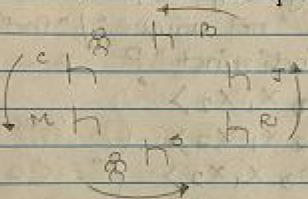
LECTURE - 04 09/07/2017

- No. of ways to sample k objects out of a set of n objects without replacement.

Permutation

$$\,^{n}P_{k} = \frac{n!}{(n-k)!} = (n)(n-1)\ldots(n-k+1)$$

→ chairs in a circle and you don't care which chair is first.



- How many ways to seat? (6!)

$$\left.\begin{array}{l} \langle B, J, R, S, M, C\rangle \\ \langle C, B, J, R, S, M\rangle \\ \langle M, C, B, J, R, S\rangle \\ \langle S, M, C, B, J, R\rangle \\ \langle R, S, M, C, B, J\rangle \\ \langle J, R, S, M, C, B\rangle \end{array}\right\} \text{"collap-sable subset"}$$

Principal of dividing out the invarience factor : $\frac{6!}{6} = \boxed{\frac{720}{6}}$

- imagine a basket of 5 flowers:
  3 orchids $\{O_1, O_2, O_3\}$ and
  2 chrysenthemums $\{X_1, X_2\}$

→ How many ways to place them in 5 flower pots? $= 5!$

→ How many ways to place them if each orchid is "not unique" / "indistinguishable" / "indistinct"?

$$\left.\begin{array}{l}\langle O_1\, O_2\, O_3\, X_1\, X_2 \rangle \\ \langle O_1\, O_3\, O_2\, X_1\, X_2 \rangle \\ \langle O_2\, O_1\, O_3\, X_1\, X_2 \rangle \\ \langle O_2\, O_3\, O_1\, X_1\, X_2 \rangle \\ \langle O_3\, O_1\, O_2\, X_1\, X_2 \rangle \\ \langle O_3\, O_2\, O_1\, X_1\, X_2 \rangle \end{array}\right\} = \langle O\, O\, O\, X_1\, X_2 \rangle$$

collapses to "1"

- no. of ways you can arrange 3 orchids for which it is $(3!)$
  Principal of dividing out the invarience factor:

$$= \frac{5!}{3!}$$

- How many ways to place them if each orchid and each chrysenthemum is indistinct?

$$\left.\begin{array}{l}\langle O_1 O_2 O_3\, X_2 X_1 \rangle \\ \langle O_1 O_3 O_2\, X_2 X_1 \rangle \\ \langle O_2 O_1 O_3\, X_2 X_1 \rangle \\ \langle O_2 O_3 O_1\, X_2 X_1 \rangle \\ \langle O_3 O_1 O_2\, X_2 X_1 \rangle \\ \langle O_3 O_2 O_1\, X_2 X_1 \rangle \end{array}\right\} \text{collapses to "1"} = \langle OOOX_2X_1 \rangle$$

Principal of dividing out the invarienc factor: $= \dfrac{5!}{3!\,2!}$

- How many ways to order the people in 6 chairs? $= 6!$

- How many ways to order the peope in 3 chairs? $= {}_6P_3$

- How many ways to order the people in 3 chairs such that the order (of thos 3) doesnot matter?

$\langle B, J, R \rangle$

$\langle B, R, J \rangle$

$\langle R, B, J \rangle$

$\langle R, J, B \rangle$

$\langle J, B, R \rangle$

$\langle J, R, B \rangle$

collapses to $\{B, J, R\}$

$$= \frac{6P_3}{3P_3} = \frac{6P_3}{3!} = \boxed{20}$$

J, B, S
J, B, R
J, B, M
J, B, C
J, S, R
J, S, M
J, S, C
J, R, M
J, R, C
J, M, C
(10)

B, S, R
B, S, M
B, S, C
B, R, M
B, R, C
B, M, C
(6)

S, R, M
S, R, C
S, M, C
(3)

R, M, C
(1)

$\approx \{J, B, S, R, M, C\}$

$10+6+3+1 = \boxed{20}$

- How many ways to seat the people 4 chairs order doesnot matter?

$$= \frac{6P_4}{4!} = \boxed{15}$$

J, B, S, R
J, B, S, M
J, B, S, C
J, B, R, M
J, B, R, C
J, B, M, C
J, S, R, M
J, S, R, C
J, S, M, C
J, R, M, C
(10)

B, S, R, M
B, S, R, C
B, S, M, C
B, R, M, C
(4)

S, R, M, C
(1)

$10+4+1 = \boxed{15}$

- How many ways to sample k items out of a set of n without replacement such that order doesnot matter?

$$\binom{n}{k} := \frac{{}_nP_k}{{}_kP_k} = \frac{{}_nP_k}{k!} = \frac{\frac{n!}{(n-k)!}}{k!}$$

$${}_nC_k :=$$

$$= \frac{n!}{(n-k)!\,k!}$$

** These are Combination - "Choose"

→ further conceptual theory of permutation.

→ choosing but not caring about the order.

* Combinotorial Indentities :

(1) $\binom{n}{1} = \frac{n!}{(n-1)!\,1!} = \boxed{n}$ $\Big\}$ "n" no. of balls

(2) $\binom{n}{n-1} = \frac{n!}{1!\,(n-1)!} = \boxed{n}$ $\Big\}$ 1 ball left for which n balls are left over?

(3) $\binom{n}{k} = \frac{n!}{\underbrace{(n-(n-k)}_{k}!\,(n-k)!} = \boxed{k}$

(4) $\binom{n}{0} = \frac{n!}{n!\,0!} = \boxed{1}$

(5) $\binom{n}{n} = \frac{n!}{n!\,n!} = \boxed{1}$

- You seat 4 people randomly out of B, J, R, S, M, C. What is the prob of Jane is seated?

$\therefore \Omega$ = all possible seating arrangements for all 4 people. The order doesnot matter and also matters.

→ Order doesnot matter :

$$P(A) = \frac{\binom{5}{3}}{\binom{6}{4}} = \boxed{\frac{2}{3}}$$

$\{B, J, R, S, M, C\}$

** if Jane seats in one : $\{J, \_, \_, \_\}$

→ Order matters :

$$P(A) = \frac{4(5P3)}{6P4} = \boxed{\frac{2}{3}}$$ divide by 4!

		$\overbrace{\quad\quad\quad}^{5P3}$		
	J	5	4	3
Chairs #	1	2	3	4
	5	J	4	3
	5	4	J	3
	5	4	3	J

4 ways in which Jane can be seated 4(5P3)

* $2^A = \{B : B \subseteq A\}$

$= \{B : B \subseteq A \text{ \& } |B| = 0\} \cup$

$\{B : B \subseteq A \text{ \& } |B| = 1\} \cup$

$\{B : B \subseteq A \text{ \& } |B| = 2\} \cup$

⋮

$\{B : B \subseteq A \text{ \& } |B| = n\}$

These are mutually exclusive and collectively exhaustive then :

$$\therefore |2^A| = |\{B : B \subseteq A \ \& \ |B| = 0\}| + \dots + |\{B : B \subseteq A \ \& \ |B| = n\}|$$

$$= \sum_{i=0}^{n} |\{B : B \subseteq A \ \& \ |B| = i\}|$$

$$= \sum_{i=0}^{n} \binom{n}{i} = 2^n$$

* $(a+b)^2 = (a+b)(a+b)$
  $= a^2 + ab + ba + b^2$ } 4 terms
  $= a^2 + 2ab + b^2$

* $(a+b)^3 = (a+b)(a+b)(a+b)$
  $=$ 8 terms as 2^3
  $= a^3 + 3a^2b + 3ab^2 + b^3$
  $(1+3+3+1 = 8)$

* $(a+b)^4 = (a+b)(a+b)(a+b)(a+b)$
  $= 2^4$ terms
  $= \sum_{i=0}^{n} \binom{4}{i} = \binom{4}{4} + \binom{4}{3} + \binom{4}{2} + \binom{4}{1} + \binom{4}{0}.$

$$= \binom{4}{4} a^4 b^0 + \binom{4}{3} a^3 b^1 + \binom{4}{2} a^2 b^2 + \binom{4}{1} a^1 b^3 + \binom{4}{0} a^0 b^4$$

* $(a+b)^n = 2^n$ terms

$$= \sum_{i=0}^{n} \binom{n}{i}$$

$$= \binom{n}{n} a^n b^0 + \binom{n}{n-1} a^{n-1} b^1 + \cdots + \binom{n}{2} a^2 b^{n-2} + \binom{n}{1} a^1 b^{n-1} + \binom{n}{0} a^0 b^n$$

$$= \sum_{i=0}^{n} \underbrace{\binom{n}{i}} a^i b^{n-i}$$

Binomial Theorem and this is the Binomial Coefficient.

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