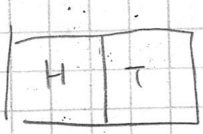
$\Omega = \{H, T\}$ Flip 3 times What is the average of the 3 ω's?

Ω

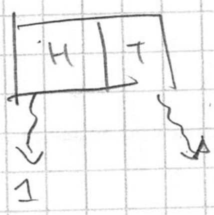


$\omega_1 = H$
$\omega_2 = T$
$\omega_3 = H$

Consider $\mathbb{1}_{\omega = H} = \begin{cases} 1 & \text{if } H \\ 0 & \text{if } T \end{cases}$

$\frac{\mathbb{1} + 0 + \mathbb{1}}{3} = 2/3$ $\quad \mathbb{1}_{\omega_1}, \mathbb{1}_{\omega_2}, \mathbb{1}_{\omega_3}$

Ω



Call this function

$\underset{\text{experimental outcomes}}{X : \Omega} \longrightarrow \underset{\text{numbers sometimes with a unit e.g. \$}}{\mathbb{R}}$

$X(H) = 1,\ X(T) = 0$ This is the def. of a "random variable" (r.v.)

$P(X = 1) = P(\{\omega : X(\omega) = 1\}) = P(\{H\}) = \frac{1}{2}$ "abuse of notation"

$\underset{\text{event space}}{P : 2^{\Omega}} \to [0, 1]$

"Support" denoted

$\text{Supp}[X] := \{x : \underbrace{P(X = x) > 0}_{\text{this } x \text{ is "possible"}}\} \subseteq \mathbb{R}$

Def: Discrete r.v. has $|\text{Supp}[X]| \leq |\mathbb{N}|$ finite or countable infinity

$\sum_{x \in \text{Supp}[X]} P(X = x) = 1$

$X : \{G, R\} \to \{0, 1\}$,
$X(G) = 0,\ X(R) = 1$

distribute as "with prob"

$X \sim \begin{cases} \$1 & \text{wp } \frac{1}{2} \\ \$0 & \text{w.p. } \frac{1}{2} \end{cases}$

$X \sim \text{Bernoulli}(\frac{1}{2}) := \begin{cases} 1 & \text{wp } \frac{1}{2} \\ 0 & \text{w.p. } \frac{1}{2} \end{cases}$ "Brand Name" $\text{Supp}[X] = \{0, 1\}$

$X \sim \text{Bernoulli}(p) := \begin{cases} 1 & \text{wp } p \\ 0 & \text{wp } 1-p \end{cases}$ $\quad \sum P(X = x) = P(X = 0) + P(X = 1) = (1 - p) + (p) = 1$ ✓

↳ "parameter" which belongs to a "parameter space" $p \in (0, 1)$

↳ parameter space is everything between 0 and 1 except zero and one. Why?

↳ if $p = 0$ then 1 D.N.E.
if $p = 1$ then 0 D.N.E.

if $p = 0$ "degenerate": the trivial r.v.

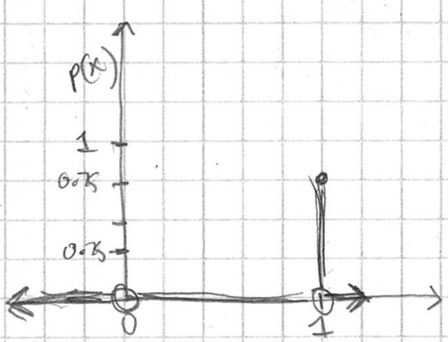
$X \sim \text{Bern}(0) = \{0 \text{ w.p. } 1 = \text{Deg}(0)$
$X \sim \text{Bern}(1) = \{1 \text{ w.p. } 1 = \text{Deg}(1)$

$X \sim \text{Deg}(c) = \{c \text{ w.p. } 1$
$c \in \mathbb{R}$
$\text{Supp}[X] = \{c\}$

$$\sum_{x \in \mathrm{Supp}[X]} P(X=x) = 1$$

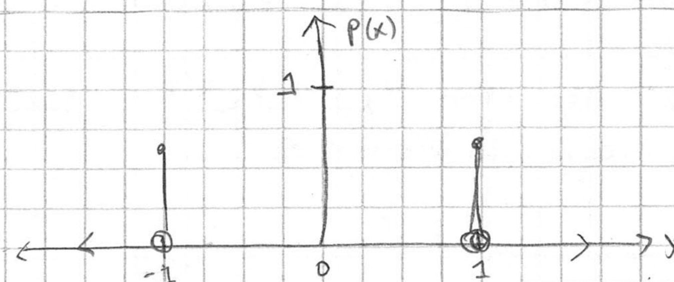
$p(x) := P(X=x)$ $\quad \sum_{x \in \mathrm{Supp}(X)} p(x) = 1$

↑ probability mass function (PMF)



$X \sim \mathrm{Bern}\left(\frac{3}{4}\right)$

$X \sim \text{Rademacher} = \begin{cases} 1 & \text{wp } \frac{1}{2} \\ -1 & \text{wp } \frac{1}{2} \end{cases}$



$X \sim \text{Uniform}(\{1, 10, 100\}) = \begin{cases} 1 & \text{w.p. } \frac{1}{3} \\ 10 & \text{w.p. } \frac{1}{3} \\ 100 & \text{w.p. } \frac{1}{3} \end{cases}$

"uniform discrete"

$X \sim \text{Uniform}(A)$

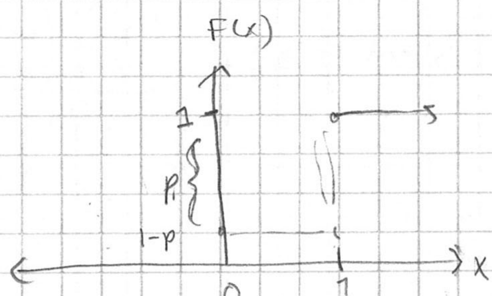
$\mathrm{Supp}[X] = A$

$A \subseteq \mathbb{R}$ but $|A| \leq |\mathbb{N}|$ & $|A| > 1$ → if A is 0 or 1, that is not a r.v. it is the "degenerate" variable

$F(x) := P(X \leq x)$

↑ cumulative distribution function (CDF)

$X \sim \mathrm{Bern}(p)$ $\quad F(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1-p & \text{if } x \in [0,1) \\ 1 & \text{if } x \geq 1 \end{cases}$



Properties of CDF

① $\lim_{x \to \infty} F(x) = 1$ ② $\lim_{x \to -\infty} F(x) = 0$ ③ $x \leq x \Rightarrow F(x) \leq F(x)$ ④ $F(x) \in (0,1]$

"monotonic increase"

$\cancel{X} \sim \text{Bern}(p) :=$ $\begin{cases} 1 & \text{w.p. } p \\ 0 & \text{w.p. } 1-p \end{cases}$

$$= p(x) = p^x (1-p)^{1-x}$$
$$p(0) = p^0 (1-p)^{1-0} = 1-p$$
$$p(1) = p^1 (1-p)^{1-1} = p$$

$$X_1 \sim \text{Bern}(p) := p^x (1-p)^{1-x} = P_{X_1}(x)$$
$$\parallel$$
$$X_2 \sim \text{Bern}(p) := p^x (1-p)^{1-x} = P_{X_2}(x)$$

Def: $X_1 \overset{d}{=} X_2$ means $P_{X_1}(x) = P_{X_2}(x)$ same PMFs or $F_{X_1}(x) = F_{X_2}(x)$

equal in distribution

same CDFs

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10 cards 4R, 6B

$$P(2R \text{ out of } 3 \text{ cards}) = \frac{\binom{4}{3}\binom{6}{1}}{\binom{10}{3}}$$

$$P(xR \text{ out of } 3 \text{ cards}) = \frac{\binom{4}{x}\binom{6}{3-x}}{\binom{10}{3}}$$

$$P(xR \text{ out of } n \text{ cards} = \frac{\binom{4}{x}\binom{6}{n-x}}{\binom{10}{n}}$$

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10 cards but K are R

$$P(xR \text{ out of } n) = \frac{\binom{K}{x}\binom{10-K}{n-x}}{\binom{10}{n}}$$

N cards, K are R

$$X \sim \text{Hypergeometric}(N, K, n) = P(x) = \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$$