10/16

10 cards, 4R 6B

$$P\left(\text{drawing 2R from 3 cards w/o rep.}\right) = \frac{\binom{4}{2}\binom{6}{1}}{\binom{10}{3}}$$

$$P(x\text{R when drawing 3}) = \frac{\binom{4}{x}\binom{6}{3-x}}{\binom{10}{3}}$$

$$P(x\text{ R when drawing } m) \; \frac{\binom{4}{x}\binom{6}{m-x}}{\binom{10}{m}}$$

→ K red

$$P(x\text{R when drawing } n) = \frac{\binom{K}{x}\binom{10-K}{n-x}}{\binom{10}{n}}$$

→ N cards, k red

$$P(x\text{R in } m) = \frac{\binom{k}{x}\binom{N-k}{n-x}}{\binom{N}{n}}$$

$$X \sim \text{Hypergeometric}(n, K, N) := \underbrace{\frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}}$$

$$p(x) := P(X=x)$$

Supp[X]

$X \sim \text{Bern}(p) := \underbrace{p^x(1-p)^{1-x}}_{p(x)}$

$\text{Supp}[X] = \{0, 1\}$

$p \in (0, 1)$

→ ex. 100 students 53 are female

$P\left(\text{6 are female from 8}\right) =$

$(n, K, N) = \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$

→ ex. 100 students, 53 female

P(6 are female choosing 8)

$X \sim \text{Hypergeometric}(8, 53, 100) = \frac{\binom{53}{x}\binom{47}{8-x}}{\binom{100}{8}}$

$n, K, N$

$P(X=6) = P(6) = \frac{\binom{53}{6}\binom{47}{2}}{\binom{100}{8}}$

$N=0$? this would be a degenerate case.

$N=0 \rightarrow n=0, K=0$

$N=1$? $\rightarrow K \in \{0,1\}$

$n=1$ ($n=0$ is a degenerate case)

$N=2$? $\rightarrow K \in \{0,1,2\}$

when $K=0$ or $K=N$, those are degenerate cases.

$n \in \{0,1,2\}$

only non-degerate case.

$X \sim \text{Hypergeometric}(1,1,2) = \frac{\binom{1}{x}\binom{1}{1-x}}{\binom{2}{1}}$ Bern(1/

$n, K, N$

$\text{Supp}[X] = \{0,1\}$

"how many could be special?"

$p(1) = \frac{\binom{1}{1}\binom{1}{0}}{\binom{2}{1}} = \frac{1}{2}$

$p(0) = \frac{\binom{1}{0}\binom{1}{1}}{\binom{2}{1}} = \frac{1}{2}$

$X \sim \text{Hyper}(1, K, N) = \frac{\binom{K}{x}\binom{N-K}{1-x}}{\binom{N}{1}} = \text{Bern}\left(\frac{K}{N}\right)$

$\text{Supp}[X] = \{0,1\}$

$$p(1) = \frac{\binom{K}{1}\binom{N-K}{0}}{\binom{N}{1}} = \frac{K}{N}$$

$$p(0) = \frac{\binom{K}{0}\binom{N-K}{0}}{\binom{N}{1}} = \frac{N-K}{N} = 1 - \frac{K}{N}$$

<u>Parameter Space</u> <u>Hyper</u>

$N \in \{2, 3, \ldots\}$

$K \in \{1, 2, \ldots N-1\}$

$n \in \{1, 2, \ldots N-1\}$

<u>Support</u>

(a) $X \sim \text{Hyper}(\underset{n}{2}, \underset{K}{4}, \underset{N}{10})$, $\text{Supp}[X] = \{0, 1, 2\}$

(b) $X \sim \text{Hyper}(5, 4, 10)$, $\text{Supp}[X] = \{0, 1, 2, 3, 4\}$

(c) $X \sim \text{Hyper}(8, 4, 10)$, $\text{Supp}[X] = \{2, 3, 4\}$

$N-K=6$

(d) $X \sim \text{Hyper}(5, 7, 10)$, $\text{Supp}[X] = \{2, 3, 4, 5\}$

$N-K=3$

(a) $n < K$, $n < \underline{N-K}$, $\text{Supp}[X] = \{0, \ldots, n\}$

"non-special choices"

(b) $n \geq K$, $n < N-K$, $\text{Supp}[X] = \{0, \ldots, K\}$

(c) $n \geq \quad$, $n \geq N-K$, $\text{Supp}[X] = \{n-(N-K), K\}$

$= \{n-(N-K), \ldots K\}$

(d) $n < K$, $n \geq N-K$, $\text{Supp}[X] = \{n-(N-K), \ldots, n\}$

| | $n<K$ | $n \geq K$ |
|---|---|---|
| $n < N-K$ | $0 \ldots n$ | $0 \ldots K$ |
| $n \geq N-K$ | $n-(N-K) \ldots n$ | $n-(N-K) \ldots K$ |

$Supp[X] = \{\max\{0, n-(N-K)\} \ldots, \min\{n, K\}\}$

$X \sim Hyper(n, K, N) = \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$

$\sum_{x \in supp[X]} p(x) = 1$

→ let $p = \frac{K}{N} \Rightarrow K = pN \qquad N - pN = (1-p)N$

$X \sim Hyper(n, \overset{K}{p}, N) = \frac{\binom{pN}{x}\binom{(1-p)N}{n-x}}{\binom{N}{n}}$

"reparameterization"

$N \in \{2, \ldots\}$

$n \in \{1, \ldots, N-1\}$

$p \in \{\frac{1}{N}, \frac{2}{N}, \ldots, \frac{N-1}{N}\}$

→ Consider $p = 0.5,\ n = 6,\ N = 100$

$p(3) = \frac{\binom{50}{3}\binom{50}{3}}{\binom{100}{6}} = .3223$

$N = 1000$

$P(3) = \frac{\binom{500}{3}\binom{500}{3}}{\binom{1000}{6}} = .3134$

$N = 10{,}000$

$P(3) = \frac{\binom{5000}{3}\binom{5000}{3}}{\binom{10000}{6}} = .3126$

what is the limiting r.v

$$\lim_{N\to\infty} \text{Hyper}(n, p, N) = \lim_{N\to\infty} \frac{\binom{pN}{x}\binom{(1-p)N}{n-x}}{\binom{N}{n}}$$

$$= \lim_{N\to\infty} \frac{\frac{(pN)!}{x!\,(pN-x)!}\;\frac{((1-p)N)!}{(n-x)!\,((1-p)N-(n-x))!}}{\frac{N!}{n!\,(N-n)!}}$$

$$\underbrace{\frac{1}{x!}\cdot\frac{1}{(n-x)!}\cdot\frac{1}{\frac{1}{n!}}}_{\binom{n}{x}} = \binom{n}{x} \lim_{N\to\infty} \frac{\frac{(pN)!}{(pN-x)!}\cdot\frac{((1-p)N)!}{((1-p)N-(n-x))!}}{\underbrace{\frac{N!}{(N-n)!}}_{N(N-1)(N-2)\dots(N-n+1)}}$$