Lecture 2, side A

Special Set denoted Ω, called the "Universe", "sample space", "scope".

Ex If $\Omega = F \cup M$, then $F \subseteq \Omega$, $M \subseteq \Omega$, $F \cap \Omega = F$

$\phi \cup \Omega = \Omega$ $\quad \phi \cap \Omega = \phi$ $\quad A \setminus \Omega = \phi$ ↓ Any arbitrary set

A^c ("A-complement") $= \Omega \setminus A \Rightarrow$ everything that is not A.

$A \cup A^c = \Omega$ $\quad A \cap A^c = \phi$

$\{A_1, A_2, A_3 \cdots\}$ are "collectively exhaustive" if $A_1 \cup A_2 \cup = \bigcup_{i=1}^{\infty} A_i = \Omega$

$\{A_1, A_2, A_3 \cdots\}$ are mutually exclusive if $A_i \cap A_j = \phi \ \forall i \neq j$

Not in Test: When we say $|A| = |B|$, that mean there exist a function matches A from B that is one to one and onto

$[a,b] = \{x : x \geq a \ \& \ x \leq b\}$ $\quad (a,b) = \{x : x > a \ \& \ x < b\}$

Ordered Pair

$\langle a,b \rangle = \{\{a\}, \{a,b\}\}$

$\langle b,a \rangle = \{\{b\}, \{a,b\}\}$

$\langle a,a \rangle = \{\{a\}, \{a,a\}\} = \{\{a\}\} \neq \{a\}$

$\{a\}$ → $\{a\}$

Set/Cartesian Product $A \times B = \{\langle a,b \rangle : a \in A, b \in B\}$

If $A = \{1,2\}$, $B = \{3,4\}$, then $A \times B = \{\langle 1,3 \rangle, \langle 1,4 \rangle, \langle 2,3 \rangle \langle 2,4 \rangle\}$

↳ $|A| = 2$, $|B| = 2$ $\quad |A \times B| = 4 \Rightarrow |A \times B| = |A||B|$

$A^2 = A \times A \Rightarrow |A^2| = |A|^2 \Rightarrow |A^n| = |A|^n$

Lecture 2, Side B

Probability

Ω is now called the "experimental space" or "outcome space" and its denoted are called "outcome" and denoted w ($w \in \Omega$). When an experiment is performed one outcome is its result. For example, the coin flip experiment.

If p is the set function called "probability of w", then $\mathbb{R}$ domain range

$$p(\{H\}) = \frac{|\{H\}|}{|\Omega|} = \frac{1}{2}$$

$p: \Omega \to (0,1)$ X bad definition

$$p(\{H,T\}) = \frac{|\{H,T\}|}{|\Omega|} = \frac{2}{2} = 1$$

$p: 2^{\Omega} \to (0,1)$ ✓ good definition

Ex Die Roll Experience

$\Omega = \{1,2,3,4,5,6\}$

$$p(\text{even \#}) = \frac{|\{2,4,6\}|}{|\Omega|} = \frac{3}{6} = \frac{1}{2}$$

working definition

The pro of "event" A

$$P(A) = \frac{|A|}{|\Omega|}$$

2^{Ω} is called "event space", ~~A~~ set $A \subseteq \Omega$ is called event.

(Total possible question to ask)

$2^6 = 64$

Trival Event

$p(\phi) = 0 \quad P(\Omega) = 1$