Pdf
-2mb

8/28

Set theory (1870's)

The Fundamental units of math are the "set".

A "set" is collection of ~~ep~~ elements/objects, which are unordered and unique.

Ex.: F ::= {Jane, Mary, Susan, Dana}

"defined is"
"assigned to"

Denotes the set on the rhs (right handside)

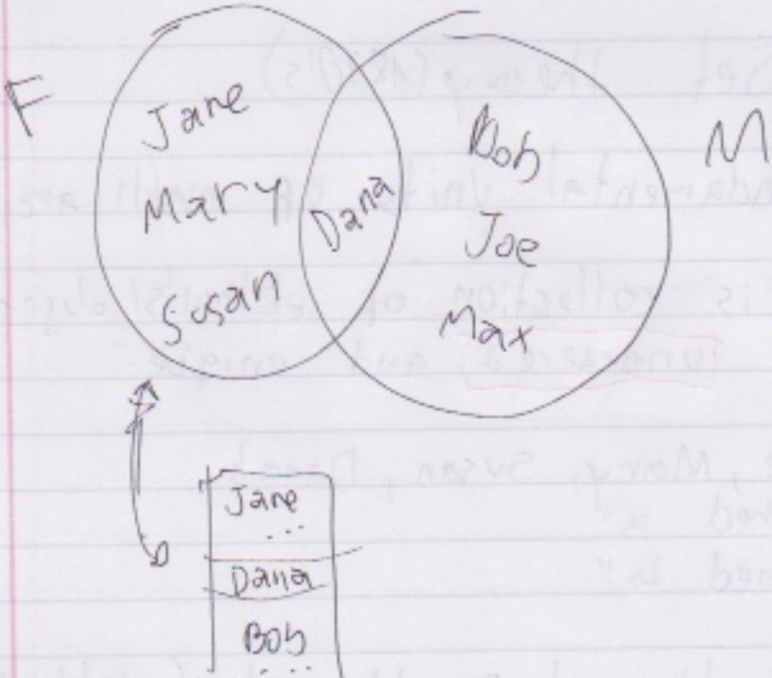
Usually we pick an descriptive letter

The braces begin and end the enumeration
comma separates elements

Ex.: M ::= {Bob, Joe, Max, Dana}

Venn Diagram examples:

Ilustration of sets and their relationships ...



Sets can have infinite elements

e.g. $\mathbb{N} ::= \{1, 2, 3, \ldots\}$ Natural #'s

ellipses is to denote the pattern continues

$\mathbb{Z} ::= \{\ldots, -2, -1, 0, 1, 2, \ldots\}$ Integrals

Operators on sets

Element operator

$\underset{\text{element}}{\underset{\uparrow}{Jane}} \in \underset{\text{set}}{\underset{\uparrow}{F}}$

Jane is an element of set F

Never write false statement

Joe $\notin$ F

Joe isn't an element of F

$\underset{\text{set}}{\underset{\uparrow}{\{Jane, Mary\}}} \underset{\text{sub set}}{\underset{\uparrow}{\subseteq}} \underset{\text{set}}{\underset{\uparrow}{F}}$

{Jane, Mary} is subset of F

All elements of set left h s. are in the set of rhs

{Joe, Mary} $\not\subseteq$ F

"Not a subset of"

let $F' := \{Jane, Mary, Susan, Dana\}$

$F = F'$? yes! true statement

set equality

means

$F \subseteq F'$ & $F' \subseteq F$

$\{Jane, Mary\} \neq F$

$\{Jane, Mary\} \subset F$

proper subset

lhs is a subset on the rhs
but the lhs $\neq$ rhs

"$\subseteq$" is "$\subset$" or "$=$"

True / False?

$\{Jane\} \subset F$ $\boxed{true}$
$\{Jane\} \in F$ False $\{Jane\} \notin F$
Jane $\in F$ true
Jane $\notin F$ true

Jane $\subset F$ ≠ within a definition the set Jane, does not "parse"

So far we have

$\in, \notin, =, \neq, \subset, \subseteq$ these are pre functions which return true or false

$\in(\text{Jane}, F) = \text{true}$

Set Functions

$F \cup M = \{$Jane, Mary, Susan, Dana, Bob, Joe, Max$\}$
Union

Union ≠ Addition it is almost addition
↳ "non-exclusive or" = "and/or"

$\{Dana\} \cup \{Dana\} = \{Dana\}$

$Dana \in M \cup F$ $\boxed{\text{true}}$

$\mathbb{N}_0 := \mathbb{N} \cup \{0\}$

$F \cap M = \{Dana\}$

↳ intersection "and"
elements in both sets

$F \cap \{Bob, Joe\} = \{\}$

$\emptyset = \{\}$ the empty or "null set"

A, B both have infinite elements.

can $A \cap B = \emptyset$?

Yes

Ex.: $A = \{0, 2, 4, \ldots\}$
$B = \{1, 3, 5, \ldots\}$

$\emptyset \subseteq F$ $\boxed{\text{"Vacuosly" true}}$

$\emptyset \in F$ $\boxed{\text{false}}$ $\emptyset \notin F$ $\boxed{\text{true}}$

Set Subtraction

$F \setminus M = \{\text{Jane}, \text{Mary}, \text{Susan}\}$

All elements in F except these in M

$\neq$

$M \setminus F = \{\text{Bob}, \text{Joe}, \text{Max}\}$

examples

If $A \cap B = \emptyset$ $\Rightarrow A \setminus B = A$

$\Rightarrow B \setminus A = B$

If $A \setminus B = \emptyset$ $A \cap B = A$

$\emptyset \setminus \emptyset = \emptyset$, $\emptyset \cap \emptyset = \emptyset$, $\emptyset \cup \emptyset = \emptyset$

$A \subseteq B \Rightarrow A \setminus B = \emptyset$

Set Builder Notations

$E ::= \{ 2n : n \in \mathbb{Z} \} = \{ \ldots, -1, -2, 0, 2, 4 \ldots \}$

new set

~~unit~~ elements $2 \cdot n$

such that

n is an integer

$2^A ::= \{ B : B \subseteq A \} = \{ \emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{1,3\}, A \}$

Power Set

e.g. let $A = \{1, 2, 3\}$

$\{1,2\}, \{2,3\}, \{1,3\}$

A size of set / cardinality

$|A|$ = # of elements of set, in A = 3

$|F \cup M| = 7$

$|2^A| = 8$

$|F \cap M| = 1$