Lecture 12

10/19/2017

$X \sim \text{Binomial}(n, p)$

$p(x) = P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$

$F(x) = P(X \leq x) = \sum_{i=0}^{x} \binom{n}{i} p^i (1-p)^{n-i}$

$F(x) = 7 + (x-3)^2$

$\min\{F(x)\} = 7$

$\max\{F(x)\} =$ Does not exist

x s.t $F(x) = \min\{F(x)\}$

$\text{argmin}\{F(x)\} = 3$ (which x to plug in to get min)

$\text{argmax}\{F(x)\} =$ D.N.E

$X_1, X_2 \ldots \overset{iid}{\sim} \text{Bern}(p)$ → potential infinite series of events

$T := \min\{t : X_t = 1\}$ 0,0,0,1 time 4 you got a 1

"stopping time"

Got one the first time $P(T=1) = p$

$P(T=2) = (1-p)(p)$

$P(T=3) = (1-p)(1-p)(p) = (1-p)^2(p)$

$\vdots$

$T \sim \text{Geometric}(p) := p(x) = P(T=x) = (1-p)^{x-1}(p)$ can go up to 1 billion

$\text{Supp}[X] = \mathbb{N}$ (1 to infinity)

$p \in (0,1)$

up to one

$$\sum_{x \in Supp(X)} p(x) = 1$$

let $i = x-1 \Rightarrow x = i+1$

want to show

$$\sum_{x=1}^{\infty} p(1-p)^{x-1} = 1 \Rightarrow \sum_{x=1}^{\infty} (1-p)^{x-1} = \frac{1}{p} \Rightarrow \sum_{i=0}^{\infty} (1-p)^{i} = \frac{1}{p}$$

$\hookrightarrow$ let $q = 1-p$ NOTE THAT $p \in (0,1) \Rightarrow q \in (0,1)$

$$\Rightarrow \sum_{i=0}^{\infty} q^i = \frac{1}{p}$$

$$S = \sum_{i=0}^{\infty} q^i = 1 + q^1 + q^2 + q^3 + q^4 + \dots$$
$$= 1 + q(1 + q + q^2 + q^3 + \dots)$$
$$S = 1 + qS$$
$$S - qS = 1$$
$$S(1-q) = 1 \Rightarrow S = \frac{1}{1-q}$$

GEOMETRIC SERIES

$$\sum_{i=0}^{\infty} q^i = \frac{1}{1-q} \text{ IF } q \in (0,1)$$

CDF

$$F(x) = P(X \le x) = \sum_{i=1}^{x} (1-p)^{i-1} p \quad \text{Hard...}$$

$$= 1 - P(X > x)$$

$$P(X > x) = (1-p)^x$$

$$\underset{1}{0} \;\; \underset{2}{0} \;\; \underset{3}{0} \;\; \underset{4}{0} \;\; \dots \;\; 0 \;\; \underset{x}{0} \;\Big|\; \underset{x+1}{\_} \;\; \underset{x+2}{\_} \;\; 1 \;\; \dots$$

$$= P(X = x+1) + P(X = x+2) + \dots$$

$$= \sum_{i=x+1}^{\infty} P(X=i) = \sum_{i=x+1}^{\infty} (1-p)^{i-1} p$$

$P(\text{Royal Flush}) = 1.53$ in a million
$= .00000153$

Play poker until we get a Royal Flush

$T \sim \text{Geometric}(.00000153)$

Whats probability I get the first Royal Flush on the 1,000,000th play?

$P(T = 1{,}000{,}000) = (.9999985)^{999999} \times .00000153$

Whats the prob. I get a royal Flush on the 1000000th time or sooner?

CDF question

$P(T \leq 1000000) = F(1000000) = 1 - (.9999985)^{1000000} \approx 77.7\%$

SN: $X \sim \text{Bern}(p)$ → while flipped in air X is abstract (all possible) → everything it could be

MODEL

$x = 1$ → element of support

~~Realization - what the model became (made real)~~

Datum: realization of a r.v - need to be in Supp(X)
Data: realizations of a r.v's
iid data ..... iid r.v's

$X \sim \text{Hyper}(n, K, N) = \dfrac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$

$X \sim \text{Hyper}(4, 3, 8)$
$\text{Supp}(X) = \{0, 1, 2, 3$

$X \sim \text{Hyper}(4, .375, 8)$

$X_1, \ldots X_8 \overset{iid}{\sim}$ Hyper (4,3,8)

$x_1 = 1$
$x_2 = 2$
$x_3 = 1$
$x_4 = 3$ $\quad \bar{x} = 1.5$
$x_5 = 1$
$x_6 = 1$
$x_7 = 1$
$x_8 = 2$

$T_n = X_1 + \ldots X_n = \sum_{i=1}^{n} X_i$

$$\bar{X}_n = \frac{X_1 + \ldots X_n}{n} = \frac{T_n}{n} = \frac{\sum_{i=1}^{n} X_i}{n}$$

average r.V

sample average $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ $\quad \bar{x}$ is a realization from $\bar{X}$

amount of heads $X \sim$ Binomial $(8, \frac{1}{2}) \rightarrow$ sampling w/ replacement
Supp$[X] = \{0, 1, \ldots 8\}$

$X_1 \ldots X_8$ iid

heads
$x_1 = 5$ $\quad \bar{x} = 3.375$
$x_2 = 4$
$x_3 = 2$
$x_4 = 5$
$x_5 = 2$
$x_6 = 3$
$x_7 = 3$
$x_8 = 3$

Keep going until heads

$X \sim \text{Geom}(\frac{1}{2})$ $\qquad$ $X_1, \ldots X_8 \overset{iid}{\sim} \text{Geom}(\frac{1}{2})$

$\text{Supp}[X] = \{1, 2, \ldots\}$

$x_1 = 1$

$x_2 = 3$ $\qquad$ $\bar{x} = 2$

3

1

1

$x_6 = 3$