MATH 241 8/28/17

Lecture #1

Set Theory (1870's)

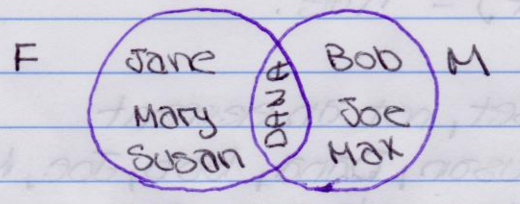
The fundamental units of math are the "set". A "set" is a collection of elements/objects which are unordered and unique.

ex: $F := \{ \text{Jane, Mary, Susan, Dana} \}$ *Note: := denotes "assigned to"

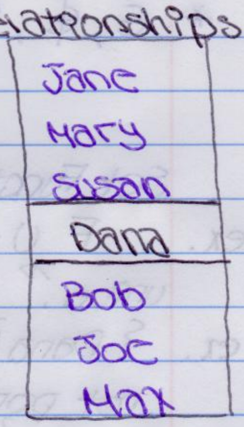
Denotes the set on the Right Hand Side. usually we pick a descriptive letter. The braces begin and end the enumeration of elements.

$M := \{ \text{Bob, Joe, Max, Dana} \}$

Venn Diagram illustrations of sets and their relationships.



or a less popular way →



- sets can have infinite elements

ex. $\mathbb{N} := \{1, 2, 3 ...\}$

denotes that the pattern continues.

ex: $\mathbb{Z} := \{..., -2, -1, 0, 1, 2 ...\}$ represents set of integers

* Operators on sets

- element operator ex. $\underset{\text{element}}{\text{Jane}} \in \underset{\text{set}}{F}$ * Read as "Jane is an element of F"

ex. $\text{Joe} \notin F$ "Joe is not an element of the set F"

- Subset → All elements of the set on the LHS are in the set on the RHS

ex. $\underset{\text{set}}{\{\text{Jane, Mary}\}} \underset{\text{subset}}{\subseteq} \underset{\text{set}}{F}$ "The set {Jane, Mary} is a subset of set F"

ex. $\{\text{Joe, Mary}\} \nsubseteq F$

"There exists atleast one element on the LHS that is not on the RHS" ex Let $F' := \{$Jane, Mary, Susan, Dana$\}$ Does $F \overset{?}{=} F'$?

Yes! set equality means $F \subseteq F'$ and $F' \subseteq F$, but, $\{$Jane, Mary$\} \neq F$

Note: order does NOT matter

- Proper subset denoted "$\subset$", the LHS is a subset of the set on the RHS but LHS $\neq$ RHS. ex. $\{$Jane, Mary$\} \subset F$

ex. $\{$Jane$\} \subset F$ → True!

ex. $\{$Jane$\} \overset{?}{\in} F$ → False, set F does NOT have SET Jane, only eleme

ex. Jane $\overset{?}{\subset} F$ → False, without a definition for set Jane, does not parse.

* $\in, \notin, \subseteq, \subset, =, \neq$ predicate functions which return T or F.
ex. $\in($Jane$, F) =$ True!

Set Functions returns only a set, not an element

ex. $F \cup M = \{$Jane, Mary, Susan, Dana, Bob, Joe, Max$\}$
union. * Note: union is NOT addition, it is "non exclusive or

ex. $\{$Dana$\} \cup \{$Dana$\} = \{$Dana$\}$, however, Dana $\cup \{$Dana$\}$ does not parse.

ex. Dana $\overset{?}{\in} M \cup F$ → True, Dana is an element of $M \cup F$

ex. $F \cap M = \{$Dana$\}$ intersection ("and") elements in both.

ex. $F \cap \{$Bob, Joe$\} = \{\}$ or $\emptyset$

ex. A, B both have infinite elements, can $A \cap B = \emptyset$?
Yes! odd numbers & even numbers

- $\emptyset \overset{?}{\subset} F$ → vacuously true
- $\emptyset \overset{?}{\in} F$ → False!

Set Subtraction $F|M$ means all elements in F except those that are in M.

- $F|M = \{$Jane, Mary, Susan$\}$
- $M|F = \{$Joe, Bob, Max$\}$

ex. if $A \cap B = \emptyset \Rightarrow A | B = A$

↗ means A and B have nothing in common

- $A \cap B = \emptyset \Rightarrow B | A = B$
- if $A | B = \emptyset$ then $A \cap B = A$
- $\emptyset | \emptyset = \emptyset$

ex. $A \subseteq B \Rightarrow A | B = \emptyset$

Set Builder Notation $\underset{\text{new set}}{E} := \{\underset{\text{with elements } 2\cdot n}{2n} \underset{\text{such that}}{:} \underset{n \text{ is an integer}}{n \in \mathbb{Z}}\} = \{\ldots -4, -2, 0, 2, 4\}$

- $2^A := \{B : B \subseteq A\} = \{\emptyset, \{1\}, \{2\}, \{3\} \{1,2\} \{2,3\} \ldots\}$

ex. Let $A = \{1, 2, 3\}$ → Power set of A

Size of set/cardinality $|A| = \#$ of elements in $A = 3$

ex. $|F \cup M| \overset{?}{=} |F| + |M|$ → False! union doesn't mean add

ex. $|F \cap M| \overset{?}{=} |F| - |M|$ → False!

$1 \neq 4 - 4$

ex. $|2^A| = 8$ because 2^3