$$\lim_{N\to\infty} \frac{\dfrac{(pN)!}{(pN-x)!}\;\dfrac{((1-p)N)!}{((1-p)N-(n-x))!}}{\dfrac{N!}{(N-n)!}}$$

$$\boxed{10/18}$$

$$\boxed{\lim_{N\to\infty} \text{Hyper}(n,p,N)} = \lim_{N\to\infty} \frac{\binom{pN}{x}\binom{(1-p)N}{n-x}}{\binom{N}{n}}$$

x terms $\to$ $(pN)\cdot(pN-1)\cdots(pN-x+1)$ $\oplus$ $\to$ n total terms

$((1-p)N)\cdot((1-p)N-1)\cdots((1-p)N-(n-x)+1)$

$\uparrow$ $(n-x)$ terms

$$= \binom{n}{x} \lim_{N\to\infty} \frac{\dfrac{(pN)!}{(pN-x)!}\;\dfrac{((1-p)N)!}{((1-p)N-(n-x))!}}{\underbrace{\dfrac{N!}{(N-n)!}}_{N\cdot(N-1)\cdot(N-2)\cdots(N-n+1)}}$$

Rule from Calculus:
$\lim f(x)g(x) = \lim f(x) \lim g(x)$

$\uparrow$ n terms

$$= \binom{n}{x} \overbrace{\lim_{N\to\infty} \underbrace{\frac{pN}{N}}_{p} \cdot \lim_{N\to\infty} \underbrace{\frac{pN-1}{N-1}}_{p} \cdot \ldots \cdot \lim_{N\to\infty} \underbrace{\frac{pN-x+1}{N-x+1}}_{p}}^{\boxed{p^x}} \cdot \overbrace{\lim_{N\to\infty} \underbrace{\frac{(1-p)N}{N-x}}_{1-p} \cdot \ldots}^{(1-p)^{n-x}}$$

$$\cdot \lim_{N\to\infty} \underbrace{\frac{(1-p)N-(n-x)+1}{N-n+1}}_{1-p} = \boxed{\binom{n}{x} p^x (1-p)^{n-x}}$$

important!

Which is like:

$$X \sim \text{Binomial}(n,p) := \binom{n}{x} p^x (1-p)^{n-x}$$

$$\text{Supp}(x) = \{0, 1, 2, \dots, n\}$$

Parameter Space

$$p \in (0,1)$$

$$n \in \mathbb{N}$$

$$x=0 \Rightarrow \binom{1}{x} = 0$$
$$x=1 \Rightarrow \binom{1}{x} = 1$$

$$X \sim \text{Binomial}(1,p) = \cancel{\binom{1}{x}} p^x (1-p)^{1-x} = \text{Bern}(p)$$

$$\text{Supp}(x) = \{0,1\}$$

Recall

$$(a+b)^n = \sum_{i=0}^{n} \binom{n}{i} a^i b^{n-i}$$

$$\sum_{x \in Supp(X)} p(x) = 1$$

$$\sum_{x=0}^{n} \binom{n}{x} p^x (1-p)^{n-x}$$

$$= \left((p) + (1-p) \right)^n = 1^n = 1 \checkmark$$

A process that either spits 1 with $\frac{1}{3}$ prob.
or 0 with $\frac{2}{3}$ prob.

$$X_1 \sim Bern\left(\frac{1}{3}\right)$$

$$X_2 \sim Bern\left(\frac{1}{3}\right)$$

$$X_1 \overset{d}{=} X_2$$

In general, if x_1 & x_2 are independent, then

(a) $P(X_1 = x_1 \mid X_2 = x_2) = P(X_1 = x_1) \quad \forall x_1 \in Supp(X_1), \forall x_2 \in Supp(X_2)$

(b) $P(X_2 = x_2 \mid X_1 = x_1) = P(X_2 = x_2)$

(c) $P(X_1 = x_1, X_2 = x_2) = P(X_1 = x_1) P(X_2 = x_2)$

Joint mass function

(independent and
independently distributive)

$X_1, X_2 \overset{ind}{\sim}$

which means

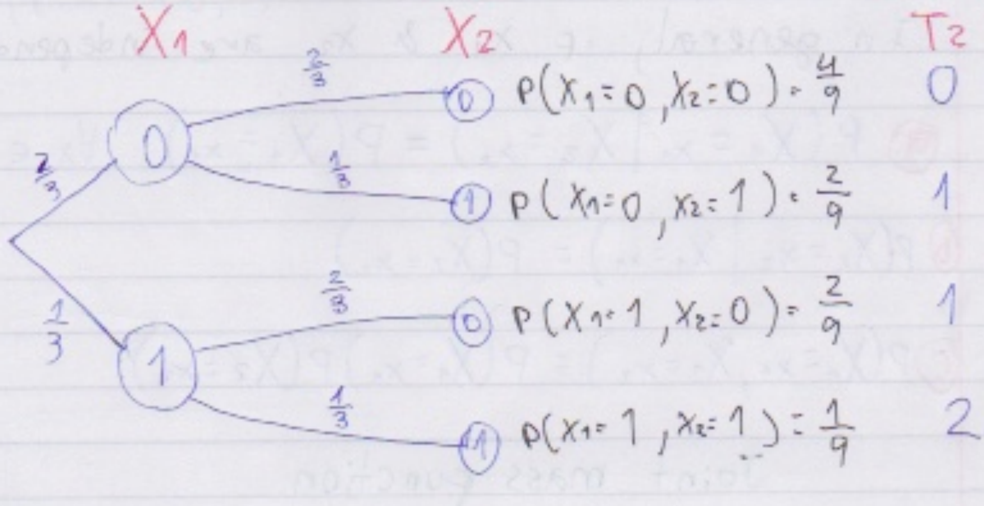
$X_1, X_2 \overset{ind}{\sim}$
AND

$X_1 \overset{d}{=} X_2$

$T_2 = X_1 + X_2$

$Supp(T_2) = \{0, 1, 2\}$

$$T_2 \sim \begin{cases} 0 & \text{wp } \frac{4}{9} \\ 1 & \text{wp } \boxed{\frac{4}{9}} \text{ it's gotta be} \\ 2 & \text{wp } \frac{1}{9} \end{cases}$$

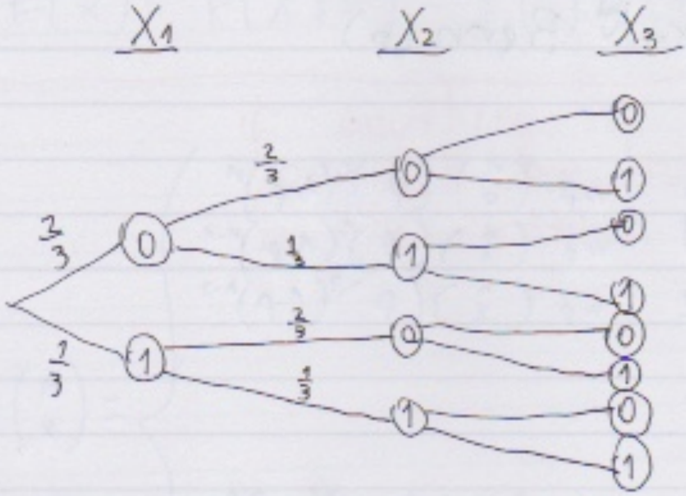


$X_1, X_2, X_3 \overset{ind}{\sim} \text{Bern}\left(\frac{1}{3}\right)$

$T_3 = X_1 + X_2 + X_3$

$\text{Supp}(T_3) = \{0, 1, 2, 3\}$

$$T_3 = \begin{cases} 0 & \text{wp} \\ 1 & \text{wp} \\ 2 & \text{wp} \\ 3 & \text{wp} \end{cases}$$



$P(X_1=x_1, X_2=x_2, X_3=x_3)$	$\boxed{T_3}$
$\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^3$	0
	1
	1
	2
	1
	2
	2
$\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^0$	3

choose p.

$P(T_3 = 1) = \binom{3}{1}\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^2$

$P(T_3 = 0) = \binom{3}{0}\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^3$

$P(T_3 = 2) = \binom{3}{2}\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^1$

$P(T_3 = 3) = \binom{3}{3}\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^0$

$$T_n = \sum_{i=1}^{n} X_i$$

$$Supp(T_n) = \{0, 1, 2, \dots, n\}$$

$$X_1, \dots, X_n \overset{ind}{\sim} Bern\left(\tfrac{1}{3}\right)$$

$$T_n \sim \begin{cases} 0 & wp \ \binom{n}{0}\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^n \\ 1 & wp \ \binom{n}{1}\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^{n-1} \\ 2 & wp \ \binom{n}{2}\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^{n-2} \\ \vdots & \\ n-1 & wp \ \binom{n}{n-1}\left(\frac{1}{3}\right)^{n-1}\left(\frac{2}{3}\right)^1 \\ n & wp \ \binom{n}{n}\left(\frac{1}{3}\right)^n\left(\frac{2}{3}\right)^0 \end{cases}$$

$$X_1, \dots, X_n \overset{ind}{\sim} Bern(p)$$

$$T_n \sim \left.\begin{cases} 0 & wp \ \binom{n}{0}(p)^0(1-p)^n \\ 1 & wp \ \binom{n}{1}(p)^1(1-p)^{n-1} \\ 2 & wp \ \binom{n}{2}(p)^2(1-p)^{n-2} \\ \vdots & \\ n-1 & wp \ \binom{n}{n-1}(p)^{n-1}(1-p)^1 \\ n & wp \ \binom{n}{n}(p)^n(1-p)^0 \end{cases}\right\} = \binom{n}{x} p^x (1-p)^{n-x} = Bin(n, p)$$

The $T \sim Bin(n, p)$ can be conceptualized by

$$T = \lim_{N \to \infty} Hyper(n, p, N)$$

or

$$T = \sum_{i=1}^{n} X_i \quad \text{s.t. } X_1, X_2, \ldots, X_n \overset{ind}{\sim} Bern(p)$$

$p(x) = P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$

$$\boxed{F(x) = P(X \le x) = \sum_{i=0}^{x} \binom{n}{i} p^i (1-p)^{n-i}}$$

if anything
you will see this
in the exam