$$X \sim \text{Bern}(p) = \underbrace{p^{x}(1-p)^{1-x}}^{\text{PMF}}$$

$\text{Supp}(X) = \{0,1\}$

$p \in (0,1)$

Q) 10 cards, 4 red

- $P\left(\begin{array}{l}\text{drawing 2 Red's}\\ \text{when sampling}\\ \text{3 cards w/o replacement}\end{array}\right) = \dfrac{\binom{4}{2}\binom{6}{1}}{\binom{10}{3}}$

- $P(x \text{ Red in } 3) = \dfrac{\binom{4}{x}\binom{6}{3-x}}{\binom{10}{3}}$ | $P(X \text{ red in } n \text{ cards}) = \dfrac{\binom{4}{x}\binom{6}{n-x}}{\binom{10}{n}}$

Q) 10 cards, K are red

$P(x \text{ R in } n \text{ cards}) = \dfrac{\binom{K}{x}\binom{10-K}{n-x}}{\binom{10}{n}}$

Q N cards, K are Red

$P(x \text{ R in sample of } n) = \dfrac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$

$X \sim \text{Hypergeometric}(n, K, N)$

$$:= p(x) = \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$$

Q) auditorium $\to$ 100 (N), 53 (K), 8 of them at random

$\Rightarrow X \sim \text{hyper}(8, 53, 100)$.

$p(6) = \dfrac{\binom{53}{6}\binom{47}{2}}{\binom{100}{8}}$

- Can $N = 0$? $\Rightarrow K = 0 \quad n = 0$ (NO!) Degenerate
- If $N = 1$? (bag with 1 marble) $K \in (0,1)$, $n \in (0,1)$ (Always generate)
- If $N = 2$? $\Rightarrow n \in (1,2) \to$ degenerate

$X \sim \text{hyper}(1,1,2) := \dfrac{\binom{1}{x}\binom{1}{1-x}}{\binom{2}{1}} = \text{Bern}\left(\tfrac{1}{2}\right)$

$\text{Supp}(X) = \{0,1\}$

$\boxed{\begin{array}{l}\text{Parameter Space}\\ N = \{2,3,\dots\},\ K = \{1,\dots,N-1\},\ n \in \{1,\dots,N\}\end{array}}$

Q) $X \sim \text{Hyper}(1, K, N)$

$\text{Supp}(X) = \{0,1\} = \dfrac{\binom{K}{x}\binom{N-K}{1-x}}{\binom{N}{1}} = \text{Bern}\left(\tfrac{K}{N}\right)$

$p(1) = \dfrac{\binom{K}{1}\binom{N-K}{0}}{N} = \dfrac{K}{N}$

$p(0) = \dfrac{\binom{K}{0}\binom{N-K}{1}}{N} = \dfrac{N-K}{N} = 1 - \dfrac{K}{N}$

Q a) $X \sim \text{Hyper}(2,4,10)$, $\text{Supp}(X) = \{0,1,2\}$

b) $X \sim \text{Hyper}(5,4,10)$, $\text{Supp}(X) = \{0,1,2,3,4\}$

c) $X \sim \text{Hyper}(8,4,10)$, $\text{Supp}(X) = \{2,3,4\}$

d) $X \sim \text{Hyper}(5,7,10)$, $\text{Supp}(X) = \{2,3,4,5\}$

1. $n < K,\ n < N-K$, $\text{Supp}(X) = \{0,\dots,n\}$
2. $n \geq K,\ n < N-K$, $\text{Supp}(X) = \{0,\dots,K\}$
3. $n \geq K,\ n \geq N-K$, $\text{Supp}(X) = \{n-(N-K),\dots,K\}$
4. $n < K,\ n \geq N-K$, $\text{Supp}(X) = \{n-(N-K),\dots,n\}$

	$n<K$	$n \geq K$
$n < N-K$	$\{0,\dots,n\}$	$\{0,\dots,K\}$
$n \geq N-K$	$\{n-(N-K),\dots\}$	$\{n-(N-K),\dots\}$

$= \underline{\text{Supp}(X)}$

$\to \{\max\{0, n-(N-K)\},\dots,\min\{K,n\}\}$

Q Let $p := \frac{K}{N} \Rightarrow K = pN$. Let $X \sim \underbrace{\text{Hyper}(n, p, N)}_{\text{reparameterization}}$

$\text{PMF} = \dfrac{\binom{pN}{x}\binom{(1-p)N}{n-x}}{\binom{N}{n}}$

Q) Let $p = 0.5, n = 6$

1. $N = 100$, $P(3) = \dfrac{\binom{50}{3}\binom{50}{3}}{\binom{100}{6}} = 0.3223$
2. $N = 1000$, $P(3) = \dfrac{\binom{500}{3}\binom{500}{3}}{\binom{1000}{6}} = .3134$
3. $N = 10000$, $P(3) = \dfrac{\binom{5000}{3}\binom{5000}{3}}{\binom{10000}{6}} = .3136$

$\lim_{N\to\infty} \text{Hyper}(n, p, N) = \lim_{n\to\infty} \dfrac{\binom{pN}{x}\binom{(1-p)N}{n-x}}{\binom{N}{n}}$

$$\lim_{N\to\infty} \text{hyp}(n,p,N) = \lim_{n\to\infty} \frac{\binom{pN}{x}\binom{(1-p)N}{n-x}}{\binom{N}{n}}$$

$$= \frac{\dfrac{(pN)!}{x!\,(pN-x)!} \cdot \dfrac{((1-p)N)!}{(n-x)!\,((1-p)N-(n-x))!}}{\dfrac{N!}{n!\,(N-n)!}}$$

$$= \lim_{N\to\infty} \frac{\dfrac{(pN)!}{(pN-x)!} \cdot \dfrac{(1-p)N!}{((1-p)N-(n-x))!}}{\dfrac{N!}{(N-n)!}}$$