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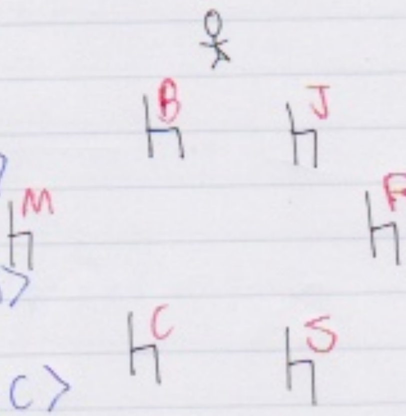
\# of ways ~~to~~ to sample K objects from a set of n without replacement

$${}_nP_K = \frac{n!}{(n-k)!}$$

B, J, R, S, C, M sitting in 6 seats.

How many ways to seat them in a circle where there's no "1st chair" or there is rotational invariance?



6 { ⟨B, J, R, S, C, M⟩ equivalent to ⟨J, R, S, C, M, B⟩ . . . ⟨M, B, J, R, S, C⟩

$6! = 720$

$\frac{6!}{6}$ ← Dividing out invariance

Imagine a basket of 5 flowers. and 5 flower pots. 3 orchids O_1, O_2, O_3 and 2 are Chrysanthemum X_1, X_2

$$\left.\begin{matrix} 5! \\ O_1\ O_2\ O_3\ X_1\ X_2 \\ O_1\ O_3\ O_2\ X_1\ X_2 \\ \vdots \\ 120 \text{ ways} \end{matrix}\right\}$$

$OOO\ X_1\ X_2$

How many flower arrangements if the orchids are "indistinct", "not unique", "indistinguishable"

$$\frac{5!}{3!}$$

same question, but chrysan... are also indistinguishable

$$\frac{5!}{3!\,2!} = 10$$

OOOXX
OOXOX
$\vdots$
XXOOO

$P(4H)$

$P(4H \text{ in } 10 \text{ coins})$

$$= \frac{|A|}{|\Omega|} = \frac{}{2^{10} =} = \frac{\frac{10!}{4!6!}}{1024} \approx .205$$

$$|\{H, T\}^{10}|$$

$$\{B, J, R, S, C, M\}$$

How many ways to sit 3 of them without replacement? ${}_6P_3$

How many ways to sit 3 of them so that their order doesn't matter?

$$\frac{{}_6P_3}{{}_3P_3} = \frac{{}_6P_3}{3!}$$

${}_nC_k :=$

$$\binom{n}{k} := \frac{{}_nP_k}{{}_kP_k} = \frac{{}_nP_k}{k!} = \frac{\frac{n!}{(n-k)!}}{n!} = \frac{n!}{(n-k)!\,k!}$$

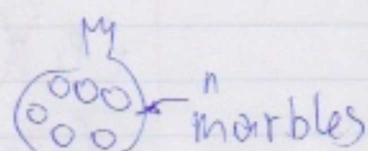
"combinations"

"n choose k"

$n \in \mathbb{N}_0$
$k \in \{0, \dots, n\}$

if n, k are inappropriate

$$\binom{n}{k} := 0$$



Identities

How many ways to pick 1 marble out of n

(1) $\binom{n}{1} = \frac{n!}{(n-1)!\,1!} = n$

(2) $\binom{n}{n-1} = \frac{n!}{(n-(n-k))!\,(n-1)!} = n$

(3) $\binom{n}{n-k} = \frac{n!}{(n-(n-k))!\,(n-k)!} = \binom{n}{k}$

(4) $\binom{n}{n} = 1$

(5) $\binom{n}{0} = 1$

$\{B, J, R, S, C, M\}$

4 of these people are seated.

What is the probability J is seated?

$$\begin{array}{cccc} \underline{J} & \underline{5} & \underline{4} & \underline{3} \\ \underline{5} & \underline{J} & \underline{4} & \underline{3} \\ \underline{5} & \underline{4} & \underline{J} & \underline{3} \\ \underline{5} & \underline{4} & \underline{3} & \underline{J} \end{array}$$

$$P(\text{Jane is seated}) = \frac{|A|}{|\Omega|} = \frac{4({}_5P_3)}{{}_6P_4} = \frac{4 \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3}}{6 \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3}} = \frac{2}{3}$$

$$= \frac{\binom{5}{3}}{\binom{6}{4}} = \frac{\frac{5 \cdot \cancel{4}^{2} \cdot \cancel{3} \cdot \cancel{2}}{\cancel{3} \cdot \cancel{2} \cdot \cancel{1}} = 10}{\frac{{}^{3}\cancel{6} \cdot 5 \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2}}{\cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}} = 15} = \frac{2}{3}$$

$\{B, \cancel{J}, R, S, C, M\}$

$\{J, ?, ?, ?\}$

les $|A| = n$

$2^A := \{B : B \subseteq A\} = \{B : B \subseteq A \;\&\; |B| = 0\} \cup$

$\{B : B \subseteq A \;\&\; |B| = 1\} \cup$

$|2^A| = 2^n$

$\{B : B \subseteq A \;\&\; |B| = 2\} \cup$

$\vdots$

$A = \{1, 2, 3\}$

$\cup \{B : B \subseteq A \;\&\; |B| = n\}$

mutually exclusive & collectively exclusive

$2^A = \{\underbrace{\{\}}_{\text{size 0}}, \underbrace{\{1\}, \{2\}, \{3\}}_{\text{size}}, \underbrace{\{1,2\}, \{2,3\}, \{1,3\}}_{\text{size 2}}, \underbrace{\{1,2,3\}}_{\text{size 3}}\}$

$|\Omega| = |A_1| + |A_2|$

$|2^A| = \sum_{i=0}^{n} |\{B : B \subseteq A \;\&\; |B| = i\}| = \sum_{i=0}^{n} \binom{n}{i} = 2^n$

$(a+b)^2 = (a+b)(a+b) = a^2 + ab + ba + b^2 = a^2 + 2ab + b^2$

$(a+b)^3 = (a+b)(a+b)(a+b) = \underbrace{\quad}_{2^3 = 8} = a^3 + 3a^2b + 3ab^2 + b^3$

$(a+b)^4 = \cdots = \underbrace{\quad}_{2^4 = 16} = \underset{1}{\binom{4}{4}} a^4 b^0 + \underset{4}{\binom{4}{3}} a^3 b^1 + \underset{6}{\binom{4}{2}} a^2 b^2 + \underset{4}{\binom{4}{1}} a^1 b^3 + \underset{1}{\binom{4}{0}} a^0 b^4$

$(a+b)^n = \binom{n}{0} a^0 b^n + \binom{n}{1} a^1 b^{n-1} + \cdots = \sum_{i=0}^{n} \binom{n}{i} a^i b^{n-i}$ Binomial Theorem