}(C_2 = 1 \mid D_2 = 0) = p(D_2 = 0 \mid C_2 = 1) = \eta$$

(b) We compute $\mathbb{P}(C_2 = 1 \mid D_2 = 0, D_3 = 1)$. We note that by the factor graph, we have the following:

$$\begin{aligned}
&\mathbb{P}(C_2 = c_2 \mid D_2 = 0, D_3 = 1) \\
&\propto \mathbb{P}(D_3 = 1 \mid D_2 = 0, C_2 = c_2) p(D_2 = 0 \mid C_2 = c) p(c_2 = c_2) && \text{(Bayes' Rule)} \\
&\propto \mathbb{P}(D_3 = 1 \mid D_2 = 0, C_2 = c_2) p(D_2 = 0 \mid C_2 = c) && (\mathbb{P}(C_2 = c_2) = \tfrac{1}{2}) \\
&\propto \mathbb{P}(D_3 = 1 \mid C_2 = c_2) p(D_2 = 0 \mid C_2 = c) && (D_3 \perp D_2 \mid C_2) \\
&\propto \left[\sum_{c_3 \in \{0,1\}} p(D_3 = 1 \mid C_3 = c_3, C_2 = c_2) p(C_3 = c_3 \mid C_2 = c_2) \right] p(D_2 = 0 \mid C_2 = c) \\
& && \text{(LOTP)} \\
&\propto \left[\sum_{c_3 \in \{0,1\}} p(D_3 = 1 \mid C_3 = c_3) p(C_3 = c_3 \mid C_2 = c_2) \right] p(D_2 = 0 \mid C_2 = c) \\
& && (D_3 \perp C_2 \mid C_3)
\end{aligned}$$

From the above and given the previous result, we compute directly the requested values. We just plug-in and lookup the corresponding conditional distributions:

$$\mathbb{P}(C_2 = 0 \mid D_2 = 0, D_3 = 1) \propto [\eta(1-\epsilon) + (1-\eta)\epsilon](1-\eta)$$
$$\mathbb{P}(C_2 = 1 \mid D_2 = 0, D_3 = 1) \propto [\eta\epsilon + (1-\eta)(1-\epsilon)]\eta$$

From the abive and the fact that we must have a valid distributions, we arrive at the following:

$$\mathbb{P}(C_2 = 1 \mid D_2 = 0, D_3 = 1) = \frac{[\eta\epsilon + (1-\eta)(1-\epsilon)]\eta}{[\eta\epsilon + (1-\eta)(1-\epsilon)]\eta + [\eta(1-\epsilon) + (1-\eta)\epsilon](1-\eta)}$$

(c) We now compute the probabilities requested where $\epsilon = 0.1$ and $\eta = 0.2$.

(i) We have:

$$\mathbb{P}(C_2 = 1 \mid D_2 = 0) = \eta = 0.2$$

and

$$\mathbb{P}(C_2 = 1 \mid D_2 = 0, D_3 = 1) = \frac{[0.2(0.1) + (0.8)(0.9)]0.2}{[0.2(0.1) + (0.8)(0.9)]0.2 + [0.2(0.9) + (0.8)(0.1)](0.8)} = 0.4157$$

(ii) By adding the second sensor reading, the probability that the car was at $C_1 = 1$ increased. This change makes sense, since the second sensor reading $D_3 = 1$ sees that car at position 1. Given that our sensor error rate is low ($\eta = 0.2$), it's likely that the car is at position 1, according to this second reading. Furthermore, since cars change positions with low probability $\epsilon = 0.2$, the probability that $C_2 = 1$ must increase. However, note that the probability is still less than 0.5. This is because our original sensor reading of $D_2 = 0$ has less room for error, so it's still more likely that the positions at $t = 2$ was 0 and that the car simply moved. This is because of our relatively low η value.

(iii) We would have to set $\epsilon = 0.5$. This is because with $\epsilon = 0.5$, at each time-step, the car has equal probabiilty of staying at the current position or alternating to a new one. As such, the sensor readings then become indendent. An additional reading of $D_3 = 1$ gives no further information about the position at $t = 2$, since the car was equally likely to have stayed or swapped positions.