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$(pN)(pN-1)\ldots(pN-x+1)$ Oct 18, 2017

$$\lim_{N\to\infty} \text{Hyper}(n,p,N) = \lim_{N\to\infty} \frac{\binom{pN}{k}\binom{(1-p)N}{n-k}}{\binom{N}{n}} = \binom{n}{x} \lim_{N\to\infty} \frac{\frac{(pN)!}{(pN-x)!}\,\frac{((1-p)N)!}{((1-p)N-(n-x))!}}{\frac{N!}{(N-n)!}} =$$

$$\frac{pN!}{(pN-x)!} = (pN)(pN-1)\ldots(pN-x+1) = x \text{ terms}$$

A permutation $\frac{N!}{(N-n)!} = \underbrace{N(N-1)(N-2)\ldots(N-n+1)}$

$$\frac{((1-p)N)!}{((1-p)N-(n-x))!} = ((1-p)N)((1-p)N-1)\ldots((1-p)N-(n-x)+1) = (n-x) \text{ terms}$$

$$\frac{N!}{(N-n)!} = N(N-1)(N-2)\ldots(N-n+1) = \cancel{N}\ n \text{ terms}$$

n terms

n terms

x terms

$$= \binom{n}{x} \underbrace{\lim_{N\to\infty}\frac{pN}{N}}_{=p}\ \underbrace{\lim_{N\to\infty}\frac{pN-1}{N-1}}_{=p}\cdots\lim_{N\to\infty}\underbrace{\frac{pN-x+1}{N-x+1}}_{p}\ \lim_{N\to\infty}\underbrace{\frac{(1-p)N}{N-x}}_{1-p}\cdots$$

n-x terms

$$\cdot \underbrace{\lim_{N\to\infty}\frac{(1-p)N-(n-x)+1}{N-n+1}}_{1-p} = \boxed{\binom{n}{x}p^x(1-p)^{n-x}}$$

Sampling with replacement

Sampling with Replacement

$X \sim \text{Binomial}(n,p) := \binom{n}{x}p^x(1-p)^{n-x}$

$\text{Supp}[x] = \{0, 1, \ldots, n\}$

param space

$p \in (0,1)$

$n \in \mathbb{N} = \{1, 2, 3, \ldots, \infty\}$

$X \sim \text{Bin}(1,p)$ its a Bern(p)

$\text{Supp}[x] = \{0,1\}$

$X \sim \text{Bin}(1,p) = \binom{1}{x}p^x(1-p)^{1-x} = \text{Bern}(p)$

$x = 0 \Rightarrow \binom{1}{x} = 0$

$x = 1 \Rightarrow \binom{1}{x} = 1$

$X \sim \text{Binomial} \Rightarrow$ we are reaching into an infinite large bag and pulling out n marbles and asking how many are successes?

$$\sum_{x \in Supp[x]} p(x) = 1$$

Recall

$$(a+b)^n = \sum_{i=0}^{n} \binom{n}{i} \cdot a^i b^{n-i}$$

$$\sum_{x=0}^{n} \binom{n}{x} p^x (1-p)^{n-x} = \left((p) + (1-p)\right)^n = 1^n = 1$$

~~Binomial Random Var~~ ~~is called~~

Binomial Random Var is named after the Binomial theorem. ~~because~~

$X_1 \sim Bern\left(\frac{1}{3}\right)$

$X_1 \stackrel{d}{=} X_2$

$X_2 \sim Bern\left(\frac{1}{3}\right)$

from conditional prob

In general if X_1 + X_2 are independent then $\boxed{\perp(X_1, X_2^{ind})}$ ~~X_1, X_2 ind~~ ~~X_1 and X_2 are ind~~

a) $P(X_1 = x_1 \mid X_2 = x_2) = P(X_1 = x_1) \quad \forall x_1 \in Supp[X_1], \forall x_2 \in Supp[X_2]$

b) $P(X_2 = x_2 \mid X_1 = x_1) = P(X_2 = x_2)$

c) $P(X_1 = x_1, X_2 = x_2) = P(X_1 = x_1) P(X_2 = x_2)$.

(Joint Mass Function) not tested on this, but may be in HW.

$X_1, X_2 \stackrel{iid}{\sim}$ which means $X_1, X_2 \stackrel{ind}{\sim}$ ← independent AND $X_1 \stackrel{d}{=} X_2$. ← identically distributed (PMF ARE The same)

X_1, X_2 are iid

$T_2 = X_1 + X_2$ = whatever X_1 and X_2 spit out, add them

$Supp[T_2] = \{0, 1, 2\}$

0+0, 0+1, 1+0, 1+1

$X_1 \sim Bern\left(\frac{1}{3}\right)$ says $1 = P\left(\frac{1}{3}\right)$, $0 = P\left(\frac{2}{3}\right)$

$$T_2 = \begin{cases} 0 & w/p \quad \frac{4}{9} = \frac{2}{3} * \frac{2}{3} \\ 1 & w/p \quad \frac{4}{9} \\ 2 & w/p \quad \frac{1}{9} \end{cases}$$

Sum has to be 1

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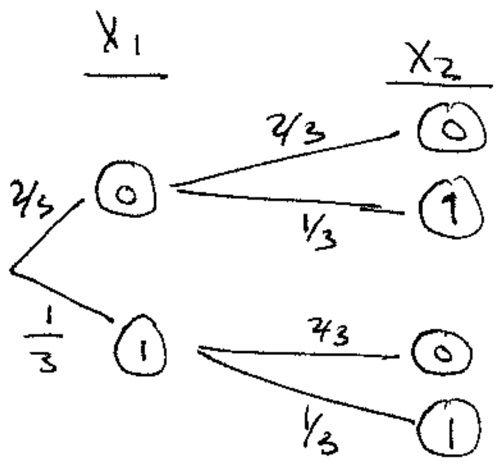
DRAWING TREES



Joint

$P(X_1=0, X_2=0) = \frac{4}{9}$ — T_2: 0

$P(X_1=0, X_0=1) = \frac{2}{9}$ — 1

$P(X_1=1, X_2=0) = \frac{2}{9}$ — 1

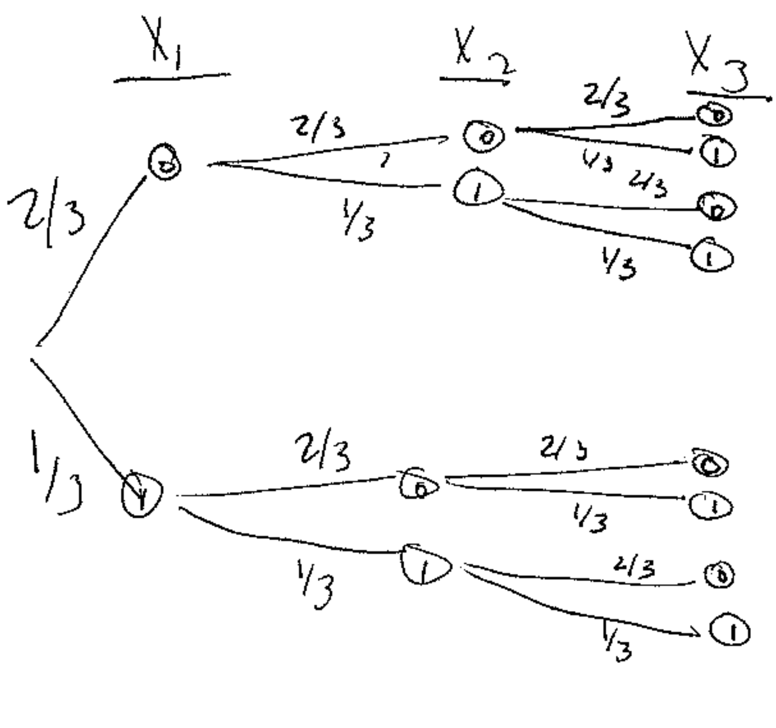
$P(X_1=1, X_2=1) = \frac{1}{9}$ — 2

$X_1, X_2, X_3 \overset{iid}{\sim} \text{Bern}(\frac{1}{3})$

$T_3 = X_1 + X_2 + X_3$

$\text{Supp}[T_3] = \{0, 1, 2, 3\}$

$$T_3 = \begin{cases} 0 & wp \\ 1 & wp \\ 2 & wp \\ 3 & wp \end{cases}$$



$P(X_1=x_1, X_2=x_2, X_3=x_3)$	T
$(\frac{1}{3})^0(\frac{2}{3})^3$	0
$(\frac{1}{3})^1(\frac{2}{3})^2$	1
$(\frac{1}{3})^1(\frac{2}{3})^2$	1
$(\frac{1}{3})^2(\frac{2}{3})^1$	2
$(\frac{1}{3})^1(\frac{2}{3})^2$	1
$(\frac{1}{3})^2(\frac{2}{3})^1$	2
$(\frac{1}{3})^2(\frac{2}{3})^1$	2
$(\frac{1}{3})^3(\frac{2}{3})^0$	3

$P(F_3 = 1) = \binom{3}{1}\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^2$

1 _ _   _ 1 _   _ _ 1

$P(T_3 = 2) = \binom{3}{2}\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^1$

1 1 _   _ 1 1   1 _ 1

$P(T_3 = 3) = \binom{3}{3}\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^0$

$P(T_3 = 0) = \binom{3}{0}\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^3$

$$T_n = \sum_{i=1}^{n} X_i \qquad \underline{X_1, X_2, \ldots, X_n \overset{iid}{\sim} \text{Bern}\left(\tfrac{1}{3}\right)}$$

$X_1, \ldots, X_n \overset{ind}{\sim} \text{Bern}(p)$

$\text{Supp}[T_n] = \{0, 1, 2, \ldots, n\}$

$$T_n \sim \begin{cases} 0 & \text{wp } \binom{n}{0}\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^n \\ 1 & \text{wp } \binom{n}{1}\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^{n-1} \\ 2 & \text{wp } \binom{n}{2}\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^{n-2} \\ \vdots & \\ n-1 & \text{wp } \binom{n}{n-1}\left(\frac{1}{3}\right)^{n-1}\left(\frac{2}{3}\right)^1 \\ n & \text{wp } \binom{n}{n}\left(\frac{1}{3}\right)^n\left(\frac{2}{3}\right)^0 \end{cases}$$

$\underline{X_1, \ldots, X_n \overset{ind}{\sim} \text{Bern}(p)}$

$$T_n \sim \begin{cases} 0 & \text{wp } \binom{n}{0}(p)^0(1-p)^n \\ 1 & \text{wp } \binom{n}{1}(p)^1(1-p)^{n-1} \\ 2 & \text{wp } \binom{n}{2}(p)^2(1-p)^{n-2} \\ \vdots & \\ n-1 & \text{wp } \binom{n}{n-1}(p)^{n-1}(1-p)^1 \\ n & \text{wp } \binom{n}{n}(p)^n(1-p)^0 \end{cases} = \binom{n}{x} p^x (1-p)^{n-x}$$

$T \sim \text{Binomial}(n,p) =$

$=$

called

$=$ ~~[illegible] (n,p)~~

The Tn Bin(n, p) can be conceptualized by

$$T = \lim_{N \to \infty} \text{Hyper}(n, p, N)$$ infinitely large bag

or

$$T = \sum_{i=1}^{n} X_i \quad \text{s.t} \quad X_1, X_2, \dots, X_n \overset{\text{iid}}{\sim} \text{Bern}(p)$$ finite bag.

$$P(x) = P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$$

$$F(x) = P(X \leq x) = \sum_{i=0}^{x} \binom{n}{i} p^i (1-p)^{n-i}$$