10/16/17. $X \sim \text{Ber}(p) := p^x (1-p)^{1-x}$.

$\text{Supp}[X] = \{0, 1\}$.

$p \in (0,1)$

100 ppl, 53 female, pull 8, $p(6f) = ?$

$$X \sim \text{Hyper}(8, 53, 100) := p(6f) = \frac{\binom{53}{6}\binom{47}{2}}{\binom{100}{8}}$$

$N = 0 \Rightarrow K = 0 \quad n = 0. \quad \times.$

$N = 1 \Rightarrow K \in \{0, 1\}$

$n \in \{\cancel{0}, 1\}$

$N = 2 \Rightarrow K \in \{\cancel{0}, 1, \cancel{2}\}$

$n \in \{0, 1, \cancel{2}\}$

$$X \sim \text{Hyper}(1, 1, 2) := \frac{\binom{1}{x}\binom{1}{1-x}}{\binom{2}{1}} = \text{Be}\left(\tfrac{1}{2}\right).$$

$\text{Supp}[X] = \{0, 1\}$

$$p(1) = \frac{\binom{1}{1}\binom{1}{0}}{\binom{2}{1}} = \frac{1}{2}.$$

$$p(0) = \frac{\binom{1}{0}\binom{1}{1}}{\binom{2}{1}} = \frac{1}{2}.$$

$N = \{2, 3, \ldots\} \quad K \in \{1, \ldots, N-1\}$.

$n \in \{1, \ldots, N-1\}$.

$$X \sim \text{Hyper}(1, K, N) = \frac{\binom{K}{x}\binom{N-K}{1-x}}{\binom{N}{1}}$$

$\text{Supp}[X] = \{0, 1\}$

$$p(1) = \frac{\binom{K}{1}\binom{N-K}{1-1}}{\binom{N}{1}} = \frac{K}{N}$$

$$p(0) = \frac{\binom{K}{0}\binom{N-K}{1-0}}{\binom{N}{1}} = \frac{N-K}{N} = 1 - \frac{K}{N}.$$

(a). $X \sim \text{Hyper}(2, 4, 10)$, $\text{Supp}(X) = \{0, 1, 2\}$.

$n < K$, $n < N-K$, $\text{Supp}[X] = \{0, \dots, n\}$.

(b) $X \sim \text{Hyper}(5, 4, 10)$, $\text{Supp}[X] = \{0, 1, 2, 3, 4\}$.

$n \geqslant K$, $n < N-K$ $\quad \text{Supp}[X] = \{0, \dots, K\}$.

(c). $X \sim \text{Hyper}(8, 4, 10)$, $\text{Supp}[X] = \{2, 3, 4\}$.

$n \geqslant K$, $n \geqslant N-K$ $\quad \text{Supp}[X] = \{n-(N-K), \dots, K\}$

(d) $X \sim \text{Hyper}(5, 7, 10)$, $\text{Supp}[X] = \{2, 3, 4, 5\}$.

$n < K$, $n \geqslant N-K$ $\quad \text{Supp}[X] = \cancel{\{2,3,4,5\}}\ \{n-(N-K), \dots, n\}$.

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| | $n < K$ | $N \geqslant K$. |
|---|---|---|
| $N < N-K$. | $\{0, \dots, n\}$ | $\{0, \dots, K\}$ |
| $N \geqslant N-K$. | $\{n-(N-K), \dots, n\}$ | $\{n-(N-K), \dots, K\}$ |

$$\text{Supp}[X] = \{\max\{0, n-(N-K)\}, \dots, \min\{K, n\}\}$$

let $p := \frac{K}{N} \Rightarrow K = pN$.

$$X \sim \text{Hyper}(n, p, N) = \frac{\binom{pN}{x}\binom{(1-p)N}{n-x}}{\binom{N}{n}}$$

$$p = \left\{ \frac{1}{N}, \frac{2}{N}, \dots, \frac{N-1}{N} \right\}.$$

let $p = 0.5$, $n = 6$.

① $N = 100$, $p(3) = \dfrac{\binom{50}{3}\binom{50}{3}}{\binom{100}{6}} = 0.3223$.

② $N = 1000$, $p(3) = \dfrac{\binom{500}{3}\binom{500}{3}}{\binom{1000}{6}} = 0.3134$.

③ $N = 10{,}000$, $p(3) = \dfrac{\binom{5000}{3}\binom{5000}{3}}{\binom{10000}{6}} = 0.3126$.

$$\therefore \lim_{N \to \infty} \text{Hyper}(n, p, N) = \lim_{N \to \infty} \frac{\binom{pN}{x}\binom{(1-p)N}{n-x}}{\binom{N}{n}}$$

$$\therefore \lim_{N \to \infty} \frac{\frac{pN!}{x!(pN-x)!} \frac{((1-p)N)!}{(n-x)!((1-p)N-(n-x))!}}{\frac{N!}{n!(N-n)!}}$$

$$= \frac{1}{x!} \cdot \frac{1}{(n-x)!} \cdot n! \lim_{N \to \infty} \frac{pN! \, ((1-p)N)! \, (N-n)!}{(pN-x)! \quad ((1-p)N-(n-x))! \cdot N!}$$