10/24/2017

Lecture 13

avg r.v $\qquad \overline{X}_n = \dfrac{X_1 + \dots X_n}{n}$

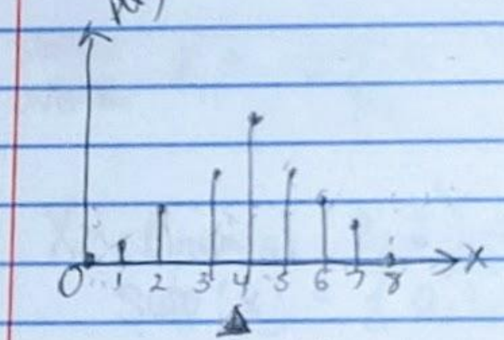
$$\bar{x}_n = \frac{x_1 + \dots x_n}{n}$$

$$T_n = X_1 + \dots + X_n$$

$$X \sim Bin(8, \tfrac{1}{2})$$

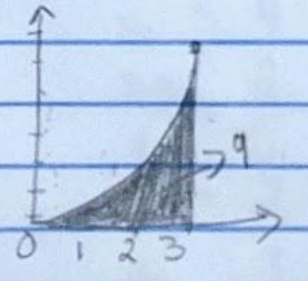
5	3	7
7	4	5
5	5	4
3	4	3
4	4	4
8	6	3

$\bar{x} = \dfrac{26+26+26}{6+6+6} = 4.33 \xrightarrow[?]{} 4$ Illustrating convergence



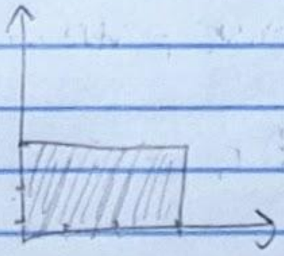
pivot point - summarize to pivot point "function of a function"

$F(x) = x^2,\ x \in [0,3]$



$$\int_A F(x)\,dx = 9$$

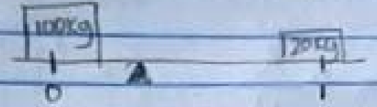
$F(x) = 3 \quad x \in A$



$$\int F(x) = 9$$

$$G[F] = \int_A F(x)\,dx = 9$$

↑ operator (function of a function)



solve for point where it will be steady

$$0 = \sum_{i \in \text{objects}} w_i (d_i - d^*) \Rightarrow \sum w_i d_i - \sum w_i d^* = 0 \Rightarrow \sum w_i d_i = d^* \sum w_i = d^*$$

$$= \frac{\sum w_i d_i}{\sum w_i}$$

$$= \frac{100 \cdot 0 + 20 \cdot 1}{100 + 20} \approx 1/6$$

$$\frac{\sum_i p_i x_i}{\sum_i p_i} = \frac{\sum_{x \in Supp[X]} x p(x)}{\underbrace{\sum_{x \in Supp(X)} p(x)}_{1}}$$

pivot point

$$\boxed{E[X] = \mu = \sum_{x \in Supp[X]} x p(x)}$$

"expectation"

$$X \sim Bin(8, \tfrac{1}{2}) = \binom{8}{x} \left(\tfrac{1}{2}\right)^x \left(\tfrac{1}{2}\right)^{8-x}$$

$$E[X] = 0 \cdot p(0) + 1 \cdot p(1) + 2p(2) + 3p(3) + 4p(4) + 5p(5) + 6p(6) + 7p(7) + 8p(8)$$

$$= 0 + 1 \cdot 0.031 + 2 \cdot .109 + 3 \cdot .219 + 4 \cdot .273 + 5 \cdot .219 + 6 \cdot .109 + 7 \cdot .031 + 8 \cdot .004$$

$$= 4$$

pivot point will be four

$X \sim Bin(187, 0.392)$

$X \sim Bin(n, p)$

$E[X]$ - would be nice to have a general formula

$$E[X] = \sum_{x=0}^{n} x \binom{n}{x} p^x (1-p)^{n-x} = n \sum_{x=0}^{n} x \frac{(n-1)!}{x (n-x)!} p^x (1-p)^{n-x} = np \sum_{x=1}^{n} \binom{n-1}{x-1} p^{x-1} (1-p)^{n-x}$$

$(n-1)-(x-1)$ "

$$= \sum_{x=1}^{n} x \binom{n}{x} p^x (1-p)^{n-x}$$

$(x-1)!$

$$\underbrace{((n-1)-(x-1))!}_{\binom{n-1}{x-1}}$$

let $m = n-1$

let $y = x-1$

$$= np \underbrace{\sum_{y=0}^{m} \binom{m}{y} p^y (1-p)^{m-y}}_{1}$$

$Y \sim Bin(n, p)$

$p(x)$

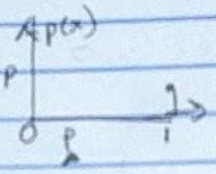
$$\boxed{E(x) = np}$$

$X \sim \text{Hyper}(n, K, N)$

$$E[X] = \sum_{x \in \text{Supp}(x)} x \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}} = \cdots = n\frac{K}{N}$$

$X \sim \text{Unif}(\{1, 10, 100\})$ $\quad E(X) = 1 \cdot \frac{1}{3} + 10 \cdot \frac{1}{3} + 100 \cdot \frac{1}{3}$

$X \sim \text{Bern}(p)$ $\quad E[X] = 0 \cdot (1-p) + 1 \cdot (p) = p$



$X \sim \text{Geom}(p) := (1-p)^{x-1} p$

$p \in (0,1)$ $\quad \text{Supp}[X] = \mathbb{N}$

$X \sim \text{Geom}(p = 0.2) = 0.8^{x-1} \cdot 0.2$

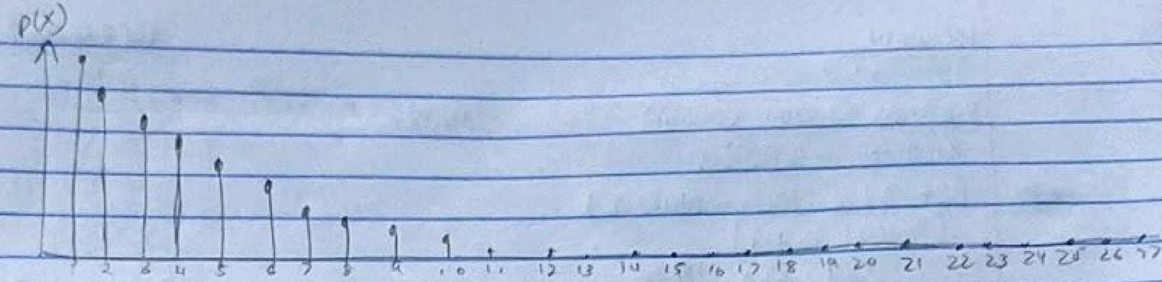
approaching 0 approaching 1

x	p(x)	F(x) ≤ 1	x	p(x)	F(x)
1	0.2	0.200	16	0.007	0.972
2	0.160	0.360	17	0.006	0.970
3	0.128	0.488	18	0.005	0.983
4	0.102	0.590	19	0.004	0.987
5	0.082	0.672	20	0.003	0.990
6	0.066	0.738	21	0.002	0.992
7	0.052	0.790	22	0.001	0.994
8	0.042	0.832	23	0.001	0.995
9	0.034	0.866	24	0.001	0.996
10	0.027	0.893	25	0.001	0.997
11	0.021	0.914	26	0.001	0.998
12	0.017	0.931	27	0.001	0.999
13	0.014	0.945			
14	0.011	0.956			
15	0.009	0.965			

"Effective Support"

$E\,\text{Supp}[X] = \{1, \ldots, 27\}$

$= \{x : p(x) > .001\}$



$$E(X) = \sum_{x=1}^{\infty} x(1-p)^{x-1} p = \sum_{y=0}^{\infty} (y+1)(1-p)^{y} = \sum_{y=1}^{\infty} y(1-p)^{y} p + \sum_{y=0}^{\infty} (1-p)^{y} p$$

$y = x-1 \Rightarrow x = y+1$ → Geometric

$$= (1-p)\sum_{y=1}^{\infty} y(1-p)^{y-1} p + \underbrace{\sum_{x=1}^{\infty} (1-p)^{x-1} p}_{1}$$

$$= \mu = (1-p)\mu + 1$$
$$\Rightarrow \mu - (1-p)\mu = 1$$
$$\Rightarrow \mu(1-(1-p)) = 1 \quad \boxed{\mu = \frac{1}{p}}$$

$Min[X] = \min\{Supp[X]\}$
$Max[X] = \max\{Supp[X]\}$
$Range[X] = Max[X] - Min[X]$
$Mode[X] = \text{argmax}\{p(x)\}$ → gives highest p(x)
$Quantile[X, p] = \text{argmin}\{F(x) \geq p\}$
100 Quantile = Percentile
$Med[X] = Q[X, 0.5]$

* Median $\neq$ E[X] expectation
* $Med[X] < E[X] \Rightarrow X$ is right-skewed
* $Med[X] > E[X] \Rightarrow X$ is left skewed

Tertiles
$Q[X, \frac{1}{3}], Q[X, \frac{2}{3}]$

Quartiles
$Q[X, \frac{1}{4}], Med[X], Q[X, \frac{3}{4}]$

Interquartile Range
$IQR(X) := Q[X, \frac{3}{4}] - Q[X, \frac{1}{4}]$

IF $|Mode[X]| = 1 \Rightarrow X$ is unimodal

Deciles
$Q[X, \frac{1}{10}], Q[X, \frac{2}{10}], \ldots Q[X, \frac{9}{10}]$

Quintiles
$Q[X, \frac{1}{5}]$ $Q[X, \frac{4}{5}]$