LECTURE 02: 08/31/2017

* Special set denoted "Ω" is called the "Universe", "sample space" or "scope".

$\therefore\ \Omega = F \cap M$ → defining my universe

Ex: $F \subseteq \Omega$ → TRUE

$M \subseteq \Omega$ → TRUE

$A \cap \Omega = A$

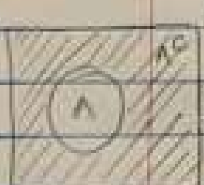
$\phi \cup \Omega = \Omega$

$A \cup \Omega = \Omega$

$\phi \cap \Omega = \phi$

$A \setminus \Omega = \phi$ → subtraction

* Complement of a set :-



A^c is the set of all objects in the universe that are not elements of A.

$\therefore\ A^c := \Omega \setminus A$

$\therefore\ A \cup A^c = \Omega$ → collectively exhaustive

$\therefore\ A \cap A^c = \phi$ → mutually exclusive

$\{A_1, A_2, A_3 \ldots\}$ are collectively exhaustive if,

$\therefore A_1 \cup A_2 \cup \ldots = \bigcup_{i=1}^{\infty} A_i = \Omega$

$\{A_1, A_2, A_3 \ldots\}$ are mutually exclusive if,

$\therefore A_i \cap A_j = \phi \ \forall i \neq j$

* Countable and Uncountable Sets :-

A set is Countable if it has the same cardinality as some subset of the set of natural numbers.

Otherwise, they are Uncountable.

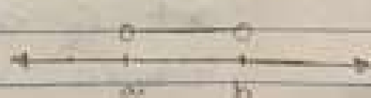
* Number Line set Notation :-

$[a, b] = \{x : x \geq a \ \& \ x \leq b\}$

$(a, b) = \{x : x > a \ \& \ x < b\}$

[a, b]

"filled bubble"



(a, b)

"empty bubble"

* Ordered Pair :-

$$\langle a,b\rangle := \{\{a\},\{a,b\}\}$$
$$\langle b,a\rangle := \{\{b\},\{a,b\}\}$$
$$\langle a,a\rangle := \{\{a\},\{a,a\}\}$$
$$= \{\{a\}\} \neq \{a\}$$

* Set / Cartesian Product :-

$$A \times B := \{\langle a,b\rangle : a \in A,\ b \in B\}$$

Ex: $A = \{1,2\}$ $B = \{3,4\}$

$$\therefore A \times B = \{\langle 1,3\rangle, \langle 1,4\rangle, \langle 2,3\rangle, \langle 2,4\rangle\}$$

$$\therefore |A \times B| = |A| \cdot |B|$$
$$= 2 \cdot 2 = 4$$

So, for sets A and B, the Cartesian Product $(A \times B)$ is the set of all ordered pairs (a,b) where $a \in A$ and $b \in B$.

* ## Exponentiation of Sets :-

For nonnegative integers A and n, the power A^n is the number of functions from a set of n elements to a set of A elements.

$\therefore |A^n| = |A|^n$

Ex: $A^2 := A \times A$

$|A^2| = |A|^2$

* ## Probability :-

Ω is now called the "experimental space" or "outcome space" and its elements are called "outcomes" and denoted as "ω", little omega ($\omega \in \Omega$)

** When an experiment is performed one outcome is its result.

Ex: The Coin Toss Experiment

$\Omega, w_i \to$ experimenting

$$\therefore \Omega = \underbrace{\{H, T\}}_{\text{all the possible outcomes for this experiment}}$$

Big omega

** universe are the experimental space.

** $|H| \to$ this is not the set but the element

"P" is the set function called "Probability of".

$$\therefore P(H) = \frac{1}{2} \to |\Omega|$$

$$\therefore P(\{H\}) = \frac{|\{H\}|}{|\Omega|} \to \text{size of all things that can happen}$$

$$\therefore P(H \text{ or } T) = \boxed{1}$$

$$P(\{H, T\}) = \frac{|\{H, T\}|}{|\Omega|}$$

$$= \frac{2}{2} = \boxed{1}$$

** Good Definition:-

$$\therefore P : 2^{\Omega} \to [0, 1]$$

Ex: The Die Roll Experiment

$\therefore \Omega = \{1, 2, 3, 4, 5, 6\}$

$\therefore P(\text{even }\#) = \boxed{\frac{1}{2}}$

$= \frac{|\{2, 4, 6\}|}{6}$

$= \frac{3}{6} = \boxed{\frac{1}{2}}$

** size of domain has the "2^n" number of questions / problems So, die roll experiment has $2^6 = 64$ question / problems

* Working Definition :-

The prob. of "event" A is :

$\therefore P(A) := \frac{|A|}{|\Omega|}$

$\therefore 2^{\Omega}$ is called "event space". A set $A \subseteq \Omega$ is called an "Event".

Let, Ω' be outcome space for 2 coin flips

$\therefore \Omega' = \{ \langle H, H \rangle, \langle H, T \rangle, \langle T, H \rangle, \langle T, T \rangle \}$

$= |\Omega'| = 4 = 2^2$

$\therefore P(\underbrace{\text{at least one H}}_{\text{set A}}) = \boxed{\frac{3}{4}}$

$= \frac{|\{ \langle H, H \rangle, \langle H, T \rangle, \langle T, H \rangle \}|}{|\Omega|}$

$= \boxed{\frac{3}{4}}$

$\therefore P(\text{just one H}) = \boxed{\frac{1}{2}}$

$= \frac{|\{ \langle H, T \rangle, \langle T, H \rangle \}|}{|\Omega|}$

$= \frac{2}{4} = \boxed{\frac{1}{2}}$

* Trivial Events :-

$\therefore P(\phi) = 0, \quad P(\Omega) = 1$

x ——— x ——— x