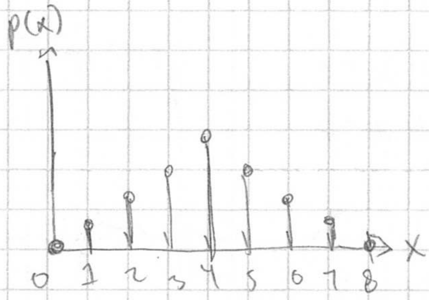
avg $\overline{X}_n := \dfrac{X_1 + \cdots + X_n}{n}$ $\quad T_n = X_1 + \cdots X_n$

r.v.

$X \sim Bin\left(8, \frac{1}{2}\right)$

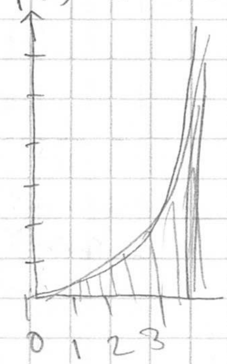
5	3	7
7	4	5
5	5	4
3	4	3
4	4	4
8	6	3

$\overline{X} = \dfrac{26+26+26}{6+6+6} = 4.33 \xrightarrow{?} 4$

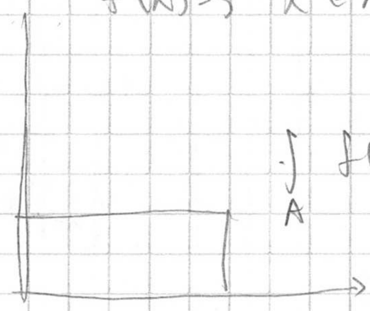


pivot pt.

$f(x) = x^2 \quad x \in \overset{A}{[0,3]}$



$f(x) = 3 \quad x \in A$



$\int_A f(x) = 9$

$\frac{d}{dx}\left[f(x)\right] \quad x = 4$

$G[f] = \int f(x)\,dx = 9$

operator (function of a function)

$0 =$

$\sum_{i \in \text{objects}} w_i (d_i - d^*) \Rightarrow \sum w_i d_i - \sum w_i d^* = 0$

$\Rightarrow \sum w_i d_i = d^* \sum w_i \Rightarrow d^*$

$= \dfrac{\sum w_i d_i}{\sum w_i} = \dfrac{100 \cdot 0 + 20 \cdot 1}{100+20} \approx 0.10$

$\dfrac{\sum p_i x_i}{\sum p_i} = \dfrac{\sum_{x \in Supp[x]} x\,p(x)}{\underbrace{\sum_{x \in Supp[x]} p(x)}_{1}}$

$E[x] = \mu = \sum_{x \in Supp[x]} x\,p(x)$

"expectation"

$X \sim Bin\left(8, \frac{1}{2}\right) = \binom{8}{x}\left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{8-x}$

$E[x] = 0 \cdot p(0) + 1 \cdot p(1) + 2p(2) + 3p(3) + 4p(4) + 5p(5) + 6p(6) + 7p(7) + 8p(8)$

$= 0 + 1 * 0.031 + 2 * 0.109 + 3 * 0.219 + 4 * 0.273 + 5 * 0.219 + 6 * 0.109 + 7 * 0.31 + 8 * 0.004 = 4$

$X \sim Bin(187, 0.392)$

$X \sim Bin(n, p)$

$E[X]$

$$E[X] = \sum_{x=0}^{n} x \binom{n}{x} p^x (1-p)^{n-x} = \sum_{x=1}^{n} x \binom{n}{x} p^x (1-p)^{n-x} =$$

$$= n \sum_{x=1}^{n} \cancel{x} \frac{(n-1)!}{\underbrace{\cancel{x!}}_{(x-1)!} (n-x)!} p^x (1-p)^{n-x} = np \sum_{x=1}^{n} \binom{n-1}{x-1} p^{x-1} (1-p)^{n-x}$$

$$(n-x)! \to \underbrace{((n-1)-(x-1))!}_{\binom{n-1}{x-1}}$$

let $m = n-1$ $\quad y = 0 \ldots n-1$

let $y = x-1$

$$= np \underbrace{\sum_{y=0}^{m} \binom{m}{y} p^y (1-p)^{m-y}}_{1}$$

$Y \sim Bin(m,p)$

$p(y)$

$$\boxed{E[X] = np}$$

$X \sim Hyper(n, K, N)$

$$E[X] = \sum_{x \in Supp[X]} x \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}} = \ldots = n\frac{K}{N}$$

$X \sim Unif([1, 10, 100])$ $\quad E[X] = 1 \cdot \frac{1}{3} + 10 \cdot \frac{1}{3} + 100 \cdot \frac{1}{3}$

$X \sim Bern(p)$

$E[X] = 0 \cdot (1-p) + 1(p) = p$

$X \sim Geometric(p) := (1-p)^{x-1} p$

$p \in (0,1)$ $\quad supp[X] = \mathbb{N}$

$X \sim \text{Geometric}(p=0.2) = 0.8^{x-1} \times 0.2$

x	p(x)	E(x)
1	0.200	0.200
2	0.160	0.360
3	0.128	0.488
4	0.102	0.590
5	0.082	0.672
6	0.066	0.738
7	0.052	0.790
8	0.042	0.832
9	0.034	0.866
10	0.027	0.893
11	0.021	0.914
12	0.017	0.931
13	0.014	0.945
14	0.011	0.956
15	0.009	0.965
16	0.007	0.972
17	0.006	0.978
18	0.005	0.983
19	0.004	0.987
20	0.003	0.990
21	0.002	0.992
22	0.001	0.994
23	0.001	0.995
24	0.001	0.996
25	0.001	0.997

pivot pt = 5

"Effective Support"

$E\ \text{supp}[X] = \{1, \ldots, 27\}$

$= \{x : p(x) > 0.001\}$

(n, K, N)

$$E(x) = \sum_{x=1}^{\infty} x(1-p)^{x-1} p = \sum_{y=0}^{\infty} (y+1)(1-p)^y p = \sum_{y=1}^{\infty} y(1-p)^y p + \sum_{y=0}^{\infty} (1-p)^y p$$

$$= \underbrace{(1-p)\sum_{y=1}^{\infty} y(1-p)^{y-1} p}_{E[x]} + \underbrace{\sum_{x=1}^{\infty} (1-p)^{x-1} p}_{1}$$

$\Rightarrow \mu = (1-p)\mu + 1$

$\Rightarrow \mu - (1-p)\mu = 1$

$\Rightarrow \mu(1-(1-p)) = 1 \longrightarrow \boxed{\mu = \frac{1}{p}}$

$\mathrm{Min}[X] = \min\{\mathrm{Supp}[X]\}$

$\mathrm{Max}[X] = \max\{\mathrm{Supp}[X]$

$\mathrm{Range}[X] = \max[X] - \min[X]$

$\mathrm{Mode}[X] = \arg\max\{p(x)\}$

$\mathrm{Quantile}[X, p] = \arg\min\{F(x) \geq p\}$

$100 \times$ Quantile = Percentile

$\mathrm{Med}[X] := Q[X, 0.5]$

median

$E(X)$

$\mathrm{med}[X] < E(X) \Rightarrow X$ is right-sk

$\mathrm{med}[X] > E(X) \Rightarrow X$ " left "

Tertiles

$Q[X, \frac{1}{3}], Q[X, \frac{2}{3}]$

Quartiles

$Q[X, \frac{1}{4}], \mathrm{Med}(X), Q[X, \frac{3}{4}]$

Deciles

$Q[X, \frac{1}{10}], Q[X, \frac{2}{10}], \ldots, Q[X, \frac{10}{10}]$

Quintiles

$Q[X, \frac{1}{5}]$

$IQR[X] := Q[X, \frac{3}{4}] - Q[X, \frac{1}{4}]$

inter