Lecture:-6

a) * Long run frequency defination of probability

$$P(A) := \lim_{n\to\infty} \frac{1}{4} \sum_{i=1}^{n} \mathbb{1}_{a_i \in A}$$

b) * Propensity theory of probability states that objects have an inherited disposition of ω or the other. Usually, for the most random experiment, we don't know how to calculate the propensity of ω's. Also, its not general.

Both a & b are called "objective" or function of physical reality.

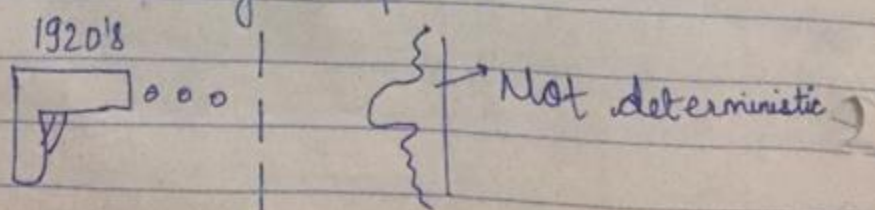
* Subjective theory of probability

There are multiple answers due to non objective as everyone use their own evidence to come up with their own evidence.

Probabitity is degree of belief and there is no defination of probability that is globally accepted.

Q: What is randomness?

A/c to Lapace, randomness is an illusion due to lack of information and ignorance to do necessary computation.



②

## Mathematical theory of probability

Lets assume $\exists\, \Omega \neq \phi$. P is a function set such that

a) $P(\Omega) = 1$ b) $P(A) \geq 0 \;\forall A \subseteq \Omega$

c) If $A_1, A_2, \ldots$ disjoint $\Rightarrow P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i)$

we can add probability of disjoint

**Theorem I**

$P(A) = 1 - P(A^c)$

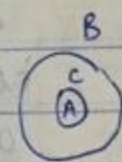
$\Omega = A \cup A^c \rightarrow$ set theory

$P(\Omega) = P(A \cup A^c) \rightarrow$ defn of function

$P(\Omega) = P(A) \cdot P(A^c) \rightarrow$ from ©

$1 = P(A) + P(A^c) \rightarrow$ from ①

$\underline{P(A) = 1 - P(A^c)}$




$C := B \setminus A$

**Theorem 2**

$\boxed{P(\phi) = 0}$

$P(\Omega) = 1 - P(\phi^c) \rightarrow$ Thm①

$= 1 - P(\Omega) \rightarrow$ set theory

$= 1 - 1 \rightarrow$ from ①

$= 0$

**Theorem 3:**

$\boxed{A \subseteq B \Rightarrow P(A) \leq P(B)}$

$P(B) = P(A \cup C) = P(A) + P(C)$

$P(B) - P(A) = P(C) \geq 0 \rightarrow$ from ②

$P(B) - P(A) \geq 0$

$P(B) \geq P(A)$

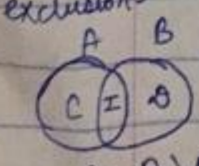
laws of inclusion exclusion } **Theorem 4**: $\underline{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$

$P(A \cup B) = P(C) + P(D) + P(I)$

$= (P(A) - P(I)) + (P(B) - P(I)) -$ from ① & ⑪

$= P(A) + P(B) - + P(I)$

$P(A \cap B)$




$P(A) = P(C) + P(I)$ - ①

$P(B) = P(D) + P(I)$ - ⑪

$C = A \setminus B$

$D = B \setminus A$

$I = A \cap B$

<u>Theorem 5</u>: $|\Omega| < \infty$ if $\forall_i P(\{\omega_i\}) = \frac{1}{|\Omega|}$ $\forall_A P(A) = \frac{|A|}{|\Omega|}$

let $|A| = n$

if $A = \{\omega_1, \omega_2, \dots, \omega_n\}$ it is also,

$$A = \{\omega_1\} \cup \{\omega_2\} \cup \dots \cup \{\omega_n\}$$

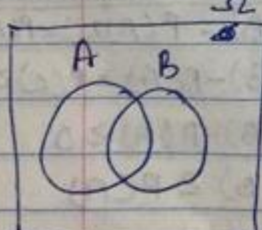
$$P(A) = P(\{\omega_1\}) \cup P(\{\omega_2\}) \cup \dots \cup (\{\omega_n\}) \quad -① $$

$$= \frac{1}{|\Omega|} + \frac{1}{|\Omega|} + \dots + \frac{1}{|\Omega|} = \frac{n}{|\Omega|} = \frac{|A|}{|\Omega|}$$

Q) lets imagine there are $n = 1000$ ppl. If 200 are smokers (i.e A: smokes) and if 60 have lung cancer (i.e B: lung cancer) and suppose 36 are smokers and have lung cancer ($A \cap B$).

Sol$^n$, lets assume $P(A) = 0.2$

$P(B) = 0.06$, $P(A \cap B) = 0.036$




Q) What is P(lung cancer)

$\because P(B) = 0.06 \left(\frac{60}{1000}\right)$ //

Q) What is P(lung cancer among smokers)

$\hookrightarrow P(B | A) = \frac{36}{200} = \left[\frac{P(B)}{P(A)}\right]$

(given / "conditional on") $= 0.18$ //