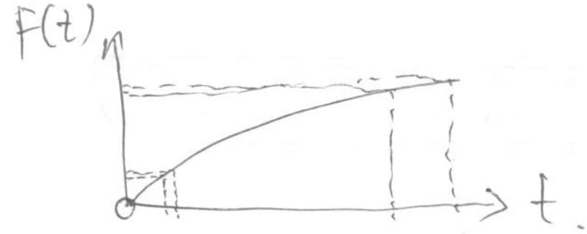
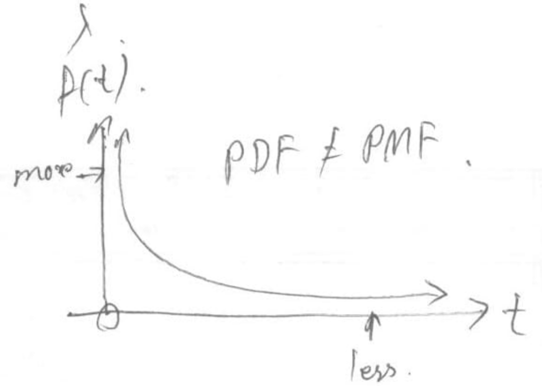
17. $F(t) = 1 - e^{-\lambda t}$.





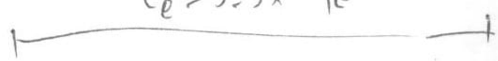
PDF ≠ PMF.

$F'(t) = \lambda e^{-\lambda t}$

P of Density $\Rightarrow$ $Supp(T) = (0, \infty)$.

$|Supp(T)| = |\mathbb{R}| > |\mathbb{N}|$ (not discrete).

$t_e = 5.3 \times 10^{-44}$.



$P(3) = P(T = 3.00000\bar{0})$.

~~Part~~ $P(3) \approx P(T \in [2.995\bar{0}, 3.004\bar{9}])$

$= F(3.004\bar{9}) - F(2.995\bar{0}) > 0$.

$\therefore P(T \in [a,b]) = F(b) - F(a) = \int_a^b f(t)\,dt$.

if $\lambda = 2 \Rightarrow f(t) = 2e^{-2t}$.

$f(1) = 2e^{-2(1)} \approx 0.27 \neq P(1) = 0$.

$f(0.1) = 2e^{-2(0.1)} \approx 1.63 \neq P(0.1) = 0$.

$\frac{f(0.1)}{f(1.0)} \approx \frac{1.63}{0.27} \approx 6$.

$$\lim_{\varepsilon \to 0} \frac{P(T = [0.1, 0.1+\varepsilon])}{P(T \in [1.0, 1.0+\varepsilon])} \cdot \frac{\varepsilon}{\varepsilon}$$

$$= \frac{\displaystyle\lim_{\varepsilon \to 0} \frac{F(0.1+\varepsilon) - F(0.1)}{\varepsilon}}{\displaystyle\lim_{\varepsilon \to 0} \frac{F(1+\varepsilon) - F(1)}{\varepsilon}} = \frac{f(0.1)}{f(1.0)} \ \#$$

$$P(T \in (-\infty, \infty)) = F(\infty) - F(-\infty) = \int_{-\infty}^{\infty} f(t)\,dt = 1.$$

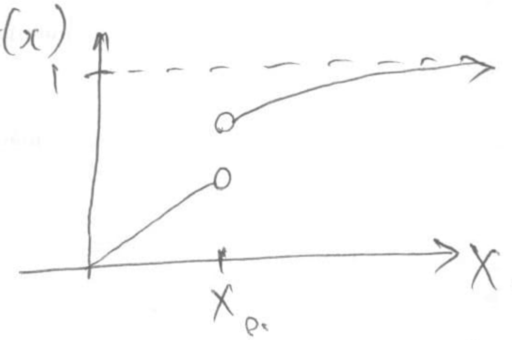
$$= P(T \in (0, \infty)) = F(\infty) - F(0)$$

$$= \int_{0}^{\infty} f(t)\,dt \Rightarrow \int_{Supp[X]} f(x)\,dx = 1.$$

Def: of continuous R.V X.

(a) $|Supp[X]| = |\mathbb{R}| > |\mathbb{N}|$ not discrete.

(b) $F(x)$ is a valid CDF with no discontinuations.

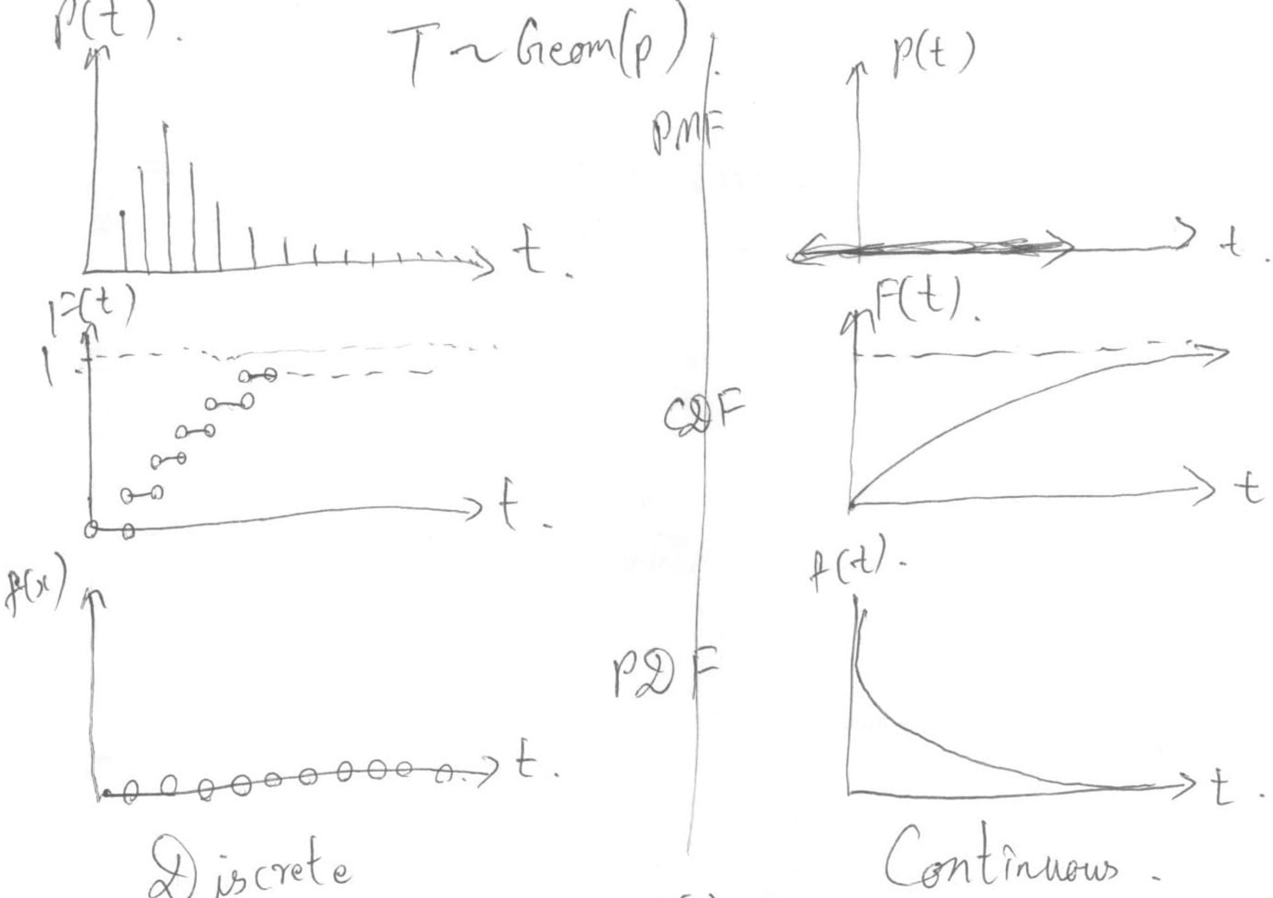


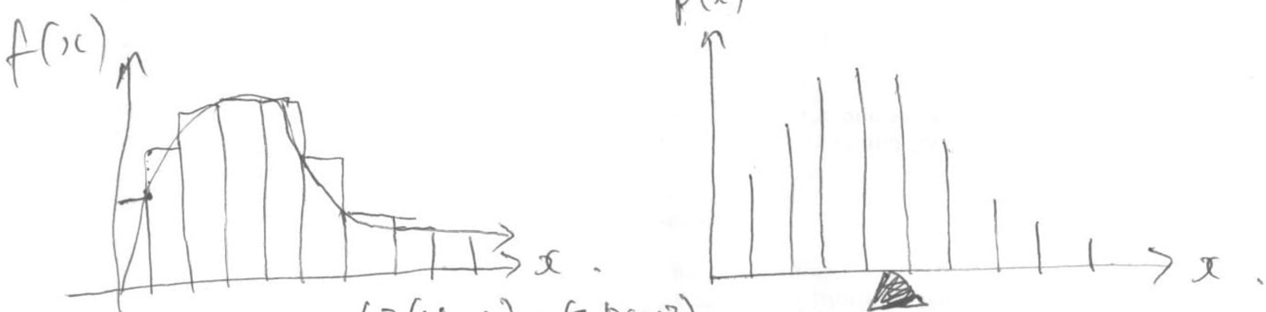
(c) PMF DNE $P(X) = 0 \ \forall x$.

(d) $f(x)$ is the PDF

① $f(x) \geq 0 \ \forall x$ $\quad F'(x) = f(x)$.

② $\int_{Supp[X]} f(x) = 1$.





$$E[X] = \sum_{Supp[X]} x \left(F(X+\varepsilon) + F(X)\right)$$

$$E[g(x)] = \int_{Supp[X]} g(X) f(X)\, dx.$$

$$Var[X] = E[(X-\mu)^2] = \int_{Supp[X]} (X-\mu)^2 f(x)\, dx.$$

$$E\left(\sum X_i\right) = \sum E[X_i] = n\mu.$$

$$E(aX+c) = a\mu + c.$$

$$Var\left[\sum X_i\right] = \sum Var(X_i) = n\sigma^2.$$

$$\mathrm{Var}[aX+c] = a^2\sigma^2.$$

$$SE[aX, \mu] = |a|\sigma$$

$$X \sim \mathrm{Exp}(\lambda) := \lambda e^{-\lambda t}.$$

$$\mathrm{Supp}[X] = (0,\infty)$$

$$\lambda \in (0,\infty)$$

$$E[X] = \int_{\mathrm{Supp}[X]} x f(x)\,dx = \int_0^\infty x\lambda e^{-\lambda x}\,dx = \lambda\int_0^\infty x e^{-\lambda x}\,dx = \frac{1}{\lambda}$$

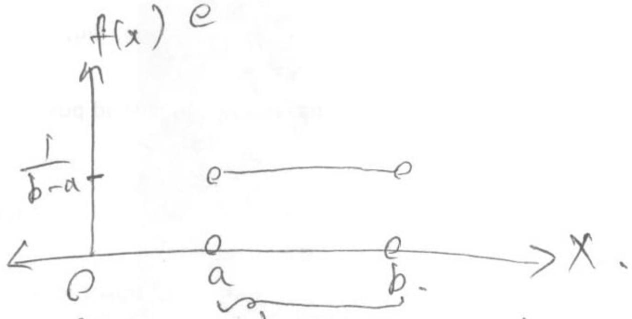
<u>Exponential is Memoryless.</u>

$$F(x) = 1 - e^{\lambda x}, \quad P(X>x) = 1 - F(x) = e^{-\lambda X}.$$

$$P(X>a+b \mid X>b) = \frac{P(X>a+b \;\&\; X>b)}{P(X>b)}.$$

$$= \frac{P(X>a+b)}{P(X>b)} = \frac{e^{-\lambda(a+b)}}{e^{-\lambda b}} = e^{-\lambda a} = P(X>a).$$

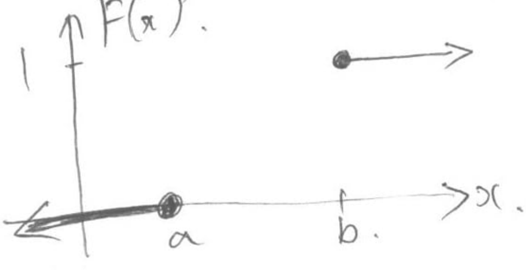
<u>$X \sim U(a,b)$</u>



$$\mathrm{Supp}[X] = (a,b).$$

$X \sim U(a,b) := \frac{1}{b-a}$, Parameter Space $a \in \mathbb{R}$ $b \in \mathbb{R}$ <u>$b>a$</u>.

$$F(X) = \int f(x)\,dx + c = \int \frac{1}{b-a}\,dx + c = \frac{x}{b-a} + c.$$



$$F(b) = 1 \Rightarrow \frac{b}{b-a} + c = 1 \Rightarrow c = 1 - \frac{b}{b-a}.$$

$$= \frac{-a}{b-a}.$$