Lecture - 9

$\Omega = \{H, T\}$

$n = 3$.

$\boxed{H \mid T \mid}$ $w_1 = H,\ w_2 = T;\ w_3 = H$

1 0

$X = \mathbb{1}_{w=H} = \begin{cases} 1 & \text{if } w = H \\ 0 & \text{if } w = T \end{cases}$

$\bar{x} = \frac{1+0+1}{3} = \frac{2}{3}$

↳ $Supp(X) = \{0, 1\}$

There is a function X s.t.

$X: \Omega \to \mathbb{R}$

is called a "random variable" (r.v)

$X(H) = 1$ | $P(X=1) = P(\{w: X(w)=1\}) = P(\{H\}) = \frac{1}{2}$

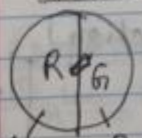
$X(T) = 0$ | shorthand or abuse of notation

Recall: $P: 2^{\Omega} \to [0,1]$

The "support" of r.v X is $Supp[X] = \{x : P(X=x) > 0\} \subseteq \mathbb{R}$

Def^n of "Discrete r.v" is one such that $|Supp(X)| \le |\mathbb{N}|$ i.e finite or ~~count~~ infinite

Spinner



$X(Red) = 1$ $P(X=1) = \frac{1}{2}$

$X(Green) = 0$ $P(X=0) = \frac{1}{2}$

Convenient Notation

$X \sim \begin{cases} 1 & wp\ \frac{1}{2} \\ 0 & wp\ \frac{1}{2} \end{cases}$

the random v. X is distributed as ... with probability

Std. Bernoulli

$X \sim \text{Bernouilli}\left(\frac{1}{2}\right) = \begin{cases} 1 & wp\ \frac{1}{2} \\ 0, & wp\ \frac{1}{2} \end{cases}$

$X \sim Bern(p) := \begin{cases} 1 & wp\ p \\ 0 & wp\ 1-p \end{cases}$ | p = "parameter". Its value is an element $\in$ "parameter space".

→ Parameter "tunes" or "customizes" the model.

$p \in (0,1)$ (not including 0 & 1)

Degenerate

$X \sim Deg(c) := \{c\ wp\ 1\}$

$Supp(X) = \{i\}, i \in \mathbb{R}$

$X \sim Bern(0) = Deg(0)$

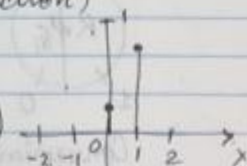
$X \sim Bern(1) = Deg(1)$

$p(x) = P(X = x)$

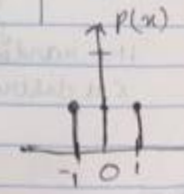
PMF (Probability Mass function)

$p: \mathbb{R} \to [0,1]$

$\sum_{x \in Supp(X)} p(x) = 1$ | $X \sim Bern\left(\frac{3}{4}\right)$

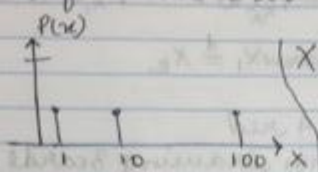


$X \sim \text{Rademacher} := \begin{cases} 1 & wp\ \frac{1}{2} \\ -1 & wp\ \frac{1}{2} \end{cases}$



$X \sim unif(\{1, 10, 100\}) = \begin{cases} 1 & wp\ \frac{1}{3} \\ 10 & wp\ \frac{1}{3} \\ 100 & wp\ \frac{1}{3} \end{cases}$

uniform discrete r.v

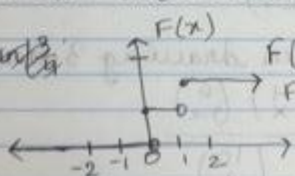


$X \sim Unif(A)$, $Supp[X] = A$, Param. space $A \in 2^{\mathbb{R}}$

$F(x) := P(X \le x)$

cumulative distribution function (CDF)

~~$X \sim Bern(\frac{3}{4})$~~



$F(-17) = 0$

$F(0) = \frac{1}{4}$

$F(\frac{1}{\ }) = \frac{1}{4}$

Properties of the CDF

① $\lim_{x \to \infty} F(x) = 1$

② $\lim_{x \to -\infty} F(x) = 0$

③ $x \le y \Rightarrow F(x) \le F(y)$

$X \sim Bern(p) := \overset{= P(x)}{\begin{cases} 1 & wp\ p \\ 0 & wp\ 1-p \end{cases}}$

$X \sim Bern(p) = p^x (1-p)^{1-x} = p(x)$

Def: X_1 & X_2 are "identically distributed" if a) $p_{X_1}(x) = p_{X_2}(x)$ or b) $F_{X_1}(x) = F_{X_2}(x)$

& it is denoted $X_1 \overset{d}{=} X_2$

Q) 10 cards → 4 red

$P(2R$ when drawing 3 cards w/o replacement$)$

$= \frac{\binom{4}{2}\binom{6}{1}}{\binom{10}{3}}$

$P(xR$ when drawing 3 cards$)$

$= \frac{\binom{4}{x}\binom{6}{3-x}}{\binom{10}{3}}$

$P(xR$ when drawing n cards $'')$

$= \frac{\binom{4}{x}\binom{6}{n-x}}{\binom{10}{n}}$

Q 10 cards → K red)

$P(xR \ldots n) = \frac{\binom{K}{x}\binom{10-K}{n-x}}{\binom{10}{n}}$ $X \sim$ Hyper-geometric $(n, K, N) := p(x)$

Q (N cards, K are red)

$P(xR \ldots n) = \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$