MAT241 - 11/1/17 - Lecture

Best of #7 \$1

$$X := \begin{cases} \$35 & wp \ 1/38 \\ -\$1 & wp \ 37/38 \end{cases} \Rightarrow \mu = -\$0.053$$

$$\sigma^2 = (35 - (-0.053))^2 \frac{1}{38} + (-1 - (-0.053))^2 \frac{37}{38} = 33.209 \ \$^2$$

$$\sigma^2 = \sqrt{33.207} = \$5.79$$

Bet on Black \$1

$$X \sim \begin{cases} \$1 & wp \ 18/38 \\ -\$1 & wp \ 20/38 \end{cases} \Rightarrow \mu = \$0.053$$

$$\sigma^2 = (1 - (-0.053))^2 \frac{18}{38} + (-1 - (-0.053))^2 \frac{20}{38} = 0.997 \ \$^2$$

$$\sigma = \sqrt{.997} \approx \$1$$

$$\mu = E[x] = \sum_{x \in supp[x]} x \, p(x)$$

$$\sigma^2 = var[x] = \sum_{x \in supp[x]} (x - \mu)^2 \, p(x)$$

$$\overline{X}_A \to \mu$$

Law of large #'s

$$\overline{X}_B \to \mu$$

~~stand~~ Standard deviation or Standard Error

$$\sigma = SE(x) = SD[x] := \sqrt{var[x]}$$

$T_2 = X_1 + X_2 \quad E[T_2] = \sum_{t \in supp[t]} t\, p(t)$ impractical

$$E[g(X)] = \sum_{x \in supp(X)} g(x) p(x)$$

Joint mass function

$$E[g(X_1, X_2)] = \sum_{x_1 \in supp(X_1)} \sum_{x_2 \in Supp[X_2]} g(x_1, x_2)\, p(x_1, x_2)$$

$$E[X_1 + X_2] = \sum_{x_1} \sum_{x_2} (x_1 + x_2)\, p(x_1, x_2) = \sum_{x_1} \sum_{x_2} x_1\, p(x_1, x_2) + \sum_{x_2} \sum_{x_1} x_2\, p(x_1, x_2)$$

$$= \sum_{x_1} x_1 \sum_{x_2} p(x_1, x_2) + \sum_{x_2} x_2 \sum_{x_1} p(x_1, x_2)$$

X_1, X_2 are independent

$$\Rightarrow p(x_1, x_2) = p(x_1)\, p(x_2)$$

$$E[X_1 + X_2] = \sum_{x_1} x_1 \sum_{x_2} p(x_1)\, p(x_2) + \sum_{x_2} x_2 \sum_{x_1} p(x_1)\, p(x_2)$$

$$= \underbrace{\sum_{x_1} x_1 p(x_1)}_{E[X_1]} \underbrace{\sum_{x_2} p(x_2)}_{1} + \underbrace{\sum_{x_2} x_2 p(x_2)}_{E[X_2]} \underbrace{\sum_{x_1} p(x_1)}_{1}$$

Ex

$Supp[X_1] = \{1, 7, 9\}$

$Supp[X_2] = \{5, 23, 88\}$

X_2 \ X_1	1	7	9	
5	$\frac{1}{15}$	$1/3$	$\frac{2}{15}$	$16/30$
23	$1/30$	$1/10$	$1/30$	$5/30$
88	$1/30$	$1/5$	$1/5$	$9/30$
	$4/30$	$19/30$	$7/30$	1

$P(X_1, X_2)$

$p(X_2 = 5)$

$p(X_2 = 23)$

$p(X_2 = 88)$

$p(X_1 = 1)$ $p(X_1 = 7)$ $p(X_1 = 9)$ → PMF

$$\sum_{x_1} \sum_{x_2} p(X_1, X_2) = 1$$

$$X_1 \sim \begin{cases} 1 & \text{wp } 4/30 \\ 7 & \text{wp } 19/30 \\ 9 & \text{wp } 7/30 \end{cases} \qquad X_2 \sim \begin{cases} 5 & \text{wp } 16/30 \\ 23 & \text{wp } 5/30 \\ 88 & \text{wp } 9/30 \end{cases}$$

X_1, X_2 independent? No

$$P(X_1 = 1, X_2 = 5) = P(X_1 = 1)\, P(X_2 = 5)$$

$$\frac{1}{5} \neq \frac{4}{30} \frac{16}{30}$$

Marging out X_1

$$\sum_{x_1} p(x_1, x_2) = p(x_2) \qquad \int_{\mathbb{R}} f(x)dx = ? \qquad \int_{\mathbb{R}} f(x,y)\, dy = g(x)$$

identically distributive

$$E[T_n] = \sum_{i=1}^{n} E[X_i] \overset{!}{=} n\mu$$

identically distributive

$$E[\bar{x}_n] = E\left[\frac{1}{n} T_n\right] = \frac{1}{n} E[T_n] = \frac{1}{n} n\mu = \boxed{\mu}$$

$X \sim \text{Hyper}(n, K, N)$

$$E[X] = \sum_{x \in \text{supp}(X)} x \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$$

$$X = X_1 + X_2 + \dots + X_n$$

identically distributed only

$X_1 \sim \text{Bern}\left(\frac{K}{N}\right)$

$X_2 \sim \text{Bern}\left(\frac{K}{N}\right)$

$\vdots$

$X_n \sim \text{Bern}\left(\frac{K}{N}\right)$

$$X_2 \mid X_1 = x_1 \sim \text{Bern}\left(\frac{K - x_1}{N - 1}\right)$$

$$\boxed{E[X] = n\frac{K}{N}}$$

$$\mathrm{Var}[X] = E[(X-\mu)^2]$$

$$= E[X^2 - 2\mu X + \mu^2]$$

$$\sum_x \mu^2 p(x) = \mu^2 \sum_x p(x) = \mu^2 \cdot 1 = \mu^2$$

$$= E[X^2] + E[-2\mu X] + E[\mu^2]$$

$$= E[X^2] \underbrace{- 2\mu \underbrace{E[X]}_{\mu}}_{-2\mu^2} + \mu^2$$

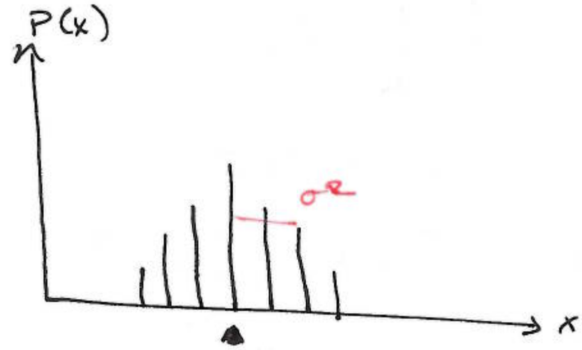
$$\boxed{\begin{aligned} \sigma^2 = \mathrm{var}[X] &= E[X^2] - \mu^2 \\ E[X^2] &= \sigma^2 + \mu^2 \end{aligned}}$$

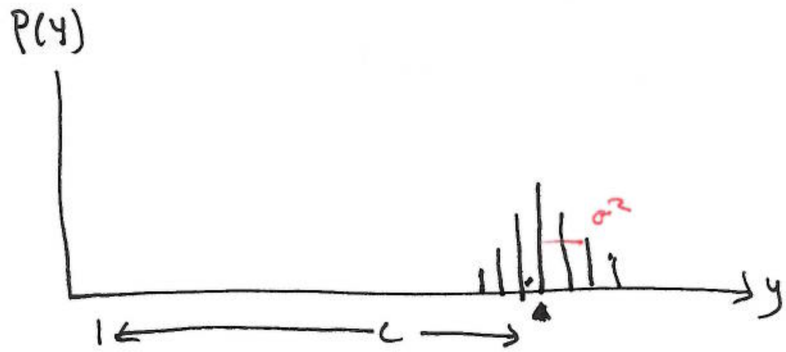
$Y = aX + c \qquad a, c \in \mathbb{R}$

$E(Y) = a\,E[X] + c$

$\mathrm{Var}[Y] = a^2 \sigma^2$

$SE[Y] = |a|\,\sigma$





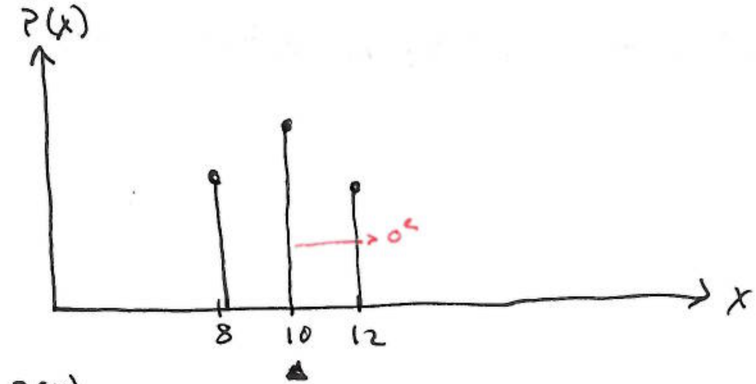
$$\mathrm{var}[X+c] = E[((X+c) - E[X+c])^2]$$

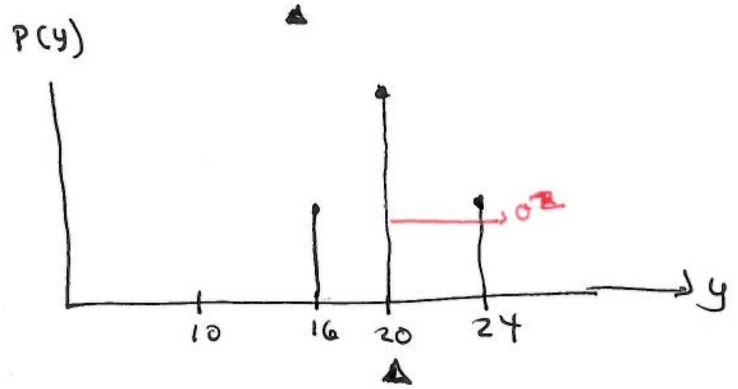
$$= E[((X+\cancel{c}) - (\mu + \cancel{c}))^2]$$

$$= E[(X-\mu)^2]$$

$$= \sigma^2$$

$y = ax^4$





$y = 2x \qquad Var[y] = 4\sigma^2$

$$var[ax] = E[(aX - \underbrace{E[ax]}_{a\mu})^2)$$

$$= E[(aX - a\mu)^2] = E[(a(X - \mu))^2]$$

$$= E[a^2(X-\mu)^2] = a^2 E[(X-\mu)^2] = a^2\sigma^2$$