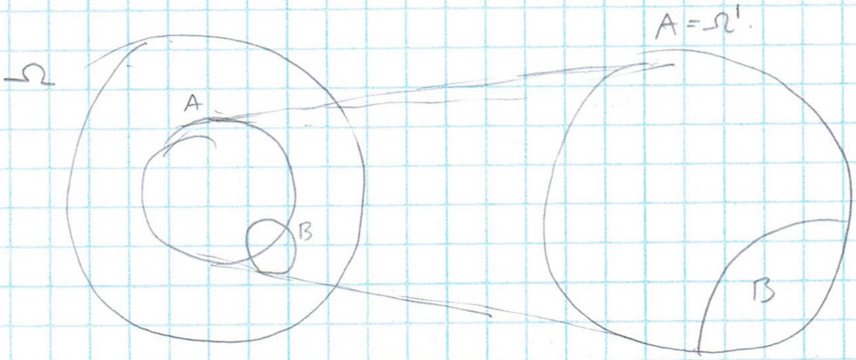
$P(A) = 0.2$ ← smoking

$P(B) = 0.06$ ← lung cancer

$P(AB) = 0.036$

$\uparrow$

$A \cap B$




$P(B) = 0.06$

$P(B|A) =$

"given"

"conditional"

"or"

P(lung cancer among smokers)

new universe

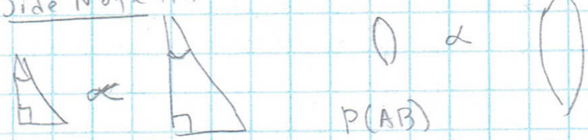
$P(B|A) = (\text{scale})\, P(AB)$

$= \dfrac{P(\Omega)}{P(A)} P(AB)$

$\Rightarrow P(B|A) = \underbrace{\dfrac{1}{P(A)}}_{\text{zoom}} P(AB)$

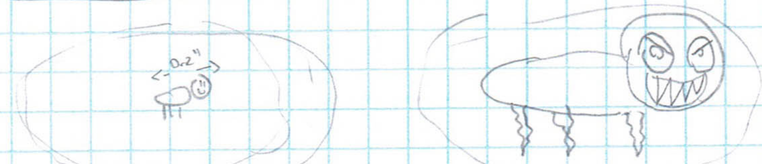
$$\boxed{P(B|A) = \frac{P(A|B)}{P(A)}}$$ Def of conditional Probability

Side Note #1



P(AB) & P(B|A)

Side Note #2




← 1" →    ← 0.5" →

$\text{Zoom} = \dfrac{\text{Prior Scope size}}{\text{New Scope Size}}$  $\dfrac{1}{0.5} = 2$

$P(A|B) = \dfrac{P(AB)}{P(B)} \Rightarrow P(AB) = P(A|B)P(B)$

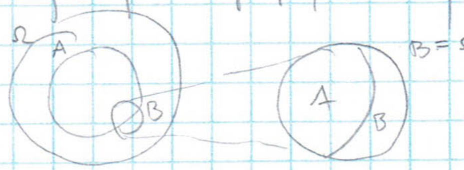
$$\boxed{P(B|A) = \frac{P(A|B)\,P(B)}{P(A)}}$$ Bayes Rule, 1763

Going Back to Lung Cancer

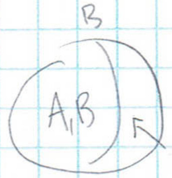
$$\boxed{P(B|A) = \frac{P(AB)}{P(A)} = \frac{0.036}{0.2} = 0.18}$$

---

$P(A|B) = \dfrac{P(AB)}{P(B)} = \dfrac{.036}{.06} = 0.6$  Among Lung cancer people, how many of them smoke?




$P\left(\begin{matrix}\text{lung cancer}\\ \text{among non-}\\ \text{smokers}\end{matrix}\right) = P(B|A^c) = \dfrac{P(B, A^c)}{P(A^c)} = \dfrac{\quad}{1 - P(A)} = \dfrac{0.024}{0.8} = 0.03$




$P(B) = P(AB) + P(A^cB)$

$\Rightarrow P(A^cB) = P(B) - P(AB)$

$= 0.06 - 0.036 = 0.024$

Risk Ratio $\left\{ \dfrac{P(B|A)}{P(B|A^c)} = \dfrac{0.18}{0.03} = 6 \right.$ Translates to: "Smokers are 6 times more likely to have lung cancer"

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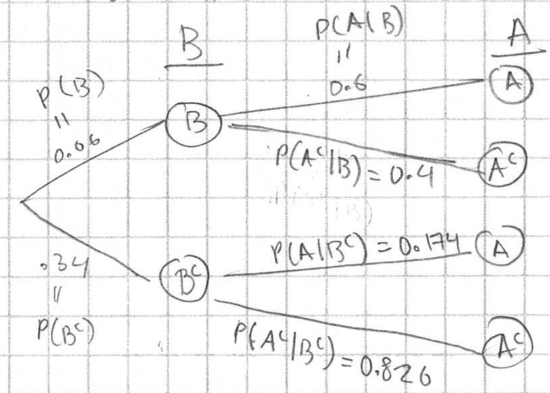
How many more questions can we ask about probability

Tree Illustration




$$P(A|B^c) = \frac{P(AB^c)}{P(B^c)} = \frac{0.164}{0.94} = 0.174$$

Joint Outcomes $= P(B)\,P(A|B)$

$P(AB) = 0.036$

$P(A^cB) = 0.024$

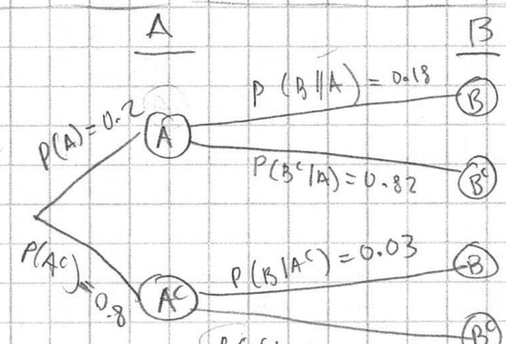
$P(AB^c) = 0.164$

$P(A^cB^c) = 0.776$

mutually exclusive / collectively exhaustive

$1 = \Omega$

---




"prob of non-smoker not having lung cancer?"

Joint Outcomes

$P(AB) = 0.036$

$P(AB^c) = 0.164$

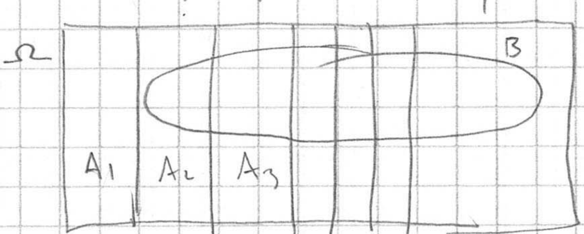
$P(A^cB) = 0.024$

$P(A^cB^c) = 0.776$

$1$

---

Consider $A_1, A_2, \ldots$ are mutually exclusive & collectively exhaustive and event $B$




Law of Total Probability

$$\begin{aligned} P(B) &= P(B \cap \Omega) \\ &= P(B \cap (A_1 \cup A_2 \cup \cdots)) \\ &= P((B \cap A_1) \cup (B \cap A_2) \cup (\ldots)) \\ &= P(B \cap A_1) + P(B \cap A_2) \end{aligned}$$

$A_k$ $B \cap A_i$, $B \cap A_j$ mutually excl?

$(B \cap A_i) \cap (B \cap A_j) \overset{?}{=} \phi$

$= \underbrace{B \cap B}_{B} \cap \underbrace{A_i \cap A_j}_{\phi} = \phi$ ✓

$$\boxed{P(B) = \sum_{i=1}^{\infty} P(BA_i)}$$

Assume girls & boys births are equally likely

P( If you have 2 kids and 1 is a girl, the other is a girl)

$\Omega = \{BB, BG, GB, GG\}$

$$P(\{GG\} \mid \{GG, BG, GB\}) = \frac{\overbrace{P(\{GG\} \cap \{GG, GB, BG\})}^{P(\{GG\})}}{P(\{GG, BG, GB\})} = \frac{1/4}{3/4} = \frac{1}{3}$$

Side Note #3

$\Omega^2 =$

| | |
|---|---|
| ⟨B, B⟩ | ⟨B, G⟩ |
| ⟨G, B⟩ | ⟨G, G⟩ |

with order

without order

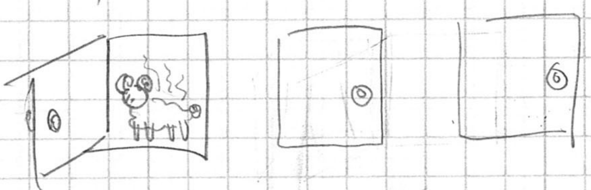
$\Omega = \{BB, GG, BG\}$

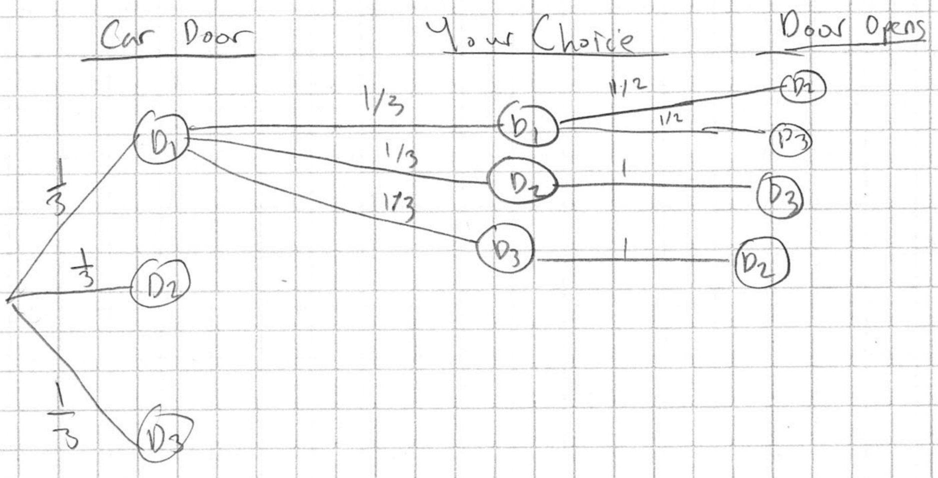
$P(BB) = 1/4$

$P(GG) = 1/4$

$P(BG) = 1/2$

"Smelly Goat Car Show Game" the MOVIE






| Switch (W/L) | P(W\|D1) |
|---|---|
| L | 1/6 |
| L | 1/6 |
| W | 1/3 |
| W | 1/3 |

$P(W \mid D_1) = \boxed{\frac{2}{3}}$