C

Lecture 23 Math 201 12/6/17

DECISION

TRUTH	Retain H_0	Reject H_0
H_0 true	✓	Type I error
H_0 false	Type II error	✓

$\hat{p}$'s happen!

$P(\text{Type I error}) := P(\text{Reject } H_0 \mid H_0 \text{ true}) = \alpha$

$P(\text{Type II error}) := P(\text{Retain } H_0 \mid H_0 \text{ false}) = \ldots$

next class

$\text{Power} := 1 - P(\text{Type II error}) = P(\text{Reject } H_0 \mid H_0 \text{ false}) = \ldots$

find a new theory

P(Type I err) ↑ ⇒ P(Type II err) ↓

P(Type I err) ↓ ⇒ P(Type II err) ↑

whack-a-mole!

Clinical trial

H_0: drug does not work (want to be sure it does)

H_a: drug works

decision: release drug

Type I err: reject H_0 if true: release drug which doesn't work to market. Cost?

Type II error: accept H_0 when false: fail to release working drug to market. Cost?

Alarm System

H_0: No fire

H_a: Fire

Decision: alarm

Type I err: "false alarm", Cost?

Type II err: fire but no alarm, Cost?

α should be high here!

Court

H_0: Innocent

H_a: Guilty

Decision: punish or not

Type I err: punish innocent person

Type II err: guilty person goes free

α?

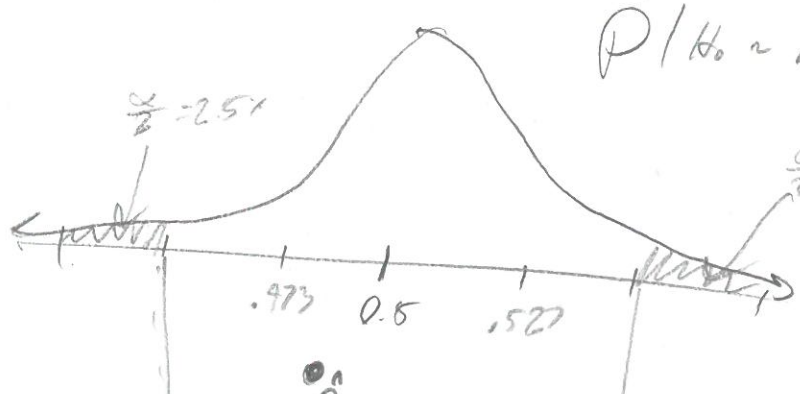
Recently, we ran a hypothesis test for equal human gender proportion $p := P(\text{Male})$

$H_0: p = 0.5$ (default position) $\leftarrow$ we don't want to just willy-nilly throw out our theory

$H_1: p \neq 0.5$ (crazy theory)

So we choose $\alpha = 5\%$, sample size $n = 345$

$$\hat{p} \,|\, H_0 \sim N\left(p, \sqrt{\tfrac{p(1-p)}{n}}^{\,2}\right) = N(0.5, .0269^2)$$



Reject H_0 too extreme less than 0.5

Retain H_0 $= [0.5 \pm 2 \cdot .0269] = [.446, .554]$

Reject H_0 too extreme more than 0.5

Two Sided Test for one proportion

169 male births $\Rightarrow \hat{p} = \frac{169}{345} = .49 \in$ Retainment Region $\Rightarrow$ Fail to reject H_0 — or — Retain H_0

Decision

Finally

	Retain H_0	Reject H_0
H_0 true	✓	Type I err
H_0 false	Type II err	✓

$\alpha := P(\text{Type I err})$ you choose this

$P(\text{Type I err}) \uparrow \Rightarrow P(\text{Type II err}) \downarrow$

$P(\text{Type I err}) \downarrow \Rightarrow P(\text{Type II err}) \uparrow$

Could be here! No way to know!!

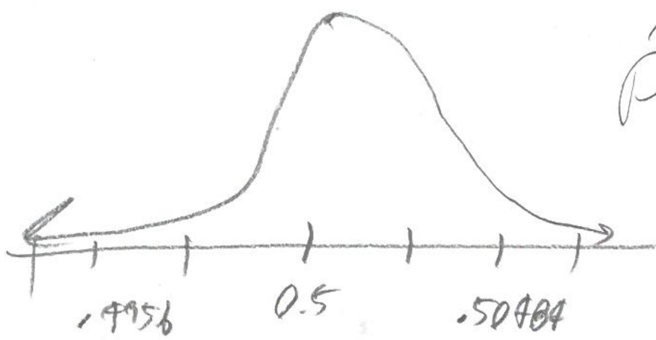
Return to M/F ratio.

$H_0: p = 0.5$

$H_a: p \neq 0.5$

$\alpha = 5\%$

In 2000 in USA $n = 4,247,000$ babies and 2,173,000 male



$$\hat{p} \sim N\left(0.5, \left(\sqrt{\frac{0.5 \cdot 0.5}{\ }}\right)^2\right)$$

Ret Region = [.49516, .50484]

$\hat{p} = 0.51165 \Rightarrow \hat{p} \notin$ Ret. Region $\Rightarrow$ Rej. H_0. There is sufficient evidence to believe gender ratio unequal. (Statistics can't figure it out)

But I retained H_0 before?
Why reject now?? $n \uparrow \Rightarrow$ more change!!!

$n \uparrow \Rightarrow P(\text{Type II err}) \downarrow$ Power $\Uparrow$ (ability to find effect / demonstrate fal. H_0)

$n \uparrow \Rightarrow P(\text{Type I err})$ no change!

Retain H_0 = Accept H_0? No

Before

$H_0: p = 0.5$ or $H_0: p = 0.500001$

$H_a: p \neq 0.5$ $H_a: p \neq 0.500001$

$\alpha = 5\%$, $n = 395$ $\alpha = 5\%$, $n = 395$

H_0 retained H_0 also retained

Accept H_0 for both is contradiction. Accept H_a for both is not a concern.

Not enough evidence to reject $p = 0.5$ or $p = 0.500001$ or so many others

All science works this way. We have a theory until we see enough evidence that it is bunk.

We don't throw theories away willy-nilly though.

We are conservative. Hence α is small.

I) UFO's & H_0: Aliens Don't Exist H_a: Aliens Do Exist α low	II) H_0: Aliens Don't Exist ← "traditional" H_a: Aliens Do exist α high
III) H_0: Aliens Do Exist H_a: Aliens Don't Exist α low	H_0: Aliens Do Exist H_a: Aliens Don't Exist α high