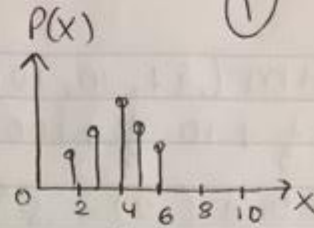
Lecture :- 13

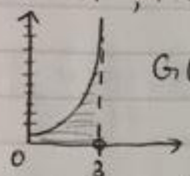
1

$X \sim Bin(8, \frac{1}{2})$



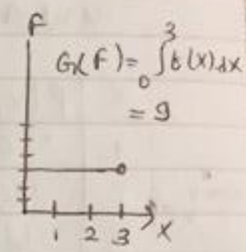
$$\bar{X} = \frac{31+34+31}{10+10+10} = 3.167$$

$P(X) = x^2,\ x \in A = [0,3]$



$G(F)$ = operator" function of a function"

$$\int_0^3 f(x)\,dx = 9$$



$$M = \frac{\sum_i P(x_i)\, x_i}{\sum_i P(x_i)} = \frac{\sum_{x \in supp(x)} x\, P(x)}{\sum_{x \in supp(x)} P(x)} = E[\overset{m}{x}] \rightarrow \text{"expectation"}$$

$$\sum_{x \in supp(x)} x\, P(x) \qquad \boxed{\bar{x} = E(x)}$$

$E(x) = 0 \cdot P(0) + 1 \cdot P(1) + 2 \cdot P(2) + 3 \cdot P(3) + 4\,p(4) + 5 \cdot P(5) + 6 \cdot p(6)$
$\quad + 7 \cdot p(7) + 8 \cdot p(8)$
$= 0 + .031 + 2 \cdot .109 + 3 \cdot .219 + 4 \cdot .273 + 5 \cdot .273 + 6 \cdot .109 + 7 \cdot .031 + 8 \cdot .004$
$= 4$

$X \sim Binomial\,(8, 376, 0.38279)$

$X \sim Binomial\,(n, p),\ E(x) = \boxed{np}$ → expectation for binomial r.v.

$$E(x) = \sum_{x=0}^{n} x \binom{n}{x} \cdot p^x (1-p)^{n-x}$$
$$= n \cdot p \sum_{x=1}^{n} x \frac{(n-1)!}{x!(n-x)!} \cdot p^{x-1} (1-p)^{n-x} = \boxed{np}$$

$X \sim hyper(n, k, N)$

$$E(x) = \frac{\sum_{supp(x)} x \binom{K}{x} \binom{N-K}{n-x}}{\binom{N}{n}} = n \cdot \frac{K}{N} \leftarrow \text{"np"}$$

②

$X \sim \text{uniform}(\{1, 10, 100\})$

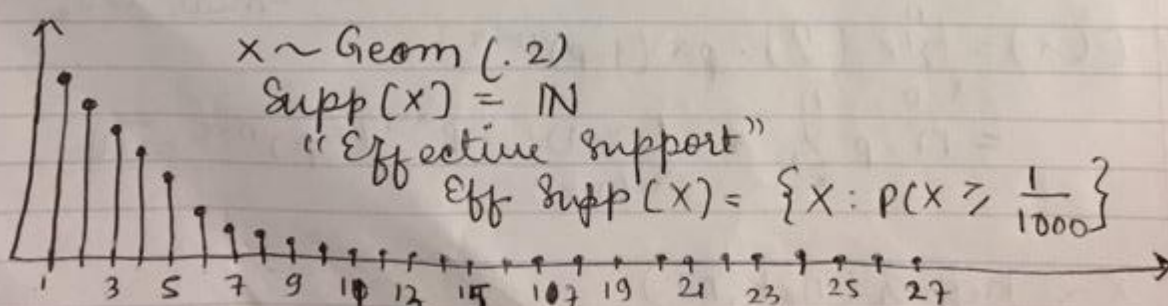
$E(X) = 1 \cdot \frac{1}{3} + 10 \cdot \frac{1}{3} + 100 \cdot \frac{1}{3} = \frac{111}{3}$

$X \sim \text{Geometric}(.2) = .8^{x-1} \cdot (.2) = 0.2 \quad \text{Supp}(X) = \mathbb{N}$

(how many times till we succeed)

X	P(x)	F(X)
1	.200	.200
2	.160	.360
3	.128	.488
4	.102	.590
5	.082	.672
6	.066	.733
7	.052	.790
8	.042	.832
9	.034	.866
10	.027	.893
11	.021	.914
12	.017	.971
13	.014	.995

X	P(X)	F(x)
14	.011	.956
15	.009	.965
16	.007	.572
17	.006	.978
18	5	.983
19	4	.987
20	3	.990
21	2	.992
22	1	.994
23	1	.995
24	1	.996
25	1	.997
26	1	.998
27	1	.999



$X \sim \text{Geom}(.2)$

$\text{Supp}(X) = \mathbb{N}$

"Effective support"

$\text{Eff Supp}(X) = \{X : P(X \geq \frac{1}{1000}\}$

$X \sim \text{Geom}(p)$

$$E(X) = \sum_{x=1}^{\infty} x(1-p)^{x-1}p = \sum_{y=0}^{\infty} (y+1)(1-p)^{y}p$$

③

Let $y = x - 1$

$$= \sum_{y=0}^{\infty} y(1-p)^y p + \sum_{y=0}^{\infty} (1-p)^{x-1} p$$

$$\mu = (1-p)\mu + 1$$

$$\mu(1-(1-p)) = 1$$

$$\mu p = 1 \Rightarrow \mu = \boxed{1/p}$$

$X \sim Geom(.2)$

$$E(x) = \frac{1}{.2} = 5$$

Mode (x) = argmax $\{p(x)\}$

min(x) = min $\{$supp$(x)\}$

max(x) = max $\{$supp$(x)\}$

Range (x) = Max (x) - Min(x)

$Q[x, 0.8] = 8$

$Q[x, 0.4] = 3$ $\quad$ $Q[x, 0.1] = 1$

$Q[x, .99] = 20$

Quantile $[x, p)$ = argmin $\{f(x) \geq p\}$

$\therefore Q(x, p)$

Median $[x] = Q[x, .5]$

$IQR[x] = Q[x, .75] - Q[x, .25]$

inner Quartile range

Textiles	Quartiles	Quintiles
$Q[x, \frac{1}{3}]$	$Q[x, \frac{1}{4}]$	$Q[x, \frac{1}{5}]$
$Q[x, \frac{2}{3}]$	Med(x)	$Q[x, \frac{2}{5}]$
	$Q[Y, \frac{2}{4}]$	$Q[x, \frac{3}{5}]$
		$Q[x, \frac{4}{5}]$

4

$K = 3,\ D = 4$

$$E(w_1 \ldots w_K) = -\sum_{n \in C_1} \log y^1(x_n) - \sum_{n \in C_2} \log y^2(x_n)$$

$$= -\sum_{n=1}^{50} \log y^1(x_n) - \sum_{n=51}^{100} \log y^2(x_n) - \sum_{n=101}^{150} \log y^3(x_n)$$

$$y^1(x_n) = \frac{e^{w_1^T x_n}}{e^{w_1^T x_n} + e^{w_2^T x_n} + e^{w_3^T x_n}}$$

$$\frac{\delta E}{\delta w_1} = -\sum_{n \in C_1} \frac{\delta \log y^1(x_n)}{\delta w_1} - \sum_{n \in C_2}$$