8/29/2017

Set theory (1970's)

- Sets are fundamental units of all of mathematics

A set is a collection of elements which are UNORDERED and UNIQUE

elements

comma-separate

For example: $F := \{ \text{Jane}, \text{Mary}, \text{Susan}, \text{Dana} \}$

assigned to / defined as

braces begin and end enumeration of elements

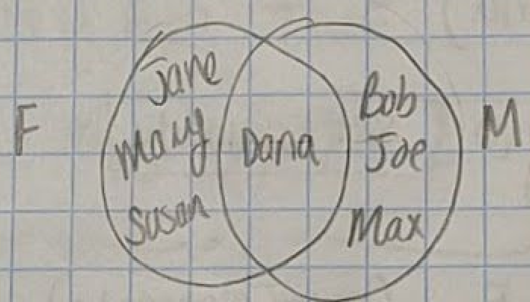
denotes the set on the rhs (right hand side)

- An appropriate notation for the set → descriptive letter

Another set: $M := \{ \text{Bob}, \text{Joe}, \text{Max}, \text{Dana} \}$

Venn Diagram Example

- Illustration of sets and their relationships



Sets can have infinite elements

e.g. $\mathbb{N} := \{ 1, 2, 3 \ldots \}$

Natural numbers

ellipses is to denote continuation of a pattern

$\mathbb{Z} := \{ \ldots, -2, -1, 0, 1, 2 \ldots \}$

Integers

Operators on Sets

Element operator

Jane $\in$ F "Jane is an element of set F"

↑ element ↑ set

Bob $\notin$ F "Bob is not an element of set F"

{Jane, Mary} $\subseteq$ F The set {Jane, Mary} is a subset of F

↑ set ↑ set

SUBSET (possible for equality)

Def: All elements in the set on the LHS are in the set on the rhs

{Bob, Jane} $\nsubseteq$ F "Not a subset of F"

F' := {Jane, mary, susan, Dona}

F' = F ? True

$\boxed{\text{set equality}}$

if $A=B \rightarrow A \subseteq B + B \subseteq A$

↳ means $F \subseteq F'$ and $F' \subseteq F$ (same elements)

{Jane, mary} $\neq$ F

{Jane, mary} $\subset$ F "LHS is a subset on the rhs but the LHS $\neq$ RHS"

↑ A "proper subset"

$A \subset B \rightarrow A \subseteq B$ but $A \neq B$

"$\subseteq$" is "$\subset$" or "="

True of False?

{Jane} $\subset$ F TRUE

{Jane} $\in$ F False "set Jane is not an element of set F" {Jane} $\notin$ F

Jane $\in$ F TRUE

Jane $\notin$ F TRUE

didn't define Jane

★ Jane $\subset$ F without a definition for the set Jane, does not "parse"

So far we have

$\in, \notin, =, \neq, \subset, \subseteq$ these are predicate functions which return True or False

$\in(\text{Jane}, F) = \text{TRUE}$

Set Functions

$F \cup M = \{\text{Jane, mary, Susan, Dana, Bob, Joe, Max}\}$

$\uparrow$ union

• combines all elements in both sets, but no duplicates "UNIQUE"

union $\neq$ Addition, it's almost addition

$\hookrightarrow$ "and/or" $\Rightarrow$ non exclusive or

★ Female name or Male name or Both ★

$\{\text{Dana}\} \cup \{\text{Dana}\} = \{\text{Dana}\}$

$\text{Dana} \in M \cup F \rightarrow \text{TRUE}$

$\mathbb{N}_0 = \mathbb{N} \cup \{0\} = \{0, 1, 2, \ldots\} \Rightarrow$ Natural #

$F \cap M = \{\text{Dana}\}$

$\downarrow$ Set Intersection "and"
elements in both sets

$F \cap \{\text{Bob}, \text{Joe}\} = \{\ \}$

$\emptyset = \{\ \}$ empty set or the "null set"

A, B have infinite elements

can $A \cap B = \emptyset$ TRUE

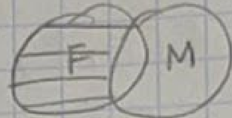
$A = \{0, 2, 4, 6, \ldots\}$
$B = \{1, 3, 5, \ldots\}$

if $A \cap B = \emptyset$
$\Rightarrow$ A, B are mutually exclusive or "disjoin"

$\emptyset \subseteq F$? $\boxed{\text{Vacuously TRUE}}$ "empty set is a subset of every set"

$\emptyset \in F$? $\boxed{\text{False}}$ $\quad \{\emptyset\} \in F$ $\boxed{\text{True}}$

Set Subtraction



$F \setminus M = \{\text{Jane, mary, Susan}\}$

↑ All elements in F except those elements of M

$\neq$

$M \setminus F = \{\text{Bob, Joe, Max}\}$

For ex:

If $A \cap B = \emptyset$ what is $A \setminus B = A$ A B

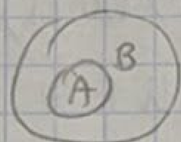
$B \setminus A = B$

If $A \setminus B = \emptyset \rightarrow A \cap B = A$

$A \overset{?}{=} B$

↓

not true, B could be bigger



$\emptyset \setminus \emptyset = \emptyset$

$\emptyset \cap \emptyset = \emptyset$

$\emptyset \cup \emptyset = \emptyset$

$A \subseteq B \Rightarrow A \setminus B = \emptyset$

Set Building Notation

$E := \{2n : n \in \mathbb{Z}\} = \{\ldots -6, -4, -2, 0, 2, 4, 6 \ldots\}$

→ n is an integer

new set — with elements 2n — such that

"E is a new set with elements 2n such that n is an integer"

Power set

"2^A is defined to be the set B such that B is a subset of A"

$2^A := \{B : B \subseteq A\} = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, A\}$

e.g. let $A = \{1, 2, 3\}$

Size of Set (cardinality)

$|A| = 3$ $\quad$ $|A|$ = # of elements of set

↓

absolute value sign

$|2^A| = 8$

$$|F \cup M| = |F| + |M|$$
$$7 \neq 4 + 4$$