10/23

Previously we saw:

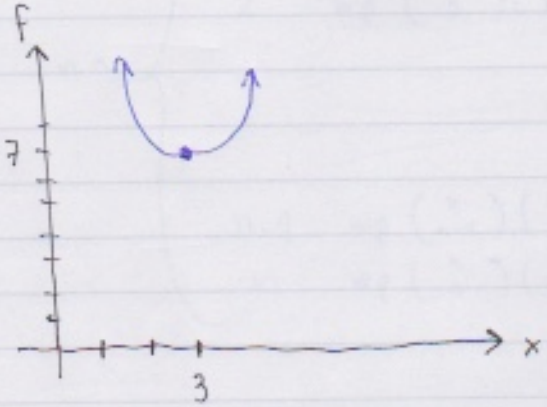
$X_1, X_2, \dots, X_n \overset{iid}{\sim} \text{Bern}(p)$

$T = X_1 + \dots + X_n \sim \text{Bin}(n, p)$

$X_1, X_2, \dots \overset{iid}{\sim} \text{Bern}(p)$

Detour from subject:

$f(x) = 7 + (x-3)^2$



$\min\{f(x)\} = 7$
$\max\{f(x)\} = \text{d.n.e.}$
$\operatorname{argmin}\{f(x)\} = 3$
x s.t. $f(x) = \min\{f(x)\}$
$\operatorname{argmax}\{f(x)\} = \text{d.n.e}$

Back on track

$T = \min\{t : X_t = 1\}$

↑ discrete time

0 0 0 0 0 1
t:1 t:2 t:3 t:4 t:5 t:6

$P(T=1) = p$
$P(T=2) = (1-p)\,p$
$P(T=3) = (1-p)^2\,p$
$\vdots$
$P(T=x) = (1-p)^{x-1}\,p$

$T \sim \text{Geometric}(p) := (1-p)^{x-1}\,p$

$\text{Supp}[x] = \mathbb{N}$ (0 not included)

$p \in (0,1)$ ↑ not inclusive

let $i = x-1 \Rightarrow x = i+1$

$\sum_{x \in \text{Supp}(x)} p(x) = 1$

$\sum_{x=1}^{\infty} (1-p)^{x-1} p = 1 \Rightarrow \sum_{x=1}^{\infty} (1-p)^{x-1} = \frac{1}{p} \Rightarrow \sum_{i=0}^{\infty} (1-p)^{i} = \frac{1}{p}$

$q \in (0,1)$

let $q = 1 - p$

$$S = \sum_{i=0}^{\infty} q^i = 1 + q + q^2 + q^3 + \cdots$$

$$= 1 + q(1 + q + q^2 + \cdots)$$

$$S = 1 + qS$$

$$\Rightarrow S - qS = 1$$

$$\Rightarrow S(1-q) = 1$$

$$\Rightarrow S = \frac{1}{1-q}$$

$$\sum_{i=0}^{\infty} (1-p)^i = \frac{1}{p} \checkmark$$

$$\frac{1}{1-(1-p)} = \frac{1}{p}$$

$$F(x) = P(X \leq x) = \sum_{i=1}^{x} p(i)$$

$$= \sum_{i=1}^{x-1} (1-p)^{i-1} p$$

$$F(x) = 1 - P(X > x) = \quad 1 - (1-p)^x$$

PMF Question

(1) Play poker until a royal flush

$$P(\text{royal flush}) = \frac{4}{\binom{52}{5}} = .00000153$$

a) Create a r.v. model for first the get the RF
$X \sim \text{Geometric}(.00000153)$

b) What's the prob. you get it on the 1,000,000th hand?
$P(X = 1{,}000{,}000) = (1 - .00000153)^{999,999} \cdot (.00000153)$

c) What's the prob. you get it before the 1,000,001st hand?
$P(X < 1{,}000{,}001) = P(X \leq 1{,}000{,}000) = 1 - (1 - .00000153)^{1{,}000{,}000}$
$\approx 77.7\%$

USING CDF

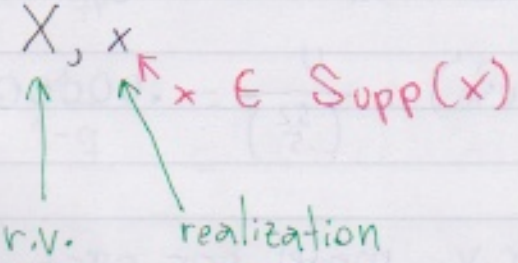
Data Datum

$X \sim \text{Bern}(\frac{1}{2})$

$p(x) := P(X = x)$ — PMF

$P(X=1) = \frac{1}{2}$

$P(X=0) = \frac{1}{2}$



datum: a realized r.v.
data: realized (r.v.)

- iid
  identity distributive without replacement

Experiment in class

$X \sim \text{Hypergeometric}(n=4, k=3, N=8)$

$\text{Supp}(x) = \{0,1,2,3\}$

$X_1, \dots, X_8 \overset{iid}{\sim} \text{Hyper}(4,3,8)$

independent

$X_1 = 1$
$X_2 = 1$
$X_3 = 2$
$X_4 = 2$
$X_5 = 2$
$X_6 = 1$
$X_7 = 1$
$X_8 = 1$

$\bar{X} = \frac{4}{8} = 1.375$ (2nd Experiment)
Heads or Tails

$X \sim \text{Binomial}(n=8, p=\frac{1}{2})$

$\text{Supp}(x) = \{0,1,\dots,8\}$

$\bar{x} = \frac{41}{9} = 4.55$

$X_1 = 6$
$X_2 = 4$
$X_3 = 3$
$X_4 = 2$
$X_5 = 5$
$X_6 = 7$
$X_7 = 5$
$X_8 = 6$
$X_9 = 3$

3rd Experiment

$X \sim$ Geometric $\left(\frac{3}{8}\right)$

$P(X=10) = \left(\frac{5}{8}\right)^9 \frac{3}{8} = 0.005$

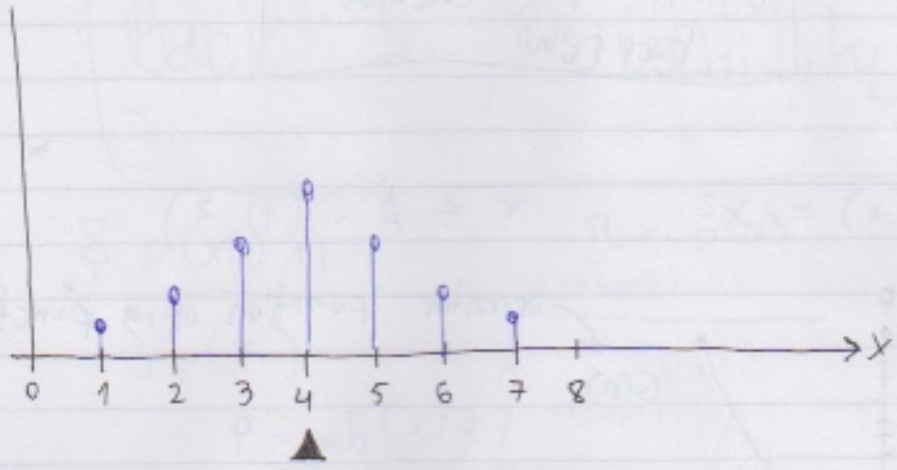
$X_1 = 10$
$X_2 = 2$
$X_3 = 1$
$X_4 = 2$
$X_5 = 1$
$X_6 = 2$

---

sum r.v. $T_n = X_1 + \cdots + X_n = \sum_{i=1}^{n} X_i$

average r.v. $\overline{X}_n = \frac{X_1 + \cdots + X_n}{n} = \frac{1}{n} T_n = \frac{1}{n} \sum_{i=1}^{n} X$.

next section..


0
1
2
3
4
5
6
7
8
X