Lecture :- 15

1

$\sigma = S.E(X) = \sqrt{Var(X)}$

$\mu = E(X) = \sum_{x \in supp(x)} x p(x)$

$\sigma^2 = Var[X] = E(x-\mu)^2 = \sum_{x \in supp(x)} (x-\mu)^2 \cdot p(x)$

Q) Bet on #7 \$1 $\mu = -0.053$

$$X_7 \sim \begin{cases} \$35 & \text{wp } \frac{1}{38} \\ -1 & \text{wp } 37/38 \end{cases}$$

$$\sigma^2 = (35 - 0.053)^2 \frac{1}{38} + (-1 - (-0.053))^2 \frac{37}{38} = 33.207 \$^2$$

Q) Bet on black \$1

$$X_B \sim \begin{cases} \$1 & \text{wp } 18/38 \\ -\$1 & \text{wp } 20/38 \end{cases} \Rightarrow \mu = -0.053$$

$$\sigma^2 = (1 - (-0.053))^2 \left(\frac{18}{38}\right) + (-1 - (-0.053))^2 \left(\frac{20}{38}\right) = 0.997 \$^2$$

$\bar{X}_7 \to \mu$, $\bar{X}_B \to \mu$ which goes faster?

The r.v. with the smallest variance

$\lim_{n \to \infty} \frac{1}{n} = 0$ $\lim_{n \to \infty} \frac{1}{e^n} = 0$

$Var[X_7] = (35 - (-0.053))^2 \frac{1}{38} + (-1 - (-0.053))^2 \frac{37}{38} = 33.207 \$^2$

$Var[X_B] = 0.997 \$^2$

Standard error:

$\sqrt{Var[X_7]} = \sqrt{33.207 \$^2} = \$5.79 \Rightarrow SD[X_7] = S.E(X_7) = \sigma$

$\sqrt{Var(X_B)} = \sqrt{0.997 \$^2} = \$1.00 \Rightarrow SD(X_B) = S.E[X_B] = \sigma$

$T_2 = X_1 + X_2$, $E[T] = \sum_{t \in supp(T)} t \cdot p(t) \leftarrow ?$

X_1 & X_2 are independent, $P(X_1 X_2) = P(X_1) \cdot P(X_2)$

$$E(X_1 + X_2] = \sum_{x_1} x_1 \sum_{x_2} P(X_1) \cdot P(X_2) + \sum_{x_2} x_2 \sum_{x_1} P(X_1) \cdot P(X_2)$$

$$= \underbrace{\sum_{x_1} x_1 P(X_1)}_{E(X_1]} \cdot \sum_{x_2} P(X_2) + \underbrace{\sum_{x_2} x_2 P(X_2)}_{E(X_2]} \cdot \sum_{y_1} P(X_1) = E(X_1] + E(X_2]$$

X_1, X_2

$Supp[X_1] = \{1, 7, 9\}$
$Supp[X_2] = \{5, 23, 88\}$

$X_1 \sim \begin{cases} 1 \text{ wp } 4/30 \\ 7 \text{ wp } 19/30 \\ 9 \text{ wp } 7/30 \end{cases}$

		X_1: 1	7	19	$P(X_1, X_2)$	
X_2	5	$\frac{1}{15}$	$\frac{1}{3}$	$\frac{2}{15}$	$16/30$	$P(X_2=5)$
	23	$\frac{1}{30}$	$\frac{1}{10}$	$\frac{1}{30}$	$5/30$	$P(X_2=23)$
	88	$\frac{1}{30}$	$\frac{1}{5}$	$\frac{1}{15}$	$\frac{9}{30}$	$P(X_2=88)$
		$4/30$	$19/30$	$7/30$	$1 = ?$	
		$P(X_1=1)$	$P(X_1=7)$	$P(X_1=19)$	(all possible prob. of (X_1))	

"marginaling out"

$\sum_{X_2} P(X_1, X_2) = P(X_2)$

$\sum_{X_1} P(X_1, X_2) = P(X_1)$

X_1, X_2 are independent? $\boxed{No}$

$P(X_1 = 1, X_2 = 5) = P(X_1 = 1) \cdot P(X_2 = 5)$

$\frac{1}{15} \neq \frac{4}{30} \cdot \frac{16}{30}$

$E(T_n) = \sum_{i=1}^{n} E[X_i]$

$E[\bar{X}_n] = E[\frac{1}{n} T_n] = \frac{1}{n} E(T_n)$
$= \frac{1}{n} \cdot n\mu = \boxed{\mu}$

$X \sim Hyper(n, k, N)$, $E(X) = \sum_{x \in Supp(X)} \frac{x \binom{k}{x} \binom{N-k}{n-x}}{\binom{N}{n}} = \ldots = \frac{k}{nN}$

$X = X_1 + X_2 + X_3 + \ldots + X_n$

$\left.\begin{array}{l} X_1 \sim Bern\left(\frac{k}{N}\right) \\ X_2 \sim Bern\left(\frac{k}{N}\right) \\ \vdots \\ X_n \sim Bern\left(\frac{k}{N}\right) \end{array}\right\}$ identically distributive

$\Rightarrow E(X) = \sum E[X_i] = n \cdot \mu = n \cdot \frac{k}{N}$

$X_1, \ldots, X_n$ are all independent

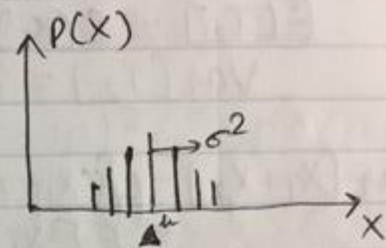
$Var[X] = E[(X-\mu)^2] = E(X^2 - 2\mu X + \mu^2) = E(X^2) + E(-2\mu X) + E(\mu^2)$

$= E(X^2) - \underbrace{2\mu \underbrace{E(X)}_{\mu}}_{-2\mu^2} + \mu^2$

$\therefore \sigma^2 = E(X^2) - \mu^2$

$E(X^2) = \sigma^2 + \mu^2$ //

$$\sum_x \mu^2 \cdot P(x) = \mu^2 \cdot \sum_x P(x) = \mu^2 \cdot 1 = \mu^2$$

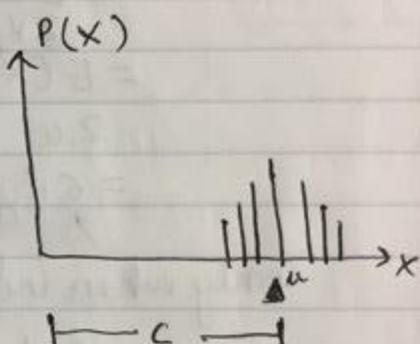


$\#$ $Y = aX + c$, $a, c \in \mathbb{R}$

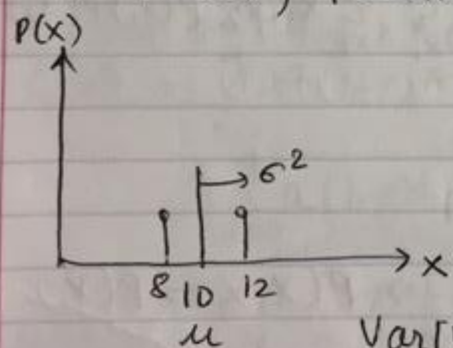
$E[Y] = aE[X] + c$

$Var[Y] = Var[X + c] = \sigma^2$

$$Var(X+c] = E[((X+c) - (X+c))^2]$$
$$= E[((X+c) - (\mu + c))^2]$$
$$= E[(X - \mu^2)]$$
$$= \sigma^2$$



$\cdot\ Y = aX$, $Y = 2X$



$Var[Y] = 4\sigma^2$

$$Var(aX] = E[(aX - a\mu)^2]$$
$$= E[\mu(X - \mu)^2]$$
$$= E(a^2(X-\mu)^2]$$
$$= a^2 E[(X - \mu)^2]$$
$$= a^2 \sigma^2 ,$$

$\because Y = aX + c$,

$Var[aX + c] = a^2 \sigma^2$

$S.E[Y] = |a| \sigma$,