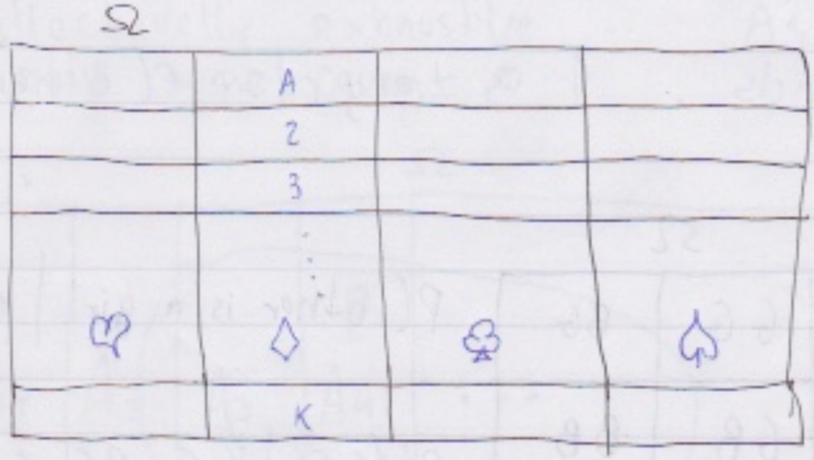
9/27



$P(A|\Omega)$

$\parallel$

$P(A) = 4/52 = 1/13$

$P(A|\heartsuit) = 1/13$

$P(A|\heartsuit) = p(A)$

information

"informationally irrelevant"

P(IBM stock ↑ today | it rains today in Buenos Aires

= P(IBM stock ↑ today)

Definition

Events A, B are independent if

$$\boxed{\begin{gathered} P(A \mid B) = P(A) \\ \text{or} \\ P(B) = P(B \mid A) \end{gathered}}$$

$$P(A \mid B) = \frac{P(AB)}{P(B)} \underset{\substack{\uparrow \\ \text{def} \\ \text{cond} \\ \text{prob.}}}{=} P(A) \underset{\substack{\uparrow \\ \text{if} \\ \text{indep}}}{\Rightarrow} P(AB) = P(A)P(B)$$

If $A_1, A_2, \dots$ are independent

$$P(A_1, A_2, \dots) = \boxed{P\left(\bigcap_{i=1}^{\infty} A_i\right) = \prod_{i=1}^{\infty} P(A_i)}$$

Mult Rule

$P(H_2 \mid H_1) = P(H_2) = 1/2$

$P(H_1, H_2, \dots, H_{10}) = \frac{1}{|\Omega|^{10}} = \left(\frac{1}{2}\right)^{10} = \frac{1}{2^{10}}$

A^c: no double 6

$\underbrace{P(\geq 1 \text{ double-6 in 24 2-die rolls})}_{A} < \frac{1}{2}$

P(1 double 6) + P(2 double 6) + ... + P(24 double 6)

= 1 - P(no double 6 in 24 rolls)

= 1 - P(no double 6)^{24}

$= 1 - \left(\frac{35}{36}\right)^{24} \approx .4114$

P(no double 6 in 2 die)
= 1 - P(double-6)
= 1 - P(6) P(6)
$= 1 - \left(\frac{1}{6}\right)^2 = \frac{35}{36}$

Definition

A, B are dependent events if.

$P(A|B) \neq P(A)$

or $P(B|A) \neq P(B)$

or $P(A,B) \neq P(A)P(B)$

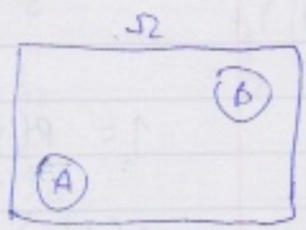
P(Q64 is late) < P(Q64 is late | Major snowstorm)

P(Q64 is late) > P(Q64 is late | Perfect weather No traffic)

P(l.c) < P(l.c. | smoking)

A, B are disjoint and empty.

Are A, B intependent?

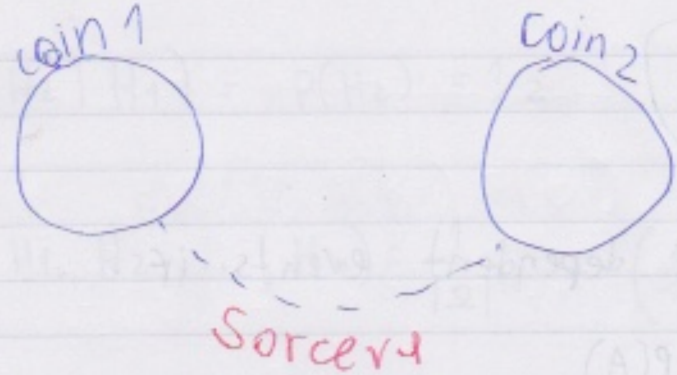


$0 = P(A|B) \neq P(A) > 0$

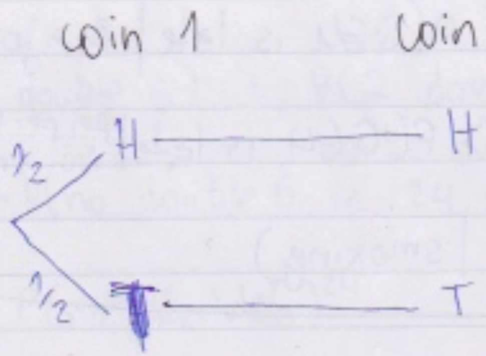
$\Rightarrow$ dependent

$P(H|T) = 0 \neq 1/2 = P(H)$

Disjoint = Ultimate dependence



If coin 1 H $\Rightarrow$ coin 2 H
If coin 1 T $\Rightarrow$ coin 2 T

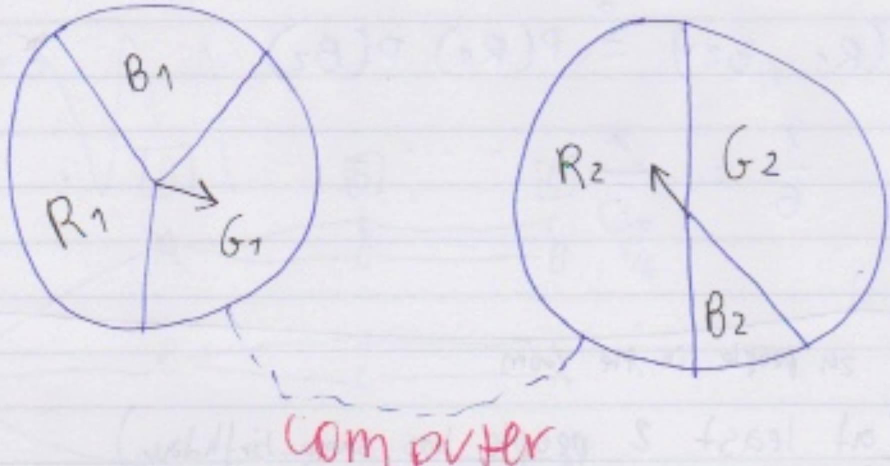


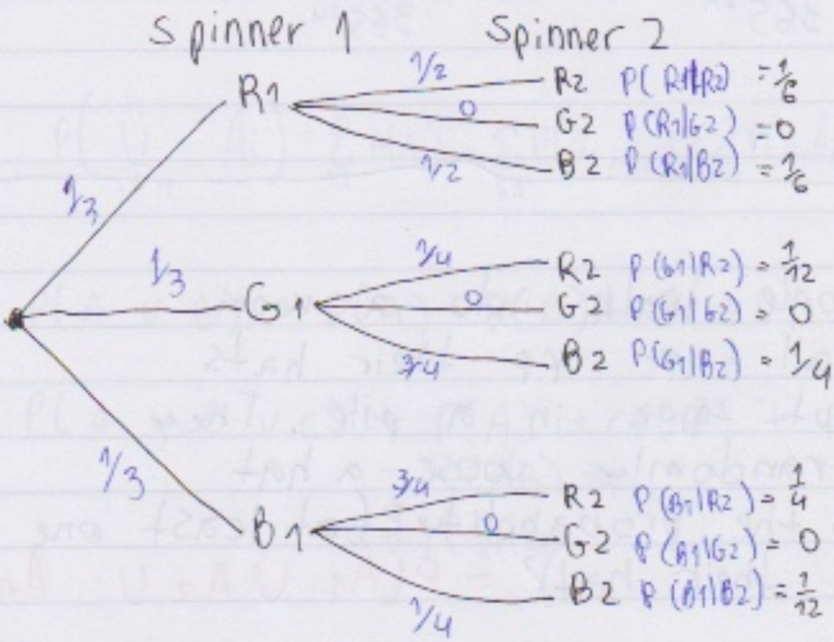
$P(HH) = \frac{1}{2}$

$P(TT) = \frac{1}{2}$

$$1 = P(H_2 \mid H_1) \neq P(H_2) = \frac{1}{2}$$

$$P(H_2) = \underbrace{P(H_2, H_1)}_{\frac{1}{2}} + \underbrace{P(H_2, T_1)}_{0}$$





$P(R_2) = P(R_1, R_2) + P(G_1, R_2) + P(B_1, R_2)$

$= \frac{1}{6} + \frac{1}{12} + \frac{1}{4}$

Is R_1, R_2 independent? YES

$P(R_1, R_2) \stackrel{?}{=} P(R_1)\,P(R_2)$

$\frac{1}{6} = \frac{1}{3} \cdot \frac{1}{2}$

Are R_1, G_2 independent

$P(R_1, B_2) \overset{?}{=} P(R_1)\, P(B_2)$

$\frac{1}{6} = \frac{1}{3} \cdot \ldots\ldots$

24 people in the room

P(at least 2 people has same birthday)

$= P(\text{one pair bdays}) + P(2 \text{ pair bdays}) + P\left(\binom{24}{2} \text{ bdai pairs}\right)$

$= 1 - P(\text{no one share bday}) = \frac{|A|}{|\Omega|}$

$= 1 - \frac{365 \cdot 364 \cdot 363 \cdots}{365^{24}} = 1 - \frac{{}_{365}P_{24}}{365^{24}}$

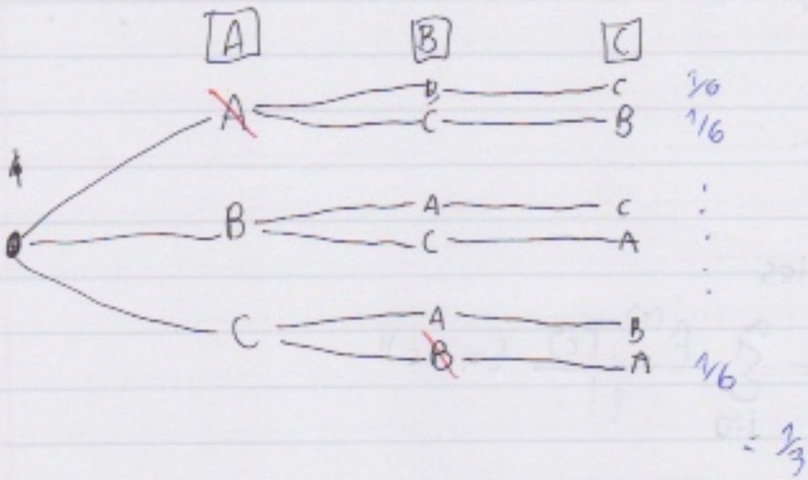
n people walk into a room
they all take off their hats
and put them in a pile. They
then randomly choose a hat
What's the probability P(at least one person
picked their hat) $= P(A_1 \cup A_2 \cup \cdots A_n)$

$=$

$= 1 - e^{-1} \approx 70\% \approx \frac{2}{3}$

$= 1 - P(\text{no one gets hat})$

$n = 3$



$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{\text{all}} P(A_i) - \sum_{\text{all}} P(A_i, A_j) + \sum P(A_i A_j A_k) + \cdots\cdots + (-1)^n P\left(\bigcap_{i=1}^{n} A_i\right)$$

$$P(A \cup B) = P(A) + P(B) - P(AB)$$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C)$$
$$- P(AB) \cdots$$
$$+ P(ABC$$

$$P(A_1) = \frac{1\,(n-1)(n-2)\cdots}{n!} = \frac{(n-1)!}{n!}$$

$$P(A_4) = \frac{(n-1)(n-2)\cdots}{n!} = \frac{(n-1)!}{n!}$$

$P(A_2, A_4) = \frac{(n-2)!(n-3)! \cdots}{n!} = \frac{(n-2)!}{n!}$

$P(A_2, A_3, A_1) = \ldots$

$\binom{n}{3} \ldots$

Taylor Series

$$f(x) = \sum_{i=0}^{\infty} \frac{f^{(i)}(c)}{i!}(x-c)^i$$

$$f(x) = \sum_{i=0}^{\infty} \frac{f^{(i)}(0)}{i!} x^i$$

$$= \frac{f(0)}{0!} + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \cdots$$

$$= e^x = \frac{1}{0!} + \frac{1}{1!}x + \frac{1}{2!}x^2 + \cdots$$

$$= \sum_{i=0}^{\infty} \frac{x^i}{i!}$$

$$e^1 = \boxed{e := \sum_{i=0}^{\infty} \frac{1}{i!}}$$