$$\boxed{11/8}$$

Review

Limiting PMF

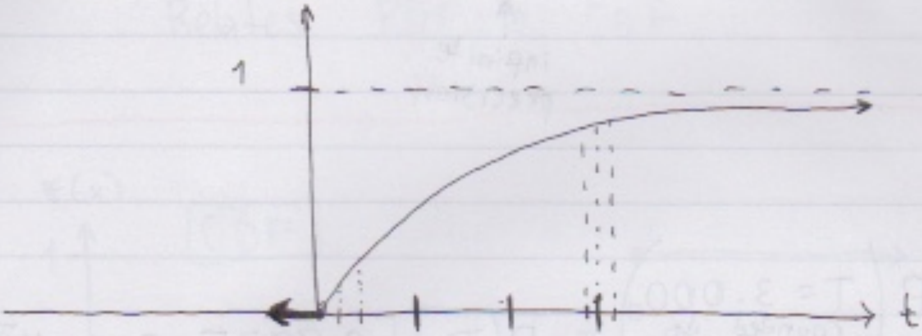
$p(t) = 0 \ \forall t$

not a PMF

limiting CDF $\boxed{F(t) = 1 - e^{-\lambda t}}$

$\text{Supp}[T] = (0, \infty)$

$$\boxed{X \sim Geom(p) = \left(1-\frac{\lambda}{n}\right)^{nt} \frac{\lambda}{n} \qquad \lambda = np}$$

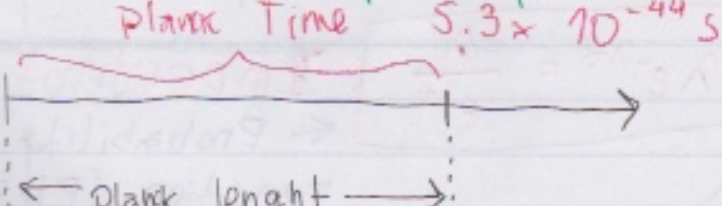


$Supp[T] = (0, \infty)$

$\Rightarrow |Supp[T]| = |\mathbb{R}| > |\mathbb{N}|$

$\Rightarrow$ T is not a Discrete r.v.

Out of topic...



1.62×10^{-35} m

Prob stopping at $t = 3s$

$P(3) = P(T=3) = 0$

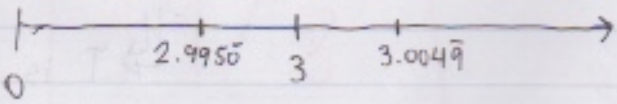
$P(T = 3.00000\ldots) = 0$

infinite precision

$$P\begin{pmatrix} T = 3.000 \\ \text{rounded to the nearest ms} \end{pmatrix} = P(T \in [2.995\bar{0}, 3.004\bar{9}])$$

$$\approx P(T < 3.004\bar{9}) \cdot P(T \leq 2..995\bar{0})$$

$$= F(3.0047) \cdot F(2.995\bar{0}) > 0$$

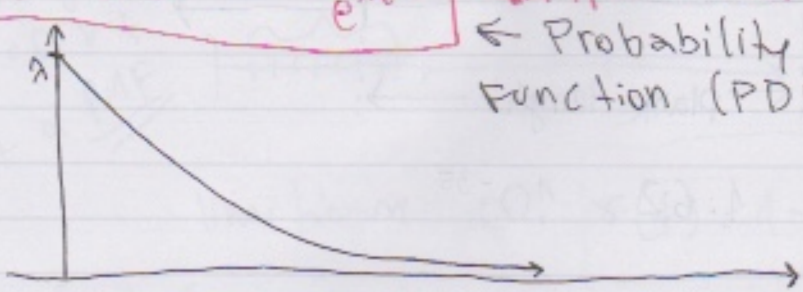


Back to topic:

$$\boxed{f(t) = F'(t) = \lambda e^{-\lambda t} = \frac{\lambda}{e^{\lambda t}}}$$

Important

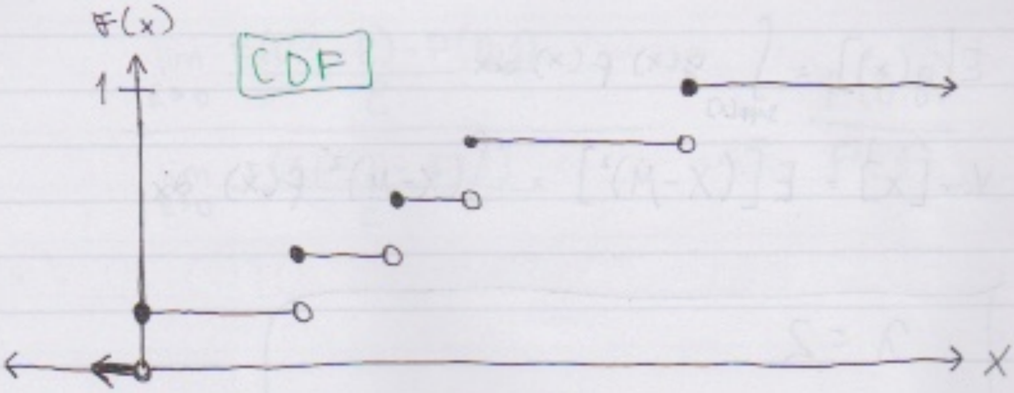
← Probability Density Function (PDF)



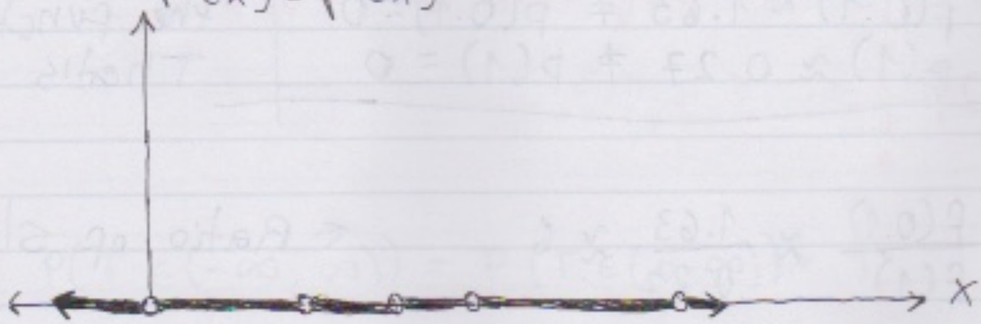
$$P\left(T \in [a,b]\right) = F(b) - F(a) = \int_a^b f(t)\,dt.$$

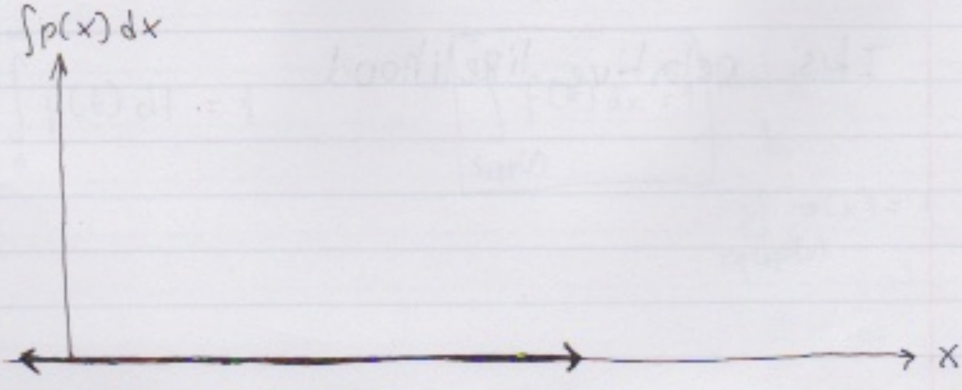
Fundamental Theory of Calculus

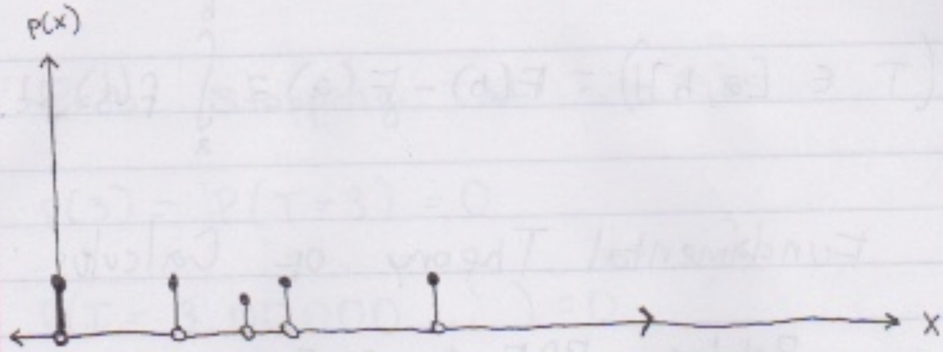
Relates PDF & CDF



$F'(x) = f(x)$ Derivative of $\boxed{\text{CDF}}$







Assume

$$E[g(x)] = \int_{supp[x]} g(x)\, f(x)\, dx$$

$$Var[x] = E[(X-\mu)^2] = \int (x-\mu)^2 f(x)\, dx$$

$\lambda = 2$

$f(t) = 2e^{-2t}$

$f(0.1) \approx 1.63 \neq p(0.1) = 0$

$f(1) \approx 0.27 \neq p(1) = 0$

← The slope of the function. That's it!

$$\frac{f(0.1)}{f(1)} \approx \frac{1.63}{0.27} \approx 6$$

← Ratio of slope

It's relative likelihood

Proving time...

$$\lim_{\varepsilon \to 0} \frac{P(T \in [0.1, 0.1+\varepsilon])}{P(T \in [1, 1+\varepsilon])}$$

$$\lim_{\varepsilon \to 0} \frac{F(0.1+\varepsilon) - F(0.1)}{F(1+\varepsilon) - F(1)}$$

$$\frac{\lim_{\varepsilon \to 0} \frac{F(0.1+\varepsilon) - F(0.1)}{\varepsilon}}{\lim_{\varepsilon \to 0} \frac{F(1+\varepsilon) - F(1)}{\varepsilon}} = \frac{f(0.1)}{f(1)}$$

Back to this guy... $P(T \in [a,b]) = F(b) - F(a)$

$$= \int_a^b f(t)\,dt$$

$$P(T \in (-\infty, \infty)) = P(T \in (0, \infty)) = 1$$

$$\Rightarrow \int_0^\infty f(t)\,dt = 1$$

$$\boxed{\int_{Supp(X)} f(x)\,dx = 1}$$ Analogous to $\sum_{x \in Supp(X)} p(x) = 1$

(enough Calculus. ...)

Continuous R.V. X

Definition:

① $|Supp[X]| = |\mathbb{R}|$

② $F(x)$ no a valid CDF without no jumps

③ PMF doesn't exist

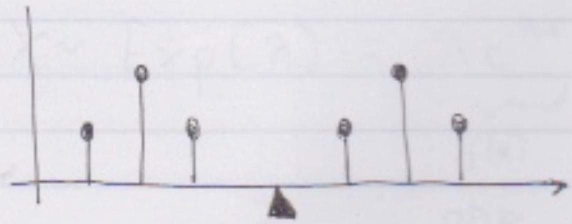
④ PDF exists (i) $\int_{Supp[X]} f(x)dx = 1$ (ii) $f(x) \geq 0 \ \forall x$

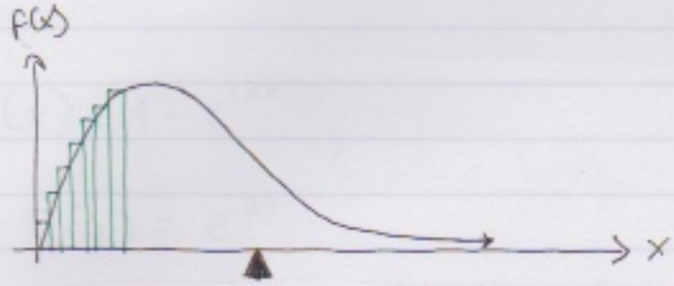
$X_1 \stackrel{d}{=} X_2$

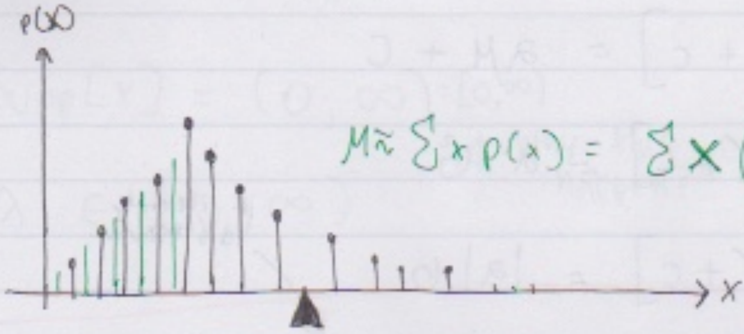
$\Rightarrow f_{X_1}(x) = f_{X_2}(x)$

or

$F_{X_1}(x) = F_{X_2}(x)$

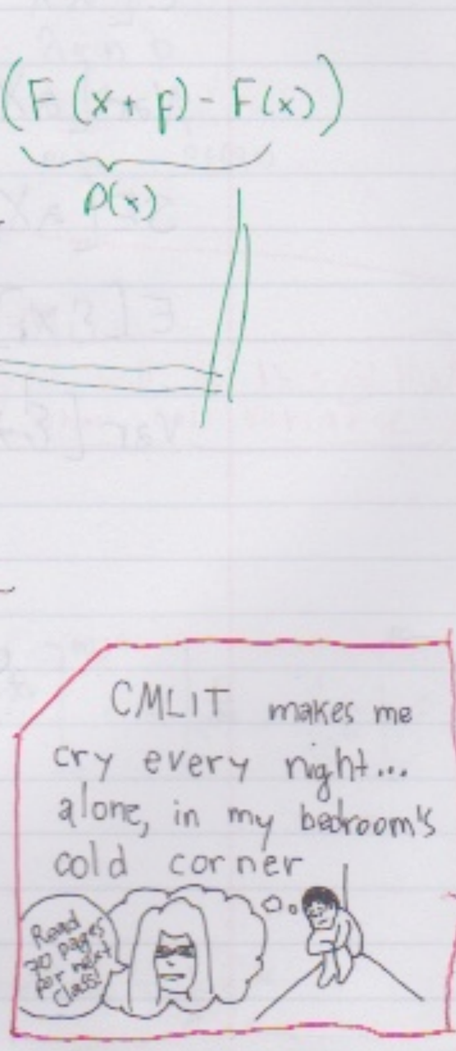






$$\mu \approx \sum x\, p(x) = \sum x \underbrace{\left(F(x+\rho) - F(x)\right)}_{p(x)}$$

$$\boxed{\mu = E[x] = \int_{Supp(x)} x\, f(x)\, dx}$$



Assume $E[g$

$$E[aX + c] = a\mu + c$$

$$Var[aX + c] = a^2 \sigma^2$$

$$SE[aX + c] = |a| \sigma$$

if identity distributive

$$E[\Sigma X_i] = \Sigma \; E[X_i] = n m$$

$$var[\Sigma X_i] = \Sigma \, Var[X_i] = n \sigma^2$$

independence of $x_1, x_2, \cdots, x_n$

iid

exponential

$$X \sim \text{Exp}(\lambda) := \underbrace{\lambda e^{-\lambda x}}_{f(x)}$$

PDF

$$F(x) = 1 - e^{-\lambda x}$$

$$1 - F(x) = e^{-\lambda x}$$

$$\text{Supp}[x] = (0, \infty) = [0, \infty)$$ ← why this happen?

$$\lambda \in (0, \infty)$$

$\lambda = n\,p$, $n \in \mathbb{N}$, $p \in (0,1)$

Let's talk about expectation

$$E[x] = \int_0^\infty x \; \lambda e^{-\lambda x}\, dx$$

you will do this @ HW when calc. variance

$\int u\,dv = uv - \int v\,du$

$$= \lambda \int_0^\infty \underset{u}{x}\, \underbrace{e^{-\lambda x}\, dx}_{dv} = \cancel{\lambda}\left[(x)\left(-\frac{1}{\cancel{\lambda}} e^{-\lambda x}\right) - \frac{1}{\lambda^2} e^{-\lambda x}\right]_0^\infty = -\left[\frac{x}{e^{\lambda x}} + \frac{1}{\lambda e^{\lambda x}}\right]_0^\infty =$$

$du = dx$

$$v = \int e^{-\lambda x} dx = -\frac{1}{\lambda} e^{-\lambda x} \qquad \int v\, du = \int -\frac{1}{\lambda} e^{-\lambda x} dx = \frac{1}{\lambda^2} e^{-\lambda x}$$

$$= -\left((0+0) - \left(0 + \frac{1}{\lambda}\right)\right) = \boxed{\frac{1}{\lambda}}$$

$$P(X > a) = P(X > a+b \mid X > b) =$$

$$= \frac{P(X > a+b \cap X > b)}{P(X > b)} = \frac{P(X > a+b)}{P(X > b)}$$

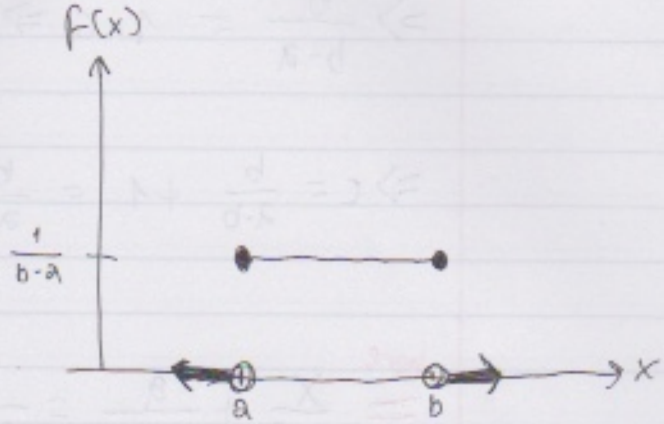
$$= \frac{\overset{e^{-\lambda a} \quad e^{-\lambda b}}{e^{-\lambda(a+b)}}}{\cancel{e^{-\lambda b}}} = P(X > a)$$

Exponential
is Memoryless

$$\boxed{X \sim \underset{\text{uniform}}{U}(a,b) = \frac{1}{b-a}}$$

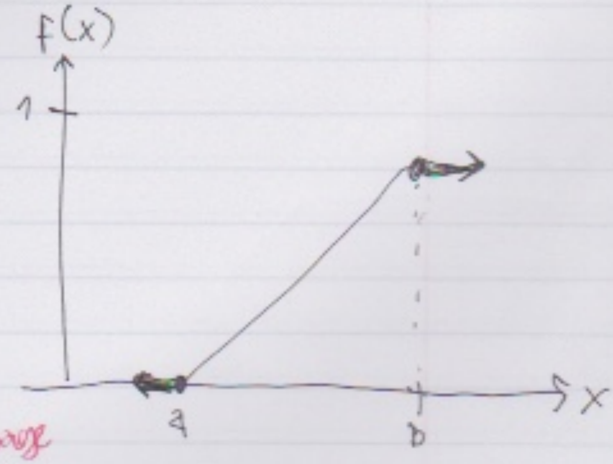
$Supp[X] = (a,b)$

$a \in \mathbb{R}$ but $a < b$
$b \in \mathbb{R}$



$$\int_{Supp[X]} f(x)\,dx = \int_a^b \frac{1}{b-a}\,dx = \frac{1}{b-a}\int_a^b dx$$

$=$



$$F(x) = \int f(x)\,dx + C$$

$$= \int \frac{1}{b-a}\,dx + C = \frac{X}{b-a} + C =$$

look next page

9:

$$F(b) = 1 \Rightarrow \frac{b}{b-a} + c = 1$$

$$\Rightarrow \frac{b}{b-a} = 1 - c \Rightarrow \frac{b}{a-b} = c - 1$$

$$\Rightarrow c = \frac{b}{a-b} + 1 = \frac{b}{a-b} + \frac{a-b}{a-b} = \frac{a}{a-b} = c$$

here

$$\Rightarrow \frac{x}{b-a} + \frac{a}{a-b} = \frac{x}{b-a} + \frac{-a}{b-a} = \boxed{\frac{x-a}{b-a}}$$