21. You take 100 random steps

$$X_1, \dots, X_{100} \overset{iid}{\sim} \begin{cases} 1 & w.p\ \frac{1}{2} \\ -1 & w.p\ \frac{1}{2} \end{cases}$$

what is P of you are more than 10 steps away from where you started.

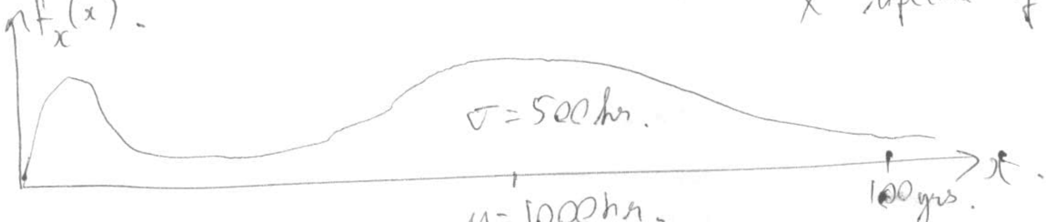
$$T = X_1 + \dots + X_{100}.$$

$$P(|T| > 10) = P(T > 10 \text{ or } T < -10).$$
$$= P(T > 10) + P(T < -10).$$

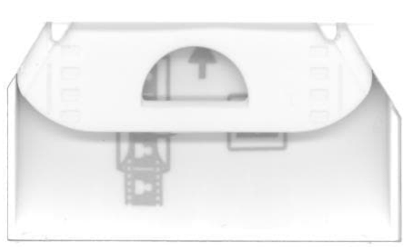
By CLT, $T \approx N(n\mu, (\sigma\sqrt{n})^2)$
$$= N(100 \cdot 0,\ (1\sqrt{100})^2).$$
$$= N(0, 100).$$

$$P(|T| > 10) = P(T > 10) + P(T < -10)$$
$$= P\left(\frac{T-0}{10} > \frac{10-10}{10}\right) + P\left(\frac{T-0}{10} < \frac{-10-0}{10}\right)$$
$$= P(Z > 1) + P(Z < -1) = 0.16 + 0.16 = 0.32.$$

X lifetime of 1 lightbulb.



You get 50 light bulbs. What is the P average. Lifetime is more than 1300 hr?



$P(\overline{X} > 1300)$

By CLT, $\overline{X} \approx N\left(\mu, \left(\frac{\sigma}{\sqrt{n}}\right)^2\right)$

$= N\left(1000, \left(\frac{500}{\sqrt{50}}\right)^2\right) = N(1000, 70.7)$

$P(\overline{X} > 1300) = P\left(\frac{\overline{X} - 1000}{70.7} > \frac{1300 - 1000}{70.7}\right)$

$\approx P(Z > 4.24) \approx 0$. if beyond $3 \Rightarrow$ is 0.

Shipments are late 2% of the time, what is P in 10,000 orders more than 3% are late?

$X_1, \ldots, X_{10,000} \overset{iid}{\sim} \text{Bern}(0.02)$.

Take average $P(\overline{X} > 0.03)$

By CLT, $\overline{X} \approx N\left(\mu, \left(\frac{\sigma}{\sqrt{n}}\right)^2\right) = N\left(0.02, \left(\frac{0.14}{\sqrt{10,000}}\right)^2\right)$

$= N(0.02, 0.0014^2)$

$P(\overline{X} > 0.03) = P\left(\frac{\overline{X} - 0.02}{0.0014} > \frac{0.03 - 0.02}{0.0014}\right)$

$\approx P\left(Z > \frac{100}{14}\right) \approx P(Z > 7.14) \approx 0$.

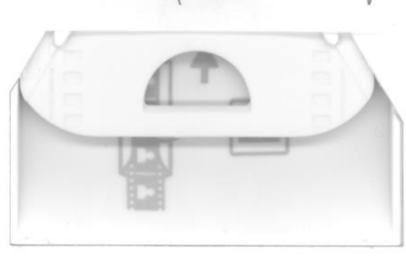
$\overline{X}$ is the R.V of avg.

$\overline{x}$ is a realization.

$\overline{x} = \frac{1+1+0+0+0}{5} = 0.4 = \hat{p}$ $\quad \overline{x} \in [0,1]$

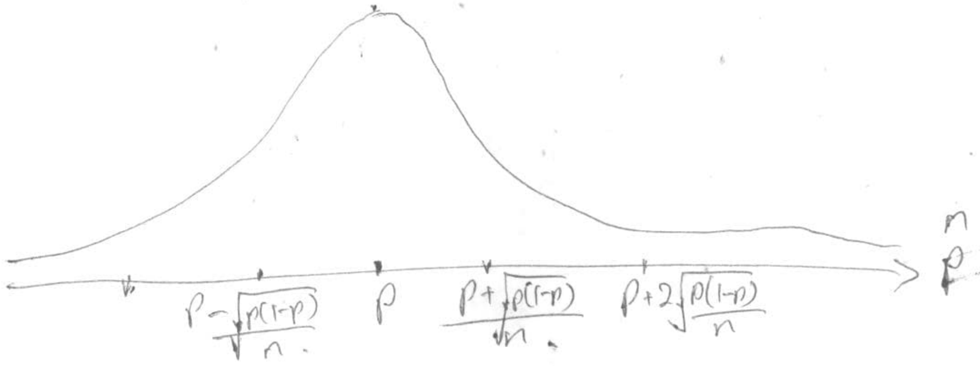
$\hat{p}$ is a sample "proportion".

$\hat{P}$ that is the R.V of the proportion.



$$\bar{X} \overset{d}{\approx} N\left(\mu, \left(\frac{\sigma}{\sqrt{n}}\right)^2\right)$$

$$\hat{p} \approx N\left(p, \left(\sqrt{\frac{p(1-p)}{n}}\right)^2\right)$$



Statistical Infrance C Statistics.

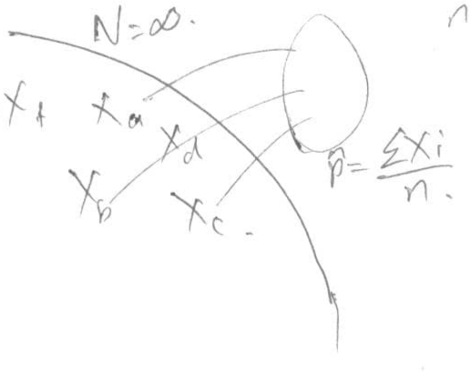
3 Goals:

I Estimate P (best guess) $\hat{p}$.

II Create a range / Window of likely value for P.

III Test theories about P

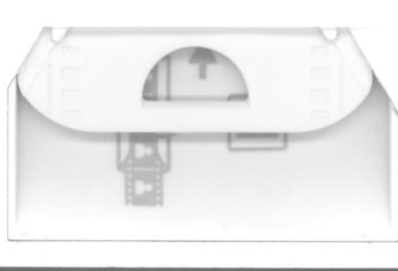
population. Sample C population.



$n << N$.
but large enough.

"Simple random sample".

Otherwise bias.



What is the ..

$$p\left(p \in \left[\hat{p} \pm \sqrt{\frac{p(1-p)}{n}}\right]\right) = p\left(\hat{p} - \sqrt{\frac{p(1-p)}{n}} \leq p \leq \hat{p} + \sqrt{\frac{p(1-p)}{n}}\right)$$

$$= p\left(-\sqrt{\frac{p(1-p)}{n}} \leq p - \hat{p} \leq \sqrt{\frac{p(1-p)}{n}}\right).$$

$$= p\left(-1 \leq \frac{p - \hat{p}}{\sqrt{\frac{p(1-p)}{n}}} \leq 1\right)$$

$$= p(-1 \leq -Z \leq 1)$$

$$= p(-1 \geq Z \geq 1) = p(Z \in (-1, 1)) = 0.68.$$

if $Z \in (-n, n)$ then $\hat{p} \pm \left(n\sqrt{\frac{p(1-p)}{n}}\right)$

Define $Z_{\frac{\alpha}{2}} := F_Z^{-1}\left(1 - \frac{\alpha}{2}\right)$

$\alpha = 0.05 \Rightarrow \frac{\alpha}{2} = 0.025.$

$1 - \alpha = 0.975$

$Z_{2.5\%} = 2.$

$$p\left(p \in \left[\hat{p} \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{p(1-p)}{n}}\right]\right)$$

$$= p\left(Z \in \left(-Z_{\frac{\alpha}{2}}, Z_{\frac{\alpha}{2}}\right)\right)$$

$$= F\left(Z_{\frac{\alpha}{2}}\right) - F\left(-Z_{\frac{\alpha}{2}}\right) = \left(1 - \frac{\alpha}{2}\right) - \left(\frac{\alpha}{2}\right) = 1 - \alpha.$$