10/11/17.

$\Omega = \{H, T\}$.

$n = 3$.
$\omega_1 = H$
$\omega_2 = T$
$\omega_3 = H$

$$\mathbb{1}_{\omega = H} = \begin{cases} 1 & \text{if } \omega = H \\ 0 & \text{if } \omega = T \end{cases}$$

$\mathrm{Supp}[X] = \{0, 1\}$.

There is a function X such that $X : \Omega \to \mathbb{R}$ is called a random variable (R.V)

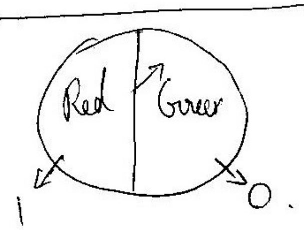
$X(H) = 1$

$X(T) = 0$.

$P(X=1) = P(\{\omega : X(\omega) = 1\}) = P(\{H\}) = \frac{1}{2}$.

$\mathrm{Supp}(X) = \{x : P(X=x) > 0\} \subseteq \mathbb{R}$

Def: "Discrete R.V" is one such that $|\mathrm{Supp}[X]| \leq |\mathbb{N}|$ i.e finite or countabily infinite.



$X(\text{Red}) = 1$
$X(\text{Green}) = 0$
$P(X=1) = \frac{1}{2}$.
$P(X=0) = \frac{1}{2}$.

Convienient Notation.

$$X \sim \begin{cases} 1 & \text{wp} \quad \frac{1}{2} \\ 0 & \text{wp} \quad \frac{1}{2}. \end{cases}$$

distributed as   values   with Probability

$$X \sim \text{Bernoiulli}\left(\tfrac{1}{2}\right) := \begin{cases} 1 & \text{wp } \frac{1}{2} \\ 0 & \text{wp } \frac{1}{2}. \end{cases}$$

$$X \sim \text{Bern}(p) := \begin{cases} 1 & \text{wp} \quad p \\ 0 & \text{wp} \quad 1-p. \end{cases}$$

$$X \sim \text{Bern}(p) := p^{x}(1-p)^{1-x} = p(x)$$

p is called "parameter", its value is an element $\in$ "parameter space".

$$p \in (0,1) \quad \underline{\underline{\text{not } [0,1]}}$$

$$X \sim \text{Deg}(c) := \{ C \quad \text{w.p} \quad 1.$$

$$\text{Supp}[X] = \{c\} \qquad c \in \mathbb{R}$$

$$X \sim \text{Bern}(0) = \text{Deg}(0).$$

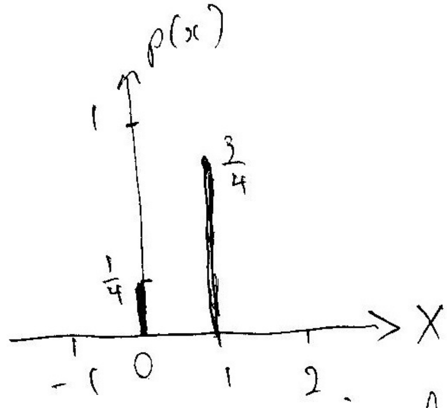
$$X \sim \text{Bern}(1) = \text{Deg}(1).$$

$p(x) := P(X = x)$ <u>p</u>robability of <u>m</u>ass <u>f</u>unction (PMF)

$$p : \mathbb{R} \to (0,1)$$

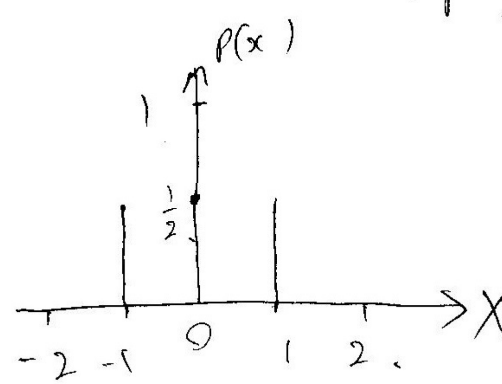
$$\sum_{X \in \text{Supp}[X]} p(x) = 1$$

$$X \sim \text{Bern}\left(\tfrac{3}{4}\right)$$



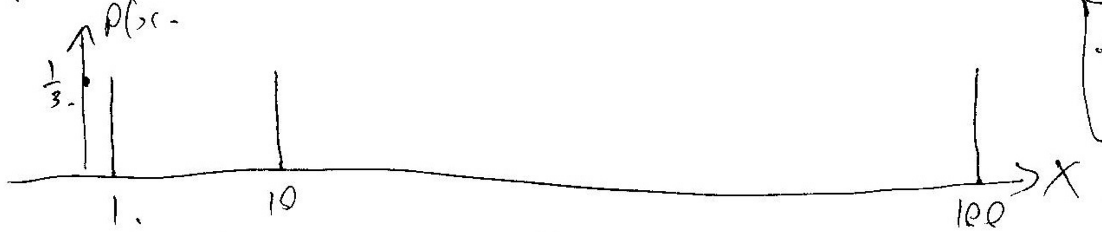
0 everywhere else.

$$X \sim \text{Rademacher} := \begin{cases} 1 & \text{w.p} \quad \tfrac{1}{2} \\ -1 & \text{w.p} \quad \tfrac{1}{2}. \end{cases}$$



$$X \sim \text{Unif}(\{1, 10, 100\}) = \begin{cases} 1 & \text{w.p } \frac{1}{3}. \\ 10 & \text{w.p } \frac{1}{3}. \\ 100 & \text{w.p } \frac{1}{3}. \end{cases}$$

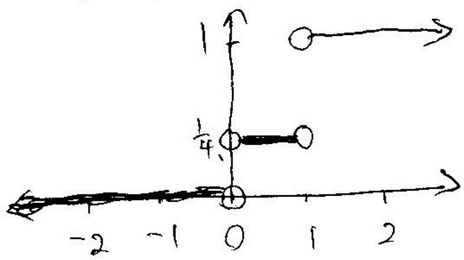
Uniform discrete R.V.



size = 3
3 is finite

$X \sim \text{Unif}(A)$, $\text{Supp}[X] = A$

$F(x) := P(X \leq x)$. Cummulative Distribution Function. (CDF)



$F(-7) = 0$.
$F(0) = \frac{1}{4}$
$F(\frac{1}{2}) = \frac{1}{4}$.
$F(1) = \frac{1}{4} + \frac{3}{4} = 1$.
$F(2) = 1$.

Properties of CDF

1. $\lim_{x \to \infty} F(x) = 1$.
2. $\lim_{x \to -\infty} F(x) = 0$.
3. $x \leq y \Rightarrow F(x) \leq F(y)$. "monotic Increase"

Def: X_1 & X_2 are "identically distributed" if
(a) $P_{X_1}(x) = P_{X_2}(x)$ or (b) $F_{X_1}(x) = F_{X_2}(x)$. and it is denoted $X_1 \overset{d}{=} X_2$.

10 cards, 4 reds.

$$P(2\ R \text{ when drawing } 3 \text{ cards } w.R) = \frac{\binom{4}{2}\binom{6}{1}}{\binom{10}{3}}$$

$$P(x\ R \quad '' \quad 3 \cdots \quad) = \frac{\binom{4}{x}\binom{6}{3-x}}{\binom{10}{3}}$$

$$P(x\ R \quad \cdots '' \quad n \quad '' \quad) = \frac{\binom{4}{x}\binom{6}{n-x}}{\binom{10}{n}}$$

10 cards, K reds.

$$P(x\ R \quad '' \quad n \quad '' \quad) = \frac{\binom{K}{x}\binom{10-K}{n-x}}{\binom{10}{n}}$$

N cards, K reds.

$$P(x\ R \quad '' \quad n \quad '' \quad) = \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$$

$$X \sim \text{Hyper Geometric}(n, K, N) := p(x).$$