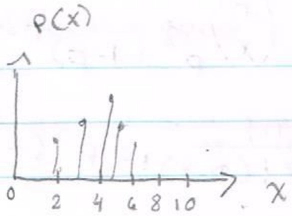
Math 241 Lecture 13 Oct 25^{th}



$X \sim (\text{Bin}(8, \tfrac{1}{2})$

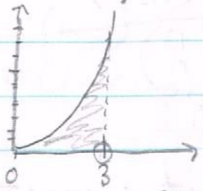
pivot point

a good metric/summary that explains the distribu

$\bar{x} = \dfrac{31 + 34 + 31}{10 + 10 + 10}$

$= \dfrac{95}{30} = 3.167$

$f(x) = x^2, \quad x \in A = [0,3]$



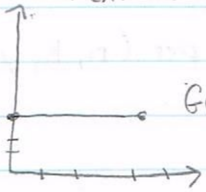
$G(f) \quad \int_0^3 f(x)\,dx = 9$

operator

"function of a function"

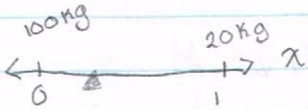
Not on exam

$f(x) = 3 \quad , x \in A$



$G(f) = \int_0^3 f(x)\,dx = 9$

$= \int_0^3 dx\,[f(x)]$



$\sum_{\text{objects}} w_i (d - d^*) = 0 \Rightarrow \sum w_i d_i - \sum w_i d^* = 0$

$\Rightarrow \sum w_i d_i = d^* \sum w_i$

$\Rightarrow d^* = \dfrac{\sum w_i d_i}{\sum w_i}$

$\mu = \dfrac{\sum_i p(x_i) x_i}{\sum_i p(x_i)} = \dfrac{\sum_{x \in \text{supp}[X]} x\,p(x)}{\sum_{x \in \text{supp}[X]} p(x)} = \boxed{\underbrace{\sum_{x \in \text{supp}[X]} x\,p(x)}_{E[X]}}$

"expectation"

$\bar{x} \Rightarrow E[X]$

$E[X] = 0\,p(0) + 1\,p(1) + 2p(2) + 3p(3) + 4p(4) + 5p(5) + 6p(6) + 7p(7) + 8p(8)$

$\binom{8}{1}\left(\tfrac{1}{2}\right)^1 \left(1 - \tfrac{1}{2}\right)$

$= 0 + .031 + 2 \cdot .109 + 3 \cdot .219 + 4 \cdot .273 + 5 \cdot .219 + 6 \cdot .109 + 7 \cdot .031 + 8 \cdot .$

$= \boxed{4}$

$X \sim \text{Bin}(8{,}976,\ 0.38279)$

$X \sim \text{Bin}(n, p), \quad E[X] =$

ossible proof

$$E(x) = \sum_{x=0}^{n} x \binom{n}{x} p^x (1-p)^{n-x}$$

$$= \sum_{x=1}^{n} x \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

$$= np \sum_{x=1}^{n} \binom{n-1}{x-1} p^{x-1} (1-p)^{n-x} = \sum_{y=0}^{n-1} \binom{n-1}{y} p^y (1-p)^{\overset{n-1-y}{\shortparallel}\, n-y-1}$$

$$= np \underbrace{\sum_{y=0}^{m} \binom{m}{x} p^y (1-p)^{m-y}}_{1} = p(x) = Bin(m, p)$$

$$= \boxed{np}$$

$x \sim Hyper(n, k, N)$ $\quad E[x] = \sum_{x \in supp\{x\}} x \frac{\binom{k}{x}\binom{N-k}{n-x}}{\binom{N}{n}}$

$$= \ldots = n \frac{k}{N}$$

$x \sim Uniform(\{1, 10, 100\})$ $\quad E\{x\} = 1 \cdot \frac{1}{3} + 10 \cdot \frac{1}{3} + 100 \cdot \frac{1}{3}$

$$= \frac{111}{3}$$

$x \sim$ Geometric $(.2) = 0.8^{x-1} \cdot 0.2$ $\quad supp\{x\} = \mathbb{N}$

(how many times till we succeed)

x	p(x)	F(x)
1	.200	.200
2	.160	.360
3	.128	.488
4	.102	.590
5	.082	.672
6	.066	.733
7	.052	.790
8	.042	.832
9	.034	.866
10	.027	.893
11	.021	.914
12	.017	.971
13	.014	.945
14	.011	.956
15	.009	.965

x		
16	.007	.972
17	.006	.978
18	5	.983
19	4	.987
20	3	.990
21	2	.992
22	1	.994
23	1	.995
24	1	.996
25	1	.997
26	1	.998
27	1	.999

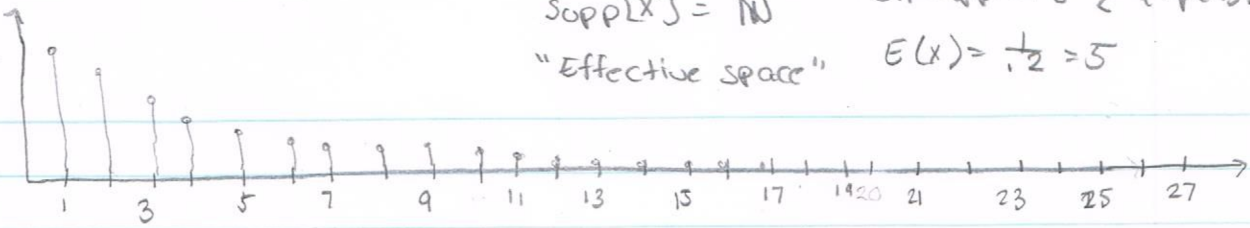
$X \sim Geom(.2)$

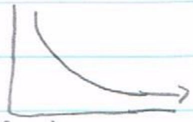
$Supp[X] = \mathbb{N}$

"Effective space"

$Eff\,Supp[X] = \{x : p(x) \geq \frac{1}{1000}\}$

$E(x) = \frac{1}{.2} = 5$





$\int_1^{\infty} \frac{1}{x^2}\,dx < \infty$

$X \sim Geom(p)$

$\mu = E[X] = \sum_{x=1}^{\infty} x(1-p)^{x-1}p = \sum_{y=0}^{\infty}(y+1)(1-p)^y p =$

let $y = x-1$

$= \underbrace{\sum_{y=0}^{\infty} y(1-p)^y p}_{m} + \underbrace{\sum_{y=0}^{\infty} (1-p)^{x-1} p}$

$\mu = (1-p)\mu + 1$

$\mu(1-(1-p)) = 1$

$\mu p = 1 \Rightarrow \boxed{\mu = \frac{1}{p}}$

Mode $(X) = \text{argmax}\{p(x)\}$

$Min(X) = m$ Supp $[X]$

$Max(X) = max(Supp[X])$

$Range(X) = Max[X] - Min[X]$

$Quantile[X, p] = \text{argmin}\{F(x) \geq$

$Q(X, p)$

$Q[X, 0.8] = 8$

first number that is greater than 0.8 $F(x)$

$Q[X, 0.4] = 3$

$Q[X, 0.99] = 20$

$Q[X, 0.1] = 1$

$Median[X] = Q[X, .5]$

$IQR[X] = Q[X, .75] - Q[X, .25]$

inter quartile range

Tertiles

$Q[X, 1/3]$

$Q[X, 2/3]$

$Q[X, 1/5]$

$Q[X, 2/5]$

$Q[X, 3/5]$

$Q[X, 4/5]$

De

$Q[X, 1/10]$

$Q[X, 2/10]$

⋮

$Q[X, 9/10]$

Quartiles

$Q(X, 1/4)$

Med (X)

$Q(X, 3/4)$

$Med[X] < E[X] \Rightarrow$ skewed right

$Med[X] > E(X) \Rightarrow$ skewed left

$Mode(X) = 1 \Rightarrow X$ is unimodel