5

Poker experiment

52 cards R := { A K Q J 10 9 8 7 6 5 4 3 2 } (ranks) |R| = 13

|Deck|=52 S := { Hearts Spades Clubs Diamonds } (suits) |S| = 4

5 cards draw Poker Deck = R x S

- draw 5 cards s.t. order does NOT matter

$P(A) = \frac{|A|}{|\Omega|}$ working def for prob. of A

$P(Win) = \frac{|Win|}{|\Omega|} = \frac{|Win|}{\binom{52}{5}}$ = 2,598,960

if Ω is finite

Duplicates - NO
Order - NO

Royal Flush

- what is the probability of 'Royal Flush' : (10 J Q K A - same suit) ?

$P(RF) = \frac{|RF|}{\binom{52}{5}} = \frac{\binom{4}{1}\binom{1}{1}}{\binom{52}{5}}$

choose suit $\binom{4}{1}$ choose rank 10 $\binom{1}{1}$

only one choice left 10-H J-H Q-H K-H A-H

there is only ONE RF

Straight Flush

- what is the probability of 'Straight Flush' : (2 3 4 5 6 - same suit) ?

Five cards in numerical order, all of identical suits

A 2 3 4 5
2 3 4 5 6
3 4 5 6 7
4 5 6 7 8
5 6 7 8 9
6 7 8 9 10
7 8 9 10 J
8 9 10 J Q
9 10 J Q K
~~10 J Q K A~~

— 9

can NOT start with J Q K

lets exclude Royal Flush

$P(SF) = \frac{|SF|}{\binom{52}{5}} = \frac{\binom{4}{1}\binom{9}{1}}{\binom{52}{5}}$

choose suit $\binom{4}{1}$ choose rank $\binom{9}{1}$

must start/choose with one of these 9 cards and the remaining cards are already chosen

or explicitly eliminating RF

$P(SF) = \frac{|SF|}{\binom{52}{5}} = \frac{\binom{4}{1}\left[\binom{10}{1} - \binom{1}{1}\right]}{\binom{52}{5}}$

Flush

- what is the probability of 'Flush' : (2 5 7 9 A - same suit) ?

Five cards of the same suit

$P(F) = \frac{|F|}{\binom{52}{5}} = \frac{\binom{4}{1}\binom{13}{5}}{\binom{52}{5}}$

or eliminate SF & RF

$P(F) = \frac{|F|}{\binom{52}{5}} = \frac{\binom{4}{1}\left[\binom{13}{5} - \binom{9}{1} - \binom{1}{1}\right]}{\binom{52}{5}}$

choose suit $\binom{4}{1}$ choose rank $\binom{13}{5}$

we must choose 5 cards and it is irrelevant which card we choose 1st, 2nd, ...

Full House

- what is the probability of 'Full House' : (8 8 8 K K) ?

Three cards of the same rank ; Two cards of the same rank (any rank / any rank except the former)

— 13 — 12

$P(FH) = \frac{|FH|}{\binom{52}{5}} = \frac{\binom{13}{1}\binom{4}{3}\binom{12}{1}\binom{4}{2}}{\binom{52}{5}}$

3 chosen ranks must have 3 of 4 available suits

2 chosen ranks must have 2 of 4 available suits

choose rank $\binom{13}{1}$ choose suit $\binom{4}{3}$ choose rank $\binom{12}{1}$ choose suit $\binom{4}{2}$

must start/choose with one of these 13 cards and the remaining 2 cards are already chosen

must start/choose with one of these 12 cards and the remaining card is already chosen

Three of a Kind

- what is the probability of 'Three of a Kind' : (K K K 3 2) ?

Three cards of the same rank, and two unrelated side cards. - suits irrelevant

— 13

$P(4K) = \frac{|3K|}{\binom{52}{5}} = \frac{\binom{13}{1}\binom{4}{3}\ \binom{12}{2}\binom{4}{1}^2}{\binom{52}{5}}$

3 chosen ranks must have 3 of 4 available suits

choose rank $\binom{13}{1}$ choose suit $\binom{4}{3}$ choose rank $\binom{12}{2}$ choose suit $\binom{4}{1}$ choose suit $\binom{4}{1}$

2 side cards have a choice of 4 suits each

must start/choose with one of these 13 cards and the remaining 2 cards are already chosen

out of the 12 remaining ranks choose 2

Four of a Kind

- what is the probability of 'Four of a Kind' : (8 8 8 8 5) ?

Four cards of the same rank, and one side card

— 13

$P(4K) = \frac{|4K|}{\binom{52}{5}} = \frac{\binom{13}{1}\binom{48}{1}}{\binom{52}{5}}$ = 0.000240096

choose rank $\binom{13}{1}$ then just choose any of the remaining 48 cards $\binom{48}{1}$ or $\binom{12}{1}\binom{4}{1}$

52-4 = 48

must start/choose with one of these 13 cards and the remaining 3 cards are already chosen

OR equivalent

$P(4K) = \frac{|4K|}{\binom{52}{5}} = \frac{\binom{13}{1}\binom{4}{4}\ \binom{12}{1}\binom{4}{1}}{\binom{52}{5}}$ = 0.000240096

4 chosen ranks are from the 4 available suits

the side card has a choice of 4 suits

choose rank $\binom{13}{1}$ choose suit $\binom{4}{4}$ choose rank $\binom{12}{1}$ choose suit $\binom{4}{1}$

must start/choose with one of these 13 cards and the remaining 3 cards are already chosen

out of the 12 remaining ranks choose 1

Two Pair

- what is the probability of 'Two Pair' : (8 8 J J 2 - any suit) ?

Two cards of a same rank ; Two cards of a same rank, and one side card.

$P(S) = \frac{|2P|}{\binom{52}{5}} = \frac{\binom{13}{2}\binom{4}{2}^2\ \binom{11}{1}\binom{4}{1}}{\binom{52}{5}}$ = 0.04753902

2 chosen ranks are from the 4 available suits - done twice

choose rank $\binom{13}{2}$ choose suit $\binom{4}{2}^2$ choose rank $\binom{11}{1}$ choose suit $\binom{4}{1}$

must start/choose with two of these 13 ranks and the matching cards are already chosen

out of the 11 remaining ranks choose 1

chose 1 of the 4 suits

wrong way !!

2 chosen ranks are from the 4 available suits

2 chosen ranks are from the 4 available suits

choose rank $\binom{13}{1}$ choose suit $\binom{4}{2}$ choose rank $\binom{12}{1}$ choose suit $\binom{4}{2}$ choose rank $\binom{11}{1}$ choose suit $\binom{4}{1}$

must start/choose with one of these 13 cards and the matching card is already chosen

out of the 12 remaining ranks choose 1 and the matching one is already chosen - BUT two 8s are still there !!

out of the 11 remaining ranks choose 1

chose 1 of the 4 suits

$P(S) = \frac{|2P|}{\binom{52}{5}} = \frac{\binom{13}{1}\binom{4}{2}\ \binom{12}{1}\binom{4}{2}\ \binom{11}{1}\binom{4}{1}}{\binom{52}{5}}$ = 0.09507803

Straight

- what is the probability of 'Straight' : (2 3 4 5 6 - any suit) ?

Five cards in sequence.

A 2 3 4 5
2 3 4 5 6
3 4 5 6 7
4 5 6 7 8
5 6 7 8 9
6 7 8 9 10
7 8 9 10 J
8 9 10 J Q
9 10 J Q K
10 J Q K A

— 10

we can't start with J Q K

choose rank $\binom{10}{1}$ choose suit

must start/choose with one of these 10 cards and the remaining 4 cards are already chosen

$\binom{4}{1}\binom{4}{1}\binom{4}{1}\binom{4}{1}\binom{4}{1}$ or $\binom{4}{1}^5$

must choose a suit for our card - and this must be done 5 times since there is 5 cards

eliminate SF & RF

$P(S) = \frac{|S|}{\binom{52}{5}} = \frac{\binom{10}{1}\binom{4}{1}^5 - \binom{9}{1}\binom{4}{1} - \binom{1}{1}\binom{4}{1}}{\binom{52}{5}}$

LRF & Propensity are objective and deal with a physical word independent of humans - no observer effect

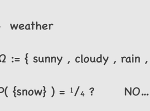
what is Probability

working definition of probability is valid only if

$P(A) = \frac{|A|}{|\Omega|}$ if $\forall \omega\ P(\{\omega\}) = \frac{1}{|\Omega|}$

all events are equally likely

- e.g.



$\Omega := \{ R,G,B \}$

$P(\{R\}) = \frac{|\{R\}|}{|\Omega|} = 1/3$? NO...

- e.g. - weather

  $\Omega := \{$ sunny , cloudy , rain , snow $\}$

  $P(\{snow\}) = 1/4$? NO...

- e.g. - presidential election

  $\Omega := \{$ Trump , Clinton $\}$

  $P(\{Trump\}) = 1/2$? NO...

- e.g. - coin experiment

  $P(H) = P(T) = 1/2$? Are you sure ?

if $\forall \omega\ P(\omega) \neq \frac{1}{|\Omega|}$, we need a better definition of probability

The word probability has been used in a variety of ways since it was first applied to the mathematical study of games of chance. Does probability measure the real, physical tendency of something to occur or is it a measure of how strongly one believes it will occur, or does it draw on both these elements? In answering such questions, mathematicians interpret the probability values of probability theory. There are two broad categories of probability interpretations which can be called "physical" and "evidential" probabilities.
Physical probabilities, which are also called objective or frequency probabilities, are associated with random physical systems such as roulette wheels, rolling dice and radioactive atoms. In such systems, a given type of event (such as a die yielding a six) tends to occur at a persistent rate, or "relative frequency", in a long run of trials. Physical probabilities either explain, or are invoked to explain, these stable frequencies. The two main kinds of theory of physical probability are frequentist accounts (such as those of Venn, Reichenbach and von Mises) and propensity accounts (such as those of Popper, Miller, Giere and Fetzer).

- wikipedia.org

Long Run Frequency

Long Run Frequency Definition is the most popular. It relies on a large number of trials in order to approximate a probability of a given event.

indicator function -> Bool

$\mathbb{1}_{\omega \in A} := \{ 1$ if $\omega \in A$; 0 if $\omega \notin A$

$\lim_{n\to\infty} \frac{\sum_{i=1}^{n} \mathbb{1}_{\omega_i \in A}}{n}$

Frequentist probability or frequentism is a standard interpretation of probability; it defines an event's probability as the limit of its relative frequency in a large number of trials.

- wikipedia.org

problem:

$n \neq \infty$ $P(A) \approx 1/n \sum_{i=1}^{n} \mathbb{1}_{\omega_i \in A}$ thus we only get an approximation of probability

problem:

non-general P(Trump) - we cant run this 1000,000 times - only one chance to see what happens

problem:

experiments are non repeatable. 1000,000 coin flips will not produce identical results every time.

Propensity Definition

propensity - an inclination or natural tendency to behave in a particular way

Objects have an inherent propensity to behave one way or another - this reduces/induces LRF

U_{238} - half life 4.5 billion years , what happens at half-life ?
how do we compute this ? - time to blow up is random
there is an inherent propensity in U_{238} to blow up

problem:

uncomputable

problem:

non-general P(Trump) - what is the propensity of Trump winning the election especially since he never done it before. Does Trump has an inherent property to be a president ?

The propensity theory of probability is one interpretation of the concept of probability. Theorists who adopt this interpretation think of probability as a physical propensity, or disposition, or tendency of a given type of physical situation to yield an outcome of a certain kind, or to yield a long run relative frequency of such an outcome.
Propensities are not relative frequencies, but purported causes of the observed stable relative frequencies. Propensities are invoked to explain why repeating a certain kind of experiment will generate a given outcome type at a persistent rate. A central aspect of this explanation is the law of large numbers. This law, which is a consequence of the axioms of probability, says that if (for example) a coin is tossed repeatedly many times, in such a way that its probability of landing heads is the same on each toss, and the outcomes are probabilistically independent, then the relative frequency of heads will (with high probability) be close to the probability of heads on each single toss. This law suggests that stable long-run frequencies are a manifestation of invariant single-case probabilities. Frequentists are unable to take this approach, since relative frequencies do not exist for single tosses of a coin, but only for large ensembles or collectives. Hence, these single-case probabilities are known as propensities or chances.
In addition to explaining the emergence of stable relative frequencies, the idea of propensity is motivated by the desire to make sense of single-case probability attributions in quantum mechanics, such as the probability of decay of a particular atom at a particular time.

- wikipedia.org

Logical Theory

Given the same evidence , do all of the people agree on a probability of an event equally ?

It is widely recognized that the term "probability" is sometimes used in contexts where it has nothing to do with physical randomness.[citation needed] Consider, for example, the claim that the extinction of the dinosaurs was probably caused by a large meteorite hitting the earth. Statements such as "Hypothesis H is probably true" have been interpreted to mean that the (presently available) empirical evidence (E, say) supports H to a high degree. This degree of support of H by E has been called the logical probability of H given E, or the epistemic probability of H given E, or the inductive probability of H given E.
The differences between these interpretations are rather small, and may seem inconsequential. One of the main points of disagreement lies in the relation between probability and belief. Logical probabilities are conceived (for example in Keynes' Treatise on Probability[12]) to be objective, logical relations between propositions (or sentences), and hence not to depend in any way upon belief. They are degrees of (partial) entailment, or degrees of logical consequence, not degrees of belief. (They do, nevertheless, dictate proper degrees of belief, as is discussed below.) Frank P. Ramsey, on the other hand, was skeptical about the existence of such objective logical relations and argued that (evidential) probability is "the logic of partial belief".[10] (p 157) In other words, Ramsey held that epistemic probabilities simply are degrees of rational belief, rather than being logical relations that merely constrain degrees of rational belief.
Another point of disagreement concerns the uniqueness of evidential probability, relative to a given state of knowledge. Rudolf Carnap held, for example, that logical principles always determine a unique logical probability for any statement, relative to any body of evidence.[citation needed] Ramsey, by contrast, thought that while degrees of belief are subject to some rational constraints (such as, but not limited to, the axioms of probability) these constraints usually do not determine a unique value.[citation needed] Rational people, in other words, may differ somewhat in their degrees of belief, even if they all have the same information.

- wikipedia.org

Subjective Theory

Subjective probability is a probability derived from an individual's personal judgment about whether a specific outcome is likely to occur. It contains no formal calculations and only reflects the subject's opinions and past experience. Subjective probabilities differ from person to person, and they contains a high degree of personal bias.

problem:

everyone has different opinions, thus there is no Grand Truth

In particular, we find the usual admiration for Newtonian mechanics, and the consequent belief in *universal determinism*. Indeed, Laplace's *Philosophical Essay on Probabilities* of 1814 gives one of the most famous formulations of the thesis of universal determinism. This is the formulation involving what is known as *Laplace's demon*. I will expound it in the next section.

Universal determinism and Laplace's demon

Laplace writes:

> We ought then to regard the present state of the universe as the effect of its anterior state and as the cause of the one which is to follow. Given for one instant an intelligence which could comprehend all the forces by which nature is animated and the respective situation of the beings who compose it – an intelligence sufficiently vast to submit these data to analysis – it would embrace in the same formula the movements of the greatest bodies of the universe and those of the lightest atom; for it, nothing would be uncertain and the future, as the past would be present to its eyes. (1814: 4)

The vast intelligence here described has come to be known as Laplace's demon. The idea is obviously founded on that of a human scientist (perhaps Laplace himself) using Newtonian mechanics to calculate the future paths of planets and comets. Extrapolating from this success, it was natural to suppose that a sufficiently vast intelligence could calculate the entire future course of the universe. Laplace himself relates his vast intelligence to human successes in astronomy. As he says:

> The human mind offers, in the perfection which it has been able to give to astronomy, a feeble idea of this intelligence. Its discoveries in mechanics and geometry, added to that of universal gravity, have enabled it to comprehend in the same analytical expressions the past and future states of the system of the world. (Laplace 1814: 4)

- thus you can predict Head or Tails
- probability is a result of your ignorance as if you do not know everything about the universe

VS.

1920s - Quantum Mechanics - Double-slit experiment: you can never predict where the electron is

- randomness is embedded in the reality

conclusion:
we do not know yet how to define probability

6

we don't really know what probability is ...
let's use mathematics to define probability

Probability - set function

Axioms of Probability:
1 $P(A) \geq 0$ for all $A \subseteq \Omega$
2 $P(\Omega) = 1$
3 if $A \cap B = \varnothing$, then $P(A \cup B) = P(A) + P(B)$

Theorem I

Theorem II

Theorem III

Theorem IV

smokers & lung cancer

n = 1000 people

A := 200 smokers $P(A) = \frac{200}{1000} = 0.2$

B := 60 has lung cancer $P(B) = \frac{60}{1000} = 0.06$

AB := 36 smokers & lung cancer $P(AB) = \frac{36}{1000} = 0.036$

what is the probability of lung cancer:
- among smokers
- given smoking
- conditional on smoking

beginning size / ending size → $\frac{P(\Omega)}{P(A)} = \frac{1}{P(A)}$ or $\frac{P(\Omega)}{P(\Omega')} = \frac{1}{P(\Omega')}$

$P(B|A) := \frac{P(BA)}{P(A)}$

Def. of Conditional Probability

$P(B|A) = \frac{P(BA)}{P(A)} = \frac{0.36}{0.2} = 0.18$

- probability of lung cancer given a person is a smoker

- probability of a smoker given a lung cancer

$P(B|A) := \frac{P(A|B)\ P(B)}{P(A)}$

Bayes' Rule - (1765)
aka Bayes' Theorem
aka Bayes' Law

lets flip the variables

- probability of a lung cancer , given didn't smoke

$P(B|A^c) = \frac{P(BA^c)}{P(A^c)} = \frac{0.024}{0.8} = 0.03$

$P(B) = P(BA) + P(BA^c)$
$P(BA^c) = P(B) - P(BA)$
$= 0.06 - 0.036$
$= 0.024$

RISK RATIO

probability of a lung cancer given smoking

$\frac{P(B|A)}{P(B|A^c)} = \frac{0.18}{0.03} = 6$

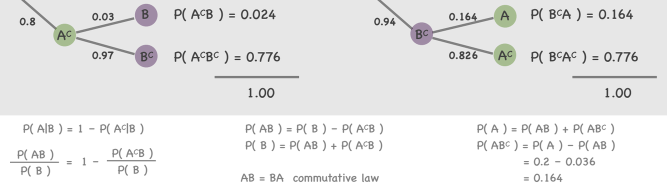
probability of a lung cancer given NOT smoking

while the previous stats weren't portentous enough this one is: smokers are 6 times more likely to develop lung cancer

how many possible probability calculations are there for this problem ?

Probability Tree

cause -> effect Inverting the Tree effect -> cause



$P(A|B) = 1 - P(A^c|B)$

$\frac{P(AB)}{P(B)} = 1 - \frac{P(A^cB)}{P(B)}$

$P(AB) = P(B) - P(A^cB)$
$P(B) = P(AB) + P(A^cB)$

AB = BA commutative law

$P(A) = P(AB) + P(AB^c)$
$P(AB^c) = P(A) - P(AB)$
$= 0.2 - 0.036$
$= 0.164$

interesting $P(A^cB^c)$ - probability of a NOT smoker given a NOT lung-cancer

Total Probability of an Event

consider event B and disjoint (mutually exclusive) and collectively exhaustive events $A_1\ A_2\ A_3$ ...

$P(B) = P(B \cap \Omega)$
$= P(B \cap (A_1 \cup A_2 \cup A_3 \cup \ldots))$ mutually exclusive & collective exhaustive
$= P((B\cap A_1) \cup (B\cap A_2) \cup (B\cap A_3) \cup \ldots)$ distributive law
$= P(B\cap A_1) + P(B\cap A_2) + P(B\cap A_3) + \ldots$

yes they are mutually exclusive thus we can add them

$\forall i\neq j\ (B\cap A_i) \cap (B\cap A_j) = \varnothing$
$= B \cap (A_i \cap A_j)$
$= B \cap \varnothing$
$= \varnothing$

clearly $A_i \cap A_j = \varnothing$

$P(B) = \sum_{i=1}^{\infty} P(B\cap A_i)$

Law of Total Probability

if $A_1 = A$, $A_2 = A^c$

then $P(B) = P(B\cap A) + P(B\cap A^c)$

$P(B) = \sum_{i=1}^{\infty} P(B|A_i)\ P(A_i)$

$P(A|B) := \frac{P(B|A)\ P(A)}{P(B)}$

$P(A|B) := \frac{P(B|A)\ P(A)}{\sum_{i=1}^{\infty} P(B|A_i)\ P(A_i)}$

Bayes' Theorem

self-note:
imagine the area of A_i=5 and B=12

A_2	1/5 · 5	+	
A_3	2/5 · 5	+	
A_4	3/5 · 5	+	
A_5	3/5 · 5	+	
A_6	2/5 · 5	+	
A_7	1/5 · 5	=	12=B

adding all individual pieces of B eventuates in recomposition of B

i.e.
B takes 1/5 of A_2
B takes 2/5 of A_3
etc...

ratios are for illustrative purposes only

Boy / Girl

You have two kids. One is a Girl

GG	GB
BG	BB

$P(\text{other is Girl}) = 1/2$?

$P(\{GG\} \mid \{GG, GB, BG\}) = \frac{|\{GG\}|}{3} = 1/3$!!!

Conditional Probability - super weird

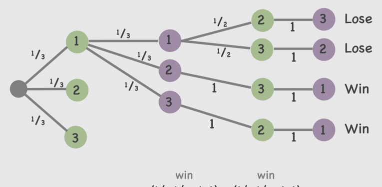
when dealing with conditional probability be VERY careful !!

Conditional Probability

Monte Hall Game

There are three doors. Behind one of the doors is a new car. In search of the car, the player picks a door, say 1. The game host then opens one of the other doors, say 3, to reveal a goat and offers to let the player pick door 2 instead of door 1.

should the player switch doors ?



$P(\text{Win} \mid \text{Car in } 1) = \frac{(1/3 \cdot 1/3 \cdot 1 \cdot 1) + (1/3 \cdot 1/3 \cdot 1 \cdot 1)}{1/3} = 2/3 \neq 1/2$

it pays to switch

self-note:
think of these two loses as one since they originate from the same node
1/3 chances of losing

if you switch, your chances of winning are much higher

2/3 chances of winning

also note that these ratios switch if you stick with your original choice, giving you only 1/3 chances of winning

7

A,B : independent events

A,B : dependent events

custom spinner system

shared birthday

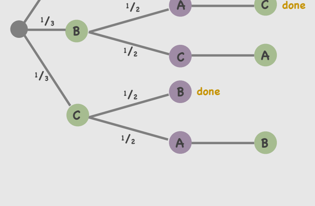
what is the probability that out of 49 people at least 2 are sharing the same birthday ?

hats problem

There are n-people in a party. m people are wearing hats which are deposited in the coatcheck area. After the party is over the hats are randomly assigned among the n people ? What is the probability that the owners do not get their hats back ?

P(no-one gets their hat) = 1 - P(someone got their hat back)
= 1 - [P(1 gets hat) + P(2 ppl got hats) + ... + P(n ppl got hats)]

difficult to solve ...



$1/3 + 1/6 + 1/6 = 2/3$

tree (brute force) is a bad strategy - non general

A_1 = person 1 got their hat
A_2 = person 2 got their hat vs 2 ppl got their hat
A_3 = person 3 got their hat
. . .
A_n = person n got their hat

$1 - P(A_1 \cup A_2 \cup \ldots \cup A_n)$ - at least one person gets their hat

$1 - P(\bigcup_{i=1}^{n} A_i)$ n-ppl randomly administer their hats

General Inclusion Exclusion Formula

$P(A\cup B) = P(A) + P(B) - P(AB)$

$P(\bigcup_{i=1}^{2} A_i) = \sum_{i=1}^{2} P(A_i) - P(\bigcap_{i=1}^{2} A_i)$

lets make it general

$P(\bigcup_{i=1}^{n} A_i) = \sum_{i=1}^{n} P(A_i) - P(\bigcap_{i=1}^{n} A_i)$

$= \sum_{i=1}^{n} P(A_i) - \sum_{i\neq j}^{n} P(A_i\cap A_j) + \sum_{i\neq j\neq k}^{n} P(A_i\cap A_j\cap A_k) - + - + \ldots (-1)^{n+1} P(\bigcap_{i=1}^{n} A_i)$

$\binom{n}{3}\frac{(n-3)!}{n!} = \frac{n!}{(n-3)!3!}\frac{(n-3)!}{n!} = \frac{1}{3!}$

$P(A_1) = 1/n$
$P(A_2) = 1/n$
. . .
$P(A_n) = 1/n$ → $\sum_{i=1}^{n} P(A_i) = 1$

assume A_1&A_2 got their hats

$P(A_1\cap A_2) = \frac{1\ \ 1\ \ n-2\ \ n-3\ \ldots}{n!} = \frac{(n-2)!}{n!}$

$P(A_1\cap A_3) = \frac{1\ \ n-2\ \ 1\ \ n-3\ \ldots}{n!} = \frac{(n-2)!}{n!}$

assume A_1&A_3 got their hats

cancelation x-mass

$\sum_{i\neq j}^{n} P(A_i\cap A_j) = \sum_{i\neq j}^{n} \frac{(n-2)!}{n!} = \binom{n}{2}\frac{(n-2)!}{n!} = \frac{n!}{(n-2)!2!}\frac{(n-2)!}{n!} = \frac{1}{2!}$

$= 1 - 1/2! + 1/3! - 1/4! + \ldots$

$= 1 - 1/e \approx 0.3678 \approx 1/3$ QED - at least 1 person gets their hat

$f(x) = \sum_{i=0}^{n} \frac{f^{(i)}(0)}{i!} x^i$

Taylor Series

$e^x = e^0 + \frac{e^0 x}{1!} + \frac{e^0 x^2}{2!} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \ldots$

$e^{-1} = 1 - 1 + 1/2! - 1/3!$

$1-e^{-1} = 1 - 1/2! + 1/3! - 1/4! + \ldots$

$1-p = 1-e^{-1}$

$p = e^{-1} \approx 0.3678 \approx 1/3$ QED - zero ppl get their hat

ans: no matter how many people in a room P(zero ppl got their hat) = $e^{-1} \approx 0.3678 \approx 1/3$