MAT241 Rodrigo Cuaracchi 10/15/17

10 cards, 4R 6B

$$P\left(\begin{array}{c}\text{drawing}\\ 2\ R \text{ in } 3 \text{ cards}\\ \text{without replacement}\end{array}\right) = \frac{\binom{4}{2}\binom{6}{1}}{\binom{10}{3}}$$

$$P(x\ R \text{ in } 3) = \frac{\binom{4}{x}\binom{6}{3-x}}{\binom{10}{3}}$$

$$P(x\ R \text{ in } n \text{ cards}) = \frac{\binom{4}{x}\binom{6}{n-x}}{\binom{10}{n}}$$

10 cards, K R

$$P(x\ R \text{ in } n) = \frac{\binom{K}{x}\binom{10-K}{n-x}}{\binom{10}{n}}$$

N cards, K R

$$P(x\ R \text{ in } n) = \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$$

$$X - \text{hypergeometric}(n, K, N) = \underbrace{\frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}}_{p(x) = P(X=x)}$$

Support

$\text{Supp}(x)$

Suppor to X - Bernoulli $= \{0,1\}$

$p \in (0,1)$

parameter space

Ex

100 students, 53 are female, choose 8, what is the probability have women

X is free variable

$$X - \text{Hypergeometric}(8, 53, 100) = \frac{\binom{53}{x}\binom{47}{8-x}}{\binom{100}{8}} =$$

$$P(X=6) = p(6) = \frac{\binom{53}{6}\binom{47}{2}}{\binom{100}{8}}$$

$N = 0 \;? \Rightarrow n = 0$ Degen
$\Rightarrow K = 0$

$N = 1 \Rightarrow K \in \{0, 1\}$ Degen
$\Rightarrow n \in \{0, 1\}$

$N = 2 \Rightarrow K \in \{0, 1, 2\}$ first not Degen case
$\Rightarrow n \in \{0, 1, 2\}$

$$x\text{-hyper}(1, 1, 2) = \frac{\binom{1}{x}\binom{1}{1-x}}{\binom{2}{1}} = \text{Bern}\left(\tfrac{1}{2}\right)$$

$$P(1) = P(X = 1) = \frac{\binom{1}{1}\binom{1}{0}}{\binom{2}{1}} = \frac{1}{2}$$

$$P(0) = \frac{\binom{1}{0}\binom{1}{1}}{\binom{2}{1}} = \frac{1}{2}$$

Picking — K of them are balls — N balls in bag

$$x\text{-hyper}(1, K, N) = \frac{\binom{K}{x}\binom{N-K}{1-x}}{\binom{N}{1}} = \text{Bern}\left(\frac{K}{N}\right)$$

$\text{Supp}[x] = \{0, 1\}$

$$P(1) = \frac{\binom{K}{1}\binom{N-K}{1-x}}{\binom{N}{1}} = \frac{\binom{K}{1}\binom{N-K}{0}}{\binom{N}{1}} = \frac{K}{N}$$

$$P(0) = \frac{\binom{K}{0}\binom{N-K}{1}}{\binom{n}{1}} = \frac{N-K}{N} = 1 - \frac{K}{N}$$

Param Space Hyper

$N \in \{2, 3, \ldots\}$
$K \in \{1, 2, \ldots, N-1\}$
$n \in \{1, 2, \ldots, N-1\}$

a) $x\text{-Hyper}(2, 4, 10)$, $\text{Supp}[x] = \{0, 1, 2\}$

b) $x\text{-Hyper}(5, 4, 10)$, $\text{Supp}[x] = \{0, 1, 2, 3, 4\}$

Note Supp[x] can't have 5 because there are only 4 ~~balls~~ special balls

c) $x\text{-Hyper}(8, 4, 10)$, $\text{Supp}[x] = \{2, 3, 4\}$

d) $x\text{-Hyper}(5, 7, 10)$, $\text{Supp}(x) = \{2, 3, 4, 5\}$

Generalized

*a) $n < K, \; n < N-K \qquad Supp(X) = \{0, \dots, n\}$

*b) $n > K, \; n < N-K \qquad Supp[x] = \{0, \dots, K\}$

*c) $n \geq K, \; n \geq N-K \qquad Supp[x] = \{n-(N-K), \dots, K\}$

*d) $n < K, \; n > N-K \qquad Supp[x] = \{n-(N-K), \dots, n\}$

	$n < K$	$n \geq K$
$n < N-K$	$0 \dots n$	$0 \dots K$
$n \geq N-K$	$n-(n-K) \dots n$	$n-(N-K) \dots K$

$$Supp(x) = \{\max\{0, n-(n-K)\}, \dots, \min\{n, K\}\}$$

x - Hyper

$$\sum_{x \in Supp(x)} p(x) = 1$$

let $p = \frac{K}{N} \Rightarrow K = pN$

$$X \sim Hyper(n, p, N) = \frac{\binom{pN}{x}\binom{(1-p)N}{n-x}}{\binom{N}{n}}$$

reparameterization

$N \in \{2, \dots\}$

$n \in \{1, \dots, N-1\}$

$p \in \{\frac{1}{N}, \frac{2}{N}, \dots, \frac{N-1}{N}\}$

Consider $p = 0.5, \; n = 6, \; N = 100$

$$p(3) = \frac{\binom{50}{3}\binom{50}{3}}{\binom{100}{6}} = .3223$$

$N = 1000$

$$p(3) = \frac{\binom{500}{3}\binom{500}{3}}{\binom{1000}{6}} = .3134$$

$N = 10,000$

$$p(3) = \frac{\binom{5000}{3}\binom{5000}{3}}{\binom{1000}{6}} = .3126$$

what is the limiting Random variable

$$\cancel{\lim_{N\to\infty} \text{Hyper}(p,k,N)} = \lim_{N\to\infty} \text{Hyper}(n,p,N) = \lim_{N\to\infty} \frac{\binom{pN}{x}\binom{(1-p)N}{n-x}}{\binom{N}{n}} =$$

$$= \lim_{N\to\infty} \frac{\frac{(pN)!}{x!\,(pN-x)!}\,\frac{((1-p)N)!}{(n-x)!\,((1-p)N-(n-x))!}}{\frac{N!}{n!\,(N-n)!}} = \underbrace{\frac{1}{x!}\cdot\frac{1}{(n-x)!}\cdot\frac{1}{\frac{1}{n!}}}_{\binom{n}{x}} =$$

$$\lim_{N\to\infty} \frac{\frac{(pN)!}{(pN-x)!}\,\frac{((1-p)N)!}{((1-p)N-(n-x))!}}{\frac{N!}{(N-n)!}}$$