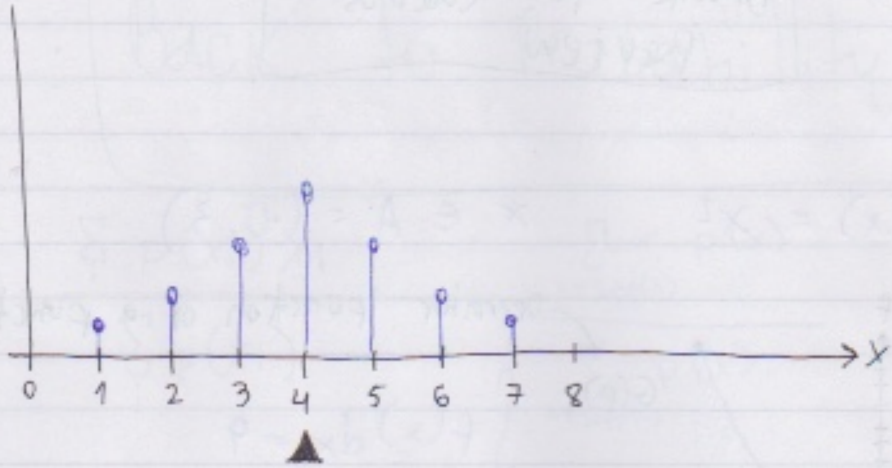
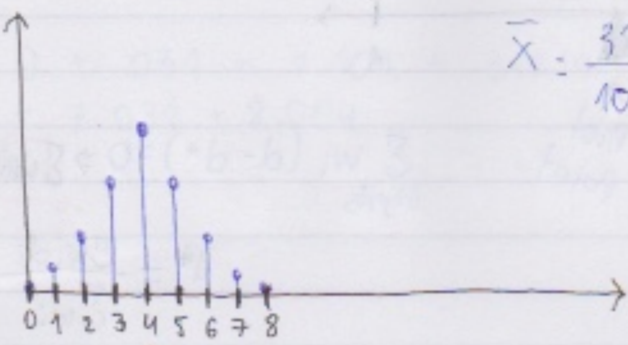


10/25

1st Experiment

$X \sim \text{Binomial}\left(8, \frac{1}{2}\right)$

PMF



Pivot Point

$$\bar{x} = \frac{31 + 32 + 31}{10 + 10 + 10}$$

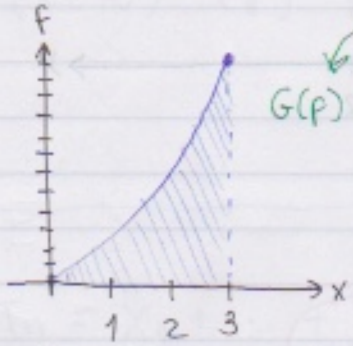
$$= 3.167$$

should be about 4

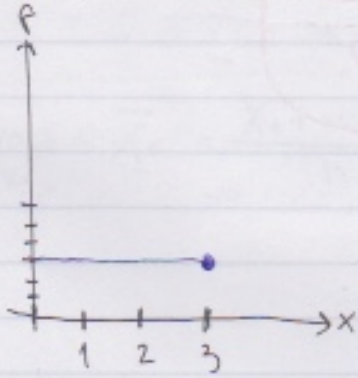
Break for Calculus Review

$f(x) = x^2 \quad , \quad x \in A = (0,3)$

Operator "function of a function"

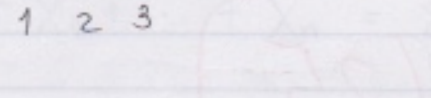


$G(p) \quad \int_0^3 f(x)\,dx = 9$



$G(p) = \int_0^3 f(x)\,dx = 9$

Break for High School Phys.



Pivot point

$\sum_{\text{Objects}} w_i (d - d^*) = 0 \Rightarrow \sum w_i d_i - \sum w_i d^* = 0$

$d^* = \frac{\sum w_i d_i}{\sum w_i}$

Back to Probability

$$M = \frac{\sum_i p(x_i) x_i}{\sum_i p(x_i)} = \frac{\sum_{x \in Supp(x)} x\, p(x)}{\sum_{x \in Supp(x)} p(x)}$$

$$\boxed{\begin{gathered} \overset{M}{\parallel} \\ = E[x] \\ \sum_{x \in Supp(x)} x\, p(x) \end{gathered}}$$

$\leftarrow$ "expectation"

$$\boxed{\bar{x} \to E(x)}$$

$$E(x) = 0\,p(0) + 1\,\underset{\binom{8}{1}\left(\frac{1}{2}\right)^1\left(1-\frac{1}{2}\right)^7 \cdots}{p(1)} + 2p(2) + 3p(3) + 4p(4) +$$

$$5p(5) + 6p(6) + 7p(7) + 8p(8)$$

$$= 0 + .031 + 2.109 + 3.219 + 4.273 + 5.213 + 6.109 + 7.031 + 8.004$$

$$= \boxed{4}$$

$X \sim \text{Binomial}(8, 376, 0.38279)$

$X \sim \text{Binomial}(n, p)$, $E(x) = \boxed{np}$

Expectation for Binomial r.v.

We're not responsible for this proof...

$$E(x) = \sum_{x=0}^{n} x \binom{n}{x} p^x (1-p)^{n-x}$$

$$= np \sum_{x=1}^{n} \cancel{x} \frac{(n-1)!}{\cancel{x!}(n-x)!} p^{x-1} (1-p)^{n-x}$$

$(x-1)!$

$$\underbrace{((n-1)-(x-1))!}_{\binom{n-1}{x-1}}$$

$$= np \sum_{x=1}^{n} \binom{n-1}{x-1} p^{x-1} (1-p)^{n-x}$$

let $y = x-1$

$$= np \sum_{y=0}^{n-1} \binom{n-1}{y} p^y (1-p)^{n-y-1}$$

let $M = n-1$

$$= np \sum_{y=0}^{M} \binom{M}{x} p^y (1-p)^{M-y} = \boxed{np}$$

$X \sim \text{Hyper}(n, K, N)$

$$E(x) = \boxed{\sum_{\text{supp}(x)} x \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}} = \boxed{n \frac{K}{N}}$$

"n p"

$X \sim \text{Uniform}(\{1, 10, 100\})$

$$E(x) = 1 \cdot \frac{1}{3} + 10 \cdot \frac{1}{3} + 100 \cdot \frac{1}{3}$$

$$= \boxed{\frac{111}{3}}$$

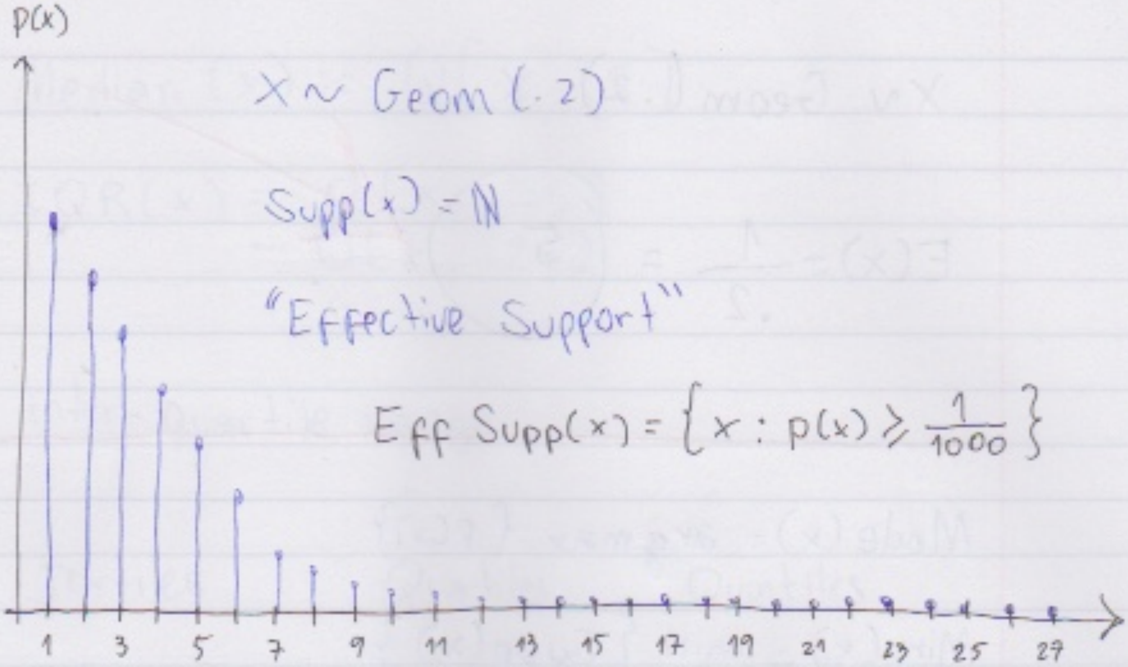
(how many times until you succeed)

$X \sim \text{Geometric}(.2) = .8^{x-1} \cdot .2$

$\boxed{\text{Supp}(x) = \mathbb{N}}$

x	p(x)	F(x)
1	.200	.200
2	.160	.360
3	.128	.488
4	.102	.590
5	.082	.672
6	.066	.738
7	.052	.790
8	.042	.832
9	.034	.866
10	.027	.893
11	.021	.914
12	.017	.931
13	.014	.945
14	.011	.956
15	.009	.965
16	.007	.972
17	.006	.978
18	.005	.983
19	.004	.987
20	.003	.990
21	.002	.992
22	.001	.994
23	.001	.995
24	.001	.996

x	p(x)	F(x)
25	.001	.997
26	.001	.998
27	.001	.999
⋮	⋮	⋮
	same	same



$X \sim \text{Geom}(.2)$

$\text{Supp}(x) = \mathbb{N}$

"Effective Support"

$\text{Eff Supp}(x) = \{x : p(x) \geq \frac{1}{1000}\}$

$X \sim \text{Geom}(p)$

$$E(x) = \sum_{x=1}^{\infty} x(1-p)^{x-1} p$$

Don't need to know this proof

$$= \sum_{y=0}^{\infty} (y+1)(1-p)^y p = (1-p)\underbrace{\sum_{\cancel{y=0}\; y=1}^{\infty} y(1-p)^y p}_{M} + \underbrace{\sum_{\cancel{y=0}\; x=1}^{\infty} (1-p)^{\cancel{y}\; x-1} p}_{1}$$

$\Rightarrow M = (1-p)M + 1$

$\Rightarrow M(1-(1-p)) = 1$

$\Rightarrow Mp = 1$

$\Rightarrow \boxed{M = \frac{1}{p}}$

$X \sim Geom(.2)$

$E(x) = \frac{1}{.2} = 5$

$Mode(x) = \arg\max \{p(x)\}$

$Min(x) = \min\{Supp(x)\}$
$Max(x) = \max\{Supp(x)\}$
$Range(x) = Max(x) - Min(x)$

$Q[x, 0.8] = 8$ (first number greater than .8)
$Q[x, 0.4] = 3$ $\quad Q[x, 0.1] = 1$
$Q[x, .99] = 20$
$Quantile(x, p) = \arg\min\{F(x) \geq p\}$

$\boxed{Q(X, p)}$

$\text{Median}(x) := Q[x, .5]$

$\text{IQR}(x) = Q[x, .75] - Q[x, .25]$

inter-quartile range

Textiles	Quartiles	Quintiles
$Q[x, \frac{1}{3}]$	$Q[x, \frac{1}{4}]$	$Q[x, \frac{1}{5}]$
$Q[x, \frac{2}{3}]$	$\text{Med}(x)$	$Q[x, \frac{2}{5}]$
	$Q[x, \frac{2}{4}]$	$Q[x, \frac{3}{5}]$
		$Q[x, \frac{4}{5}]$

Deciles

$Q[x, \frac{1}{10}]$
$Q[x, \frac{2}{10}]$
$\vdots$
$Q[x, \frac{9}{10}]$

$|\text{Mode}(x)| \cdot 1 \Rightarrow X$ is unimodal