08/31/2017

More on Set Theory with Power Sets (Cardinality)

$2^A = \{B : B \subseteq A\} = \{\overset{\text{OFF}}{\emptyset}, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \overset{\text{ON}}{A}\}$

$A = \{1,2,3\}$ $|A| = 3$ $|2^A| = 8 = 2^3$

Special set denoted Ω

- This is called the "universe", or "sample space", or "scope"

$\Omega = F \cup M$

"defining my universe" (everything you care about)

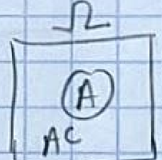
Ex:

$F \subseteq \Omega$? yes $F \cap \Omega$? $= F$ $\emptyset \cup \Omega = \Omega$ $A \setminus \Omega = \emptyset$

$M \subseteq \Omega$? yes $F \cup \Omega$? $= \Omega$ $\emptyset \cap \Omega = \emptyset$

A^c "A complement" $= \Omega \setminus A$

everything that is not A



$A \cup A^c = \Omega$

$\underbrace{A \cap A^c}_{} = \emptyset$

mutually exclusive/disjoined

$\{A_1, A_2, A_3 \ldots\}$ are collectively exhausted if

$A_1 \cup A_2 \cup \ldots = \Omega$

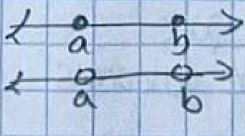
$= \bigcup_{i=1}^{\infty} A_i = \Omega$

$\{A_1, A_2, A_3 \ldots\}$ are mutually exclusive if

$A_i \cup A_j = \emptyset \quad \forall i \neq j$

$[a,b] = \{x : x \geq a \text{ and } x \leq b\}$

$(a,b) = \{x : x > a \text{ and } x\ b\}$



ordered pair impose order

$\langle a,b \rangle := \{\{a\}, \{a,b\}\}$

$\neq \langle b,a \rangle := \{\{b\}, \{a,b\}\}$

$\langle a,a \rangle = \{\{a\}, \{a,a\}\}$

$= \{\{a\}\} \neq \{a\}$

$\{a,a\} = \{a\}$

Set/Cartesian Product

$A \times B = \{ \langle a,b \rangle : a \in A, b \in B \}$

$A = \{1,2\}$, $B = \{3,4\}$

$A \times B = \{ \langle 1,3 \rangle, \langle 1,4 \rangle, \langle 2,3 \rangle, \langle 2,4 \rangle \}$

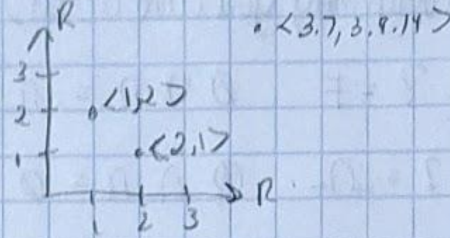
$|A \times B| = 4$ elements
$|A| = 2 \quad |B| = 2$

$|A \times B| = |A|\,|B|$
In general $|A_1 \times A_2 \times \dots A_3| = \prod_{i=1}^{n} |A_i|$

Cartesian Product

$A^2 = A \times A$
$|A^2| = |A|^2$

$|A^n| = |A|^n$



$\mathbb{R} \times \mathbb{R}$

Probability

Ω is now called the "experimental space" or "outcome space" and its elements are called "outcomes" and denoted ω (little omega)
($\omega \in \Omega$)

When an experiment is performed, one outcome is its result
For example, the coin toss experiment

$\Omega = \{ \underset{\omega_1}{H}, \underset{\omega_2}{T} \}$ only one outcome

universe is now the experiment space
experimental space has all the outcomes

"P" is the set function called "probability of"

$P(H) = \frac{1}{2}$

$\frac{|H|}{\text{size of } \Omega = |\Omega|}$ → can't do because H is an element not a set

remember $\Omega = \{H, T\}$

Is this a good definition?

$P: \Omega \to (0,1)$ No. Not a good definition, only describes elements of a set, therefore it can only solve the probability of 1 element at a time

$\Rightarrow$ BAD

$P(\{H\}) = \frac{|\{H\}|}{|\Omega|}$

$P(\{H,T\}) = \frac{|\{H,T\}|}{|\Omega|} = \frac{2}{2} = 1$

$\Downarrow$

$P(H \text{ or } T)$

GOOD DEFINITION

$P: 2^{\Omega} \to [0,1]$ including all set $\{H\}, \{T\}$

(powerset)

Die Roll Experiment

$\Omega = \{1,2,3,4,5,6\}$

event not outcome

$P = (\text{even } \#) = \frac{|\{2,4,6\}|}{|\Omega|} = \frac{3}{6} = \frac{1}{2}$

outcome includes all event - single outcome

\# of questions $P = 2^{\Omega} = 2^6 = 64$ questions

★ Working Definition of Probability: The probability of "event A" is $P(A) := \frac{|A|}{|\Omega|}$ ★

2^{Ω} is called "event space". A set $A \subseteq \Omega$ is called an event

let Ω' be an outcome space for 2 coin flips

$\Omega' = \Omega^2 = \{\langle H,H\rangle, \langle H,T\rangle, \langle T,H\rangle, \langle T,T\rangle\}$, $|\Omega'| = 4$

$P(\underset{A}{\text{of at least one } H}) = \frac{|\langle H,H\rangle, \langle H,T\rangle, \langle T,H\rangle|}{|\Omega'|} = \frac{3}{4}$

event space $2^{\Omega} = 2^4 = 16$

$P(\text{one } H) = \frac{|\langle H,T\rangle, \langle T,H\rangle|}{\Omega} = \frac{2}{4}$

Trivial Events

$P(\emptyset) = 0$, $P(\Omega) = 1$