Math 241 Lecture 08

52 cards

$P(A) = \frac{4}{52} = \frac{1}{13}$ $\quad P(A \mid \heartsuit) = \frac{1}{13}$ (out of the universe of just ♡'s, pick A)

$P(A \mid \heartsuit) = P(A)$ (numerically)

information

"informationally irrelevant"

$P($IBM stock ↑ today $\mid \underbrace{\text{it rains today in Buenos Aires}}_{\text{not relevant}}) = P($IBM stock ↑ today$)$

Def. events A, B are independent if

$P(A \mid B) = P(A)$ or $P(B \mid A) = P(B)$

$$\underbrace{P(A \mid B) = \frac{P(AB)}{P(B)}}_{\text{def. of cond. prob.}} \underbrace{= P(A)}_{\text{if independent}} \Rightarrow P(AB) = P(A)P(B)$$

If $A_1, A_2, \ldots$ are independent

$$P(A_1, A_2, \ldots) = \boxed{P\left(\bigcap_{i=1}^{\infty} A_i\right) = \prod_{i=1}^{\infty} P(A_i) \quad \text{mult Rule}}$$

$P(H_2 \mid H_1) = P(H_2) = \frac{1}{2}$

$P(H_1 H_2 \ldots H_{10}) = \frac{1}{|\Omega|^{10}} = \left(\frac{1}{2}\right)^{10}$

$= \frac{1}{2^{10}}$

$P(\geq 1$ double 6 in 24 2-die rolls$) < \frac{1}{2}$

$= P(1$ double $6) + P(2$ double $6) + \ldots + P(24$ double $6)$

$= 1 - P($no double 6 in 24 rolls$)$

$= 1 - P($no double $6)^{24}$

$= 1 - P(6)P(6)$

$= 1 - \left(\frac{1}{6}\right)^{6} = \frac{35}{36}$

Def A, B are depedent events if

$P(A|B) \neq P(A)$ or $P(B|A) \neq P(B)$ or $P(A, B) \neq P(A)P(B)$

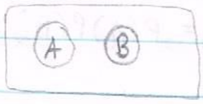
$P(\text{Q64 is late})$

$P(\text{Q64 is late} \mid \text{snow storm})$

$\underset{\text{all cases}}{P(\text{Q64 is late})} > P(\text{Q64 is late} \mid \text{perfect weather, no traffic})$

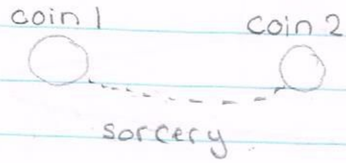
$P(\text{l.c.}) < P(\text{l.c.} \mid \text{smoking})$

A, B are disjoint and not empty. Are A, B independent?



$0 = P(A|B) \neq P(A) > 0 \Rightarrow$ Dependent

$P(H|T) \neq \frac{1}{2} = P(H)$ (disjoint)



if coin 1 H $\Rightarrow$ coin 2 H

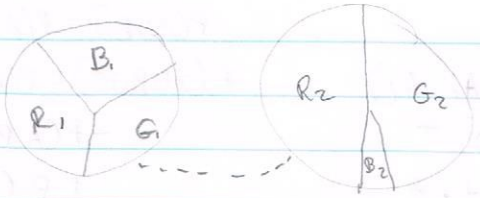
if coin 1 T $\Rightarrow$ coin 2 T

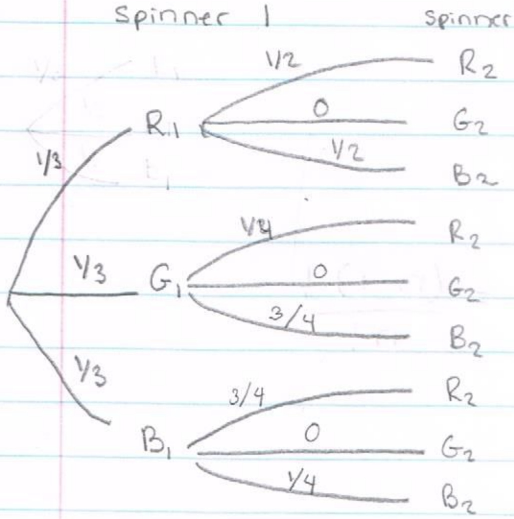
coin 1 coin 2

$\frac{1}{2}$ H $\xrightarrow{1}$ H $\qquad P(HH) = \frac{1}{2}$ $\qquad P(H_2 | H_1) \overset{?}{=} P(H_2) = \frac{1}{2}$

$\frac{1}{2}$ T $\xrightarrow{1}$ T $\qquad P(TT) = \frac{1}{2}$

$P(H_2) = \underset{1/1}{P(H_2, H_1)} + \underset{0}{P(H_2, T_1)}$





joint

$= \frac{1}{6}$

$= 0$

$= \frac{1}{6}$

$= \frac{1}{12}$

$= 0$

$= \frac{1}{4}$

$= \frac{1}{4}$

$= 0$

$= \frac{1}{12}$

1

$$P(R_2) = P(R_1, R_2) + P(G_1, R_2) + P(B_1, R_2)$$
$$= \frac{1}{6} + \frac{1}{12} + \frac{1}{4}$$
$$= \frac{1}{2}$$

Are R_1, R_2 independent? YES

$$P(R_1, R_2) \overset{?}{=} P(R_1)\, P(R_2)$$
$$\frac{1}{6} = \frac{1}{3} \cdot \frac{1}{2}$$

Are $R_1\, B_2$ independent?

$$P(R_1, B_2) \overset{?}{=} P(R_1)\, P(B_2)$$
$$\frac{1}{6} = \frac{1}{3} \cdot \frac{1}{2}$$

P (at least 2 people share bdays) = P(one pair bdays) + P (2 pair bdays) + ...

$P(\binom{24}{2}$ bday pairs$)$ = 1 − P (no one shares bdays) $= \frac{|A|}{|\Omega|}$

$$= 1 - \frac{365P24}{365^{24}}$$

$$\begin{array}{r} 365 \\ -24 \\ \hline \end{array}$$

n people walk into a room, they all take off their hats and put them in a pile. They then all randomly choose a hat. What is the probability that one person got their hat?

1 − P (no one gets their hat)

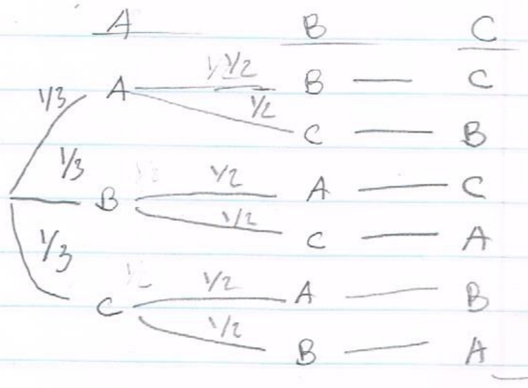
Let A_1 : person 1 got their hat

A_2 : " " 2 " "

⋮

A_n " " n " "

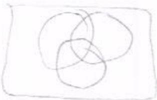
$n = 3$



$\frac{2}{6} = 1/3$

$1 - e^{1}\ x$

$\frac{2}{3}$ $\frac{1}{3}$ no one



$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B \cup C) = P(A) + P(B) + P(C)$
$- P(AB) - P(AC) - P(BC)$
$+ P(ABC)$

$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum P(A_i) - \sum P(A_i, A_j) + \sum P(A_i A_j A_k) - \sum P(A_i A_j A_k) + - + - + (-1)^n\, P\left(\bigcap_{i=1}^{n} A_i\right)$$

$$P(A_1) = \frac{1 \cdot (n-1) \cdot (n-2)}{n!} = \frac{(n-1)!}{n!}$$

$$P(A_4) = \frac{(n-1)(n-2)(n-3)(n-4) \ldots}{n!} = \frac{(n-1)!}{n!}$$

$$P(A_2, A_i) = \frac{(n-2)!}{n!}$$

$$\binom{n}{2}\left(\frac{n-2}{n!}\right) = \frac{n!}{(n-2)!\,2!} \cdot \frac{(n-2)!}{n!} = \frac{1}{2!}$$

$$P(A_2 A_3 A_5) = \frac{(n-3)!}{n!}$$

$$\binom{n}{3}\frac{(n-3)!}{n!} = \frac{1}{3!}$$

$$e^{1} = \boxed{e^{1} = \sum_{i=0}^{\infty} \frac{1}{i!}} \qquad e^{-1} = \frac{1}{0!} - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!}$$

$$= 1 - 1 + \frac{1}{2!}$$

$$1 - e^{-1} = 1 - \frac{1}{2!} +$$