LECTURE - 6 :-

09/14/2017

(I) Long Run Frequency / Limiting Freq.

Def. :-

$$\therefore P(A) := \lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} \mathbb{1}_{\omega_i \in A}$$

(II) Karl Popper, 1957

Objects have inherent disposition towards outcomes "propensity"

↳ Propensity induces the long run freq.

Radioactive U238

P(U238 atom explodes < 4.5 Billion years)

= $\frac{1}{2}$

→ can be calculated explicity of you underground question reduces.

Prob: (1) For most random experiments, we don't know how to calculate the propensitions of ω's.

(2) Not general

P(OJ Simpson guilty)

I, II are objectivist theorem.

(III) Subjectivist Def. :-

Everyone uses their own evidence, biases, institution to come up with their own outcome of upcoming.

$P_{Adam}(H) = 0.5$, P(Newton's F = ma is true)

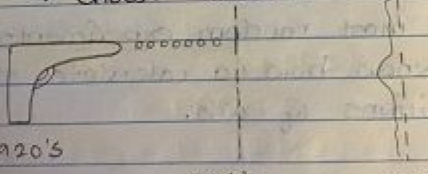
Ramsey, 1926
De Finetti, 1928

Conclusion:-

No acceptions def. of probability.

What are Randomers?

↳ Choose $\omega \in \Omega$



Quantum Mechanics ⇒ Universe is Random

1920's

double slit

screen

Kolmagon 1930's $\rightarrow$ Mathematical Def. of Prob.

Assume $\exists \Omega \neq \emptyset$ P is a set function satisfying the 3 conditions:

(a) $P(\Omega) = 1$

(b) $\forall A \subseteq \Omega \quad P(A) \geq 0$

(c) If $A_1, A_2, \ldots$ are disjoint

$$\Rightarrow P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i)$$

* Theorem I :-

$$\boxed{P(A) = 1 - P(A^c)}$$

$\Omega = A \cup A^c$ and $\{A, A^c\}$ are disjoint

$P(\Omega) = P(A \cup A^c)$

$\Rightarrow P(\Omega) = P(A) + P(A^c)$ by (c)

$\Rightarrow 1 = P(A) + P(A^c)$ by (a)

$\therefore P(A) = 1 - P(A^c)$ ---- (I)

* Theorem II :-

$$\boxed{P(\emptyset) = 0}$$

$\therefore P(\emptyset) = 1 - P(\emptyset^c)$ by theorem I

$= 1 - P(\Omega)$ set theory

$= 1 - 1$ by (a) (II)

$= 0$

* ## Theorem III :-

the size of A has to be the size of B

$$\boxed{A \subseteq B \Rightarrow P(A) \leq P(B)}$$

If, $C = B \setminus A$ and $C \cap A = \emptyset$ then,

$$P(B) = P(A \cup C) = P(A) + P(C) \quad \text{set theory}$$

$$\Rightarrow P(B) - P(A) = P(C) \geq 0 \quad \text{by } (b)$$

$$\Rightarrow P(B) - P(A) \geq 0$$

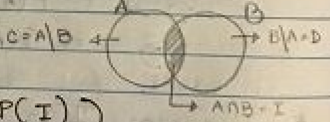
$$\Rightarrow P(B) \geq P(A)$$

$$\therefore P(A) \leq P(B) \quad \text{--- } (III)$$

* ## Theorem IV :-

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

Laws of Inclusion-Exclusion



$$\left.\begin{aligned} P(C \cup I) &= P(C) + P(I) \\ &= P(A) \\ P(D \cup I) &= P(D) + P(I) \\ &= P(B) \end{aligned}\right\} \text{by } (c)$$

$$\therefore P(A \cup B) = P(C \cup D \cup I)$$
$$= P(C) + P(D) + P(I) \quad \text{by } C$$
$$= (P(A) - P(I)) + P(I) + (P(B) - P(I))$$
$$= P(A) + P(B) - P(I)$$
$$= P(A) + P(B) - P(A \cap B) \quad \text{---} \; IV$$

* Theorem V :-

$$\boxed{|\Omega| < \infty \text{ if } P(\omega_i) = \frac{1}{|\Omega|} \; \forall \omega_i \Rightarrow P(A) = \frac{|A|}{|\Omega|}}$$

Let $n = |A| < \infty$

since $A \subseteq \Omega \Rightarrow |A| \leq |\Omega|$

$A = \{\omega_i, \omega_2, \dots \omega_n\}$

$$A = \{\omega_1\} \cup \{\omega_2\} \cup \dots \cup \{\omega_n\}$$
$$\Rightarrow P(A) = P\left(\bigcup_{i=1} \{\omega_i\}\right)$$
$$= \sum_{i=1}^{n} P(\{\omega_i\}) \quad \text{by } C$$
$$= \sum_{i=1}^{n} \frac{1}{|\Omega|} = \frac{n}{|\Omega|} = \frac{|A|}{|\Omega|} \quad \text{---} \; V$$