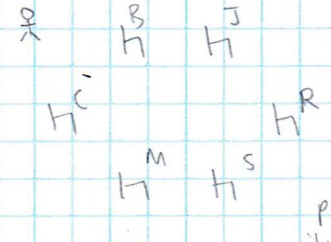
\# ways to sample k objects out of a set of n objects without replacement?

"permutations" $_nP_K = \frac{n!}{(n-k)!} = (n)(n-1)\cdots(n-k+1)$

$\overline{B, J, R, S, C, M}$

Chairs in a circle and you don't care which chair is first



How many ways to seat?

$\boxed{\frac{6!}{6}}$

principle of dividing out the invariabl

$\left.\begin{array}{l} \langle B, J, R, S, M, C\rangle \\ \langle C, B, J, R, S, M\rangle \\ \langle M, C, B, J, R, S\rangle \\ \langle S, M, C, B, J, R\rangle \\ \langle R, S, M, C, B, J\rangle \\ \langle J, R, S, M, C, B\rangle \end{array}\right\}$ "collapsable set"

Imagine a basket of 5 flowers: 3 $\langle O_1, O_2, O_3\rangle$ and 2 chrysanthemums $\{X_1, X_2\}$

How many ways to place them in 5 pots? $5! = 12$

" " " " " " if each orchid is "not unique"/"indistinguishable"/"indistinct" and each chrysanthemum is indistinct? $\frac{5!}{3!}$

$P(4H \text{ in } 10 \text{ coin tosses}) = \frac{|A|}{|\Omega|} = \frac{\frac{10!}{4!6!}}{2^{10}} \approx 0.205$

$2^{10} = 1024$

How many ways to order 10 flips? 10!

4H, 6T

$\{B, J, R\}$ $\langle B, J, R\rangle, \langle B, R, J\rangle, \langle R, B, J\rangle, \langle R, J, B\rangle, \langle J, B, R\rangle, \langle J, R, B\rangle$

$\{B, J, R, S, C, M\}$

1 How many ways to order than 6 chairs? 6!

2 " " " " " 3 chairs? $_6P_3$

3 " " " " " " such that the order (of these 3) does not matter? $\frac{_6P_3}{_3P_3} = \frac{_6P_3}{3!} = \frac{\cancel{6}\cdot 5\cdot 4}{\cancel{3\cdot 2}} = 20$

How many ways to seat in 4 chairs, such that order does not matter?

$\frac{_6P_4}{4!} = \frac{\overset{3}{\cancel{6}}\cdot 5\cdot \cancel{4}}{\cancel{4}\cdot\cancel{3}\cdot\cancel{2}} = 15$

How many ways to sample k items out of a set of n without replacement s.t. order does not matter?

$$\binom{n}{k} := \frac{{}_nP_k}{{}_kP_k} = \frac{{}_nP_k}{k!} = \frac{\frac{n!}{(n-k)!}}{k!} = \frac{n!}{(n-k)!\,k!}$$

${}_nC_k :=$

combinations "choose"

Combinatorial Identities

① $\binom{n}{1} = \frac{n!}{(n-1)!\,1!} = n$

② $\binom{n}{k-1} = \frac{n!}{1!\,(n-1)!}\ n$

⑤ $\binom{n}{n} = 1$

③ $\binom{n}{k} = \binom{n}{n-k} = \frac{n!}{\underbrace{(n-(n-k))!}_{k}\,(n-k)!}$

④ $\binom{n}{0} = \frac{n!}{n!\,0!} = 1$

B, J, R, S, M, C

You seat 4 people randomly

What is the prob. Jane is seated?

2 WAYS TO APPROACH THIS

① NOT ORDERED

② ORDERED

① $P(A) = \frac{|A|}{|\Omega|} = \frac{\binom{5}{3}}{\binom{6}{4}} = \frac{2}{3}$ 5 choose 3

$\{B, \cancel{J}, R, S, M, C\}$ $\{J, \underset{1}{\_\_}, \underset{2}{\_\_}, \underset{3}{\_\_}\}$

5 options 3 chairs

② $P(A) = \frac{4\,({}_5P_3)}{{}_6P_4} = \frac{4\,(\cancel{5}\cdot\cancel{4}\cdot\cancel{3})}{6\cdot\cancel{5}\cdot\cancel{4}\cdot\cancel{3}} = \frac{2}{3}$

Chair #			
J	5	4	3
5	J	4	3
5	4	J	3
5	4	3	J

$2^A = \{B : B \subseteq A\}$

$= \{B : B \subseteq A \ \&\ |B| = 0\} \cup$
$\{B : B \subseteq A \ \&\ |B| = 1\} \cup$
$\vdots$
$\{B : B \subseteq A \ \&\ |B| = 4\}.$

$A = \{1, 2, 3\}$

$2^A = \{\underbrace{\emptyset}_{\text{size } 0}, \underbrace{\{1\}, \{2\}, \{3\}}_{\text{size } 1}, \underbrace{\{1,2\}, \{1,3\}, \{2,3\}}_{\text{size } 2}, \underbrace{A}_{\text{size } 3}\}$

mutually exclusive & collectively exhaustive.

$|A| = n$

$$2^n = |2^A| = |\{B : B \subseteq A \;\&\; |B| = 0\}| + \cdots + |\{B : B \subseteq A \;\&\; |B| = n\}|$$

$$= \sum_{i=0}^{n} |\{B : B \subseteq A \;\&\; |B| = i\}| = \boxed{\sum_{i=0}^{n} \binom{n}{i} = 2^n}$$

$$(a+b)^2 = (a+b)(a+b) = \underbrace{a^2 + ab + ba + b^2}_{4 \text{ terms}} = a^2 + 2ab + b^2$$

$$(a+b)^3 = (a+b)(a+b)(a+b) = \underbrace{\qquad\qquad}_{8 \text{ terms}} = a^3 + 3a^2b + 3ab^2 + b^3$$

$\vdots$

$$(a+b)^n = \underbrace{\qquad\qquad}_{2^n \text{ TERMS} = \sum_{i=0}^{n} \binom{n}{i}} = \binom{n}{n} a^n b^0 + \binom{n}{n-1} a^{n-1} b^1 + \cdots + \binom{n}{2} a^2 b^{n-2} + \binom{n}{1} a^1 b^{n-1} + \binom{n}{0} a^0 b^n$$

$$= \sum_{i=0}^{n} \binom{n}{i} a^i b^{n-i}$$ Binomial Theorem

↳ Binomial Coefficient