$\boxed{1/16}$ Review...

$$\boxed{Var[x] = E[(X-\mu)^2]}$$

$$E[x^2] = \sigma^2 + \mu^2$$

$$Var[aX + c] = a^2 \sigma^2$$

$$SE[aX + c] = |a|\sigma$$

$$T_2 = X_1 + X_2$$

Recall

$$E[T_2] = E[X_1] + E[X_2]$$

$$Var[T_2] = E[((X_1 + X_2) - (\mu_1 + \mu_2))^2]$$

$$= E[X_1^2 + X_2^2 + \mu_1^2 + \mu_2^2 + 2X_1X_2 + 2\mu_1\mu_2 - 2\mu_1 X_1$$
$$- 2\mu_2 X_1 - 2\mu_1 X_2 - 2\mu_2 X_2]$$

$$= \underbrace{E[X_1^2]}_{\sigma_1^2 + \cancel{\mu_1^2}} + \underbrace{E[X_2^2]}_{\sigma_2^2 + \cancel{\mu_2^2}} + \mu_1^2 + \mu_2^2 + 2E[X_1 X_2]$$
$$+ 2\mu_1\mu_2 - 2\mu_1^2 - 2\mu_1\mu_2 - 2\mu_1\mu_2 - 2\mu_2^2$$

$$= \sigma_1^2 + \sigma_2^2 + 2(E[X_1 X_2] - \mu_1\mu_2)$$

Covariance → "$Cov[X_1, X_2]$"

$$= \sigma_1^2 + \sigma_2^2 + 2\,Cov[X_1, X_2]$$

Assume X_1, X_2 are independent

$$E[X_1, X_2] = \sum_{x_1}\sum_{x_2} x_1 x_2\, p(x_1, x_2) = \sum_{x_1}\sum_{x_2} x_1 x_2\, p(x_1)\,p(x_2) = \sum_{x_1} x_1\, p(x_1) \sum_{x_2} x_2\, p(x_2) = \mu_1\mu_2$$

$$\Rightarrow Cov[X_1, X_2] = E[X_1 X_2] - \mu_1\mu_2 \underset{\text{independent}}{=} \mu_1\mu_2 - \mu_1\mu_2 = \boxed{0}$$

$$Var[X_1 + \dots + X_n] \overset{\text{iid}}{=} \sum_{i=1}^{n} Var[X_i] \overset{\text{if iid}}{=} n\sigma^2$$

$$E[\bar{x}] = E\left[\frac{1}{n}(X_1 + \dots + X_2)\right] = \frac{1}{n}E[\Sigma X_i] =$$
$$\frac{1}{n}\, n\mu = \boxed{\mu}$$

if iid

$$Var[\bar{x}] = Var\left[\frac{1}{n}(X_1 + \dots + X_n)\right] \overset{\text{indep.}}{=} \frac{1}{n^2}\Sigma Var[X_i]$$
$$= \frac{1}{n^2}\, n\sigma^2 = \boxed{\frac{\sigma^2}{n}}$$

$$SE[\bar{x}] = \dots \boxed{\frac{\sigma}{\sqrt{n}}}$$

Important

$X = X_1 + \dots + X_n$ s.t.

$X \sim \text{Binomial}(n, p)$ $\quad X_1, \dots, X_n \overset{iid}{\sim} \text{Bern}(p)$

$E[X] = np$

$$Var[X] = \sum_{x=0}^{n} (x - np)^2 \binom{n}{x} p^x (1-p)^{n-x} = \dots$$

$$= Var[X_1 + \dots + X_n] = n\sigma^2 = np(1-p)$$

$$= SE[X] = \sqrt{np(1-p)}$$

$X \sim \text{Geometric}(p) = (1-p)^{x-1} p$

What is μ of a geometric? $= \left(\frac{1}{p}\right)^2$

$$Var[X] = E[X^2] - \underbrace{\mu^2}_{= \left(\frac{1}{p}\right)^2}, \quad E[X^2] = \sum_{x=1}^{\infty} x^2 (1-p)^{x-1} p$$

let $y = x - 1 \Rightarrow x = y + 1$

$$= \sum_{y=0}^{\infty} (y+1)^2 (1-p)^y p = \sum_{y=0}^{\infty} y^2 (1-p)^y + p + 2\sum_{y=0}^{\infty} y (1-p)^y p + p\sum_{y=0}^{\infty} (1-p)^y$$

$$= \underbrace{(1-p)\sum_{y=1}^{\infty} y^2 (1-p)^{y-1} p}_{E[X^2]} + 2(1-p) \underbrace{\sum_{y=1}^{\infty} y (1-p)^{y-1} p}_{\mu = \frac{1}{p}} + \underbrace{p \underbrace{\sum_{y=0}^{\infty} (1-p)^y}_{\frac{1}{p}}}_{1}$$

$\boxed{\text{iid means independent}}$

$$\Rightarrow E[X^2] = (1-p)E[X^2] + \frac{2(1-p)}{p} + 1$$

$$\Rightarrow E[X^2] - (1-p)E[X^2] = \frac{2(1-p)+p}{p}$$

$$E[X^2](p) = \frac{2-p}{p} \Rightarrow \boxed{E[X^2] = \frac{2-p}{p^2}}$$

$X \sim Hyper(n, K, N)$

$Var[X] =$ Hard

$X \sim Geom(p)$

$p(X=7) = (1-p)^6 p$

$p(X=17) = (1-p)^{16} p$

$p(X=17 \mid X > 10)$

0 0 0 0 0 0 0 0 0 0 _ _ _ _ _ _ _

1 10 17

restarting geometric

$= P(X=7) = (1-p)^6 p$

CDF formula $= (1-(1-p)^{10})$

$$P(X=17 \mid X>10) = \boxed{\frac{P(X=17 \cap X>10)}{P(X>10)}}$$

$$= \frac{P(X=17)}{P(X>10)} = \frac{(1-p)^{\cancel{16}\,6}\,p}{\cancel{(1-p)^{10}}} = \boxed{(1-p)^6\,p}$$

$$P(X>10) = 1-F(X \le 10) = 1-(1-(1-p)^{10}) = (1-p)^{10}$$

W.T.S.

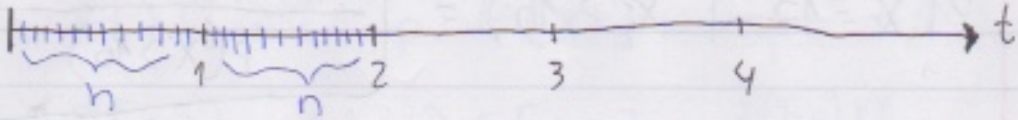
$$P(X=a) = P(X=a+b \mid X>b) = \frac{P(X=a+b \cap X>b)}{P(X>b)}$$

$$= \frac{P(X=a+b)}{P(X>b)} = \frac{(1-p)^{\cancel{a+b-1}\,a-1}\,p}{\cancel{(1-p)^b}} = P(X=a)$$

Memoryless property of the Geometric

$X \sim Geom(p)$

let's say $n=10$



Midterm 2

$p(x) = (1-p)^{x-1}\, p$, $p(t) = (1-p)^{t-1}\, p$ (in seconds)

Run iid Bern(p)'s at every $\frac{1}{n}$ the period

$p(t) = (1-p)^{nt-1}\, p$

$t = 0.7s$

Final

Jam infinite experiments into every time period

$$\lambda = np \Rightarrow \boxed{p = \frac{\lambda}{n}}$$

$$\lim_{n\to\infty} p(t) = \lim_{n\to\infty} \left(1 - \frac{\lambda}{n}\right)^{nt-1} \frac{\lambda}{n}$$

$$= \lim_{n\to\infty} \left(1 - \frac{\lambda}{n}\right)^{nt-1} \qquad \underbrace{\lim_{n\to\infty} \frac{\lambda}{n} = 0}_{0}$$

Limiting PMF $p(t) = 0 \;\; \forall t$

$\sum_{t \in Supp(t)} p(t) = 0 \Rightarrow p(t)$ is not a PMF because you're not using a discrete number of experiment. You're using continuous

$$\lim_{n\to\infty} F(t) = \lim_{n\to\infty} 1-\left(1-\frac{\lambda}{n}\right)^{nt}$$

$$= 1 - \lim_{n\to\infty}\left(1-\frac{\lambda}{n}\right)^{nt} = 1-\left(\lim_{n\to\infty}\left(1-\frac{\lambda}{n}\right)^{n}\right)^{t}$$

$$= \boxed{1-e^{-\lambda t} = F(t)}$$

limiting CDF

$$\boxed{e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n}}$$

$X \sim Geom(p)$

$F(x) = 1-(1-p)^{x}$

Let $c = nx \Rightarrow n = \frac{c}{x}$

$$e^{x} = \left(\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n}\right)^{x} = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{nx} = \lim_{c\to\infty}\left(1+\frac{x}{c}\right)^{c}$$

$\rightarrow$ CDF valid?

$\lim_{t\to\infty} F(t) = 1$ ✓

$\lim_{t\to-\infty} F(t) = F(0) = 0$ ✓

because support begins @ 0

$F'(t) = \underbrace{\lambda}_{\underbrace{n}_{+} \cdot \underbrace{p}_{+}} \underbrace{e^{-\lambda t}}_{+} \geq 0 \Rightarrow$ monotonically increasing

$\Rightarrow F(t)$ is a CDF!