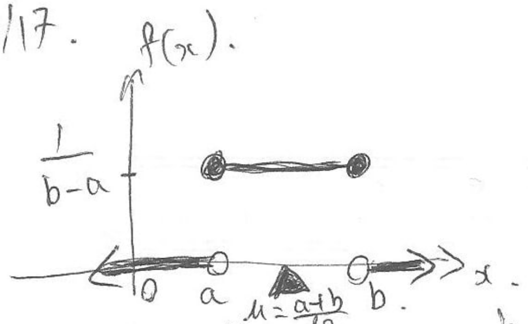
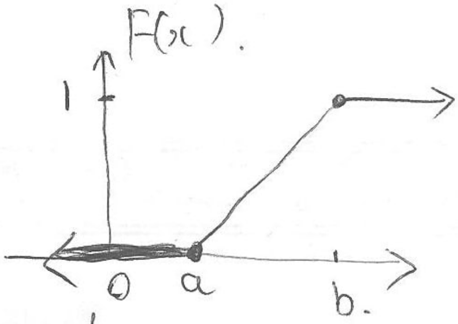
11/20/17.





$$E(x) = \int_{Supp(X)} x f(x)\, dx = \int_a^b x \frac{1}{b-a}\, dx.$$

$$= \frac{1}{b-a} \left[\frac{x^2}{2}\right]_a^b$$

$$= \frac{b^2 - a^2}{2(b-a)} = \frac{(b+a)\cancel{(b-a)}}{2\cancel{(b-a)}}$$

$$\therefore \mu = \frac{b+a}{2}$$

$Med[X] = Quantile[X, \frac{1}{2}]$ $\quad X \sim U(a,b) = \frac{1}{b-a}.$

$\Leftarrow F^{-1}(p)$ is for continuous R.V. $\quad F(x) = \frac{x-a}{b-a}.$

Inverse CDF

$$P = F(x) = \frac{x-a}{b-a}.$$

$$x = p(b-a) + a = F^{-1}(p).$$

$$\therefore Med[X] = Quantile[X, \tfrac{1}{2}] = \frac{1}{2}(b-a) + a.$$

$$= \frac{b}{2} + \frac{a}{2} = \frac{a+b}{2}$$

$$\sigma^2 = Var[X] = E[(X-\mu)^2] = E[X^2] - \cancel{\mu^2} \Rightarrow \left(\frac{a+b}{2}\right)^2.$$

$$E[X^2] = \int_{Supp[X]} x^2 f(x)\, dx = \int_a^b x^2 \frac{1}{b-a}\, dx = \frac{1}{b-a}\left[\frac{x^3}{3}\right]_a^b$$

$$= \frac{1}{b-a} \frac{b^3 - a^3}{3}$$

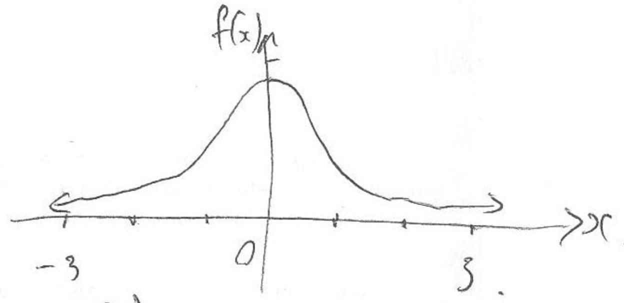
$$= \frac{1}{\cancel{(b-a)}} \frac{\cancel{(b-a)}(b^2+ab+a^2)}{3} = \frac{b^2+ab+a^2}{3}.$$

$$\sigma^2 = \frac{b^2+ab+a^2}{3} - \frac{a^2+2ab+b^2}{4}.$$

$$= \frac{b^2-2ab+a^2}{12} = \frac{(b-a)^2}{12}.$$

$$\sigma = SE[X] = \frac{b-a}{\sqrt{12}}.$$

$$Z \sim N(0,1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$



①. $f(x) > 0$ the $Supp(Z) = \mathbb{R} = (-\infty, \infty)$.

②. $\int_{Supp(X)} f(x)\,dx = 1$.

$$1 = \int_{\mathbb{R}} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}\,dx.$$

$$\Rightarrow \int_{\mathbb{R}} e^{-\frac{x^2}{2}}\,dx = \sqrt{2\pi}$$

let $u = \frac{x}{\sqrt{2}} \Rightarrow u^2 = \frac{x^2}{2}$.

$du = \frac{1}{\sqrt{2}}dx \Rightarrow dx = \sqrt{2}\,du$.

$$\Rightarrow \int_{\mathbb{R}} e^{-u^2}\sqrt{2}\,du = \sqrt{2\pi}.$$

$$\therefore \left(\int_{\mathbb{R}} e^{-u^2} du\right)^2 = \pi$$

$$E[Z] = \int_{Supp[X]} x f(x)\,dx = \int_{\mathbb{R}} x \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}\,dx$$

let $u = \frac{x^2}{2}$

$du = x\,dx$.

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x\, e^{-u} \frac{du}{x}.$$

$$= \frac{1}{\sqrt{2\pi}}\; -1.\left[e^{-u}\right]_{-\infty}^{\infty}.$$

$$\mu = \frac{1}{\sqrt{2\pi}} \left(-1(0-0)\right) = 0.$$

$$\sigma^2 = Var[Z] = E[Z^2] - \cancel{\mu^2}^{\,0} = \int_{Supp[X]} x^2 f(x)\,dx.$$

$$= \int_{\mathbb{R}} x^2 \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}\,dx = \frac{1}{\sqrt{2\pi}} \left\{ \left[x^2 e^{-\frac{x^2}{2}}\right]_{-\infty}^{\infty} - \int_{-\infty}^{\infty} e^{-\frac{x^2}{2}} (2x)\,dx \right\}$$

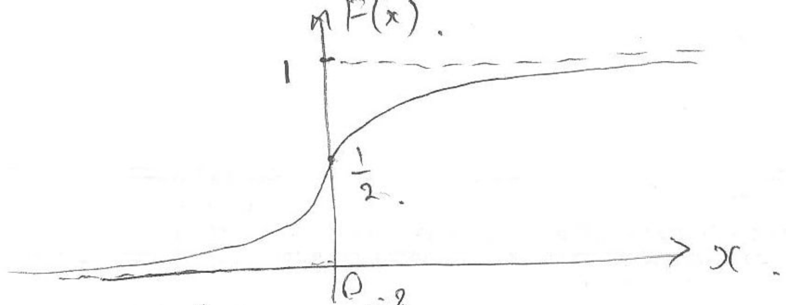
$$\cancel{= \frac{1}{\sqrt{2\pi}} \left\{ \left[x^2 e^{-\frac{x^2}{2}} - 2\left(x e^{x} - e^{x}\right)\right]_{-\infty}^{\infty} \right\}}$$

$$\cancel{= \frac{1}{\sqrt{2\pi}} \left[x^2 e^{-\frac{x^2}{2}} - 2x e^{x} + 2e^{x}\right]_{-\infty}^{\infty}}$$

$$= \frac{1}{\sqrt{2\pi}} \left[x^2 e^{-\frac{x^2}{2}} + 2 e^{-\frac{x^2}{2}}\right]_{-\infty}^{\infty}$$

$$= \frac{1}{\sqrt{2\pi}} \left[(x^2+2) e^{-\frac{x^2}{2}}\right]_{-\infty}^{\infty}.$$

$$= 0\ ?$$



$$F(x) = \int \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx.$$

$$\left.\begin{aligned} P(Z \in [-1,1]) &\approx 0.68 \\ P(Z \in [-2,2]) &\approx 0.95 \\ P(Z \in [-3,3]) &\approx 0.997 \end{aligned}\right\} \text{Empirical Rule, } 3\sigma \text{ Rule.}$$

$$E[Z] = 0, \quad SE[Z] = 1.$$

$$X = \sigma Z + \mu.$$

$$E[X] = \sigma \overset{0}{E[Z]} + \mu = \mu.$$

$$Var[X] = \sigma^2 \overset{1}{Var[Z]} = \sigma^2 \Rightarrow SE[X] = \sigma$$

$$F_X(x) = P(X \leq x) = P(\sigma Z + \mu \leq x)$$

$$= P\left(Z \leq \frac{x-\mu}{\sigma}\right) = F_Z\left(\frac{x-\mu}{\sigma}\right)$$

$$f_X(x) = F'(x) = \frac{d}{dx}\left[F_Z\left(\frac{x-\mu}{\sigma}\right)\right]$$

$$= \frac{1}{\sigma} f_Z\left(\frac{x-\mu}{\sigma}\right) = \frac{1}{\sigma}\frac{1}{\sqrt{2\pi}} e^{-\frac{\left(\frac{x-\mu}{\sigma}\right)^2}{2}}.$$

$$= \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(X-\mu)^2}.$$

$$\Rightarrow N(m, \sigma^2) \quad \text{normal R.V.}$$

Prob: Male height in America is distributed normally with mean $70'' = 5'10''$ and standard error $4''$. What is the probability that a random male is taller than $78'' = 6'6''$?

Ans: $X \sim N(70'', 4''^2)$

$$P(X \geq 78) = P\left(\frac{X-70}{4} \geq \frac{78-70}{4}\right)$$

$$P(Z \geq 2) = 2.5\%.$$

Standard Normal