

$$\sigma = \frac{\sqrt{1-p}}{p} = \frac{\sqrt{\frac{1}{2}}}{\frac{1}{2}} = 1.414$$

$$X_1, \dots, X_{30} \stackrel{iid}{\sim} \text{Geome}(\frac{1}{2}) \Rightarrow \mu = \frac{1}{\frac{1}{2}} = 2$$

what is the prob. the avg wait time is more than 2.75?

$$P(\bar{X} \geq 2.75) = P\left(\frac{\bar{X} - 2}{\frac{0.258}{\sqrt{30}}} > \frac{2.75 - 2}{\frac{0.258}{\sqrt{30}}}\right) = P(Z > 3) \approx 0.0045$$

we know $n=30$ is "large"

$$\bar{X} \stackrel{d}{\approx} N\left(\mu, \left(\frac{\sigma}{\sqrt{n}}\right)^2\right) = N\left(2, \left(\frac{1.414}{\sqrt{30}}\right)^2\right)$$

$$N = \left(2, (0.258)^2\right)$$

Take 100 steps with prob forward and backward being $\frac{1}{2}$

$$X \stackrel{iid}{\sim} \begin{cases} +1 \text{ wp } \frac{1}{2} \\ -1 \text{ wp } \frac{1}{2} \end{cases} \Rightarrow \mu = 0, \sigma^2 = 1 \Rightarrow \sigma = 1$$

What is prob you are more than 10 steps away from starting point after 100 steps.

$$X_1, \dots, X_{100} \stackrel{iid}{\sim} \begin{cases} +1 \text{ wp } \frac{1}{2} \\ -1 \text{ wp } \frac{1}{2} \end{cases}$$

when x is larger $\lim_{x \rightarrow \infty} \frac{1}{x} = 0$ i.e. $\frac{1}{100,000} = 0$

$$T = X_1 + \dots + X_{100} \stackrel{d}{\approx} N(\mu n, (\sigma \sqrt{n})^2)$$

$$T = X_1 + \dots + X_{100} \quad P(|T| > 10)$$

$$P(|T| > 10) = P(T > 10 \text{ or } T < -10)$$

$$= P(T > 10) + P(T < -10)$$

By CLT $T \sim N(\mu n, (\sigma \sqrt{n})^2)$

$$= N(100 \cdot 0, (1 \sqrt{100})^2) = N(0, 10^2)$$

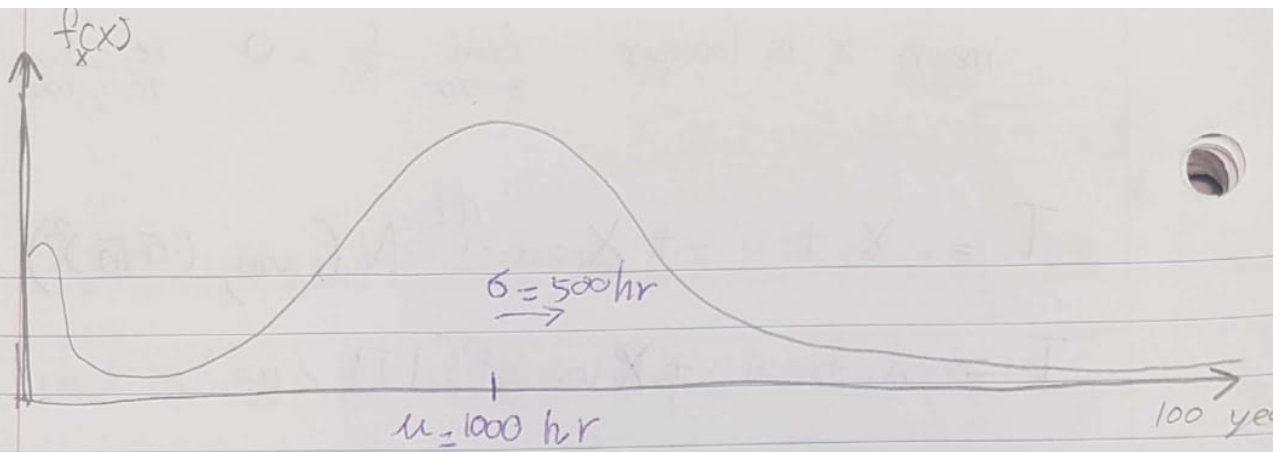
Δ

$$= P\left(\frac{T-0}{10} > \frac{10-0}{10}\right) + P\left(\frac{T-0}{10} < \frac{-10-0}{10}\right)$$

$$= P(Z > 1) + P(Z < -1)$$

$$= 16 + 16 = 32$$

Problem
#2



X is lifetime of a lightbulb

You get $(50)^n$ light bulbs. What is probability any lightbulb is more than 1300 hr?

$P(\bar{X} > 1300)$: \bar{X} is avg of 50 lightbulbs

By clt

$$\bar{X} \approx N\left(\mu, \left(\frac{\sigma}{\sqrt{n}}\right)^2\right)$$

$$= N\left(1000, \left(\frac{500}{\sqrt{50}}\right)^2\right) = N\left(1000, 70.7^2\right)$$

$$P(\bar{X} > 1300) = P\left(\frac{\bar{X} - 1000}{70.7} > \frac{1300 - 1000}{70.7}\right)$$

$$\approx P(Z > 4.29) \approx 0$$

each one late 2% is $X_1, \dots, X_{10000} \stackrel{iid}{\sim} \text{Bern}(0.02)$

Shipments are late 2% of the time. What is prob in 10,000 shipment more than 3% are late?

$$P(\bar{X} > 0.03)$$

by clt $\bar{X} \approx N\left(\mu, \left(\frac{\sigma}{\sqrt{n}}\right)^2\right)$

$$\stackrel{*}{=} N\left(0.02, \left(\frac{0.14}{\sqrt{10000}}\right)^2\right) = N(0.02, 0.0014^2)$$

$$P(\bar{X} > 0.03) = P\left(\frac{\bar{X} - 0.02}{0.0014} > \frac{0.03 - 0.02}{0.0014}\right)$$

$$\approx P(Z > 7.14) \approx 0$$

\bar{X} is the r.v. of the avg \hat{p} that's the r.v. of proportion

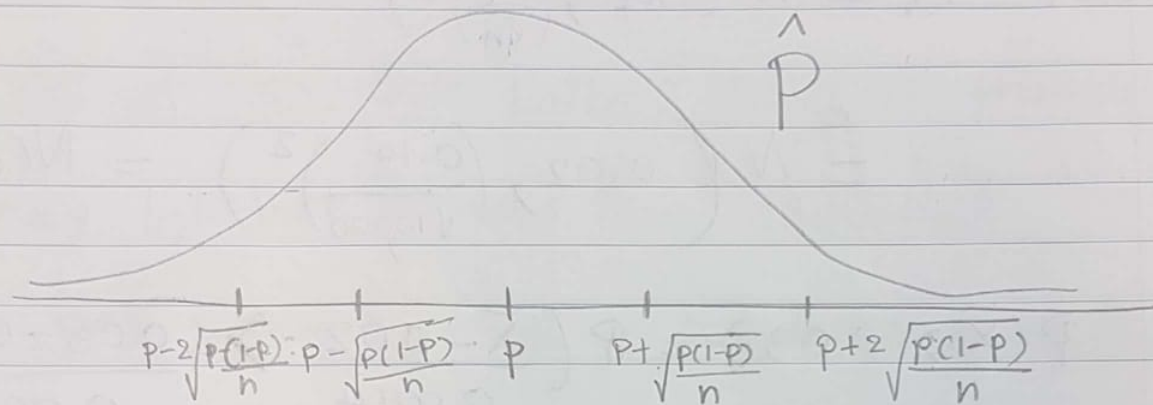
\bar{x} is a realization (sample avg) \hat{p} "estimate" sample proportion

ie $\bar{x} = \frac{1+1+0+0+0}{5} = 0.4$

$$\bar{X} \in (0, 1)$$

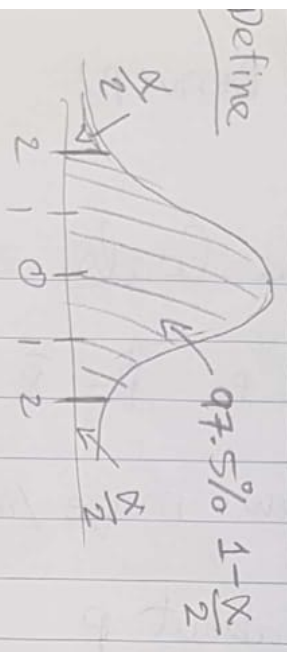
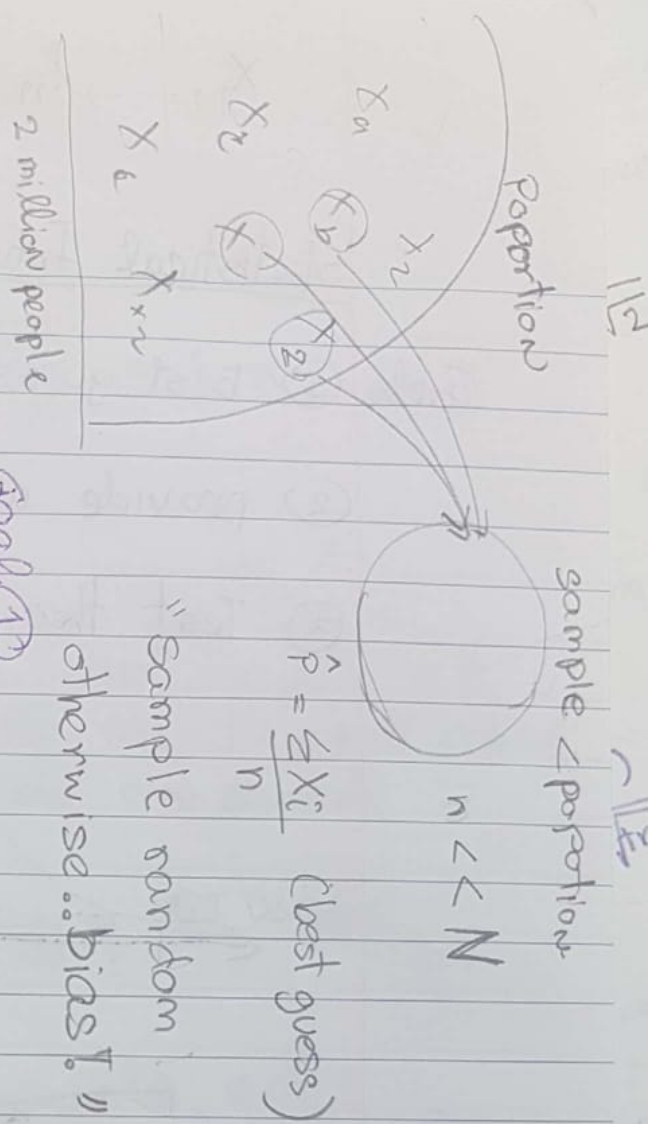
$$\bar{X} \stackrel{d}{\approx} N\left(\mu, \left(\frac{\sigma}{\sqrt{n}}\right)^2\right)$$

$$\hat{P} \approx N\left(p, \left(\sqrt{\frac{p(1-p)}{n}}\right)^2\right)$$



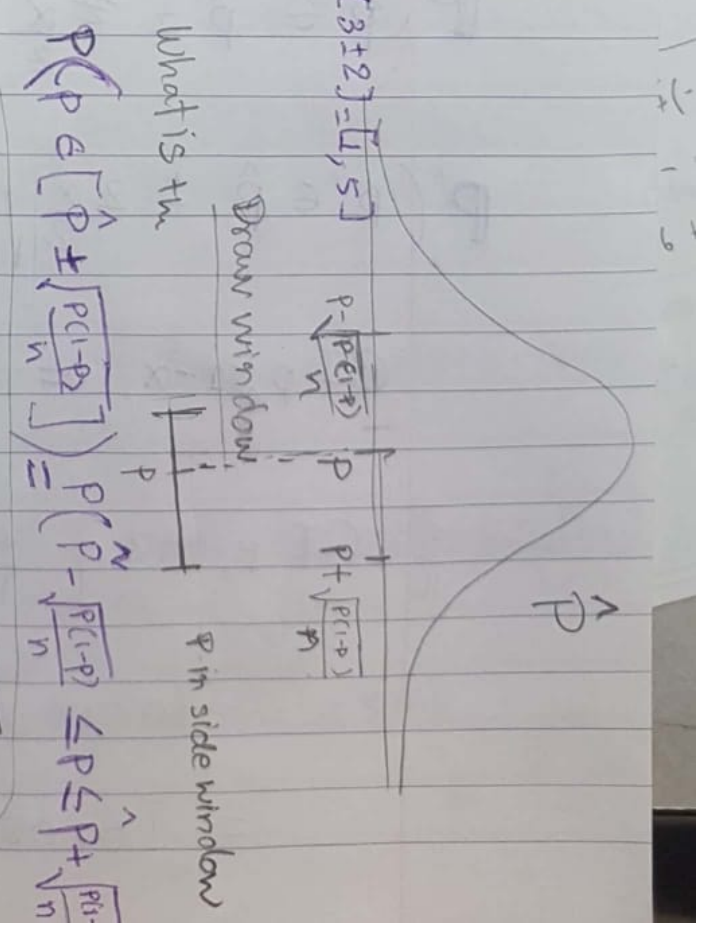
Statistical Inference Goals:

- ① Estimate p (best guess) Best guess \hat{P}
- ② Create a range/window of likely value for p ie giving rang 10-100 p is in that range
- ③ Test Theory about p (Hypotheses testing)



$$Z_{\alpha/2} = F_z^{-1}\left(1 - \frac{\alpha}{2}\right)$$

$$\alpha 5\% \Rightarrow \frac{\alpha}{2} = 2.5\% \Rightarrow 1 - \frac{\alpha}{2} = 97.5\% \Rightarrow Z_{97.5\%} = 2$$



$$= P\left(-\sqrt{\frac{p(1-p)}{n}} \leq p - \hat{p} \leq \sqrt{\frac{p(1-p)}{n}}\right)$$

$$= P(-1 \leq \frac{p - \hat{p}}{\sqrt{\frac{p(1-p)}{n}}} \leq 1)$$

$$= P(1 > Z > -1) = P(Z \in [-1, 1]) = 0.68$$

$$CI_{p, 1-\alpha} := \left[\hat{p} \pm \frac{Z_{\alpha/2}}{2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \right]$$