$\mu = E[X] = \sum_{x \in Supp[X]} x\, p(x)$

$\sigma = Var[X] = \sum_{x \in Supp[X]} (x-\mu)^2 p(x)$

Bet \$1 on #7

$X_7 \sim \begin{cases} \$35 & wp\ \frac{1}{38} \\ -\$1 & wp\ \frac{37}{38} \end{cases}$ $\mu = \$0.053$

Bet \$1 on block

$X_B \sim \begin{cases} \$1 & wp\ \frac{18}{38} \\ -\$1 & wp\ \frac{20}{38} \end{cases}$ $\mu = -\$0.053$

sigma

$\sigma = SE[X] = \sqrt{Var[X]}$

$X_7 \to \mu,\ X_B \to \mu$ which goes faster?

The r.v. with the smallest variance

$\lim_{n\to\infty} \frac{1}{n} = 0 \qquad \lim_{n\to\infty} \frac{1}{e^n} = 0$

$Var[X_7] = (35 - (-0.053))^2 \frac{1}{38} + (-1 - (-0.053))^2 \frac{37}{38} = 33.207\ \$^2$

$Var[X_B] = (1 - (-0.053))^2 \frac{18}{38} + (-1 - (-0.053))^2 \frac{20}{38} = 0.997\ \$^2$

standard error

$\sqrt{Var[X_7]} = \sqrt{33.207\,\$^2} = \$\ 5.79 \Rightarrow SD[X_7] = SE[X_7] = 6$

$\sqrt{Var[X_B]} = \sqrt{0.997\,\$^2} = \$\ 1.00 \Rightarrow SD[X_7] = SE[X_B] = 6$

$T_2 = X_1 + X_2,\ E[T] = \sum_{t \in Supp[T]} t\, p(t) \leftarrow ?$

Notes

① $T = g(X_1, X_2)$

② $E[g(X)] = \sum_{x \in Supp[X]} g(x)\, p(x)$

③ $E[g(X_1, X_2)] = \sum_{\substack{x_1 \in Supp[X_1], \\ x_2 \in Supp[X_2]}} g(x_1, x_2)\, p(x_1, x_2)$ joint PMF

$E[X_1 + X_2] = \sum_{x_1 \in Supp[X_1]} \sum_{x_2 \in Supp[X_2]} (x_1 + x_2)\, p(x_1, x_2)$

$= \sum_{x_1}\sum_{x_2} x_1 p(x_1, x_2) + \sum_{x_2}\sum_{x_1} p(x_1, x_2)$

$= \sum_{x_1} x_1 \sum_{x_2} p(x_1, x_2) + \sum_{x_2} x_2 \sum_{x_1} p(x_1, x_2)$

If X_1, X_2 independent $\Rightarrow p(x_1, x_2) = p(x_1)\, p(x_2)$

$= \sum_{x_1} x_1 \sum_{x_2} p(x_1) p(x_2) + \sum_{x_2}\sum_{x_1} p(x_1)\, p(x_2) = \underbrace{\sum_{x_1} x_1\, p(x_1)}_{E[X_1]} \underbrace{\sum_{x_2} p(x_2)}_{1} + \underbrace{\sum_{x_2} x_2 p(x_2)}_{E[X_2]} \underbrace{\sum_{x_1} p(x_1)}_{1} =$

$= E[X_1] + E[X_2]$

X_1, X_2

$supp[X_1] = \{1, 7, 19\}$

$supp[X_2] = \{5, 23, 88\}$

$$\begin{array}{c|c|c|c|c} & & & X_1 & & \\ & & 1 & 7 & 19 & p(x_1, x_2) \\ \hline X_2 & 5 & \frac{1}{15} & \frac{1}{3} & \frac{2}{15} & 16/30 \\ & 23 & \frac{1}{30} & \frac{1}{10} & \frac{1}{30} & \frac{5}{30} \\ & 88 & \frac{1}{30} & \frac{1}{5} & \frac{1}{15} & \frac{9}{30} \\ \hline & & \frac{4}{30} & \frac{19}{30} & \frac{7}{30} & 1 \end{array}$$

$p(X_2 = 5)$

$\sum_{x_1}\sum_{x_2} p(x_1, x_2) = 1$

$p(X_2 = x) = p(x_2)$

$P(X_1 = 1)$ $\quad \underbrace{}_{p(x_1)}$ ← pmf of X_1

$p(A) = P(A, B_1) + P(A, B_2) + P(A, B_3)$

$B_1 \cup B_2 \cup B_3 = \Omega$

B_1, B_2, B_3 mutex

"marginning out"

$$\sum_{x_2} p(x_1, x_2) = P(x_1)$$

$$\sum_{x_1} p(x_1, x_2) = P(x_2)$$

· Therefore

$$E[X_1 + X_2] = \dots = \dots = \dots = \sum_{x_1} x_1 p(x_1) + \sum_{x_2} x_2 p(x_2) = E[X_1] + E[X_2]$$

$$E[T_n] = \sum_{i=1}^{n} E[X_i] \quad \Big| \quad E[\bar{X}_n] = E\left[\frac{T_n}{n}\right] = \frac{1}{n} E[T_n] = \frac{1}{n}\sum_{i=1}^{n} E[X_i] =$$

$$= \frac{1}{n} n\mu = \boxed{\mu}$$

if $X_1, \dots, X_n$ identity distr.

$X \sim Hyper(n, K, N)$, $E[X] = \sum_{x \in Supp[X]} \frac{x \binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}} = \dots = \frac{K}{nN}$

$X = X_1 + X_2 + \dots + X_n$

$\implies E[X] = \sum E[X_i] = n\mu = n\frac{K}{N}$

$X_1 \sim Bern\left(\frac{K}{N}\right)$

$X_2 \sim Bern\left(\frac{K}{N}\right)$ $X_1, \dots, X_n$ are $\underline{\underline{\text{all}}}$ dependent

$\vdots$

$X_n \sim Bern\left(\frac{K}{N}\right)$ — are all the same because we do not have information on another X

$L = (X - \mu)^2$ $Var[X] :\neq E[L]$

$Var[X] = E[(X-\mu)^2] = E[X^2 - 2\mu X + \mu^2] = E[X^2] + E[-2\mu X] + E[X^2]$

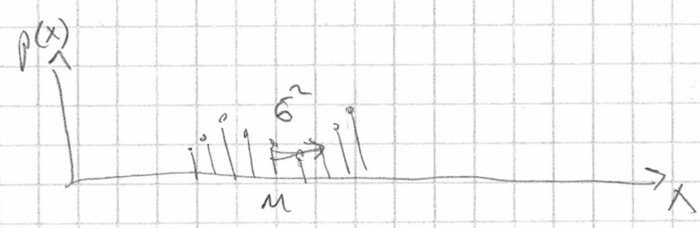
$= \underbrace{E[X^2] - 2\mu \underbrace{E[X]}_{\mu}}_{-2\mu^2} = \boxed{E[X^2] - \mu^2 = Var[X] = \sigma^2}$

$\Downarrow$

$\boxed{E[X^2] = \sigma^2 + \mu^2}$

$Y = aX + c$ $a \in \mathbb{R}, c \in \mathbb{R}$

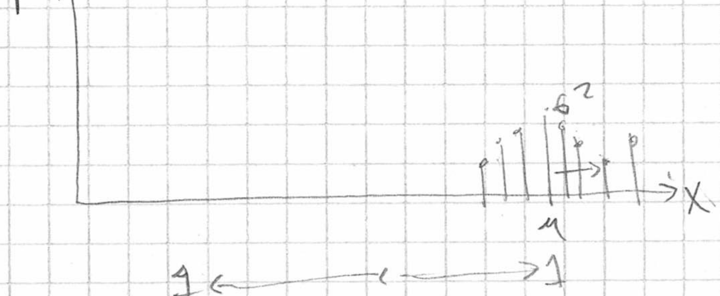
$Y = X + c$, $Var[Y] = Var[X + c] = \sigma^2$



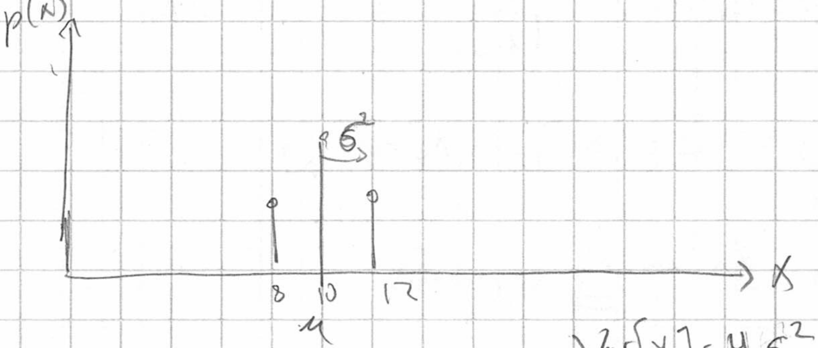
$Var[X+c] = E[((X+c) - (\mu + c))^2]$

$E[X+c] = \mu + c$ $= E[(X + \cancel{c} - \mu - \cancel{c})^2]$

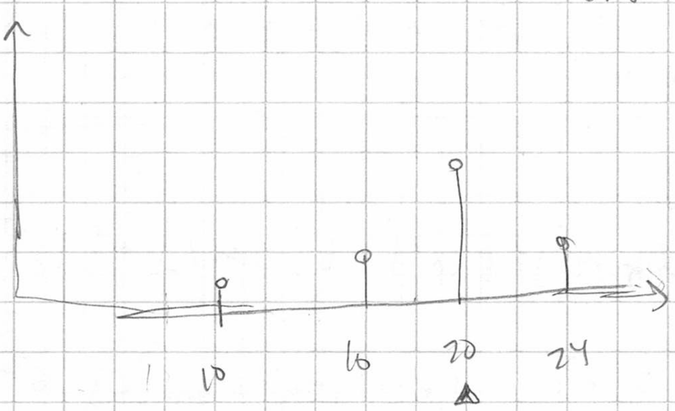
$= E[(X-\mu)^2] = \sigma^2$



$Y = aX \qquad Y = 2X$



$Var[Y] = 4\sigma^2$



$Var[aX] = E[(aX - a\mu)^2] =$

$E[\mu(x-\mu)^2]$

$= E[a^2 (X-\mu)^2]$

$= a^2 E[(X-\mu)^2]$

$= a^2 \sigma^2$

$Var[aX + c] = a^2 \sigma^2$

$SE[aX + c] = |a| \sigma$