Lecture 22

12/4/17

$X_1, \dots, X_n \overset{iid}{\sim} Bern(p)$

p is unknown

Statistical Inference — Goals

① Best guess p $\quad \hat{p} = \bar{x} = \frac{X_1 + \dots + X_n}{n} = \frac{\# 1's}{n}$

② Provide window/range/interval of likely values of p.

③ Test theories about p.

$\hat{p} \sim N\left(p, \left(\sqrt{\frac{p(1-p)}{n}}\right)^2\right)$

$P\left(p \in \left[\hat{p} \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{p(1-p)}{n}}\right]\right) = 1-\alpha$

$P\left(p \in \left[\hat{p} \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right]\right) \approx 1-\alpha$

$CI_{p, 1-\alpha} := \left[\hat{p} \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right]$

Confidence interval for parameter p with coverage $1-\alpha$

$CI_{p, 95\%} = \left[\frac{5}{16} \pm 2\sqrt{\frac{5/16(1-5/16)}{16}}\right] = [0.313 \pm 0.232] = [0.081, 0.545]$

Interpretations of CI's

① If I take many samples and compute a $\hat{p}$ for each, $1-\alpha$ proportion of the time they will cover (contain p). - Not useful

② Before obtaining the sample, $P(p \in CI_{p,1-\alpha}) = 1-\alpha$ - Not useful

③ $P\left(p \in \left[\hat{p} \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right]\right) =$ either 0 or 1 - Not useful

④ What everyone wants to say is: $P\left(p \in \left[\hat{p} \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right]\right) = 1-\alpha$

It is only true if you are a subjetivist and have specific prior information.

Do you think the proportion of babies born male $\neq$ 50%?

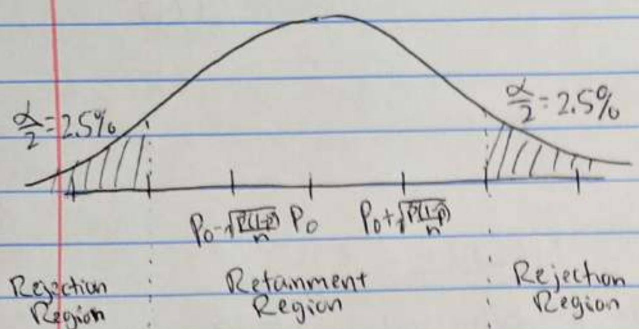
Simple model called the "null hypothesis" (H_0)

Occam's Razor: Simple model is true.

$H_0: p = p_0 = 50\%$

$H_a: p \neq p_0 = 50\%$

$\hat{p} \mid H_0$ (Assuming H_0 is true), $\alpha = 5\%$



Retainment Region $:= \left[p_0 \pm z_{\frac{\alpha}{2}} \sqrt{\frac{p_0(1-p_0)}{n}}\right]$

Rejection Region is the complement.

To test, check:

$\hat{p} \in$ Retainment Region $\Rightarrow$ Retains H_0

$\hat{p} \notin$ Retainment Region $\Rightarrow$ Reject H_0

2-sided 1-proportion hypothesis test

$n = 345$

males $= 169$

$\hat{p} = \frac{169}{345} \approx .48$

Retainment Region, $\alpha = 5\%$

$\left[.5 \pm 2\sqrt{\frac{.5(1-.5)}{345}}\right] = [.446, .554]$

Flip a coin 100 times and ask is it fair?

Situation 1: 51 heads, Fair? Yes

2: 98 heads, Fair? No

3: 61 heads, Fair?

$H_0: p = 0.5$

$H_a: p \neq 0.5$

$\hat{p} = \frac{61}{100} = .61$

Retainment Region $= \left[0.5 \pm 2\sqrt{\frac{0.5(1-0.5)}{100}}\right]$

$= [0.40, 0.60]$

$\hat{p} \notin$ Retainment Region $\Rightarrow$ Reject H_0

Mars Inc. says the proportion of blue m&m's is 20%.

$H_0: p = 0.2$

$H_a: p \neq 0.2$

$n = 206$

Retainment Region $= \left[0.2 \pm 2\sqrt{\frac{0.2 \cdot 0.8}{206}}\right] = [.144, .256)$

$\hat{p} = \frac{33}{206} = .160$

$\hat{p} \in$ Retainment Region $\Rightarrow$ Retain H_0

$\Rightarrow$ no reason to doubt their claim.