SET THEORY (invented around 1870's)

FACTS

⓪ Sets are the fundamental unit of all mathematics

① A **set** is: a collection of elements which are **unordered** and **unique**

assignment/ definition

elements

e.g $F := \{ \text{Jane}, \text{Mary}, \text{Susan}, \text{Dana} \}$ <= a set of female names

denotes l.h.s. an approximate notation for the set

begins enumeration

seperators

ends enumeration

left-hand side (l.h.s)
right-hand side (r.h.s)

$M := \{ \text{Bob}, \text{Joe}, \text{Max}, \text{Dana} \}$ <= a set of male names

② Sets can have infinite elements

e.g. $\mathbb{N} := \{1, 2, 3, \dots\}$ NATURAL NUMBERS

ellipsis denotes continuation of a pattern

$\mathbb{Z} := \{\dots, -2, -1, 0, 1, 2, \dots\}$ INTEGERS

③ **Operators on Sets**

Jane $\in$ F "Jane is an element of set F"

Bob $\notin$ F "Bob is not an element of set F"

$\{\text{Jane}, \text{Mary}\} \subseteq F$ The set on the l.h.s is a "subset" of the set on the r.h.s. set

$F' := F$ If $A = B \Rightarrow A \subseteq B$ & $B \subseteq A$

$F' := \{\text{Jane}, \text{Mary}, \text{Susan}, \text{Dana}\}$

$A \neq B$ Not $A = B$

$\{\text{Jane}, \text{Mary}\} \subset F$ A "proper subset"

$A \subset B \Rightarrow A \subseteq B$ but $A \neq B$

$\{\text{Jane}\} \subset F$ Jane $\not\subset$ F => because Jane is an undefined set

$\{\text{Jane}\} \notin F$ Jane $\in$ F

$\in, \notin, =, \neq, \subset, \subseteq$ < are predicate functions which return TRUE or FALSE

$F \cup M = \{\text{Jane}, \text{Mary}, \text{Susan}, \text{Dana}, \text{Bob}, \text{Joe}, \text{Max}\}$

"union"
"and/or"
"non-exclusive or"

combines all elements in both sets

$\{\text{Dana}\} \cup \{\text{Dana}\} = \{\text{Dana}\}$
$\text{Dana} \in M \cup F$
$\mathbb{N}_0 := \mathbb{N} \cup \{0\} = \{0, 1, 2, \dots\}$

$F \cap M = \{\text{Dana}\}$
set "intersection"
"AND"

$F \cap \{\text{Bob}, \text{Joe}\} = \{\}$

$\{\} = \emptyset$ empty set or the "null set"

A, B have infinite elements.

Can $A \cap B = \emptyset$?
$A = \{2, 4, 6, \dots\}$
$B = \{1, 3, 5, \dots\}$

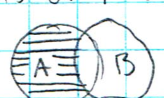
If $A \cap B = \emptyset$
$\Rightarrow$ "A, B are mutually exclusive"
or "disjoint"

$\emptyset \notin F \Leftarrow$ True

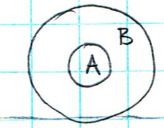
$\emptyset \subset F \Leftarrow$ True

Set Subraction

$F \setminus M = \{\text{Jane}, \text{Mary}, \text{Susan}\}$ < elements of F without the elements of M



$A \cap B = \emptyset \Rightarrow A \setminus B = A, \; B \setminus A = B$
if $A \setminus B = \emptyset \Rightarrow A \cap B = A$



$\emptyset \setminus \emptyset = \emptyset$
$\emptyset \cap \emptyset = \emptyset$
$\emptyset \cup \emptyset = \emptyset$

$E := \{2n : n \in \mathbb{Z}\} = \{\dots, -6, -4, -2, 0, 2, 4, 6, \dots\}$

"set builder notation"

E is the set of all $2n$ such that $n \in \mathbb{Z}$

Power Set $2^A := \{B : B \subseteq A\}$ $\quad A = \{1, 2, 3\}$ $\quad 2^A = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{1,3\}, A\}$

Set Size (cardinality)

$|A| = 3$ $\quad |2^A| = 8$ $\quad |F \cup M| \stackrel{?}{=} |F| + |M|$

$7 \neq 4 + 4$

absolute value sign