Ex 6 people in seating 6 chairs in a circle. How many ways?

If not a circle, then it's regular permutation. 6!. But if sit in a circle, some seating can be "collapsable set", example. <B.J.R.S.M.C> will be the same as <J.R.S.M.C.B> by put the last element to the first seat, and can be done 4 more times with same steps.

By this principle of dividing all the invarible factor. (because there are 6 of such examples, thus answer is $\frac{6!}{6}$.

Ex Ways of ordering 3 O flower and 2 X flowers if each O is indistinguishable?

$O_1 O_2 O_3 X_1 X_2$ $O_2 O_3 O_1 X_1 X_2$
$O_1 O_3 O_2 X_1 X_2$ $O_3 O_1 O_2 X_1 X_2$
$O_2 O_1 O_3 X_1 X_2$ $O_3 O_2 O_1 X_1 X_2$

$\} \; O O O X_1 X_2 \quad \frac{5!}{3!}$

Ex What if the X is also indistingushable? $\frac{5!}{3!2!}$

Ex P(4H in 10 coins flip)

since every H and T are collapsable, thus $|A| = \frac{10!}{4!6!}$ ← ways to order 10 flips / ← collapsable

$|\Omega| = 2^{10}$ ← All possible outcome.

Ex Ways of siting 3 chairs with 6 people. if these 3 people order doesn't matter?

$$\frac{6p3}{3!} = \frac{6p3}{3p3}$$

Ex Way of sitting 4 chair with 6 people, if 4 of them order doesn't matter?

$$\frac{6p4}{4!}$$

{J, B. S R, M, C}

JBSR   JBRM   JBMC
JBSM   JBRC
JBSC
JSRM   JRMC
JSRC
BSRM   BRMC
BSRC
SRMC

Ex. How many ways of choose k from n without replacement if k order doesn't matter?

$$\frac{n!}{(n-k)!k!} = \binom{n}{k} = \frac{nPk}{kPk} = \frac{nPk}{k!} = \frac{\frac{n!}{(n-k)!}}{k!}$$

Combinational Identities

$\binom{n}{1} = \frac{n!}{(n-1)!1!} = n$ ⇐ How many way to pull one ball
n ways. for each ball

$\binom{n}{n-1} = \frac{n!}{1!(n-1)!} = n$ ⇐ How many way to pull $(n-1)$ ball
still n ways. We can think like ways to leave 1 ball

$\binom{n}{k} = \binom{n}{n-k} = \frac{n!}{(n-(n-k))!(n-k)!}$ ⇐ How many ways to pull k ball

$\binom{n}{0} = \frac{n!}{n!0!} = 1$ ⇐ How many ways to pull nothing
One way, that is not to pull anything

$\binom{n}{n} = 1$ ⇐ How many ways to pull everything
One way.

Ex. 6 people with 4 random selected seat. Probability with Jane is seat?

order doesn't matter: $\dfrac{\binom{5}{3}}{\binom{6}{4}}$ ← Jane must in it, so only 5 left, and 3 seat left.
← # of way to choose 4 from 6

order doesn't matter: $\dfrac{(5P3)4}{6P4}$ ← J 5 4 3 / 5 J 4 3 / 5 4 J 3 / 5 4 3 J
← # of ways to choose 4 from 6

$$|2^A| = |\{B : B \subseteq A \ \& \ |B| = 0\}| + \cdots + |\{B : B \subseteq A \ \& \ |B| = 4\}|$$
$$= \sum_{i=0}^{n} |\{B : B \subseteq A \ \& \ |B| = i\}| = \sum_{i=0}^{4} \binom{n}{i} = 2^n$$

mutually exclusive & collective exhaustive.
$\leq$ $\geq$ $\Rightarrow =$

Ex. $A = \{1, 2, 3\}$

$2^A = \{ \underbrace{\{\}}_{\text{size 0}}, \underbrace{\{1\}, \{2\}, \{3\}}_{\text{size 1}}, \underbrace{\{1,2\}, \{1,3\}, \{2,3\}}_{\text{size 2}}, \underbrace{\{1,2,3\}}_{\text{size 3}} \}$

$$(a+b)^2 = (a+b)(a+b) = a^2+2ab+b^2$$

$$(a+b)^3 = (a+b)(a+b)(a+b) = a^3+3a^2b+3ab^2+b^3$$

$$(a+b)^n = \binom{n}{n}a^n b^0 + \binom{n}{n-1} n^{n-1} b^1 + \cdots \binom{n}{2} a^2 b^{n-2} + \binom{n}{1} a^1 b^{n-1} + \binom{n}{0} a^0 b^n$$

$$= \sum_{i=0}^{n} \binom{n}{i} a^i b^{n-i}$$

Binomial Thm. / Binomial Thm.