MAT 241 — Rodrigo Guaracho — 10/11/17

$\Omega = \{H, T\}$

$n = 3$

$\omega_1 = H$

$\omega_2 = T$

$\omega_3 = H$

$$\mathbb{I}_{\omega = H} = \begin{cases} 1 & \text{if } \omega = H \\ 0 & \text{if } \omega \neq H \end{cases}$$

"Indicator"

Generally, there is a form

$X : \Omega \to \mathbb{R}$

called a "random variable" (rv)

$\mathbb{I}_{\omega_1 = H}, \mathbb{I}_{\omega_2 = T}, \mathbb{I}_{\omega_3 = H}$

$1, \quad 0, \quad 1$

$\bar{X} = \frac{1+0+1}{3} = \frac{2}{3}$

$\text{Supp}[X] = \{0, 1\}$

the set of all things that could happen

$X(H) = 1$

$X(T) = 0$

shorthand "abuse of notation"

$P(X = 1) = P(\{\omega : X(\omega) = 1\})$

$P: 2^{\Omega} \to (0, 1)$

$$P(X = 1) = P(\{\omega : X(\omega) = 1\}) = P(\{H\}) = \frac{|A|}{|\Omega|} = \frac{1}{2}$$

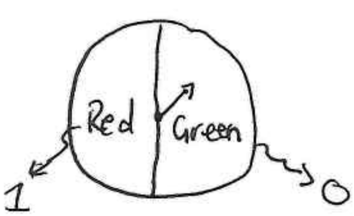
"Support". the range of the r.v (random variable) denoted: $\text{Supp}[X] = \{a : P(X = a) > 0\}$

$\text{Supp}[X] \subseteq \mathbb{R}$

Def

A discrete r.v is one ~~st~~ s.t. (such that) $|\text{Supp}(X)| \leq |\mathbb{N}|$ i.e finite or countably infinite

$\Omega = \{R, G\}$



$P(X = 1) = \frac{1}{2}$ $\quad \text{Supp}[X] = \{0, 1\}$

$P(X = 0) = \frac{1}{2}$

r.v "discrete as" "with prob"

$$X \sim \begin{cases} 1 & \text{w.p } \frac{1}{2} \\ 0 & \text{w.p } \frac{1}{2} \end{cases}$$

~~X ~ Bernoulli~~ ~~X ~ Bernoulli(1/2)~~

$$X \sim \text{Bernoulli}\left(\tfrac{1}{2}\right) := \begin{cases} 1 & \text{w.p } 1/2 \\ 0 & \text{w.p } 1/2 \end{cases}$$

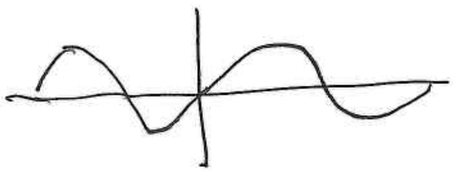
$\text{Supp}[X] = \{0, 1\}$

X is discrete.

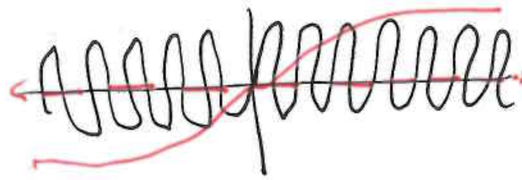
$X \sim \text{Bernoulli}(p) := \begin{cases} 1 & \text{w.p. } p \\ 0 & \text{w.p. } 1-p \end{cases}$

p is called a "paramater", a number you choose to "tune" the rv model.

$f(x) = \sin(x)$



$f(x) = \sin(ax)$ where $a \in \mathbb{R}$



if $a = 10$;

if $a = \frac{1}{10}$

$\boxed{\text{if } a = 0}$ ---

$\boxed{\text{degenerate case}}$

If you exclude $a=0$ ~~where $a \in \mathbb{R}/\{0\}$~~

$f(x) = \sin(ax)$ where $a \in \mathbb{R}/\{0\}$

Paramater space: the set where p "lives"

~~$p \in [0,1]$ $p \neq 0$ degenerate~~

$p \in (0,1)$

$p = 0$ degenerate $\Rightarrow X \sim \text{Deg}(0)$

$p = 1$ " $\Rightarrow X \sim \text{Deg}(1)$

$X \sim \underbrace{\text{Deg}}_{\text{degenerate}}(c) := \{ c \text{ w.p. } 1$ $\text{Supp}[X] = \{c\}$

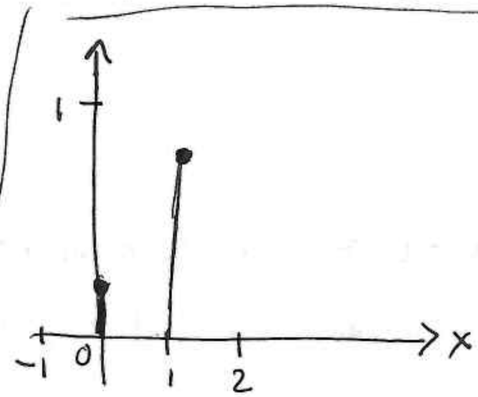
big X   lil x

$p(x) = P(X = x)$

probabality mass function (PMF)

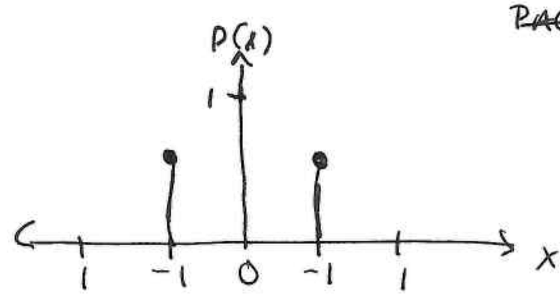
$$\sum_{x \in \text{Supp}[X]} p(x) = 1$$

$p: \mathbb{R} \to (0,1)$



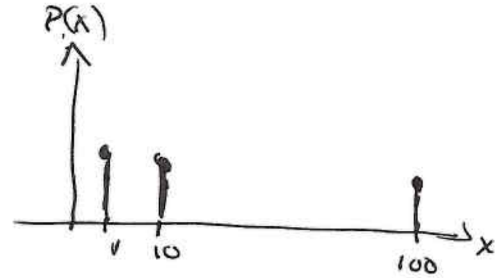
$X \sim \text{Bernoulli}(3/4)$

X - Rademacher $= \begin{cases} 1 & \text{wp } \frac{1}{2} \\ -1 & \text{wp } \frac{1}{2} \end{cases}$



X - Unif $(\overbrace{\{1, 10, 100\}}^{A}) = \begin{cases} 1 & \text{wp } \frac{1}{3} \\ 10 & \text{wp } \frac{1}{3} \\ 100 & \text{wp } \frac{1}{3} \end{cases}$

"discrete uniform"



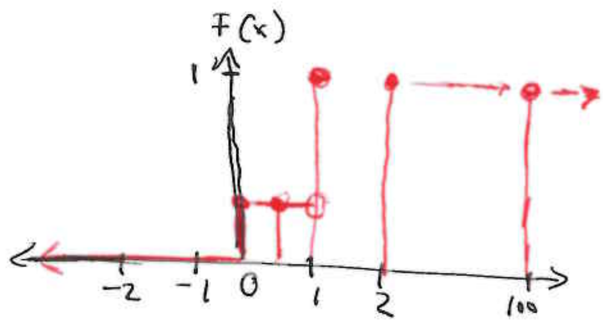
X - Unif (A) $\quad$ Supp $(X) = A$

$A \in 2^{\mathbb{R}}$ but $|A|$ is finite

$F(x) := P(X \leq x)$

"cummulative distribution function" (CDF)

$X \sim$ Bern $(3/4)$



Properties of CDF

① $\lim\limits_{x \to \infty} F(x) = 1$

② $\lim\limits_{x \to -\infty} F(x) = 0$

③ $\forall\ x \leq y\ \exists\ F(x) \leq F(x)$ monotonically increasing

X - Bern $(p) = \begin{cases} 1 & \text{wp } \frac{1}{2} \\ 0 & \text{wp } 1-p \end{cases}$

$p(x) = p^x (1-p)^{1-x}$

Def

X_1, X_2 are "identically" denoted $X_1 \stackrel{d}{=} X_2$ if

a) $p_{X_1}(x) = p_{X_2}(x)$

b) $F_{X_1}(x) = F_{X_2}(x)$

10 cards, 4R, 6B

$$P(2R \text{ when drawing } 3) = \frac{\binom{4}{2}\binom{6}{1}}{\binom{10}{3}}$$

$$P(xR \text{ when drawing } 3) = \frac{\binom{4}{x}\binom{6}{3-x}}{\binom{10}{3}}$$

$$P(x R \text{ when drawing } n) = \frac{\binom{4}{x}\binom{6}{n-x}}{\binom{10}{n}}$$

10 cards, K Red

$$P(x R \text{ when drawing } n) = \frac{\binom{K}{x}\binom{10-K}{n-x}}{\binom{10}{n}}$$

N cards, K Red

$$P(x R \text{ when drawing } n) = \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}} = pr$$

$$\boxed{X \sim \text{Hypergeometric}(n, K, N)}$$

Next class