LECTURE - 07 :-

09/19/2017

* $P(A) = 0.2$ → smoking

$P(B) = 0.06$ → lung cancer

$P(AB) / P(A \cap B) = 0.036$

• $P(B|A) = P($lung cancer among smokers$)$

↑ "given" / "conditional on"

$= (\text{scale})\, P(AB)$

Ex:

$\therefore \text{zoom} = \dfrac{\text{Prior scope size}}{\text{New scope size}} = \dfrac{1}{0.5} = \boxed{2}$

$$\therefore P(B|A) = \frac{\overset{\text{Axiom } (P(\Omega)=1)}{P(\Omega)}}{P(A)} \cdot P(AB)$$

$$= \frac{P(AB)}{P(A)} \left\{ \begin{array}{l} \text{def. of Conditional} \\ \text{Prob.} \end{array} \right.$$

** only valid for $P(A) \neq 0$

- $P(A|B) = \dfrac{P(AB)}{P(B)}$

$$\Rightarrow P(AB) = P(A|B) \cdot P(B)$$

$$\therefore P(B|A) = \frac{P(A|B) \cdot P(B)}{P(A)} \left\{ \begin{array}{l} \text{Bayes} \\ \text{Rule, 1763} \end{array} \right.$$

$$\therefore P(B|A) = \frac{P(AB)}{P(A)} = \frac{0.036}{0.2} = 0.18$$

- $P(A|B) = \dfrac{P(AB)}{P(B)} = \dfrac{0.036}{0.06} = 0.6$

<u>English:-</u>

$P(A|B)$ → What is the probability if you are a smoker and have a lung cancer?

→ Among people have lung cancer and majority of them smoked.

- $P(\text{lung cancer among non-smoker}) = P(B|A^c) = \dfrac{P(BA^c)}{P(A^c)}$

$P(B) = P(AB) + P(A^cB)$

$\Rightarrow P(A^cB) = P(B) - P(AB)$

$= 0.06 - 0.036$

$= 0.024$

$\therefore P(B|A^c) = \dfrac{0.024}{1 - P(A)} \rightarrow$ Axiom

$= \dfrac{0.024}{0.8} = 0.03.$

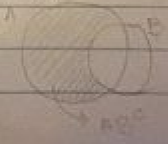
- $\dfrac{P(B|A)}{P(B|A^c)} = \dfrac{0.18}{0.03} = 6 \;\Big\}$ Risk Ratio

English:-

Increasing the chance of lung cancer "6" times by changing from a non-smoker to a smoker.

- $P(A|B^c) = \dfrac{P(AB^c)}{P(B^c)} = \dfrac{0.164}{0.94} = 0.174$

$P(A) = P(AB^c) + P(AB)$

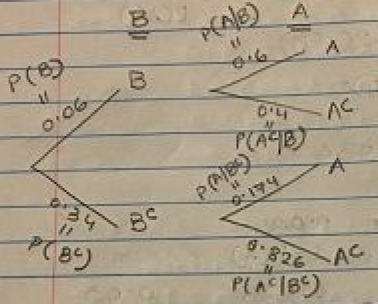


$\Rightarrow P(AB^c) = P(A) - P(AB)$

$= 0.02 - 0.036$

$= 0.164$

* **Tree Illustration :-**



Joint Outcome

$P(AB) = P(B) \cdot P(A|B)$
$= 0.036$

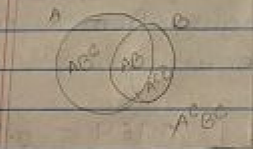
$P(A^cB) = 0.024$

$P(AB^c) = 0.164$

$P(A^cB^c) = 0.776$

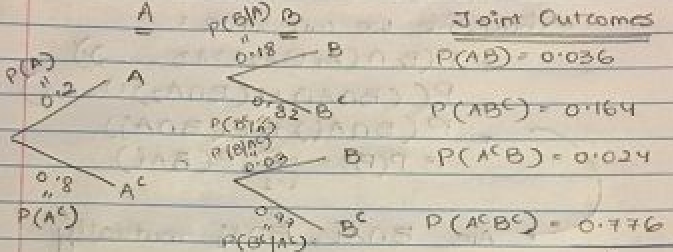
** The branches are the "conditional probabilities".

** When we add up all the joint outcomes we get "1".



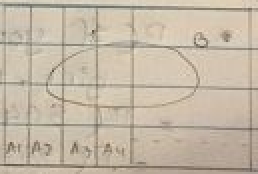
$\therefore P(AB^c) + P(AB) + P(A^cB) + P(A^cB^c) = \boxed{1}$

* Inversion Illustration:-



** $P(B^c|A^c)$ → given that the person is a non-smoker and doesnot get lung cancer.

* Consider $A_1, A_2, \ldots$ mutually exclusive and collectively exhaustive and event B.
→ "B" can be made up of all the intersections of $A_1, A_2, A_3 \ldots$
→ This is called "Law of Total Probability".



- $P(B) = P(B \cap \Omega)$
As $A_1, A_2, A_3, \ldots$ are collectively exhaustive we can write:
$$= P(B \cap (A_1 \cup A_2 \cup A_3 \ldots))$$
$$= P((B \cap A_1) \cup (B \cap A_2) \cup \ldots)$$
$$= P(B \cap A_i) + P(B \cap A_j)$$
$$\therefore P(B) = \sum_{i=1}^{\infty} P(B A_i)$$

Are $B \cap A_i$, $B \cap A_j$ mutually exclusive?

$$(B \cap A_i) \cap (B \cap A_j) \stackrel{?}{=} \emptyset$$
$$= \underbrace{B \cap B}_{B} \cap \underbrace{A_i \cap A_j}_{\emptyset} = B \cap \emptyset = \boxed{\emptyset} \checkmark$$

* Assume girl births and boy births are equally likely.

- P(If you had 2 kids and 1 is a girl, the other is a girl)
$$= P(\{GG\} \mid \{GG, BG, GB\})$$
$$= \frac{P(\{GG\} \cap \{GG, GB, BG\})}{P(\{GG, BG, GB\})}$$
$$= \frac{1/4}{3/4} = \boxed{\frac{1}{3}}$$

Here the $\Omega = \{BB, BG, GB, GG\}$

* <u>Uniform Discreet Probability Distribution Tree Illustration</u> :-