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$\Omega = \{H, T\}$

$\boxed{H \mid T}$

$n = 3$

$\omega_1 = H$

$\omega_2 = T$

$\omega_3 = H$

indicator function

$$\mathbb{1}_{\omega = H} = \begin{cases} 1 & \text{if } \omega = H \\ 0 & \text{if } \omega \neq H \end{cases}$$

$$\mathbb{1}_{\omega_1} = H, \quad \mathbb{1}_{\omega_2} = T, \quad \mathbb{1}_{\omega_3} = H$$

$$\bar{x} = \frac{1+0+1}{3} = \frac{2}{3}$$

Generally, there is a function

$$X : \Omega \to \mathbb{R}$$

called a "random variable" (r v)

short hand notation for

"abuse of notation"

$X(H) = 1$

$X(T) = 0$

$$P(X=1) \overset{\text{def}}{=} P(\{\omega : X(\omega) = 1\}) = P(\{H\}) = \frac{|A|}{|\Omega|} = \frac{1}{2}$$

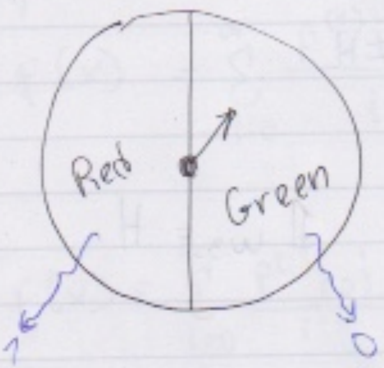
$\text{Supp}[X] = \{0, 1\}$ $\quad P: 2^{\Omega} \longrightarrow (0, 1)$

"Support" the range of r.v. (random variable)

denoted: $Supp[x] = \{x : P(X=x) > 0\} \subseteq \mathbb{R}$

Definition: A discrete r.v. is one such that $|Supp(x)| \leq |\mathbb{N}|$ i.e. finite or contably infinite.

$\Omega = \{R, G\}$



$P(x=1) = \frac{1}{2}$

$P(X=0) = \frac{1}{2}$

$Supp(x) = \{0, 1\}$

r.v. "distributive as" "with probability"

$$X \sim \begin{cases} 1 & \text{w.p. } \frac{1}{2} \\ 0 & \text{w.p. } \frac{1}{2} \end{cases}$$

useful notation:

$X \sim \text{Bernonilli}(\tfrac{1}{2}) := \begin{cases} 1 & \text{w.p. } \tfrac{1}{2} \\ 0 & \text{w.p. } \tfrac{1}{2} \end{cases}$

"Standard Beronilli"

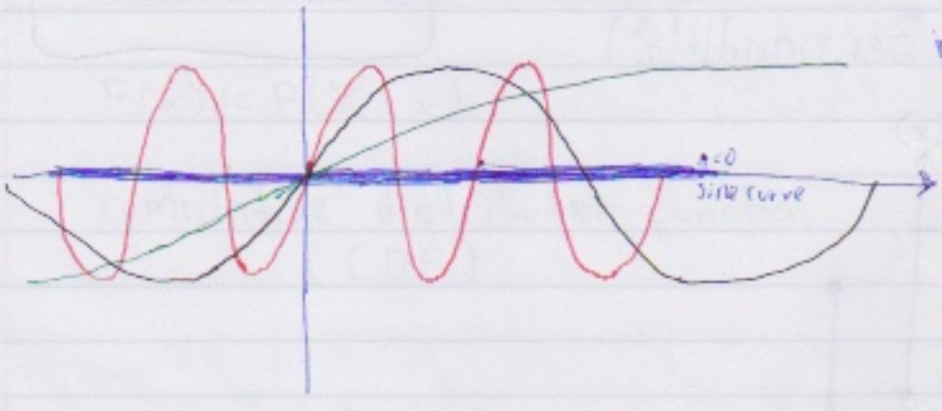
$\text{Supp}(x) = \{0, 1\}$

X is discrete

$X \sim \text{Bernonilli}(p) := \begin{cases} 1 & \text{w.p. } p \\ 0 & \text{w.p. } 1-p \end{cases}$

p is calle a parameter, a number you choose to "tune" the r.v. model.

$f(x) = \sin(ax)$ where $x \in \mathbb{R}$



if $a=0$: "degenerate case"

$$X \sim \text{Bernonilli}(p) := \begin{cases} 1 & \text{w.p. } p \\ 0 & \text{w.p. } 1-p \end{cases}$$

Parameter Space: the set where p "lives".

$p = 0$ "degenerate" $\Rightarrow X \sim \text{Deg}(0)$
$p = 1$ "degenerate" $\Rightarrow X \sim \text{Deg}(1)$

$$X \sim \text{Deg}(c) := \{\, c \ \text{ w.p. } 1$$

$$\text{Supp}(X) = \{c\}$$

little p of x

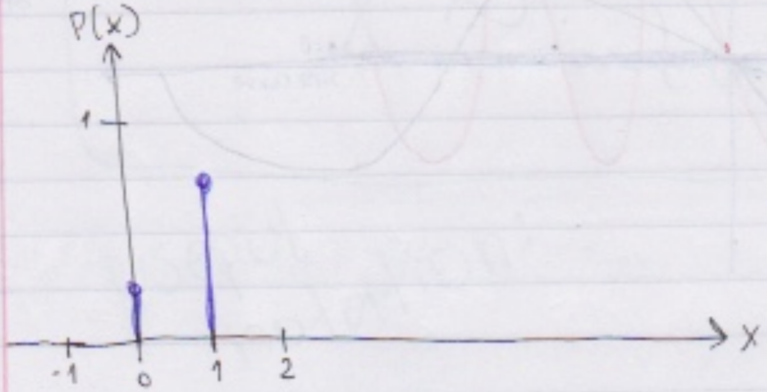
$$p(x) := P(X = x)$$

probability mass function (PMF)

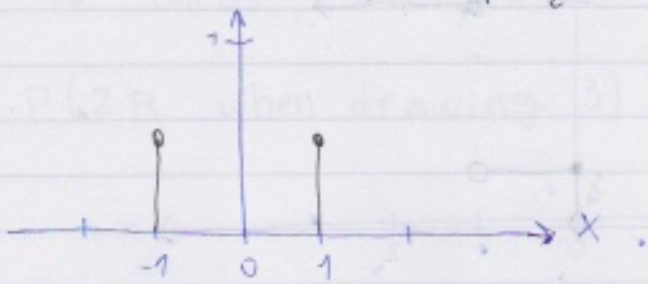
$$p: \mathbb{R} \longrightarrow (0,1)$$

$$\boxed{\sum_{x \in \text{Supp}(X)} p(x) = 1}$$

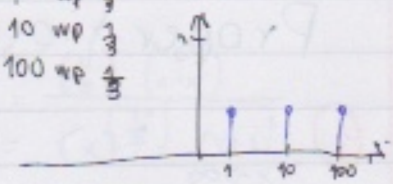
$X \sim \text{Bernonilli}\left(\frac{3}{4}\right)$



$X \sim \text{Rademacher} = \begin{cases} 1 & \text{wp } \frac{1}{2} \\ -1 & \text{wp } \frac{1}{2} \end{cases}$



$X \sim \text{Unif}(\overbrace{\{1, 10, 100\}}^{A}) = \begin{cases} 1 & \text{wp } \frac{1}{3} \\ 10 & \text{wp } \frac{1}{3} \\ 100 & \text{wp } \frac{1}{3} \end{cases}$



"discrete uniform"

$X \sim \text{Unif}(A)$ $\qquad \text{Supp}(X) = A$

$A \in 2^{\mathbb{R}}$ but $|A|$ is finite

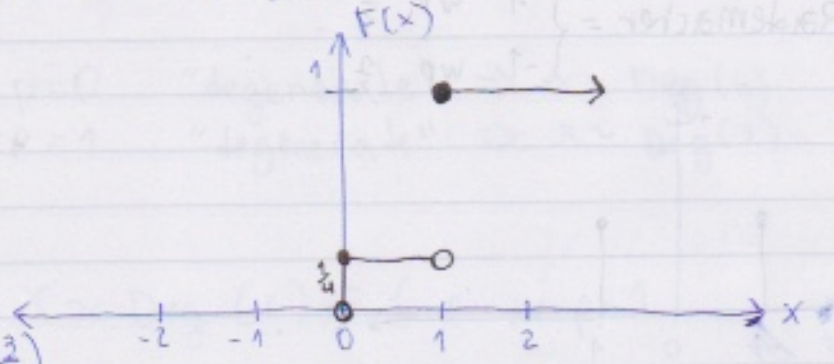
$F(x) := P(X \leq x)$

cumulative distribution function
(CDF)

$\exists x$ there exist in x (something about x)

$\forall x$ all x (something about x)

$X \sim Bern(\frac{3}{4})$



$x \sim Bern(\frac{3}{4})$

Properties of CDF

(1) $\lim_{x \to \infty} F(x) = 1$

(2) $\lim_{x \to -\infty} F(x) = 0$

(3) $x \leq y \Rightarrow F(x) \leq F(y)$ monotonically increasing

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$$X \sim Bern(P) = \begin{cases} 1 & \text{w.p. } \frac{1}{2} \\ 0 & \text{w.p. } \frac{1}{2} \end{cases}$$

$$p(x) = p^x (1-p)^{1-x}$$

Definition: X_1, X_2 are "identically distributive" denoted $X_1 \stackrel{d}{=} X_2$ if (a) $P_{X_1}(x) = P_{X_2}(x)$ (b) $F_{X_1}(x) = F_{X_2}(x)$

Example

10 cards, 4R, 6B

$$P(2R \text{ when drawing } 3) = \frac{\binom{4}{2}\binom{6}{1}}{\binom{10}{3}}$$

$$P(x R \text{ when drawing } 3) = \frac{\binom{4}{x}\binom{6}{3-x}}{\binom{10}{3}}$$

$$P(x R \text{ when drawing } n) = \frac{\binom{4}{x}\binom{6}{n-x}}{\binom{10}{n}}$$

10 cards, K Red

$$P(x R \text{ when drawing } n) = \frac{\binom{K}{x}\binom{10-K}{n-x}}{\binom{10}{n}}$$

N cards, K Red

$$P(x R \text{ when drawing } n) = \frac{\binom{K}{x}\binom{n-K}{n-x}}{\binom{N}{n}}$$