Lecture 1. Side A

Set Theory (1870's)

Sets are the fundemental unit of all of mathematics.

A set is a collection of elements which unordered & unique

$\boxed{Eg}$

assignment/definition

elements

F = { Jane, Mary, Susan, Dora }

denotes the rhs. an appropriate notation for the set

begin enn

sepration

end enn

Sets can have infinite elements

$\boxed{Eg}$ $\mathbb{N} = \{1, 2, 3, \cdots\}$ $\mathbb{Z}: \{\cdots -2, -1, 0, 1, 2, \cdots\}$

natural numbers integers

Operations on Set

Jane $\in$ F $\Rightarrow$ Jane is an element of F

Jane $\notin$ F $\Rightarrow$ Jane is not an element of F

{Jane, Mary} $\subseteq$ F $\Rightarrow$ The set or the lhs is a "subset" of the set on the rhs.

Def: all elements in the lhs set are $\in$ the rhs set

If $A = B$, then $A \subseteq B$ & $B \subseteq A$

If $A \neq B$, then $\sim(A = B)$

Proper subset

{Jane, Mary} $\subset$ F $\Rightarrow A \subset B \Rightarrow A \subseteq B$ but $A \neq B$

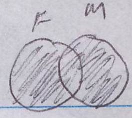
↯: {Jane} $\subset$ F Jane $\not\subset$ F {Jane} $\notin$ F Jane $\in$ F

$\in, \notin, =, \neq, \subset, \subseteq$ are predicate functions which return true or false.

$\boxed{Eg}$ $\in(\text{Jane}, F) = T$

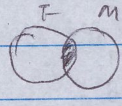
if M = { Bob, Joe, Max, Dova }, then

Lecture 1. Side B



$F \cup M = \{$Jane, Mary, Susan, Dora, Bob, Joe, Max$\}$ $\quad \cup \Rightarrow$ Union, combine all elements
"and/or" " non-exclusive or"

$F \cap M = \{$Dara$\}$ $\quad \cap \Rightarrow$ intersection, "and"



Q: A, B have infinite elements, Can $A \cap B = \varnothing$?

- Yes, eg. $A = \{2, 4, 6, 8 \cdots\}$ and $B = \{1, 3, 5, \cdots\}$

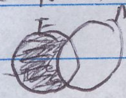
if $A \cap B = \varnothing$, then A, B are mutually exclusive or disjoint

Futhermore:

$\varnothing \subset F \qquad \varnothing \notin F$

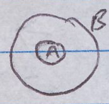
Set Subtraction.

$F \setminus M = \{$Jane, Mary, Susan$\}$ $\Rightarrow$ elements of F save the element of M.



If $A \cap B = \varnothing \Rightarrow A \setminus B = A,\ B \setminus A = B$

If $A \setminus B = \varnothing \Rightarrow A \cap B = A$



$\Downarrow$

Woudn't be true if A is a proper subset of B

$\varnothing \setminus \varnothing = \varnothing$

$\varnothing \cap \varnothing = \varnothing$

$\varnothing \cup \varnothing = \varnothing$

Set builder notation

$E := \{2n : n \in \mathbb{Z}\} = \{\cdots -6, -4, -2, 0, 2, 4, 6 \cdots\}$

$\Rightarrow$ E is the set of all $2n$ such that $n \in \mathbb{Z}$.

Power Set

$2^A := \{B : B \subseteq A\} \qquad A = \{1, 2, 3\}$ $\quad /\!/ \{1, 2, 3\}$ equivilent

$2^A = \{\varnothing, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\} \{1, 3\}, A\}$

Set Size (Cardinlity)

$|A| = 3 \qquad |2^A| = 8 = 2^3 \qquad |F \cup M| = |F| + |M|$

$7 \neq 4 + 4$