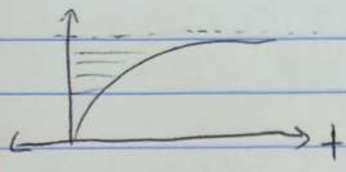
Lecture 17

11/13/17

$F(t) = 1 - e^{-\lambda t}$



$Supp(T) = (0, \infty)$

$|Supp(T)| = |\mathbb{R}| > |\mathbb{N}|$ (not discrete)

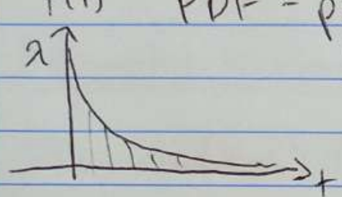
Plank length/time

$P(3) = P(T = 3.000\ldots)$

$P(3) \approx P(T \in [2.995\bar{0}, 3.004\bar{9}]) = F(3.004\bar{9}) - F(2.995\bar{0}) > 0$

$P(T < 3.004\bar{9}) - P(T < 2.995)$

$f(t)$ PDF - probability density function



$a < b$

$P(T \in [a,b]) = \underbrace{F(b) - F(a) = \int_a^b f(t)\,dt}_{\text{Fundamental Theorem of Calculus}}$

$\lambda = 2 \Rightarrow f(t) = 2e^{-2t}$

$f(1) = 2e^{-2} \approx 0.27 \neq P(1) = 0$

$f(0.1) = 2e^{-2(.1)} \approx 1.63 \neq P(0.1) = 0$

no meaning

$\dfrac{f(0.1)}{f(1)} \approx \dfrac{1.63}{0.27} \approx 6$

$$\frac{P(T = [0.1, 0.1 + \varepsilon])}{P(T = [1.0, 1.0 + \varepsilon])} = \frac{\lim_{\varepsilon \to 0} \frac{F(0.1 + \varepsilon) - F(0.1)}{\varepsilon}}{\lim_{\varepsilon \to 0} \frac{F(1 + \varepsilon) - F(1)}{\varepsilon}} = \frac{f(0.1)}{f(1)}$$

$P(T \in (-\infty, \infty)) = F(\infty) - F(-\infty) = \int_{-\infty}^{\infty} f(t)\,dt = 1$

$= P(T \in (0, \infty)) = F(\infty) - F(0) = \int_0^{\infty} f(t)\,dt \Rightarrow \int_{Supp(X)} f(x)\,dx = 1$

$\sum_{x \in Supp(X)} p(x) = 1$

Def Continuous random variable X

(a) $|Supp(X)| = |\mathbb{R}| > |\mathbb{N}|$

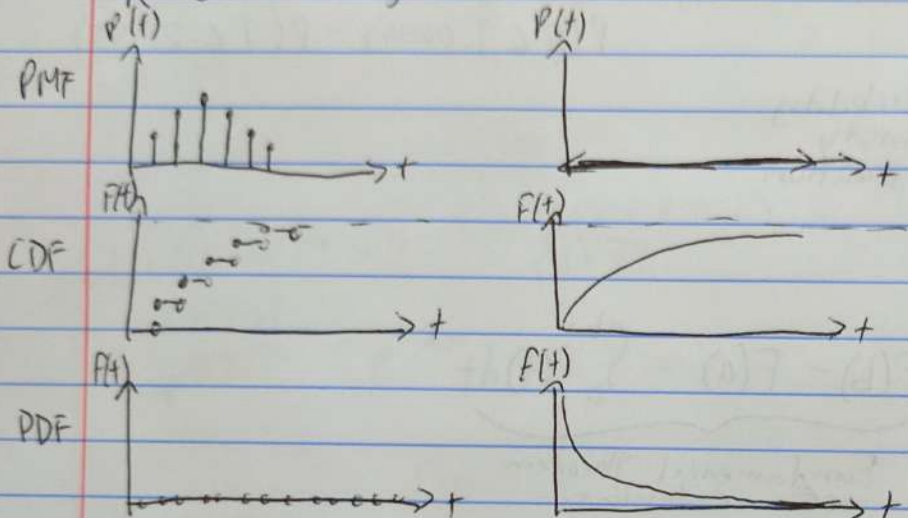
(b) $F(x)$ is a valid CDF with no discontinuity

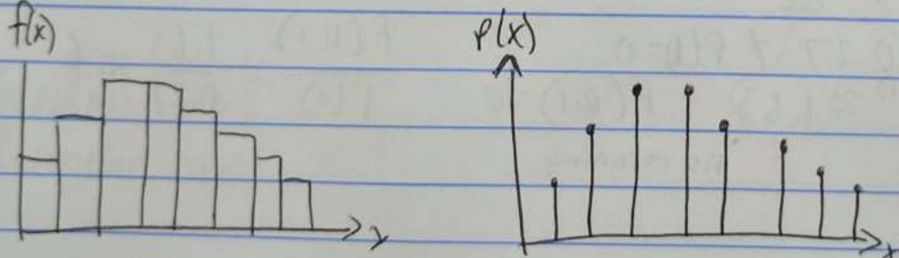
(c) PMF doesn't exist $p(x) = 0, \forall x$

(d) $f(x)$ is the PDF

① $\underset{F'(x)}{\overset{}{f(x)}} \geq 0, \forall x$ ② $\int_{supp(x)} f(x) = 1$

$X = \text{Geometric}(P)$





$$E(X) = \sum_{supp(x)} x\left(F(x+\varepsilon) - F(x)\right)$$

$$\varepsilon \to 0 \Rightarrow E(X) = \int_{supp(x)} x f(x)\,dx$$

$$E(g(x)) = \int_{supp(x)} g(x) f(x)\,dx$$

$$Var(x) = E\left[(X-\mu)^2\right] = \int_{supp(x)} (X-\mu)^2 f(x)\,dx$$

$$E(\sum X_i) = \sum E(X_i) = n\mu$$

$$E(aX+c) = a\mu + c$$

$Var(\Sigma X_i) = \Sigma Var(X_i) = n\sigma^2$

$Var(aX+c) = a^2\sigma^2$

$SE(aX+c) = |a|\sigma$

$X \sim Exp(\lambda) := \lambda e^{-\lambda x}$ - PDF

$Supp(T) = (0, \infty)$

Parameter Space: $\lambda \in (0, \infty)$

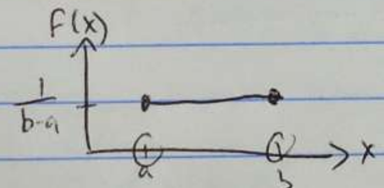
$E(X) = \int_{supp(x)} x f(x) dx = \int_0^\infty x \lambda e^{-\lambda x} = \lambda \int_0^\infty \underbrace{x}_{u} \underbrace{e^{-\lambda x} dx}_{du} \overset{IBP}{=} \dots = \frac{1}{\lambda}$

<u>Exponential is memoryless</u>

$P(X > a+b \mid X > b) = \frac{P(X > a+b \; \& \; X > b)}{P(X > b)} = \frac{P(X > a+b)}{P(X > b)}$

$= \frac{e^{-\lambda(a+b)}}{e^{-\lambda b}} = e^{-\lambda a} = P(X > a)$

$X \sim U(a,b)$



Flat line means equal probability

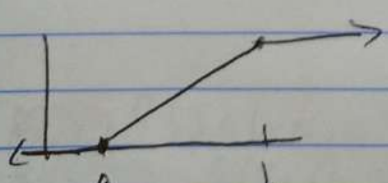
$Supp(x) = [a,b]$

Parameter Space: $a \in \mathbb{R}$

$b \in \mathbb{R}$

$a < b$

$F(x) = \int f(x) dx + c = \int \frac{1}{b-a} dx + c = \frac{x}{b-a} + c = \frac{x}{b-a} + \frac{-a}{b-a} = \frac{x-a}{b-a}$



$F(b) = 1 \Rightarrow \frac{b}{b-a} + c = 1 \Rightarrow c = 1 - \frac{b}{b-a} = \frac{b-a}{b-a} - \frac{b}{b-a} = \frac{-a}{b-a}$

$F(a) = 0$