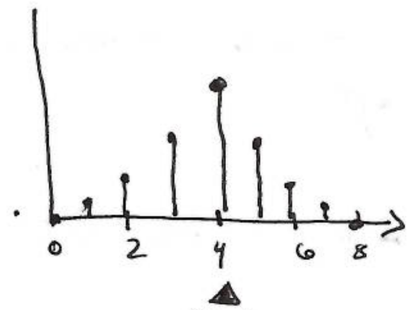
10/25/17

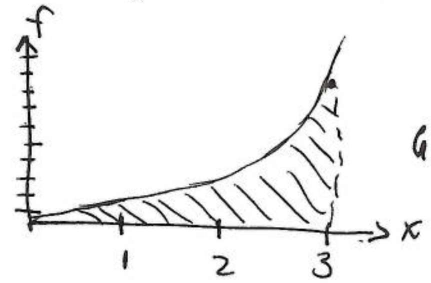
$X \sim \text{Binomial}\left(8, \frac{1}{2}\right)$



Pivot point, is a good summary that explains this distribution

Detour

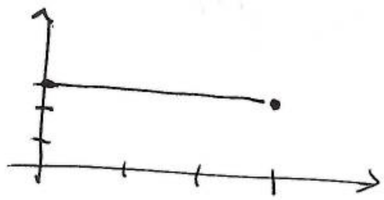
$f(x) = x^2 \quad , x \in A = [0,3)$



Operator "function of a function"

$G(f) \quad \int_0^3 f(x)\,dx = 9$

$f(x) = 3 \quad , x \in A$



$G(f) \quad \int_0^3 f(x)\,dx = 9$

Detour

$$\sum_{\text{objects}} w_i (d_i - d^*) = 0 \Rightarrow \sum w_i d_i - \sum w_i d^* = 0$$

$$\Rightarrow \sum w_i d_i = d^* \sum w_i$$

$$\Rightarrow d^* = \frac{\sum w_i d_i}{\sum w_i}$$

"Expectation"

$$M = \frac{\sum_i P(x_i)\, x_i}{\sum_i P(x_i)} = \frac{\sum_{x \in \text{Supp}[x]} x\, p(x)}{\underbrace{\sum_{x \in \text{Supp}[x]} P(x)}_{=1}} = \boxed{\begin{matrix} E[x] \\ \parallel \\ \sum_{x \in \text{Supp}[x]} x\, p(x) \end{matrix}}$$

$\overline{X} \rightarrow E[X]$

$$\overline{X} = \frac{x_1 + \dots x_n}{n}$$

$$E[X] = \underbrace{0p(0)}_{0} + \underbrace{1 \cdot p(1)}_{\binom{8}{1}\left(\frac{1}{6}\right)^1\left(1-\frac{1}{2}\right)^7} + 2p(2) + 3p(3) + 4p(4) + 5p(5) + 6p(6) + 7p(7) + 8p(8)$$

$$= 0 + .031 + 2(.109) + 3(.219) + 4\cdot(.273) + 5\cdot(.213) + 6\cdot(.109) + 7(.031)$$
$$8(.004) = \boxed{4}$$

$X \sim Bin(8{,}976,\ 0.38279)$

$X \sim Bin(n, p),\ E[X] =$

$$E[X] = \sum_{x=0}^{n} x \binom{n}{x} p^x (1-p)^{n-x}$$

$$= p\sum_{x=1}^{n} x \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x} = np \sum_{x=1}^{n} \binom{n-1}{x-1} p^{x-1}(1-p)^{n-x}$$

$(x-1)!$ $((x-1)-(n-1))!$ $\binom{n-1}{x-1}$

~~Let y =~~

Let $y = x-1$

$$= np\sum_{y=0}^{n-1} \binom{n-1}{y} p^y (1-p)^{n-y-1}$$

Let $m = n-1$

$$= np \sum_{y=0}^{m} \binom{m}{y} \underbrace{p(1-p)^{m-y}}_{Bin(m,p) = p(y)} =$$

$X \sim hyper(n, K, N)$

$$E[X] = \sum_{x \in supp[X]} x \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}} = \dots = n\left(\frac{K}{N}\right)$$

$X \sim \text{Unif}(\{1, 10, 100\})$

$E[X] = 1 \cdot \frac{1}{3} + 10 \cdot \frac{1}{3} + 100 \cdot \frac{1}{3} = \frac{111}{3}$

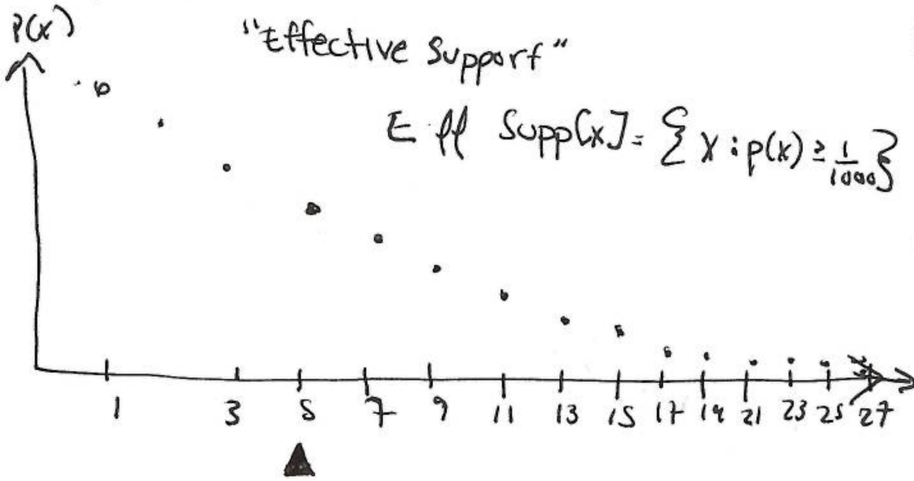
"how many times until you succeed"

$X \sim \text{Geometric}(.2) = .8^{x-1} \cdot .2$

$\text{Supp}[X] = \mathbb{N}$

"Effective Support"

$\text{Eff Supp}[X] = \{x : p(x) \geq \frac{1}{1000}\}$



X	P(x) (goes to 0)	F(x) (goes to 1)
1	.200	.200
2	.160	.360
3	.128	.488
4	.102	.590
5	.082	.672
6	.066	.738
7	.052	.790
8	.042	.832
9	.034	.866
10	.027	.893
11	.021	.914
12	.017	
13	.014	
14	.011	
15	.009	
16	.007	
17	.006	
18	.005	
19	.004	
20	.003	
21	.002	
22	.001	
23	.001	
24	.001	
25	.001	
26	.001	.998
27	.001	.999

$X \sim \text{Geom}(p)$ *don't need to know this proof*

$$E[X] = \sum_{x=1}^{\infty} x(1-p)^{x-1}p = \sum_{y=0}^{\infty}(y+1)(1-p)^y p =$$

let $y = x-1$

$$= \sum_{y=0}^{\infty} y(1-p)^y p + \sum_{y=0}^{\infty}(1-p)^y p$$

$$= \underbrace{(1-p)\sum_{y=1}^{\infty} y(1-p)^{y-1}p}_{M} + \underbrace{\sum_{x=1}^{\infty}(1-p)^{x-1}p}_{1}$$

$\Rightarrow M = (1-p)M + 1$

$\Rightarrow M(1-(1-p)) = 1$

$\Rightarrow Mp = 1 \Rightarrow M = \frac{1}{p}$

$\text{Mode}(x) = \text{argmax}\{p(x)\}$

$\text{Min}[x] = \min\{\text{supp}[x]\}$

$\max[x] = \max\{\text{supp}[x]\}$

$\text{Range}[x] = \max[x] - \min[x]$

$\text{Quantile}[X, p) = \text{argmin}\{F(v) \geq p\}$

first # greater than p.

$Q[x, p)$

$Q[x, 0.8] = 8$

$Q[x, .1] = 1$

$\cancel{Q[x]}$, $Q[x, 0.4] = 3$

$Q[x, .99] = 20$

$\text{median}[x] = Q[x, .5]$

$IQR[x] = Q[x, .75] - Q[x, .25]$

- inter-quantile range

tertiles	Quantiles	med (x)	Quintiles	Deciles
$Q[x, \frac{1}{3}]$	$Q[x, \frac{1}{4}]$	$Q[x, \frac{3}{4}]$	$Q[x, \frac{1}{5}]$	$Q[x, \frac{1}{10}]$
$Q[x, \frac{2}{3}]$	$\cancel{Q}$		$Q[x, \frac{2}{5}]$	$Q[x, \frac{2}{10}]$
$\cancel{Q}$			$Q[x, 3/5]$	$\vdots$
			$Q[x, 4/5]$	$Q[x, \frac{9}{10}]$

$|\text{Mode}[x]| = 1 \Rightarrow X$ is unimodel