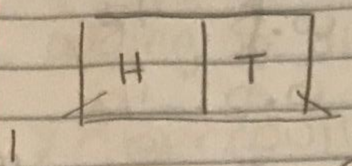
10/11

$\Omega = \{H, T\}$ $n = 3$

$\omega_1 = H$

$\omega_2 = T$

$\omega_3 = H$



$\mathbb{1}_{\omega = H} = \begin{cases} 1 & \text{if } \omega = H \\ 0 & \text{if } \omega \neq H \end{cases}$

$\mathbb{1}_{\omega_1 = H}, \mathbb{1}_{\omega_2 = T}, \mathbb{1}_{\omega_3 = H}$

$1, \quad 0, \quad 1$ $\bar{x} = \frac{1+0+1}{3} = 2/3$

Generally, there is a function
$X : \Omega \to \mathbb{R}$
called a "random variable" (rv)
$X(H) = 1$
$X(T) = 0$

shorthand for

$P(X=1) = P(\{\omega : X(\omega) = 1\}) = P(\{H\}) = 1/2$

$P : 2^{\Omega} \to (0, 1)$

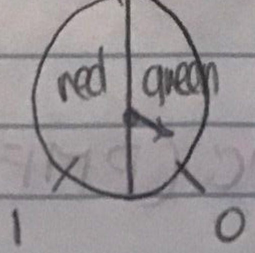
$\text{Supp}[X] = \{0, 1\}$ "set of all things that happe

"support" the range of the r.v
$\text{Supp}[X] = \{x : P(X = x) > 0\} \subseteq \mathbb{R}$
"any value that's possible"

Def: A discrete r.v is one s.t $|\text{Supp}[X]| \leq |\mathbb{N}$
i.e finite or countably infinite.

since it's finite it's discrete.

$\Omega = \{R, G\}$



$P(X=1) = 1/2$

$P(X=0) = 1/2$

$\text{Supp}[X] = \{0, 1\}$

r.v "distributed as" "with prob."

$$X \sim \begin{cases} 1 & \text{w.p } 1/2 \\ 0 & \text{w.p } 1/2 \end{cases}$$

$$X \sim \text{Bernouilli}(1/2) := \begin{cases} 1 & \text{w.p } 1/2 \\ 0 & \text{w.p } 1/2 \end{cases}$$

"standard Bernouilli"

$\text{Supp}[X] = \{0, 1\}$

X is discrete.

$$X \sim \text{Bernouilli}(P) := \begin{cases} 1 & \text{w.p } P \\ 0 & \text{w.p } 1-P \end{cases}$$

p is called a parameter, a number you choose to "tune" the r.v model

$f(x) = \sin(ax)$ where $a \in \mathbb{R} \setminus \{0\}$

a is a parameter to the sin fc.

ex: if $a = 0$

$f(x) = \sin(0) = 0$, this is a "degenerate case"

Parameter space: the set where p "lives"

$p \in (0, 1)$ "non-inclusive"

because if you include 0 or 1 you get degenerate cases. ($X \sim \text{Deg}(0)$, $X \sim \text{Deg}(1)$)

$X \sim \text{Deg}(c) := \{c \text{ w.p. } 1\}$

$\text{Supp}[X] = \{c\}$

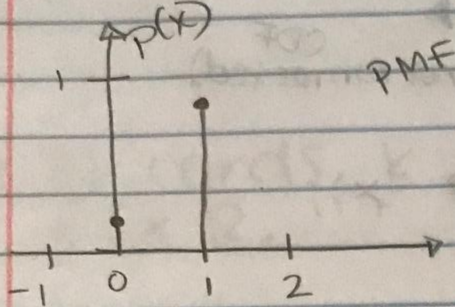
$p(x) := P(X = x)$ "prob. mass fc (PMF)

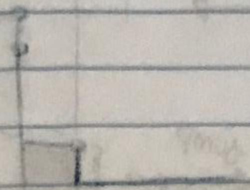
$p: \mathbb{R} \longrightarrow [0, 1]$

$\sum_{x \in Supp[X]} p(x) = 1$

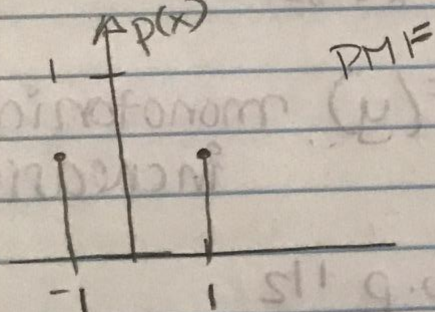
adding up all the probabilities.

$X \sim$ Bernouilli $(3/4) := \begin{cases} 1 & \text{w.p } 3/4 \\ 0 & \text{w.p } 1/4 \end{cases}$



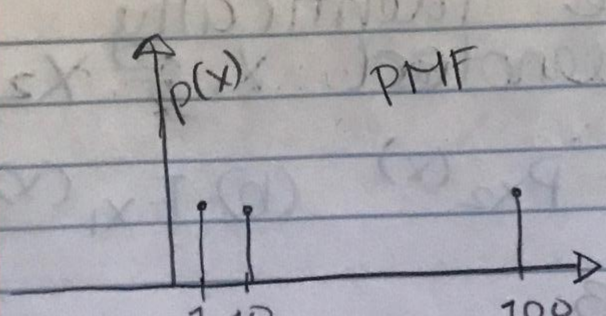


$X \sim$ Rademacher $= \begin{cases} 1 & \text{w.p. } 1/2 \\ -1 & \text{w.p } 1/2 \end{cases}$



$X \sim$ Unif $(\{1, 10, 100\}) : \begin{cases} 1 & \text{w.p } 1/3 \\ 10 & \text{w.p } 1/3 \\ 100 & \text{w.p } 1/3 \end{cases}$

"discrete uniform"



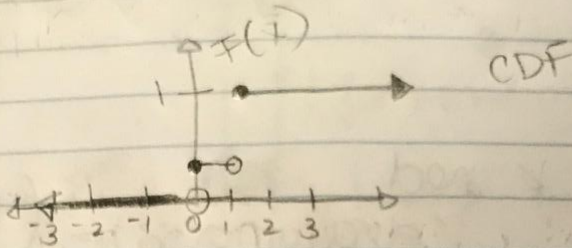
$X \sim$ Unif (A) $\quad$ Supp $[X] = A$

$A \in 2^{\mathbb{R}}$

but $|A|$ is finite.

$F(x) := P(X \leq x)$
cumulative distribution function (CDF)

$X \sim Bern(3/4) := \begin{cases} 1 & \text{w.p } 3/4 \\ 0 & \text{w.p. } 1/4 \end{cases}$



properties of CDF

① $\lim_{x \to \infty} F(x) = 1$ ② $\lim_{x \to \infty} F(x) = 0$

③ $x \leq y \rightarrow F(x) \leq F(y)$ monotonically inc.

$X \sim Bern(p) = \begin{cases} 1 & \text{w.p } 1/2 \\ 0 & \text{w.p } 1/2 \end{cases}$

$p(x) = p^x (1-p)^{1-x}$

def: X_1, X_2 are "identically distributive" denoted $X_1 \stackrel{d}{=} X_2$ if (a) $P_{X_1}(x) = P_{X_2}(x)$ (b) $F_{X_1}(x) = F_{X_2}(x)$

ex. 10 cards 4R, 6B

P(2 R when drawing 3) = $\dfrac{\binom{4}{2}\binom{6}{1}}{\binom{10}{3}}$

P(xR when drawing 3) = $\dfrac{\binom{4}{x}\binom{6}{3-x}}{\binom{10}{3}}$

$$P(x\,R \text{ when drawing } n) = \frac{\binom{4}{x}\binom{6}{n-x}}{\binom{10}{n}}$$

→ 10 cards, k red

$$P(x\,R, \text{ when drawing } n) = \frac{\binom{k}{x}\binom{10-k}{n-x}}{\binom{10}{n}}$$

→ N cards, k red

$$P(x\,R, \text{ `` '' drawing } n) = \frac{\binom{k}{x}\binom{N-k}{n-x}}{\binom{N}{n}}$$

$X \sim \text{Hypergeometric}(n, k, N)$