11/1

Best of #7 \$1

$$X_A \sim \begin{cases} \$35 & \text{wp } \frac{1}{38} \\ -\$1 & \text{wp } \frac{37}{38} \end{cases} \Rightarrow \mu = -\$0.053$$

$$\sigma^2 = (35-(-0.053))^2 \tfrac{1}{38} + (-1-(-0.053))^2 \tfrac{37}{38} = 33.207 \$^2$$

$$\sigma = \sqrt{33.207} = \$5.79$$

Bet on Black \$1

$$X_B \sim \begin{cases} \$1 & \text{wp } \frac{18}{38} \\ -\$1 & \text{wp } \frac{20}{38} \end{cases} \Rightarrow \mu = -\$0.053$$

$$\sigma^2 = (1-(-0.053))^2 \tfrac{18}{38} + (-1-(-0.053))^2 \tfrac{20}{38} = 0.997 \$^2$$

$$\sigma = \sqrt{0.997} = \$1.00$$

$$\mu = E(x) = \sum_{x \in Supp[x]} x \, p(x)$$

$$\sigma^2 = \underset{Var(x)}{} = \sum_{x \in Supp[x]} (x-\mu)^2 \, p(x)$$

$\bar{X}_A \to \mu$

Law of Large #'s

$\bar{X}_B \to \mu$

Standard deviation
or
Standard error

$$\sigma = SE(X) = SD[X] := \sqrt{Var[X]}$$

$T_2 = X_1 + X_2$ $\quad E[T_2] = \sum_{t \in Supp[X]} t \; p(t)$ ← impractical

We need a better way

$$E[g(x)] = \sum_{x \in Supp(x)} g(x)\,p(x)$$

Joint mass function

$$E[g(X_1, X_2)] = \sum_{x_1 \in Supp} \sum_{x_2 \in Supp(x)} g(x_1, x_2)\, p(x_1, x_2)$$

$$E[X_1 + X_2] = \sum_{x_1} \sum_{x_2} (x_1 + x_2)\, p(x_1, x_2)$$

$$= \sum_{x_1} \sum_{x_2} x_1\, p(x_1, x_2) + \sum_{x_2} \sum_{x_1} x_2\, p(x_1, x_2)$$

$$= \sum_{x_1} x_1 \sum_{x_2} p(x_1, x_2) + \sum_{x_2} x_2 \sum_{x_1} p(x_1, x_2)$$

$$= \sum_{x_1} x_1\, p(x_1) + \sum_{x_2} x_2\, p(x_2) = \boxed{E[X_1] + E[X_2]}$$

Important (Cheat Sheet)

Let's say X_1, X_2 are independent

$$\Rightarrow p(x_1, x_2) = p(x_1) \cdot p(x_2)$$

$$E[x_1 + x_2] = \sum_{x_1} x_1 \sum p(x_1)\, p(x_2) + \sum_{x_2} x_2 \sum_{x_1} p(x_1)\, p(x_2)$$

$$= \sum_{x_1} x_1\, p(x_1) \underbrace{\sum p(x_2)} + \sum_{x_2} x_2\, p(x_2) \underbrace{\sum_{x_1} p(x_1)} =$$

$$E[X_1] + E[X_2]$$

Example

$$Supp[X_1] = \{1, 7, 9\}$$

$$Supp[X_2] = \{5, 23, 88\}$$

X_1 → $p[X_1, X_2]$

		1	7	9		
	5	$\frac{1}{15}$	$\frac{1}{3}$	$\frac{2}{15}$	$\frac{16}{30}$	$\leftarrow p(X_2=5)$
X_2	23	$\frac{1}{30}$	$\frac{1}{10}$	$\frac{1}{30}$	$\frac{5}{30}$	$\leftarrow p(X_2=23)$
	88	$\frac{1}{30}$	$\frac{1}{5}$	$\frac{1}{15}$	$\frac{9}{30}$	$\leftarrow p(X_2=88)$
		$\frac{4}{30}$	$\frac{19}{30}$	$\frac{7}{30}$	1	
		$P(X_1=1)$	$P(X_1=7)$	$P(X_1=9)$		

$\underbrace{P(X_1=1) \quad P(X_1=7) \quad P(X_1=9)}_{} \rightarrow$ PMF

$$\sum_{x_1} \sum_{x_2} p(X_1, X_2) = 1$$

$$X_1 \sim \begin{cases} 1 & wp \ \frac{4}{30} \\ 7 & wp \ \frac{19}{30} \\ 9 & wp \ \frac{7}{30} \end{cases} \qquad X_2 \sim \begin{cases} 5 & wp \ \frac{16}{30} \\ 23 & wp \ \frac{5}{30} \\ 88 & wp \ \frac{9}{30} \end{cases}$$

$$P(X_1 = 1, X_2 = 5) \overset{?}{=} P(X_1 = 1)\, P(X_2 = 5)$$

$$\frac{1}{15} \neq \frac{4}{30} \frac{16}{30}$$

$\boxed{\text{Marging out } X_1}$

$$\sum_{x_1} p(x_1, x_2) = p(x_2)$$

$$\int_{\mathbb{R}} f(x)\,dx = 7$$

$$\int_{\mathbb{R}} f(x, y)\,dx = g(x)$$

Important

$$E[T_n] = \sum_{i=1}^{n} E[X_i] \overset{\text{under identically distributed assumption}}{=} n\mu$$

$$E[\bar{X}_n] = E\left[\frac{1}{n} T_n\right] = \frac{1}{n} E[T_n] = \frac{1}{n} n\mu = \boxed{\mu}$$

$\cancel{X} \sim Hyper(n, K, N)$

$$E(X) = \sum_{x \in Supp(X)} x \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$$

$X = X_1 + X_2 + \cdots + X_n$

identically distributive BUT Not independent

$$\begin{cases} X_1 \sim Bern\left(\frac{K}{N}\right) \\ X_2 \sim Bern\left(\frac{K}{N}\right) \\ \vdots \\ X_n \sim Bern\left(\frac{K}{N}\right) \end{cases}$$

$X_2 | X_1 = x_1 \sim Bern\left(\frac{K - x_1}{N-1}\right)$

Important

$$\boxed{E[X] = n\frac{K}{N}}$$

$$Var[X] = E\left[(X-\mu)^2\right]$$

$$= E\left[X^2 - 2\mu X + \mu^2\right]$$

$\sum_x \mu^2 p(x) = \mu^2 \sum_x p(x) = \mu^2 \cdot 1 = \mu^2$

$$= E[X^2] + E[-2\mu X] + E[\mu^2]$$

$$= E[X^2] \underbrace{- 2\mu \underbrace{E[X]}_{\mu}}_{-2\mu^2} + \mu^2$$

$$\boxed{\begin{aligned} \sigma^2 = Var[X] &= E[X^2] - \mu^2 \\ E[X^2] &= \sigma^2 + \mu^2 \end{aligned}}$$

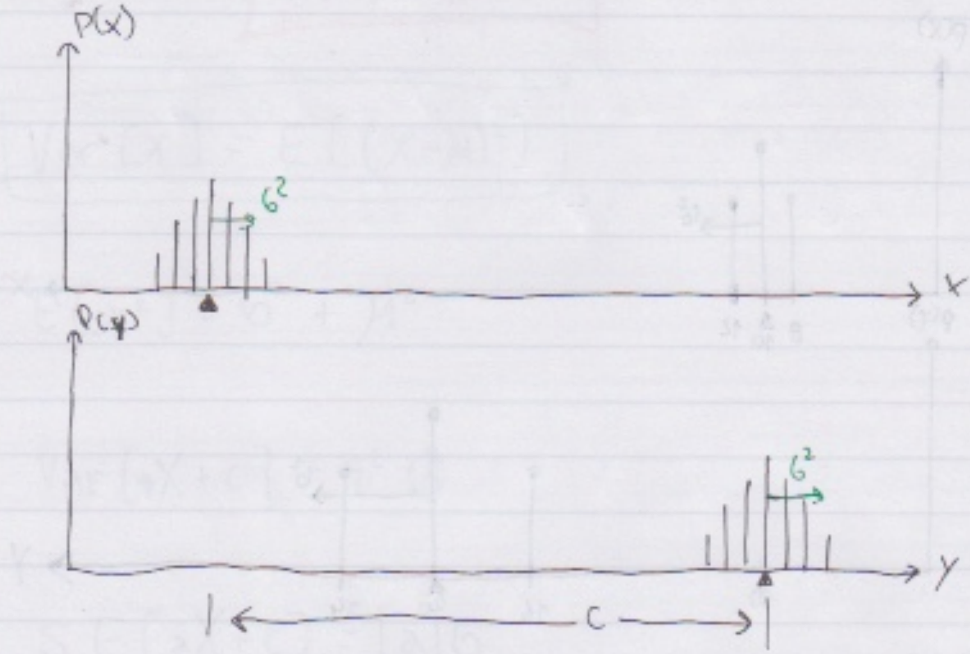
$Y = aX + c \quad , \quad a, c \in \mathbb{R}$

$E(Y) = a\,E[X] + c$

$Var[Y] = a^2 \sigma^2$

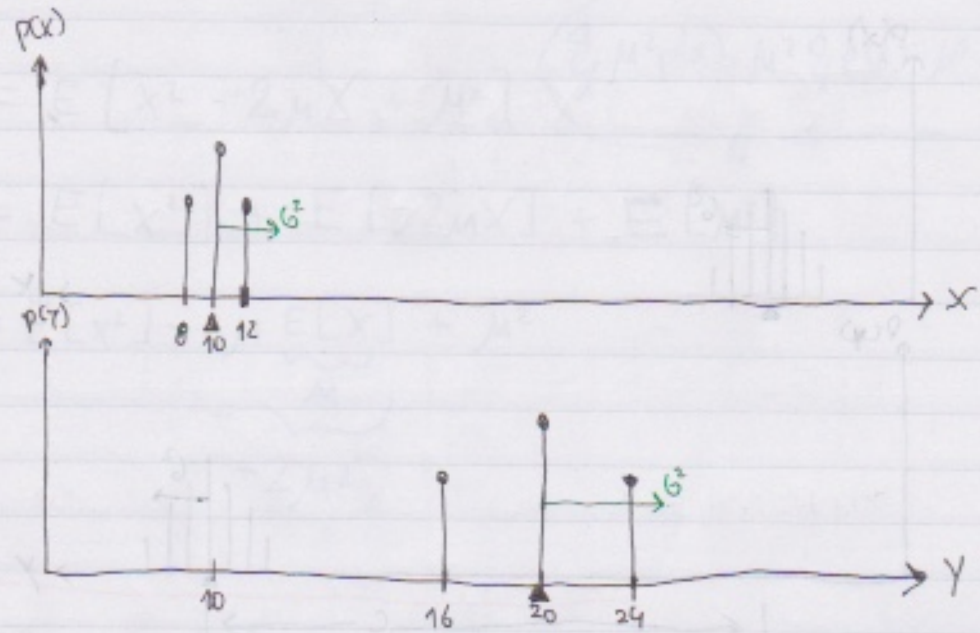
$SE[Y] = |a|\sigma$

$Y = X + c \qquad Var[Y] = \sigma^2$



$$
\begin{aligned}
Var[X+c] &= E[((X+c) - E[X+c])^2] \\
&= E[((X+\cancel{c}) - (\mu+\cancel{c}))^2] \\
&= E[(X-\mu)^2] \\
&= \sigma^2
\end{aligned}
$$

$Y = aX^1$



$Y = 2X \qquad Var[Y] = 4\sigma^2$

$$Var(aX) = E\left[(aX - \underbrace{E(aX)}_{a\mu})^2\right]$$

$$= E\left[(aX - a\mu)^2\right] = E\left[(a(X-\mu))^2\right]$$

$$= E\left[a^2(X-\mu)^2\right] = a^2 E\left[(X-\mu)^2\right] = a^2\sigma^2$$