Lecture 15 10/31/2014

$\mu = E(X) = \sum_{x \in supp(X)} x\, p(x)$

Long Run Average

$\sigma^2 = Var(X) = \sum_{x \in supp(X)} (x-\mu)^2 p(x)$ $\sigma = SE[X] = \sqrt{Var[X]}$

"just square root of avg square distance from expectation"

Bet $1 on #7

$X_7 \sim \begin{cases} \$35 & \text{w.p } 1/38 \\ -\$1 & \text{w.p } 37/38 \end{cases}$ $\mu = -\$0.053$

Bet $1 on Black

$X_B \sim \begin{cases} \$1 & \text{w.p } 18/38 \\ -\$1 & \text{w.p } 20/38 \end{cases}$ $\mu = -\$0.053$

less variable around μ

$X_7 \to \mu$, $X_B \to \mu$ Which goes faster?

The r.v with the smaller variance

"more risky"

avg square distance from mean (expectation)

$Var[X_7] = (35 - {-.053})^2 \frac{1}{38} + (-1 - {-.053})^2 \frac{37}{38} = 33.207 \$^2$

$Var[X_B] = (1 - {-.053})^2 \frac{18}{38} + (-1 - {-.053})^2 \frac{20}{38} = 0.997 \$^2$

$\sqrt{Var(X_7)} = \sqrt{33.207 \$^2} = \$5.79 = \sigma = SE[X_7] = SD[X_7]$

$\sqrt{Var[X_B]} = \sqrt{\$0.997 \$^2} = \$1.00 = \sigma = SE[X_B] = SD[X_B]$

Standard Error

$T_2 = X_1 + X_2$, $E(T) = \sum_{t \in supp(T)} t\,p(t)$ — ?

Tree Illustration is not pratical, imagine $X \to$ 100 Times

so what do we do?

$E(X_1 + X_2] = E[X_1] + E[X_2]$ if X_1 and X_2 independent + $p(x_1,x_2) = p(x_1)p(x_2)$

but what if not independent?

$\sum_{x_2} p(x_1, x_2) = p(x_1)$ — summed over x_2

"Marginning out"

Sumed over it

$\sum_{x_1} p(x_1, x_2) = p(x_2)$ — summed over x_1

Back again $\Rightarrow$ $E[X_1 + X_2] = E[X_1] + E[X_2]$

Summary: $E[T_n] = \sum_{i=1}^{n} E[X_i]$ | $E[\bar{X}_n] = E\left[\frac{T_n}{n}\right] = \frac{1}{n} E[T_n]$

Avg R.V

$= \frac{1}{n} \sum_{i=1}^{n} E[X_i] = \frac{1}{n} n\mu = \boxed{\mu}$

if $X_1 \dots X_2$ identically distributed ☆

same pmf $\Rightarrow$ expectation

variance

$X \sim Hyper(n, K, N)$, $E[X] = \sum_{x \in supp[x]} x \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}} = \dots n\frac{K}{N}$

crazy

$X = X_1 + X_2 + \dots + X_n$ $\Rightarrow E[X] = \sum E[X_i] = n\mu = n\frac{K}{N}$

$X_1 \sim Bern\left(\frac{K}{N}\right)$

$X_2 \sim BerN\left(\frac{K}{N}\right) \Rightarrow$ don't know what first ball is (X_1)

$X_1, \dots, X_2$ are all dependent

$X_n \sim Bern\left(\frac{K}{N}\right)$

think of cards on ♠A

but proved that independence/dependence doesn't matter

$C \sim \mathrm{Deg}(c) = \{c$ w.p. 1

$E[C] = \sum_{c \in supp[C]} c\,p(c) = c\,p(c) = c \cdot 1 = c$

$L = (X-\mu)^2 \qquad Var(X) := E[L]$

$Var[X] = E[(X-\mu)^2] = E[X^2 - 2\mu X + \mu^2] = E[X^2] + E[-2\mu X] + E[\mu^2]$

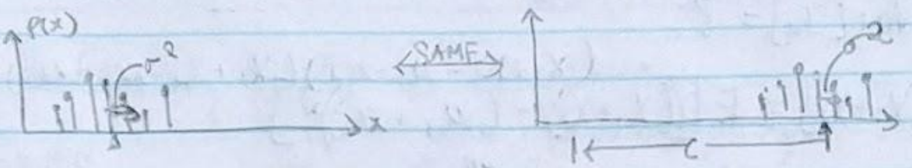
$= E[X^2] - \underbrace{2\mu \underbrace{E[X]}_{\mu}}_{-2\mu^2} + \mu^2 = E[X^2] - \mu^2 = \underset{\sigma^2}{Var(X)}$

$\boxed{E[X^2] = \sigma^2 + \mu^2}$

$Y = aX + c \quad a \in \mathbb{R}, c \in \mathbb{R}$

$a=1$

$Y = X + c, \; Var[Y] = Var[X+c] = \sigma^2$ "adding c does not affect variance"

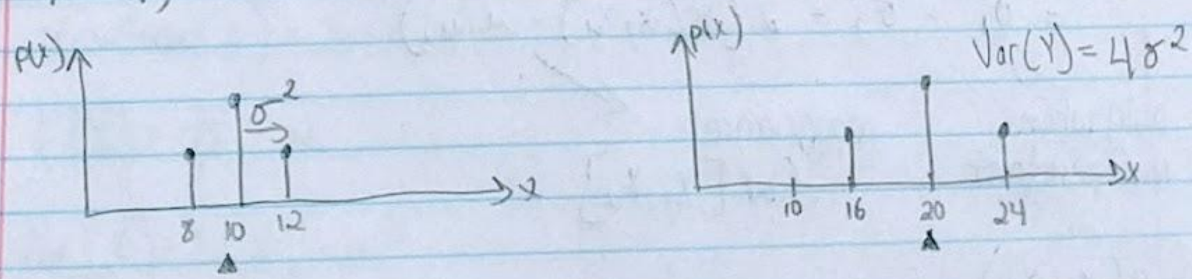


$Var[X+c] = E[((X+c) - (\mu+c))^2]$

$E(X+c) = \mu + c \quad = E[(X + \cancel{c} - \mu - \cancel{c})^2]$

$= E[(X-\mu)^2] = \sigma^2$

$Y = aX, \quad Y = 2X$



$Var[aX] = E[(aX - a\mu)^2] = E[(a(X-\mu))^2]$

$E[aX] = a\mu \qquad = E[a^2 (X-\mu)^2]$

$= a^2 E[(X-\mu)^2]$

$= a^2\sigma^2$

★ $\boxed{Var[aX+c] = a^2\sigma^2}$ ★ $\boxed{SE[aX+c] = |a|\sigma}$ ★