The binomial conceptually is

$T = \lim_{N \to \infty} \text{Hyper}(n, p, N)$

$T = X_1 + \ldots + X_n$ where $X_1, \ldots, X_n \overset{iid}{\sim} \text{Bern}(p)$

Lecture :-12

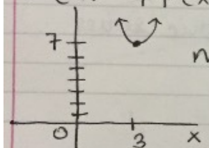
1

$X \sim \text{Binomial}(n, p)$

$p(x) = P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$

$F(X) = P(X \leq x) = \sum_{i=0}^{x} \binom{n}{i} p^i (1-p)^{n-i}$

$F(X) = 7 + (X-3)^2$



$\min\{F(x)\} = 7$

$\max\{F(x)\} = \text{D.N.E}$

x s.t $F(x) = \min\{F(x)\}$ — which x to plug in to get min

$\text{argmin}\{F(x)\} = 3$

$\text{argmax}\{F(x)\} = \text{D.N.E}$

$X_1, X_2, \ldots \overset{iid}{\sim} \text{Bern}(p) \Rightarrow$ potential infinite series of events

$T := \min\{t : X_t = 1\}$

"stopping time"

If we get one for the first time,

$P(T=1) = p$

$P(T=2) = (1-P)(P)$

$P(T=3) = (1-P)(1-P)(P) = (1-P)^2(P)$

$T \sim \text{Geometric}(p)$: $p(x) = P(T=x) = (1-P)^{x-1}(P)$ can

Supp $[X] = \mathbb{N}$ (1 to infinity) — group 1 billion

$p \in (0,1)$

↑ upto one

2

$\sum_{x \in \text{Supp}[X]} p(x) = 1$, let $i = x-1 \Rightarrow x = i+1$

We want to show that,

$\sum_{x=1}^{\infty} p(1-p)^{x-1} = 1 \Rightarrow \sum_{x=1}^{\infty} (1-p)^{x-1} = \frac{1}{p} = \sum_{i=0}^{\infty} (1-p)^i = \frac{1}{p}$

let $q = 1-p$ (NOTE: $p \in (0,1) \Rightarrow q \in (0,1)$)

$= \sum_{i=0}^{\infty} q^i = \frac{1}{p}$

$S = \sum_{i=0}^{\infty} q^i = 1 + q^1 + q^2 + q^3 + q^4 + \ldots$

$= 1 + q(1 + q + q^2 + q^3 + \ldots) = 1 + qS$

$S - qS = 1$

$S(1-q) = 1$

$S = \frac{1}{(1-q)}$ ∵ Geometric series

$\sum_{i=0}^{\infty} q^i = \frac{1}{1-q}$ if $q \in (0,1)$

CDF

$F(x) = P(X \leq x) = \sum_{i=1}^{x} (1-p)^{i-1} p$. Hard to do...

$= 1 - P(X > x)$

$P(X > x) = (1-p)^x$

$= P(X = x+1) + P(X = x+2) + \ldots$ $\quad \frac{0}{1}\ \frac{0}{2}\ \frac{0}{3}\ \frac{0}{4} \ldots \frac{0}{x} \Big| \frac{}{x+1}\ \frac{}{x+2}$

$= \sum_{i=x+1}^{\infty} P(X = 1) = \sum_{i=x+1}^{\infty} (1-p)^{i-1} p$

3

amount of heads : $X \sim \text{Binomial}(8, \frac{1}{2}) \rightarrow$ sampling w/ replacement

Supp $[X] = \{0, 1, \ldots 8\}$

$X_1 \ldots X_8 \overset{iid}{\sim}$

$X_1 = 5$

$X_2 = 4$

$X_3 = 2$

$X_4 = 5$ $\quad \bar{X} = 3.375$

$X_5 = 2$

$X_6 = 3$

$X_7 = 3$

$X_8 = 3$

Keep going until heads

$X \sim \text{Geom}(\frac{1}{2})$ $\quad X_1, \ldots X_8 \overset{iid}{\sim} \text{Geom}(\frac{1}{2})$

Supp $[X] = \{1, 2, \ldots\}$

$X_1 = 1$

$X_2 = 3$

$X_3 = 3$ $\quad \bar{X} = 2$

$X_4 = 1$

$X_5 = 1$

$X_6 = 3$