Lecture-14

black = 18 slots

Robin bet \$1 on black (Roulette in America)

Payout 1:1

$E(X) = \mu = \sum_{x \in supp(X)} x \cdot p(x)$

$X \sim \begin{cases} \$1 & \text{wp } \frac{18}{38} \\ -\$1 & \text{wp } \frac{20}{38} \end{cases}$

$\mu = (1) \cdot \frac{18}{38} + (-1) \frac{20}{38}$

$= -0.053$

long run average

Bet \$1 on #7

Payout is 35:1

$X \sim \begin{cases} 35 & \text{wp } \frac{1}{38} \\ -1 & \text{wp } \frac{37}{38} \end{cases}$ $\mu = \$0.053$

Bet on 1 - 12. Payout is 2:1

$X \sim \begin{cases} \$2 & \text{wp } \frac{12}{38} \\ -\$1 & \text{wp } \frac{26}{38} \end{cases}$ $\mu = 2 \cdot \left(\frac{12}{38}\right) - (1) \cdot \frac{26}{38}$

$= -0.053$

$\lim_{n \to \infty} T_n = \lim_{n \to \infty} n\bar{X} = -\infty$

Def:- Fair Game

$E(X) = 0$

In general
$\mu \notin Supp(X)$

Q $P(traffic) = .3$

If traffic ride takes 12 min, O.W 7min. Build r.v model for time in traffic.

$$W \sim \begin{cases} 12 & w.p.\ 0.3 \\ 7 & w.p.\ 0.7 \end{cases} \quad E(W) = 12\cdot(0.3) + 7\cdot(0.7) = 8.5 \text{ min}$$

Q Uber charges \$0.47/min. Find the PMF of B, the r.v model for cost per time.

$$B \sim \begin{cases} 12\cdot(0.4) = \$4.80 & w.p.\ 0.3 \\ 7\cdot(0.4) = 2.80 & w.p.\ 0.7 \end{cases}$$

$\mu = 4.80(0.3) + (2.80)(0.7) = \$3.40$

$B = 0.4W = g(W)$

$E(B) \stackrel{?}{=} 0.4E(W)$ (Yes)

$0.4E(W) = 0.4(12\cdot(0.3) + 7\cdot(0.7))$
$= 0.4(E(W))$

Q1 Uber charges flat \$3 to start trip. Find r-v T of the cost.

$$T \sim \begin{cases} (3) + 4.80 = \$7.80 & w.p.\ 0.3 \\ (3) + 2.80 = \$5.80 & w.p.\ 0.7 \end{cases}$$

$\mu = (7.8)(0.3) + (5.8)(0.7)$
$= 6.40$

$Y = aX + c, (a, c \in \mathbb{R})$
$E(X) = aE(X) + C$

$E[g(X)] = \sum_{x \in Supp(x)} g(x) \cdot P(X)$

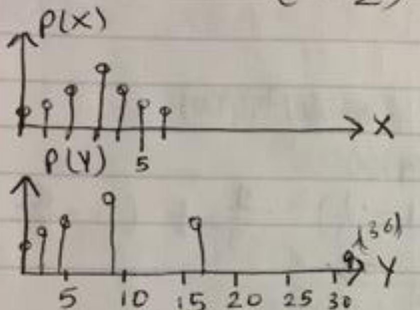
$$E[aX + c] = \sum_{x \in Supp(x)} (ax + c)\cdot p(x) = \sum_{x \in Supp(x)} ax\, P(x) + \sum c(P(x))$$

$$= a\overbrace{\sum_x x\cdot p(x)}^{E(X)} + c\overbrace{\sum_x p(x)}^{1}$$

$$\hookrightarrow \boxed{E(aX+c) = a\,E(X)+c}$$

$X \sim \text{Binom}(6, \frac{1}{2})$, $Y = X^2$ $E(Y) \overset{?}{\neq} E(X)^2$

$\because 3^2 \neq 9$

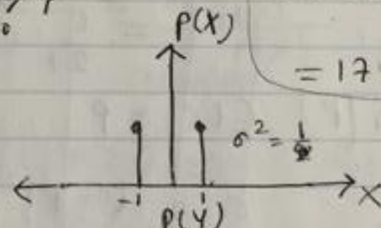


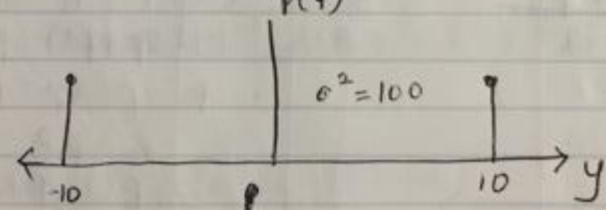
$$E(X^2) = 0\cdot p(0) + 1\cdot p(1) + 4\cdot p(2) + \dots + 36\cdot p(6)$$

$$= 0 + \binom{6}{1}\left(\tfrac{1}{2}\right)^1\left(\tfrac{1}{2}\right)^5 + 4\cdot\binom{6}{2}\left(\tfrac{1}{2}\right)^2\left(\tfrac{1}{2}\right)^4 + \dots$$

$$= 17.5$$

$X \sim \begin{cases} 1 & \text{w.p. } \frac{1}{2} \\ -1 & \text{w.p. } \frac{1}{2} \end{cases}$ $Y = 10X$



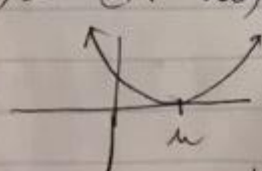


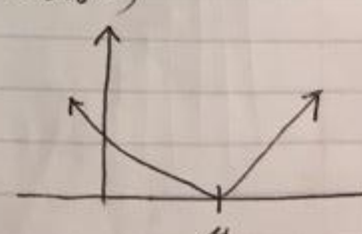
"Y is more spread-out than X"
or
"Y has more deviation than X"

Error functions (always ≥ 0)

$e(X, \mu) = |X - \mu|$ (L1 loss or L1 error)

$e(X, \mu) = (X - \mu)^2$ L2 loss



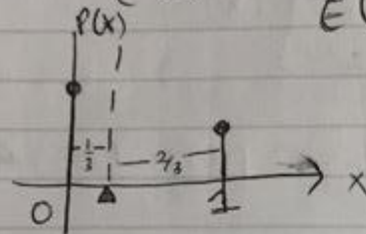


$$L = (X - \mu)^2 = g(X)$$

"Mon(18)"
exam - Wed(15)

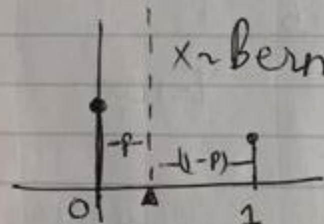
$$\sigma^2 = Var(X) = E(L) = E[(X-\mu)^2]$$
$$= \sum_{x \in Supp(X)} (x-\mu)^2 \cdot p(x)$$

Q) $X \sim Bern(\frac{1}{3})$



$$E(X) = \frac{1}{3}$$

$$\sigma^2 = \sum_{x \in Supp(X)} (x-\mu)^2 \cdot p(x)$$
$$= (1-\frac{1}{3})^2 \cdot \frac{1}{3} + (0-\frac{1}{3})^2 \cdot \frac{2}{3}$$
$$= \frac{6}{27}$$



$X \sim Bern(p)$ $E(X) = p$

$$\sigma^2 = (1-p)^2 \cdot p + (0-p)^2 \cdot (1-p)$$
$$= (1-2p+p^2) \cdot p + p^2(1-p)$$
$$= p - 2p^2 + p^3 + p^2 - p^3$$
$$= p - p^2$$
$$= p(1-p)$$