CS221 Fall 2018 Homework 7

SUNet ID: 05794739
Name: Luis Perez
Collaborators:

By turning in this assignment, I agree by the Stanford honor code and declare that all of this is my own work.

Problem 1

(a) We compute $\mathbb{P}(C_2 = 1 \mid D_2 = 0)$. We note that by the factor graph, we have the following:

$$\begin{aligned}
\mathbb{P}(C_2 = c_2 \mid D_2 = 0) &\propto p(D_2 = 0 \mid C_2 = c_2) \sum_{c_1 \in \{0,1\}} p(C_2 = c_2 \mid C_1 = c_1) p(C_1 = c_1) \\
&\propto p(D_2 = 0 \mid C_2 = c_2) \sum_{c_1 \in \{0,1\}} p(C_2 = c_2 \mid C_1 = c_1) \\
&\qquad (p(C_1 = c_1) = 0.5, \text{ which we can drop since it's just a proportionality constant}) \\
&\propto p(D_2 = 0 \mid C_2 = c_2) \\
&\qquad (\forall c_2, \textstyle\sum_{c_1} p(c_2 \mid c_1) = 1 \text{ is a valid probability distribution})
\end{aligned}$$

Note that $p(d_2 \mid c_2)$ is a valid probability distribution, so the proportionality constant is 1. Then we have:

$$\mathbb{P}(C_2 = 0 \mid D_2 = 0) = p(D_2 = 0 \mid C_2 = 0) = \eta$$
$$\mathbb{P}(C_2 = 1 \mid D_2 = 0) = p(D_2 = 0 \mid C_2 = 1) = 1 - \eta$$

(b) We compute $\mathbb{P}(C_2 = 1 \mid D_2 = 0, D_3 = 1)$. We note that by the factor graph, we have the following:

$$\begin{aligned}
\mathbb{P}(C_2 = c_2 \mid D_2 = 0, D_3 = 1) &\propto \mathbb{P}(D_2 = 0 \mid D_3 = 1, C_2 = c_2) \mathbb{P}(D_3 = 1 \mid C_2 = c_2) \mathbb{P}(C_2 = c_2) \\
&\qquad \text{(Bayes' Rule)} \\
&\propto p(D_2 = 0 \mid C_2 = c_2) \mathbb{P}(D_3 = 1 \mid C_2 = c_2) \mathbb{P}(C_2 = c_2) \\
&\qquad (D_2 \perp D_3 \mid C_2) \\
&\propto p(D_2 = 0 \mid C_2 = c_2) \sum_{c_3 \in \{0,1\}} p(D_3 = 1 \mid C_3 = c_3) \\
&\quad \sum_{c_2' \in \{0,1\}} p(C_3 = c_3 \mid C_2' = c_2') \sum_{c_1' \in \{0,1\}} p(C_2' = c_2' \mid C_1' = c_1') p(c_1') \\
&\quad \sum_{c_1 \in \{0,1\}} p(C_2 = c_2 \mid C_1 = c_1) p(c_1) \\
&=
\end{aligned}$$