Math 241 Lec 1 8/20/17

28 class days
− 2 midterms in-class
− 3 reviews
23 lectures

Probability Lec 1-20, Statistics Lec 21-23

Set Theory (1870's): fundamental units that all of mathematics is built up from — unordered collections of unique elements

separators

F := { Jane, Mary, Susan, Dana }

assignment / definition

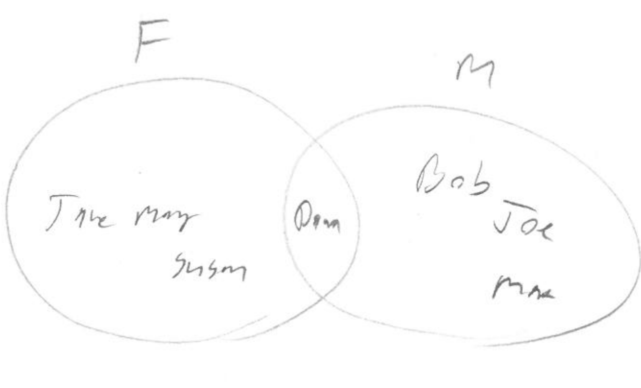
elements in the set

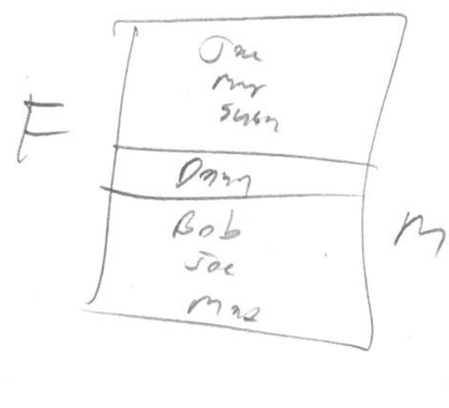
descriptive letter denoting the set

braces denote enumeration begins & ends

M := { Bob, Joe, Max, Dana }

Venn Diagram can illustrate your sets





Sets can have infinite elements e.g.

$\mathbb{N} := \{1, 2, 3, \ldots\}$ ellipses mean pattern is obvious

natural #'s, $\mathbb{N}_0 := \{0, 1, 2, \ldots\}$ natural #'s including zero

$\mathbb{Z} = \{\ldots, -1, 0, 1, \ldots\}$ (integers)

Operations on sets

Jane $\in$ F (element of) "Jane is an element in the set F"

Either this is true or it is not true. Mathematicians as a rule, do not write false statements e.g. $1 = 2$. Must never see that

Joe $\notin$ F "Joe is not an element of the set F"

$\underbrace{\{\text{Jane}, \text{Mary}\}}_{\text{set}} \subseteq \underbrace{F}_{\text{set}}$ (subset of) all elements in the set on the lhs are $\in$ the set on the rhs

let $F' := \{$Jane, Mary, Susan, Dana$\}$

$F = F'$ these two sets are equal

which is shorthand for $F \subseteq F'$ & $F' \subseteq F$. All elements...

$\{\text{Jane}, \text{Mary}\} \neq F$ these two sets are not equal. One of these is violated or both violated.

$\{\text{Jane}, \text{Mary}\} \subset F$ l.h.s. is a subset of r.h.s but l.h.s $\neq$

called "proper subset"

"$\subseteq$" same as "$\subset$" or "$=$"

$\{\text{Jane}\} \overset{?}{\subset} F$, Jane $\overset{?}{\subset} F$, $\{\text{Jane}\} \overset{?}{\subseteq} F$, Jane $\overset{?}{\subseteq} F$

$\{\text{Jane}\} \overset{?}{\in} F$, Jane $\overset{?}{\in} F$

Jane $\subset$ F does not parse e.g. $\frac{3 =}{7 + 6}$ } does not make any sense at all!

$\in, \notin, =, \neq, \subset, \subseteq$ are predicate functions returning true or false

technically $\in(\text{Jane}, F) = \text{TRUE}$

Functions that return sets? There are many "set functions"!

$F \cup M$ union, "combine" all elements, "

= { Jane, Mary, Susan, Dana } $\cup$ { Bob, Joe, Mark, Dana }

= { Jane, Susan, Mary, Dana, Bob, Joe, Mark }

Dana not listed twice!

Union $\neq$ addition ... it's a bit different

{Dana} $\cup$ {Dana} = {Dana}

singleton set

Union is "non-exclusive or" tea or coffee or both

Dana $\in M \cup F$ order of operations... set functions first so.. yes!

Dana is male or female or both

Jane $\cup$ {Jane} ? Huh...

$\mathbb{N}_0 = \mathbb{N} \cup \{0\}$

$F \cap M$ is set intersection... returns a set where elements are $\in$ both.

$F \cap M \overset{?}{=}$ Dana NO! $F \cap M = \{\text{Dana}\}$

Intersection is "AND". Dana is male and female

$F \cap \{Bob, Joe\} = \{\}$ ← empty set / null set

$\phi := \{\}$

special symbol

$A \cap B \overset{?}{=} \phi$

if A and B both have infinite elements?

$A = \{2, 4, 6, 8, \ldots\}$, $B = \{1, 3, 5, 7, \ldots\}$ $\quad A \cap B = \phi$

If $A \cap B = \phi$ .... A and B are said to be "mutually exclusive"

$\underbrace{\phi \overset{?}{\subset} F}_{\text{vacuously true}} \qquad \phi \overset{?}{\in} F$

we can subtract sets $\quad F \setminus M \quad$ all elements in F that are <u>not</u> in M

= Jan, Amy, Susan

Note $\quad F \setminus M \neq M \setminus F = \{Bob, Joe, Max\}$

If $A \cap B = \phi$ what is $A \setminus B$? $= A$, $B \setminus A$? $= B$

if $A \setminus B = \phi$ what is $A \cap B$? $= A = B$ $\quad B \cap A$? $= A = B$

$\phi \setminus \phi \overset{?}{=} \phi \qquad \phi \cap \phi = \phi \qquad \phi \cup \phi = \phi$

if $A \subseteq B$ why is $A \setminus B$? $= \phi$