$$X \sim \text{Hyper}(n, K, N) = \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$$

$$X \sim \text{Hyper}(n, p, N) = \frac{\binom{pN}{x}\binom{(1-p)N}{n-x}}{\binom{N}{n}}$$

s.t. $p := \frac{K}{N}$

$$\lim_{N\to\infty} \text{Hyper}(n,p,N) = \lim_{N\to\infty} \frac{\frac{(pN)!}{x!(pN-x)!} \cdot \frac{((1-p)N)!}{(n-x)!((1-p)N-(n-x))!}}{\frac{N!}{n!(N-n)!}}$$

$$= \underbrace{\underbrace{\frac{1}{x!}\,\frac{1}{(n-x)!}\,\frac{1}{\frac{1}{n!}}}_{\frac{n!}{x!(n-x)!}}}_{\binom{n}{x}} = \binom{n}{x} \lim_{N\to\infty} \frac{\frac{(pN)!}{(pN-x)!}\,\frac{((1-p)N)!}{((1-p)N-(n-x))!}}{\underbrace{\frac{N!}{(N-n)!}}_{N(N-1)\dots(N-n+1)}}$$

$$\frac{\overbrace{(pN)(pN-1) * \dots * (pN-x+1)}^{x \text{ terms}} \cdot \overbrace{((1-p)N)((1-p)N-1) * \dots * ((1-p)N-(n-x)+1)}^{n-x \text{ total terms}}}{\underbrace{N(N-1) * \dots * (N-n+1)}_{n \text{ total terms}}}$$

$$= \binom{n}{x} \underbrace{\lim_{N\to\infty} \frac{pN}{N}}_{p} * \underbrace{\lim_{N\to\infty} \frac{pN-1}{N-1}}_{p} * \dots * \underbrace{\lim_{N\to\infty} \frac{pN-x+1}{N-x+1}}_{p} \underbrace{\lim_{N\to\infty} \frac{(1-p)N}{N-x}}_{1-p} \underbrace{\lim_{N\to\infty} \frac{(1-p)N-1}{N-x-1}}_{1-p} * \dots$$

$$\dots * \underbrace{\lim_{N\to\infty} \frac{(1-p)N-(n-x)+1}{N-n+1}}_{1-p}$$

We have p^x and $(1-p)^{n-x}$

↳ x terms of p

$$\Rightarrow \binom{n}{x} p^x (1-p)^{n-x}$$

$$X \sim \text{Binomial}(n,p) := \binom{n}{x} p^x (1-p)^{n-x}$$

sampling n with replacement and asking "how many special balls did I guess?"

Parameter Space

$n \in \mathbb{N}$
$p \in (0,1)$

$\text{Support}[X] = \{0, 1, \dots, n\}$

We want to show

$$\sum_{x \in \text{Supp}[X]} p[X] = 1$$

$$\Rightarrow \sum_{x=0}^{n} \binom{n}{x} p^x (1-p)^{n-x} = (p + (1-p))^n$$

1 ✓
1^n
$"$

X_1 and X_2 are independent random variable's if

a) $P(X_1 = x_1 \mid X_2 = x_2) = P(X_1 = x_1)$ for all $x_1 \in Supp[X_1]$, for all $x_2 \in Supp[X_2]$

b). $P(X_2 = x_2 \mid X_1 = x_1) = P(X_2 = x_2)$

c) $P(X_1 = x_1, X_2 = x_2) = P(X_1 = x_1) P(X_2 = x_2)$

AND

JOINT MASS FUNCTION

$X_1 X_2$ are "iid" → independent and identically distributed

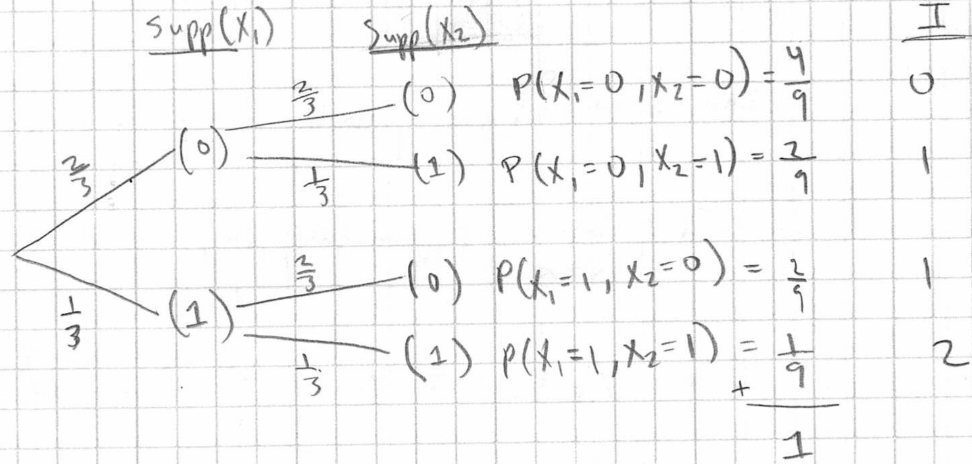
$X_1, X_2 \overset{iid}{\sim} Bern\left(\frac{1}{3}\right)$

$T_2 := X_1 + X_2$

$Supp[X_1] = \{0, 1\}$

$Supp[X_2] = \{0, 1\}$

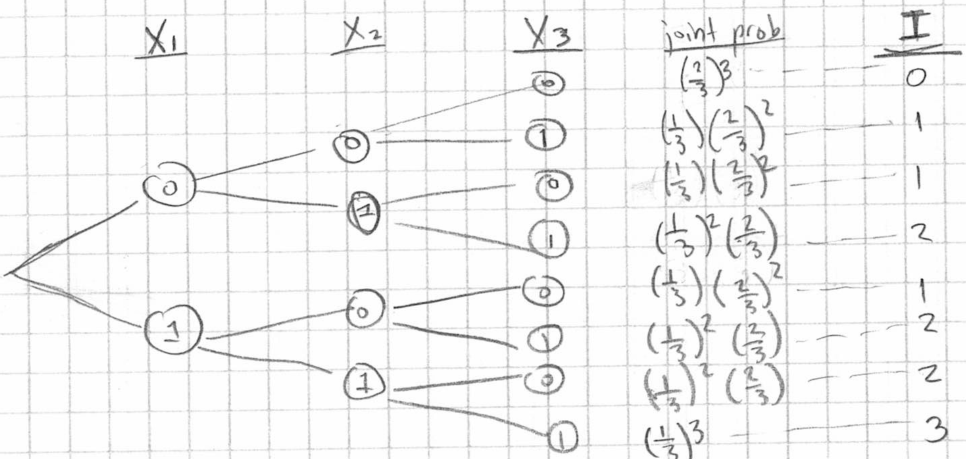
$Supp[T_2] = \{0, 1, 2\}$



$X_1, X_2, X_3 \overset{iid}{\sim} Bern\left(\frac{1}{3}\right)$

$T_3 = X_1 + X_2 + X_3$

$Supp[T_3] = \{0, 1, 2, 3\}$



$$T_3 \sim \begin{cases} 0 & wp \quad \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^3 \\ 1 & wp \quad \left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^2 = 3\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^2 \\ 2 & wp \quad \left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right) + \left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right) + \left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right) = 3\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^2 \\ 3 & wp \quad \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^0 \end{cases}$$

$X_1, X_2, \ldots, X_n \overset{iid}{\sim} \text{Bern}\left(\frac{1}{3}\right)$

$T_n = \sum_{i=1}^{n} X_i$

$\text{Supp}[T_n] = \{0, 1, \ldots, n\}$

$$T_n \sim \begin{cases} 0 & \text{wp } \binom{n}{0}(p)^0(1-p)^n \\ 1 & \text{wp } \binom{n}{1}(p)^1(1-p)^{n-1} \\ 2 & \text{wp } \binom{n}{2}(p)^2(1-p)^{n-2} \\ n-1 & \text{wp } \binom{n}{n-1}(p)^{n-1}(1-p)^1 \\ n & \text{wp } \binom{n}{n}(p)^n(1-p)^0 \end{cases}$$

$\Downarrow$

$T_n \sim \text{Binom}(n, p) := \binom{n}{x} p^x (1-p)^{n-x}$

The binomial conceptually is

$T = \lim_{N \to \infty} \text{Hyper}(n, p, N)$

$T = X_1 + \ldots + X_n$ where $X_1, \ldots, X_n \overset{iid}{\sim} \text{Bern}(p)$