Math 241 Lec 12 Oct 23^{rd}

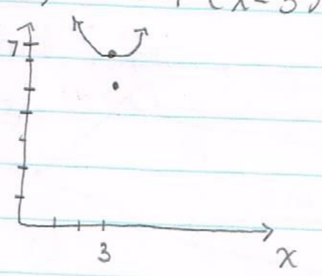
$$\begin{array}{r} 69 \\ +25 \\ \hline 94 \end{array}$$

$x_1, x_2, \dots x_n \overset{iid}{\sim} \text{Bern}(p)$

$T = x_1 + \dots + x_n \sim \text{Bern}(n, p)$

$x_1, x_2, \dots \overset{iid}{\sim} \text{Bern}(p)$

$f(x) = 7 + (x-3)^2$



$\min\{f(x)\} = 7$

$\max\{f(x)\}$ d. n. e.

$\text{argmin}\{f(x)\} = 3$

x s.t. $f(x) = \min\{f(x)\}$

$\text{arg max}\{f(x)\}$

discrete time

$T = \min\{t : x_t = 1\}$

$P(T=1) = p$

$P(T=2) = (1-p)^1 p$

$P(T=3) = (1-p)^2 p$

$\vdots$

$P(T=x) = (1-p)^{x-1} p$

$x \sim \text{geometric}(p) := (1-p)^{x-1} p$

$\text{supp}\{x\} = \mathbb{N}$

$p \in (0,1)$

Let $i = x-1 \Rightarrow x = i+1$

$\sum_{x \in \text{supp}\{x\}} P(x) = 1 \qquad \sum_{x=1}^{\infty} (1-p)^{x-1} p = 1 \Rightarrow \underbrace{\sum_{x=1}^{\infty} (1-p)^{x-1}}_{\frac{1}{1-(1-p)} = \frac{1}{p}} = \frac{1}{p}$

$\Rightarrow \sum_{i=0}^{\infty} (1-p)^i = \frac{1}{p}$

let $q = 1-p \quad q \in (0,1)$ geometric series

$S = \sum_{t=0}^{\infty} q^t = 1 + q + q^2 + q^3 + \dots$

$= 1 + q(1 + q + q^2 + \dots)$

$S = 1 + qS$

$S + S = 1$

$S(1-q) = 1 \Rightarrow S = \frac{1}{1-q}$

ative binomial
theorem

$$F(x) = P(X \le x) = \sum_{i=1}^{x} p(i) = \sum_{i=1}^{x} (1-p)^{i-1} p$$

$$F(x) = 1 - P(X > x) = 1 - (1-p)^x$$

$$\underset{1}{0} \quad \underset{2}{0} \quad \cdots \quad \underset{x-1}{0} \quad \underset{x}{0} \quad \Big| \quad \underset{x+1}{\_} \quad \underset{x+2}{\_} \quad \underset{x+3}{\_} \quad \cdots$$

$$F(2) = p(1) + p(2)$$
$$= p + (1-p)p$$
$$1-(1-p)^2 = p(1+1-p)$$
$$1-(1-p)^2 \stackrel{?}{=} p(2-p)$$
$$1-(1-2p+p^2) =$$

① Play poker until a royal flush

$$P(\text{royal flush}) = \frac{4}{\binom{52}{5}} = .00000153$$

a) create a r.v model for the first time you get the RF

b) what is the prob. you get is on the 1,000,000th hand?

c) what is the prob. you get it before the 1,000,000st hand?

$X \sim \text{geometric}(.00000153)$

$$p(x = 1{,}000{,}000) = (1-.00000153)^{999{,}999} \cdot .00000153 =$$

$$P(X < 1{,}000{,}001) = P(X \le 1{,}000{,}000) = 1-(1-.00000153)^{1{,}000{,}000}$$

$$\approx 77.7\%$$

$X \sim \text{Bern}(\tfrac{1}{2})$

$$p(x) := \underbrace{P(X = x)}_{\text{PMF}} \qquad P(X=1) = \tfrac{1}{2}$$
$$P(X=0) = \tfrac{1}{2}$$

X, x ← represents a process which spits out random variables (abstract process)

$\uparrow$

$x \in \text{Supp}[X]$

example : Flipping a coin

while it flips it is abstract and represents all possibilities

once it lands, its a little x

r.v realization

X x

$\uparrow$

$x \in \text{supp}[X]$

datum : a realized r.v.

data : realized r.v.

$X \sim$ Hypergeometric $(\overset{n}{4}, \overset{K}{3}, \overset{N}{8})$ $\quad n > K \quad supp[X] = \{0, \dots K\}$

$supp[X] = \{0, 1, 2, 3\}$

eventually, we'll see every single value in the support

$x_1 = 1$
$x_2 = 1$
$x_3 = 2$
$x_4 = 2$
$x_5 = 2$
$x_6 = 1$
$x_7 = 1$
$x_8 = 1$

$X_1, \dots, X_8 \overset{iid}{\sim}$ Hypergeometric $(4, 3, 8)$

$\bar{x} = \frac{11}{8} = 1.375$

$X \sim Bin(\overset{n}{8}, \overset{p}{\tfrac{1}{2}})$ 8 coins, $\frac{1}{2}$ is p(of Heads)

$supp[X] = \{0, 1, 2, \dots, 8\}$

$x_1 = 6$
4
3
2
5
7
$\cancel{5}$
6
$x_9 = 3$

$\bar{x} = \frac{41}{9} = 4.5\bar{5}$

$X \sim$ geometric $(\frac{3}{8})$

$P(X = 10) = (\frac{5}{8})^9 \frac{3}{8} = 0.005$

$\bar{X} = \frac{17}{6} = 2.83$

sum r.v. $\quad T_n = X_1 + \dots + X_n = \sum_{i=1}^{n} x_i$

average r.v. $\quad \bar{X}_n = \frac{X_1 + \dots + X_n}{n} = \frac{1}{n} T_n = \frac{1}{n} \sum_{i=1}^{n} x$