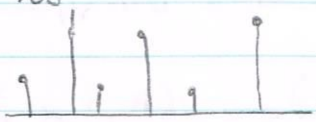
lecture Math 241 15

Bet on #7 \$1

$X_7 \sim \begin{cases} \$35 \text{ wp } 1/38 \\ \$-1 \text{ wp } 37/38 \end{cases} \Rightarrow M = -\$0.053$

Bet on Black \$1

$X \sim \begin{cases} \$1 \text{ wp } 16/38 \\ -\$1 \text{ wp } 20/38 \end{cases} \Rightarrow -\$0.053$



$\left.\begin{array}{l} \bar{X}_7 \Rightarrow M \\ \bar{X}_B \Rightarrow M \end{array}\right.$

Law of Large #'s

$M = E(X) = \sum_{x \in supp(X)} x\, p(x)$

$\sigma^2 = Var(X^2) = \sum_{x \in supp\, X} (x - M)^2 p(x)$

$= (35 - 0.053)^2 \frac{1}{38} + (-1 - -0.053)^2 \frac{37}{38}$

$= 33.207\ \$^2$

$= (1 - -0.053)^2 \left(\frac{18}{38}\right) + (-1 - -0.053)^2 \left(\frac{20}{38}\right)$

$= 0.997\ \$^2$

$\lim_{n \to \infty} \frac{1}{n} = 0$

$\lim_{n \to \infty} \frac{1}{e^n} = 0$

$\bar{X}_7 \to M$ slower than $\bar{X}_B \to M$

$\$^2$ is not a comprehensable unit so $\sigma = \sqrt{33.207\$^2} = \$5.79$

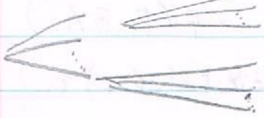
$\sigma = \sqrt{0.997\$^2} = \$1.08$

Standard deviation or standard error

$\sigma = E[X] = SD[X] := \sqrt{var[X]}$

$T_2 = X_1 + X_2 \quad E[T_2] = \sum_{t \in supp(t)} t\, p(t)$ impractical

X_1 X_2 T



* Don't need to know the following proof

$E[g(x)] = \sum_{x \in supp\{x\}} g(x) p(x)$

joint mass function

$E[g(x_1, x_2)] = \sum_{x_1 \in supp(x_1)} \sum_{x_2 \in supp(x_2)} g(x_1 x_2)\, p(x_1, x_2)$

$E[x_1 + x_2] = \sum_{x_1} \sum_{x_2} (x_1 + x_2)\, p(x_1 x_2) = \sum_{x_1} \sum_{x_2} x_1 p(x_1 x_2) + \sum_{x_2} \sum_{x_1} x_2 p(x_1 x_2)$

$= \sum_{x_1} x_1 \sum_{x_2} p(x_1 x_2) + \sum_{x_2} x_2 \sum_{x_1} p(x_1 x_2)$

X_1 X_2 independent

$\Rightarrow p(x_1\ x_2) = p(x_1)\, p(x_2)$

$E[x_1 + x_2] = \sum_{x_1} x_1 \sum_{x_2} p(x_1) p(x_2) + \sum_{x_2} x_2 \sum_{x_1} p(x_1) p(x_2)$

$= \underbrace{\sum_{x_1} x_1 p(x_1)}_{E[x_1]} \underbrace{\sum_{x_2} p(x_2)}_{1} + \underbrace{\sum_{x_2} x_2 p(x_2)}_{E[x_2]} \underbrace{\sum_{x_1} p(x_1)}_{1}$

$supp\{x_1\} = \{1, 7, 9\}$
$supp\{x_2\} = \{5, 23, 88\}$

x_1

$p(x_1, x_2)$

		1	7	9		
x_2	5	1/15	1/3	2/13	16/30	$p(x_2=5)$
	23	1/30	1/10	1/30	5/30	$p(x_2=23)$
	88	1/30	1/5	1/15	9/30	$p(x_2=88)$
		4/30	19/3	7/30	1	why 1?

$P(B) = P(B \& A_1)$
$+ P(B \& A_2)$
$+ P(B \& A_2)$
$\vdots$
$+ P(B \& A_2)$

$(x_1=1)$
$P(x_2=7)$
$p(x_1=9)$

$\sum_{x_1} \sum_{x_2} p(x_1, x_2) = 1$

All possible probabilities of (x_1)

$(x_1) = P(x_1 = 1)$

$x_1 \sim \begin{cases} 1 & wp\ 4/30 \\ 7 & wp\ 19/30 \\ 9 & wp\ 7/30 \end{cases}$

X_1, X_2 independent? $\boxed{No}$

$p(x_1 = 1, x_2 = 5) = p(x_1 = 1)\, p(x_2 = 5)$

$\frac{1}{15} \neq \frac{4}{30} \cdot \frac{16}{30}$

marging out x_1

$\sum_{x_1} p(x_1, x_2) = p(x_2)$ $\int_{\mathbb{R}} f(x)\,dx = 7$ $\int_{\mathbb{R}} f(x, y)\,dy = g(x)$

$\rightarrow \sum_{x} x_1\, p(x_1) + \sum_{x_2} x_2\, p(x_2) = E[x_1] + E[x_2]$

under identically distributed assumption

$E[T_n] = \sum_{i=1}^{n} E(x_i)$

$E[\bar{x}_n] = E[\frac{1}{n} T_n] = \frac{1}{n} E[T_n] = \frac{1}{n} n m = \boxed{\mu}$ *

$X \sim \text{Hyper}(n, k, N)$

$E[X] = \sum_{x \in \text{supp}(X)} x \dfrac{\binom{k}{x}\binom{N-k}{n-x}}{\binom{N}{n}}$

$X = X_1 + X_2 + \ldots + X_n$

$X_1 \sim \text{Bern}\left(\frac{k}{N}\right)$

identically distributive $X_2 \sim \text{Bern}\left(\frac{k}{N}\right)$ $\quad X_2 | X_1 = \underset{x_1}{0} \sim \text{Bern}\left(\frac{k - x_1}{N-1}\right)$

$\vdots$

but NOT IDEP $X_n \sim \text{Bern}\left(\frac{k}{N}\right)$ $\quad E[X] = n\frac{k}{N}$

dependent - you get info, things change

$$\begin{aligned}
\text{Var}[X] &= E[(X-\mu)^2] \\
&= E[X^2 - 2\mu X + \mu^2] \\
&= E[X^2] + E[-2\mu X] + E[\mu^2] \\
&= E[X^2] \underbrace{- 2\mu \underbrace{E[X]}_{\mu}}_{-2\mu^2} + \mu^2
\end{aligned}$$

$$\boxed{\begin{aligned}\sigma^2 = \text{Var}[X] &= E[X^2] - \mu^2 \\ E(X^2) &= \sigma^2 + \mu^2\end{aligned}}$$

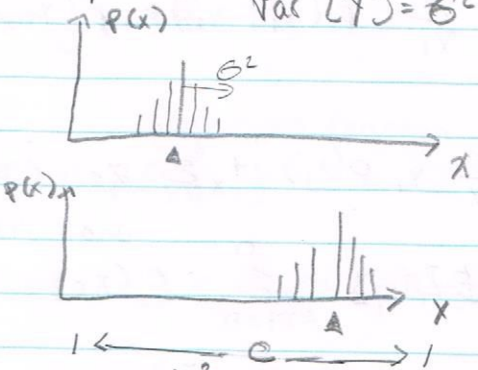
$\sum_{x} \mu^2 p(x) = \mu^2 \sum_{x} p(x) = \mu^2 \cdot 1 = \mu^2$

$Y = aX + c \,,\ a, c \in \mathbb{R}$

$E[Y] = a E[X] + c$

$\mathrm{Var}[Y] =$

$y = x + c \qquad \mathrm{Var}[Y] = \sigma^2$



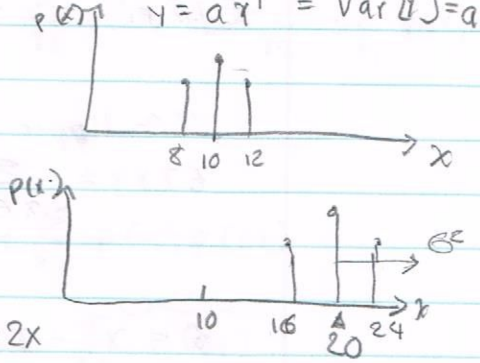
$\mathrm{Var}[X + c] = E[((X+c) - \underbrace{E(X+c)}_{m+c}))^2$

$= E[((X+c) - (\mu + c))^2]$

$= E[(X - \mu)^2]$

$= \sigma^2$

$Y = aX' = \mathrm{Var}[Y] = a^2\sigma^2$



$y = 2x$

$\mathrm{Var}[Y] = 4\sigma^2$

$\mathrm{Var}(aX) = E((aX - \underbrace{E[aX]}_{an})^2$

$= E[aX - am)^2]$

$= E[(a(X - m))^2]$

$= E[a^2(X-m)^2) = a^2 E((X-m)^2) = a^2\sigma^2$

$Y = aX + c,\ a, c \in \mathbb{R}$

$E[Y] = a E(X) + c$

$\mathrm{Var}[Y) = a^2 \sigma^2$

$S E[Y] = |a| \sigma$