Math 241 Lecture 11

Oct 18

n total terms; x terms: $(PN)(PN-1)\ldots(PN-x+1)$

n-x terms: $((1-p)N)((1-p)N-1)\ldots(1-p)N-(n-x)+1$

$$\lim \text{hyper}(n,p,N) = \lim_{N\to\infty} \frac{\binom{PN}{x}\binom{(1-p)N}{n-x}}{\binom{N}{n}} = \binom{n}{x} \lim_{N\to\infty} \frac{\frac{(PN)!}{(PN-x)!}\,\frac{((1-p)N)!}{((1-p)N-(n-x))!}}{\frac{N!}{(N-n)!}}$$

$$\frac{10!}{(10-x)!} = \underbrace{10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 4}_{7 \text{ terms}}$$

$$\underbrace{\frac{N!}{(N-n)!}}_{N(N-1)(N-2)(N-3)\ldots(N-n+1)}$$

n terms

$\lim f(x)g(x) = \lim f(x) \lim g(x)$

$$= \binom{n}{x} \underbrace{\underbrace{\lim_{N\to\infty}\frac{PN}{N}}_{p}\ \underbrace{\lim_{N\to\infty}\frac{PN-1}{N-1}}_{p}\cdots\underbrace{\lim_{N\to\infty}\frac{PN-x+1}{N-x+1}}_{p}}_{p^x}\ \underbrace{\underbrace{\lim_{N\to\infty}\frac{(1-p)N}{N-x}}_{1-p}\cdots\underbrace{\lim_{N\to\infty}\frac{(1-p)N-(n-x)+1}{N-n+1}}_{1-p}}_{(1-p)^{n-x}}$$

$$= \binom{n}{x} p^x (1-p)^{n-x}$$

$X \sim \text{Binomial}(n,p) := \binom{n}{x} p^x (1-p)^{n-x}$

$\text{supp}[X] = \{0,1,\ldots,n\}$

param space

$p \in \{0,1\}$

$n = \mathbb{N}$

$(a+b)^2 =$

$\sum \binom{n}{i} a^i b^{n-i}$

$X \sim \text{Bin}(1,p) = \binom{1}{x} p^x (1-p)^{1-x} = \text{Ber}$

$\text{supp}[X] = \{0, \}$

$\sum_{x\in \text{supp}[X]} p(x) = 1$ $\quad \sum_{x=0}^{n} \binom{n}{x} p^x (1-p)^{n-x}$ $\quad X=0$ $\quad = ((p)+(1-p))^2$

$= 1^n = 1$ ✓

$X_1, X_2 \overset{iid}{\sim}$ which means X_1, X_2, ind and $X_1 \overset{d}{=} X_2$

$X_1 \sim \text{Bern}\left(\frac{1}{3}\right)$ $\quad X_1 \overset{d}{=} X_3$

$X_2 \sim \text{Bern}\left(\frac{1}{3}\right)$

In general, if X_1 & X_2 are independent then $\forall_1 \in \text{Supp}[X_1]$

(a) $P(X_1 = x_1 \mid (X_2 = x_2) = P(X_1 = x_1)$ $\quad \forall x_2 \leftarrow \text{Supp}[X_2]$

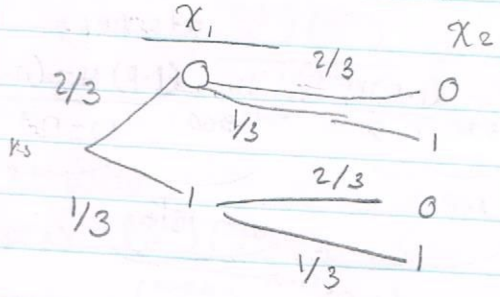
(b) $P(X_2 = x_2 \mid X_1 = x_1) = P(X_2 = x_2)$

(c) $P(X_1 = x_1, X_2 = x_2) = P(X_1 = x_1) P(X_2 \leq x_2)$

$T_2 = X_1 + X_2$

$Supp\,[T] = \{0, 1, 2\}$

$$T_2 - \begin{cases} 0 & \frac{4}{9} \\ 1 & \frac{4}{9} \\ 2 & \frac{1}{9} \end{cases}$$



$P(X_1 = 0, X_2 = 0) = \frac{4}{9}$ I

$P(X_1 = 0, X_2 = 1) = \frac{1}{9}$?   0

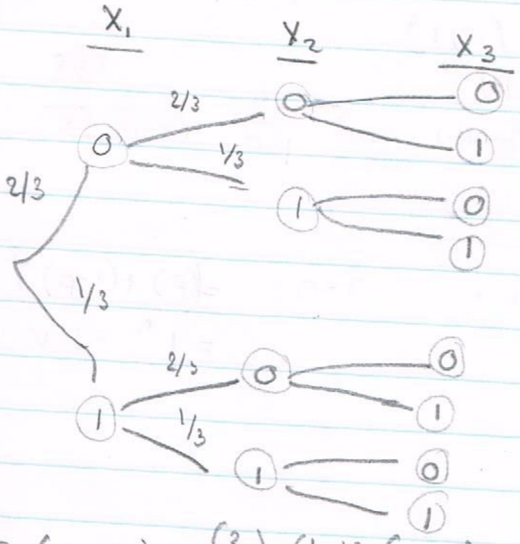
$P(X_1 = 1, X_2 = 0) = 2/9$ 1

$P(X_1 = 1, X_2 = 1) = 1/9$ 1

$\overline{}$ 1   1

$T_4 = X_1 + X_2 + X_3$

$Sup\,[T_3] = \{0, 1, 2, 3\}$



$P(X_1 = x_1, X_2 = x_2, X_3 = x_3)$

$(2/3)^3$

$(1/3)(\frac{2}{3})^2$

$(1/3)^1(2/3)^2$

$(1/3)^2(2/3)^1$

$(1/3)^1 \; (2/3)^2$

$(1/3)^2 \; (2/3)^0$ $\Leftrightarrow$

$(1/3)^2 \; (2/3)^1$

$(1/3)^? \; (\frac{2}{3})$

$P(T_3 = 0) = \binom{3}{0}\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^3$

$P(T_2 = 1) = \binom{3}{1}\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)$

$P(T_2 = 1) = \binom{3}{2}\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^1$

$P(T_3 = 3)\ \binom{3}{3}\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^0$

$$T_n = \sum_{i=1}^{n} x_i$$

$\text{supp}\{T_n\} = \{0, 1, 2, \dots n\}$

$$T_n \sim \begin{cases} 0 & \sim p \binom{n}{0} \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^n \\ 1 & \sim p \binom{n}{1} \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^{n-1} \\ 2 & \sim p \binom{n}{2} \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^{n-2} \\ \vdots & \\ n-1 & \sim p \\ n & \sim p \end{cases} \qquad = \binom{n}{x} p^x (1-p)^{n-x}$$

$= Bin(n, p)$

The $Bin(n,p)$ can be conceptualized by

$T = \lim_{N \to \infty} Hyper(n, p, N)$ infinite huge bag

or

$T = \sum_{i=1}^{n} x_i$ such that $x_1, x_2, \dots x_n \overset{iid}{\sim} Bern(p)$ finite

$p(x) = P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$

$F(x) = P(X \leq x) = \sum_{i=0}^{x} \binom{n}{i} p^i (1-p)^{n-i}$