Imagine $n=1000$ people (Ω)

200 smokers (A)

60 lung cancer (B)

36 S & l.c (AB) $A \cap B$, A,B or $A \& B$

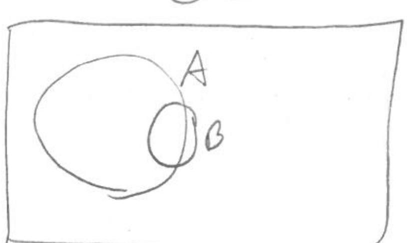
Via LRF Def...

$P(A) = \frac{200}{1000} = 0.2$

$P(B) = \frac{60}{1000} = 0.06$

$P(AB) = \frac{36}{1000} = 0.036$

Ω

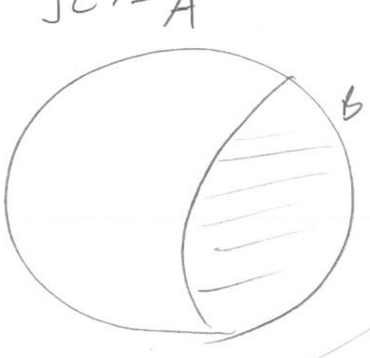


What if I want to know the "probability of l.c. only among smokers" given smoking, What is $P(\text{l.c.})$? Reversal...

$$P(B|A)$$

Then you just need to look at A and ignore the rest of the Ω.

$$\Omega' := A \subseteq \Omega$$

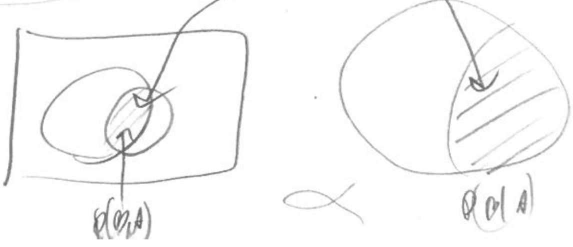


$$P(B|A) = \frac{36}{200} = 0.18$$

What if we only had prob's?

Same shape but zoomed in...

Note



just like

$P(B,A) \propto P(B|A)$

$P(B|A) = P(B,A) \cdot \text{Zoom}$

Let's take a look at zooming

Zoom: ?

$$\text{Zoom} = \frac{\text{prior scope size}}{\text{new scope size}} = \frac{1}{0.5} = 2 \qquad \text{Zoom} = \frac{P(\Omega)}{P(A)} = \frac{1}{P(A)}$$

$$\boxed{P(B|A) := \frac{P(B,A)}{P(A)}}$$ Def. of cond. prob.

$\Rightarrow P(B,A) = P(B|A)P(A)$ corollary

if $P(A|B) = \frac{P(A,B)}{P(B)} = \frac{P(B,A)}{P(B)}$

$\Rightarrow \boxed{P(A|B) = \frac{P(B|A)P(A)}{P(B)}}$ "Bayes Rule" (1763)

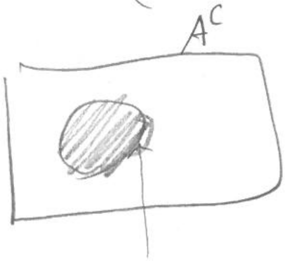
$P(B|A) = \frac{.036}{.2} = .18 \approx 20\%$ smoke is corr. with l.c.

low? high?

$P(\text{smoke}|\text{l.c.}) = P(A|B) = \frac{P(A,B)}{P(B)} = \frac{.036}{.06} = .6$ good chance he was smoker...

$$P(\text{l.c.} \mid \text{didn't smoke}) = \frac{P(B, A^c)}{P(A^c)} = \frac{?}{1-0.2} = \frac{.024}{.8} = .03$$

high? low?



$$P(B) = P(B, A) + P(B, A^c)$$

$$\Rightarrow P(B, A^c) = P(B) - P(B, A)$$

$$= .024$$

proof of this coming soon..

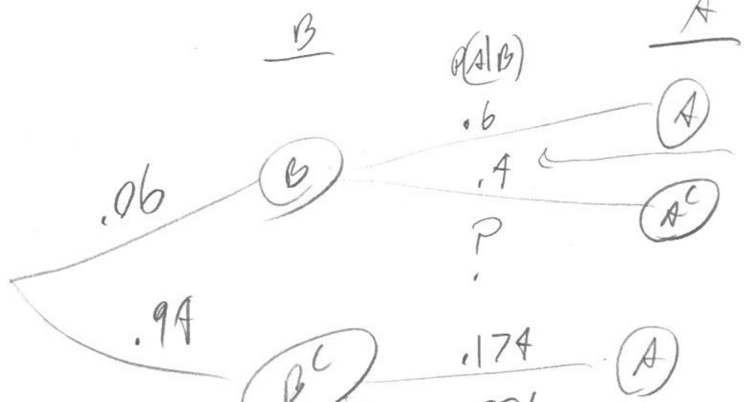
$$\frac{P(B|A)}{P(B|A^c)} = \frac{.18}{.03} = 6 \quad \text{Risk ratio}$$

$P(B|A) =$

RR = 6? 0.18?

How to make decision???

$P(B|A)$? $P(B|A^c)$? How many questions are there? So, let's build a tree to answer all questions



joint own

$P(AB) = .036$

$P(A^cB) = .024$

$P(AB^c) = .164$

$P(A^cB^c) = .776$

mut. excl., coll. exh.

$= 1$

$$P(A|B) \stackrel{?}{=} 1 - P(A^c|B)$$

$$\frac{P(AB)}{P(B)} \stackrel{?}{=} 1 - \frac{P(A^c, B)}{P(B)}$$

$$\Rightarrow P(AB) \stackrel{?}{=} P(B) - P(A^c, B)$$

$$\Rightarrow P(B) \stackrel{?}{=} P(AB) + P(A^cB) \checkmark$$

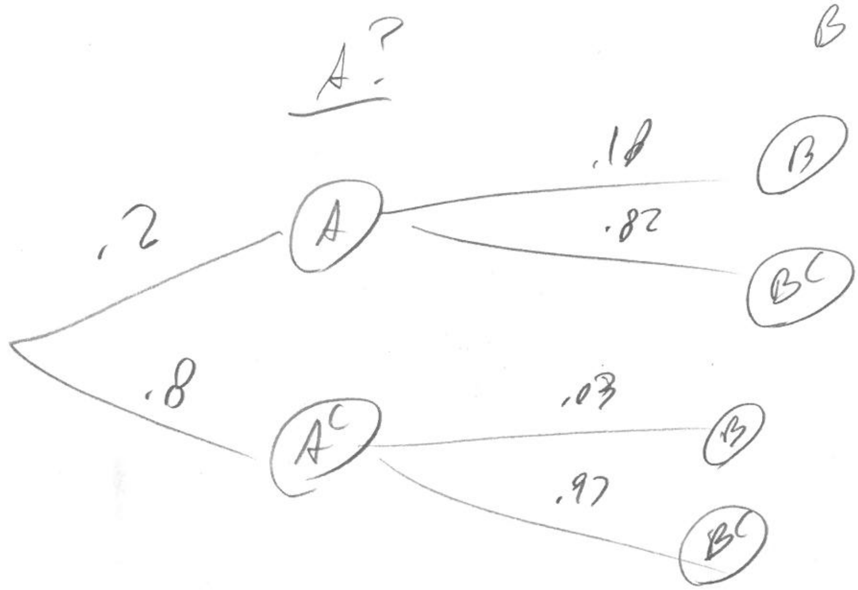
$$P(A) = P(B^c, A) + P(B, A)$$

$$.2 - .036 = .164$$

makes sense but not useful

$$P(A^c \mid B^c) = .826$$

What are we missing? $P(B|A)$... Need to "invert the tree"



$P(AB) = .036$

$P(A, B^C) = .164$

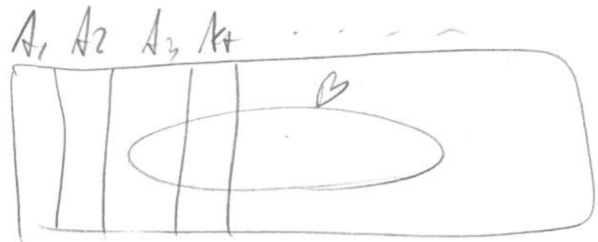
$P(A^C B) = .024$

$P(A^C B^C) = .776$

$P(B^C | A^C) = .97$

Prove that idea

Consider event B and mut. excl, coll exh. events $A_1, A_2, \ldots$



$P(B) = P(B \cap \Omega)$ Hw 1

$= P(B \cap (A_1 \cup A_2 \cup A_3 \cup \ldots))$ mut. excl. coll. exh.

$= P((B \cap A_1) \cup (B \cap A_2) \cup (B \cap A_3) \cup \ldots)$ How to prove?

Are $B \cap A_i$ and $B \cap A_j$ mut. excl?

mut. excl and

$$(B \cap A_i) \cap (B \cap A_j)$$

$$= B \cap A_i \cap A_j$$

$$= B \cap \phi = \phi$$ Hw 1 $\Rightarrow$ Yes

$$\Rightarrow \boxed{P(B) = \sum_{i=1}^{\infty} P(B, A_i)}$$ Law of Total Prob

with $A_1 = A$, $A_2 = A^c$ $\Rightarrow$ $P(B) = P(B, A) + P(B, A^c)$

$$P(B) = \sum_{i=1}^{\infty} P(B|A_i)P(A_i)$$

$$P(A_\ell|B) = \frac{P(B|A_\ell)\,P(A_\ell)}{P(B)}$$ Bayes Rule

$$P(A_\ell|B) = \frac{P(B|A_\ell)\,P(A_\ell)}{\sum_{i=1}^{\infty} P(B|A_i)P(A_i)}$$ Bayes Thm

SKIP

Cond. Prob. Super weird!!

You have two kids. One is a girl. I know that P(other is girl) $\stackrel{?}{=} \frac{1}{2}$

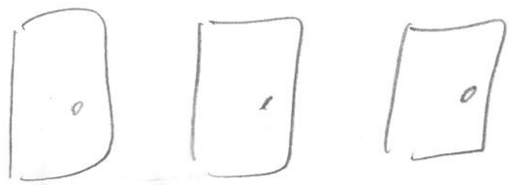
Ω

GG	GB
BG	BB

P({GG} one is a girl)

$= P(\{GG\} \mid \{GG, GB, BG\}) = \frac{|\{GG\}|}{|\{GG,GB,BG\}|} = \boxed{\frac{1}{3}}$ Crazy.....

Monte Hall



two goats and a car

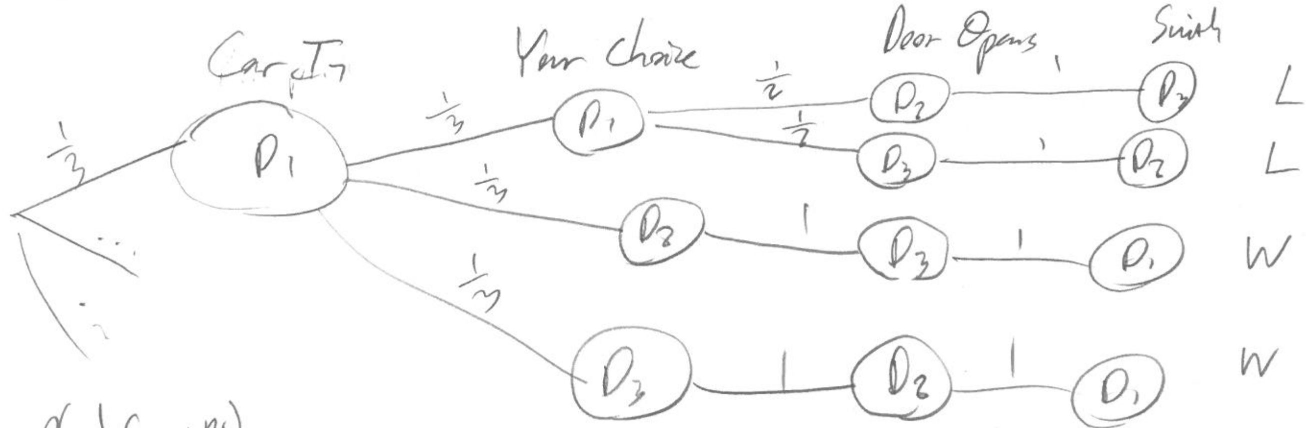
Game

(1) You pick a door

(2) Gameshow host opens one of the other two doors to show goat.

(3) You have the option of keeping your door or switching

P(winning if switch)? Draw Tree



$P(W) = P(W \mid \text{Car in } D_1) + P(W \mid \text{Car in } D_2) + P(W \mid \text{Car in } D_3)$

$P(W \mid \text{Car in } D_1) = \frac{\frac{1}{3}\left(\frac{1}{3}+\frac{1}{3}\right)}{\frac{1}{3}} = \boxed{\frac{2}{3}}$