8/30

Powerset

$$2^A := \{B : B \subseteq A\}$$

$$A = \{1,2,3\}$$

$$2^A = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{1,3\}, A\}$$

(cardinality) $|A| = 3$

$$|F/M| \stackrel{?}{=} |F| - |M|$$

$$3 \neq 4 - 4$$

$$|2^A| = 8 = 2^3 = 2^{|A|}$$

$|2^A| = 2^{|A|}$ because every element in A has 2 options. [T||F]

- Special set Ω called the "universe", "space of discourse," "scope" what you're limited to.

$$\Omega := F \cup M$$

note: $F \subseteq \Omega$ * all sets are subsets of Ω *

$M \subseteq \Omega$

$\forall A$

$A \cap \Omega = A$

$A \cup \Omega = \Omega$

$\emptyset \cup \Omega = \Omega$

$\emptyset \cap \Omega = \emptyset$

A^c "a-complement" - everything that is not A.

$$A^c := \Omega / A$$

A, A^c "collectively exhaustive!"

$(A^c)^c = A$

$A \cup A^c = \Omega$

$A \cap A^c = \emptyset$

A, A^c are "mutually exclusive" "disjoint!"

$\{A_1, A_2, \dots\}$ are mutually exclusive
if $A_i \cap A_j = \emptyset \quad \forall i \neq j$

$\{A_1, A_2, \dots\}$ are collectively exhaustive
if

$$A_1 \cup A_2 \cup \dots = \bigcup_{i=1}^{\infty} A_i = \Omega$$

$$|A| + |A^c| = |\Omega| \qquad T$$

$\mathbb{N} = \{1, 2, \dots\}$ $\quad |\mathbb{N}| = (\text{"}\infty\text{"})\ \aleph_0$
countable infinity.

$\mathbb{Z} = \{\dots, -1, 0, 1, \dots\}$ $\quad |\mathbb{Z}| = \aleph_0$

$$\boxed{\begin{array}{l} \text{def } |A| = |B| \\ \exists f : A \to B \quad 1:1 \end{array}}$$

"rational numbers"
$\mathbb{Q} := \{\frac{p}{q} : p \in \mathbb{Z}, q \in \mathbb{N}\}$ $\quad |\mathbb{Q}| = \aleph_0$

$\sqrt{2} \overset{?}{\notin} \mathbb{Q}$

"real numbers"
$\mathbb{R} := \mathbb{Q} \cup \{\text{"holes"}\}$ $\quad |\mathbb{R}| \overset{?}{=} \aleph_0$

$[a,b] := \{x : x \geq a \ \& \ x \leq b\}$
$(a,b) := \dots$

$A = (0,1)$
$|A| =$
if $|(0,1)| \neq \aleph_0$ $\quad |\mathbb{R}| = c$ "uncountable infinity"

ordered pair

$\langle a, b \rangle := \{\{a\}, \{a, b\}\}$

$A \times B := \{\langle a, b \rangle : a \in A, b \in B\}$

cartesian product

$A = \{1, 2\}, \quad B = \{3, 4\}$

$A \times B = \{\langle 1,3 \rangle, \langle 1,4 \rangle, \langle 2,3 \rangle, \langle 2,4 \rangle\}$

$|A \times B| = 4 = |A| \cdot |B|$

{same as Σ but for multiplication}

$|A_1 \times A_2 \times \ldots A_N| = \prod_{i=1}^{n} |A_i|$

$A^2 := A \times A \qquad |A^2| = |A \times A| = |A||A| = |A|^2$

$A^3 := A \times A \times A \qquad |A^n| = |A|^n$

Probability

Ω is called the "sample space" || "outcome space"

It's elements are called "outcomes"
denote the ω's.

coin toss : $\quad \Omega = \{H, T\}$ *space of all possible outcomes*

$\omega_1 = H, \; \omega_2 = T$

$P(\{H\}) = \frac{|\{H\}|}{|\Omega|}$

$P(H)$

$P: \Omega \rightarrow [0, 1]$

$2^{\Omega} = \{\emptyset, \{H\}, \{T\}, \{H, T\}\}$

set of all "events"

An event $A \subseteq \Omega$.

- all possible events are $\in Z^{\Omega}$.

Probability is a set function

$$P: Z^{\Omega} \to [0,1]$$

die roll experiment:

$$\Omega = \{1,2,3,4,5,6\}$$

$$A = \text{even \#s} = \{2,4,6\} \subseteq \Omega, \{2,4,6\} \in Z^{\Omega}$$

$$P(\emptyset) = \frac{|\emptyset|}{|\Omega|} = \frac{0}{6} = 0$$

$$P(\Omega) = \frac{|\Omega|}{|\Omega|} = 1$$

Def of $P(A) := \dfrac{|A|}{|\Omega|}$

let $A :=$ even #s $\quad P(A) = \frac{3}{6} = \frac{1}{2}$

Flip 2 coins:

$$\Omega = \{\langle H,H\rangle, \langle T,H\rangle, \langle H,T\rangle, \langle T,T\rangle\}$$

$$\Omega \times \Omega \qquad \begin{array}{c|c|c} & \multicolumn{2}{c}{\Omega} \\ \hline \Omega & \langle H,H\rangle & \langle H,T\rangle \\ & \langle T,H\rangle & \langle T,T\rangle \end{array}$$

$$P(\text{heads \& heads}) = P(\{\langle H,H\rangle\}) = \frac{|\{\langle H,H\rangle\}|}{|\Omega|} = \frac{1}{4}$$

$$P(\text{one h \& one t}) = P\left(\frac{\{\langle H,T\rangle, \langle T,H\rangle\}}{|\Omega|}\right) = \frac{2}{4} = \frac{1}{2}$$