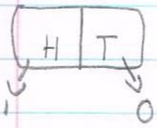
Math 241 Lecture 09 Weds, Oct 11th

$\Omega = \{H, T\}$



$n = 3$
$\omega_1 = H$
$\omega_2 = T$
$\omega_3 = H$

$\mathbb{1}_{\omega = H} = \begin{cases} 1 & \text{of } \omega = H \\ 0 & \text{of } \omega \neq H \end{cases}$

$\mathbb{1}_{\omega_1 = H}, \ \mathbb{1}_{\omega_2 = T} = H$

$\bar{x} = \frac{1+0+1}{3} = \frac{2}{3}$

Generally there is a function
$x : \Omega \to \mathbb{R}$ called a "random variable". (rv)

$x(H) = 1$
$x(T) = 0$
$\text{supp}[x] = \{0, 1\}$

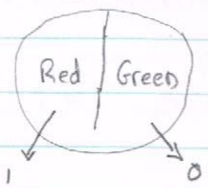
"shorthand abuse of notation"
$P(x=1) = P(\{\omega : x(\omega) = 1\}) = P(\{H\}) = \frac{|A|}{|\Omega|} = \frac{1}{2}$
$P : 2^{\Omega} \to (0,1)$

"support" the range of the random variable denoted: $\text{Supp}[x] = \{x : P(X=x) > 0\} \subseteq \mathbb{R}$

$P(x=0) = \frac{1}{2}, \ P(x=17) = 0$

Def: A discrete r.v is one s.t. $|\text{supp}[x]| \leq |\mathbb{N}|$ ie. finite or ctbly infinite

$\Omega = \{R, G\}$



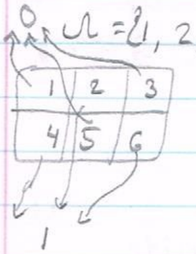
$P(x=1) = \frac{1}{2}$
$P(x=0) = \frac{1}{2}$

$\text{supp}[x] = \{0, 1\}$

r.v. "distributed as" "with prob"
$x \sim \begin{cases} 1 & \text{w.p. } \frac{1}{2} \\ 0 & \text{w.p. } \frac{1}{2} \end{cases}$

$x \sim \text{Bernoulli}(\frac{1}{2}) := \begin{cases} 1 & \text{wp } \frac{1}{2} \\ 0 & \text{wp } 1/2 \end{cases}$
"Standard Bernoulli"
$\text{supp}[x] = \{0, 1\}$
x is discrete

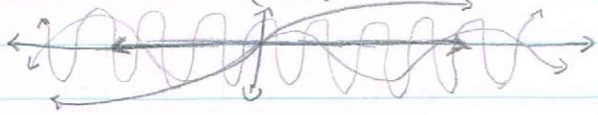
$\Omega = \{1, 2, 3, 4, 5, 6\}$



$x \sim \text{Bernoulli}(p) := \begin{cases} 1 & \text{wp } p \\ 0 & \text{wp } 1-p \end{cases}$

p is called a parameter"
a number you choose to "tune"
the r.v. model

$f(x) = \sin(ax)$ where $a \in \mathbb{R}$



if $a = 0$ "trivial / degenerate case"

parameter space: the set where p "lives"

$p \in (0, 1)$ $\qquad p = 0 \Rightarrow X \sim \text{Deg}(0)$

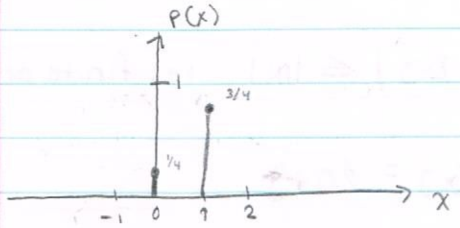
p=0 degenerate $\qquad p = 1 \Rightarrow X \sim \text{Deg}(1)$

$X \sim \overset{\text{degenerate}}{\text{Deg}}(c) := \{ c \text{ w.p. } 1$ $\qquad \text{supp}[X] = \{c\}$

$p(x) := P(X = x)$ prob. mass function (P)

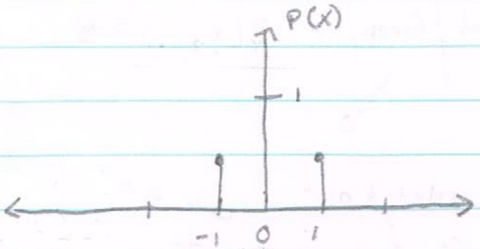
$p: \mathbb{R} \to [0, 1]$

$$\sum_{x \in \text{supp}[X]} p(x) = 1$$

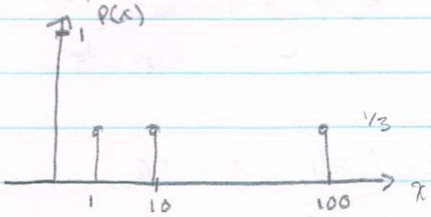


$X \sim \text{Bernonilli}\left(\frac{3}{4}\right)$

$X \sim \text{Radomacher} = \begin{cases} 1 & \text{wp } \frac{1}{2} \\ -1 & \text{wp } \frac{1}{2} \end{cases}$



$X \sim \text{unif}(\overbrace{\{1, 10, 100\}}^{A}) := \begin{cases} 1 & \text{wp } \frac{1}{3} \\ 10 & \text{wp } \frac{1}{3} \\ 100 & \text{wp } \frac{1}{3} \end{cases}$

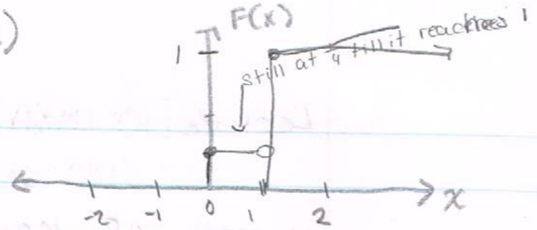


$X \sim \text{Unif}(A)$ $\qquad \text{supp}[X] = A$

$A \in 2^{\mathbb{R}}$ but $|A|$ is finite

$\exists x$ (something about x)

$\forall x$ (something about x)



$F(x) := P(X \leq x)$

commulative distribution function $\quad X \sim \text{Bern}\left(\frac{3}{4}\right)$

(CDF)

Properties of CDF

① $\lim_{x \to \infty} F(x) = 1$

② $\lim_{x \to -\infty} F(x) = 0$

③ $x \leq y \Rightarrow F(x) \leq F(y)$ montonically increasing

$X \sim \text{Bern}(p) = \begin{cases} 1 & \text{wp} \quad \frac{1}{2} \quad p \\ 0 & \text{wp} \quad 1/2 \quad 1-p \end{cases}$

$p(x) = p^x (1-p)^{1-x}$

Def: X_1, X_2 are "identically distributive denoted $X_1 \overset{\phi}{=} X_2$

of (3) $P_{X_1}(x) = P_{X_2}(x)$ $\quad$ (b) $F_{X_1}(x) = F_{X_2} x$

10 cards 4R 6B

$P(2R \text{ when drawing } 3) = \dfrac{\binom{4}{2}\binom{6}{1}}{\binom{10}{3}}$

$P(xR \ldots 3) = \dfrac{\binom{4}{x}\binom{6}{3-x}}{\binom{10}{3}}$

$P(xR \text{ when drawing } n) = \dfrac{\binom{4}{x}\binom{6}{n-x}}{\binom{10}{n}}$

10 cards k Red

$P(xR \ldots n) = \dfrac{\binom{k}{x}\binom{10-k}{n-k}}{\binom{10}{n}}$

N

N cards k Red

$P(xR \ldots n) = \dfrac{\binom{k}{x}\binom{N-k}{n-x}}{\binom{N}{n}} = P(x)$

$X \sim \text{hypegeometric}(n, k, N)$