Math 241 Lecture 22 Dec 4^{th}

Statistical inference

we don't know parameter so we want to...

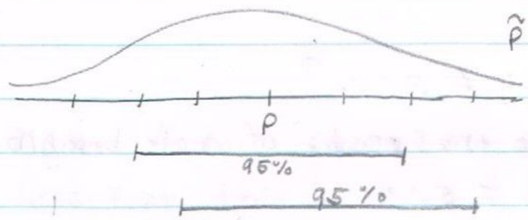
① Estimate its best guess

② Provide range of possible (likely) values

③ Test theories

$X_1, \ldots, X_n \overset{iid}{\sim} \text{Bern}(p)$

$\hat{p} = \bar{x} = \frac{x_1 + \ldots + x_n}{n} = \frac{\# \, 1\text{'s}}{n} \approx p$



$\hat{p} \sim N\left(p, \sqrt{\frac{p(1-p)}{n}}\right)$

$P\left(p \in \left[p \pm z_{\frac{\alpha}{2}} \sqrt{\frac{p(1-p)}{n}}\right]\right) = 1 - \alpha$

$P\left(p \in \left[\hat{p} \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right]\right) \approx 1-\alpha$

margin of error

$CI_{p,\,1-\alpha} := \left[\hat{p} \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right]$

confidence interval for parameter p with coverage $1-\alpha$

Interpretation of the CI

① If you sample many times and compute the CI for each, the $p \in CI$ $1-\alpha$ prob. of the time

NOT USEFUL

$CI_{p\,95\%} = [0.47, 0.57]$

$= [0.52 \pm 0.05]$

② Before you begin, your CI will contain p w.p. $1-\alpha$

NOT USEFUL

③ $P(p \in CI_{p,\,1-\alpha})$ → Deg (0) or Deg (1)

$= P(p \in [0.47, 0.57])$

everyone wants this: ④ $P(p \in CI_{p,\,1-\alpha}) = 1-\alpha$

only true if you are subjectivist with the right prior information

Do you like mushrooms?

$n = 20$ sample size (not a representative sample)

guess p: $\hat{p} = \frac{11}{20} = .55$ — means the probability that someone likes mushrooms

$\alpha = 5\%$ (95% coverage) $\Rightarrow z_{2.5\%} = 2$

$CI_{p.55\%} = \left[.55 \pm 2\sqrt{\frac{.55 - .45}{20}}\right] = [.33, .77]$

does not give inference for the population of all humans

Human sex ratio/proportion

Do you think?

P(new human baby being male) $\neq 50\%$?

No, I think its even. p is the prob. of male being born

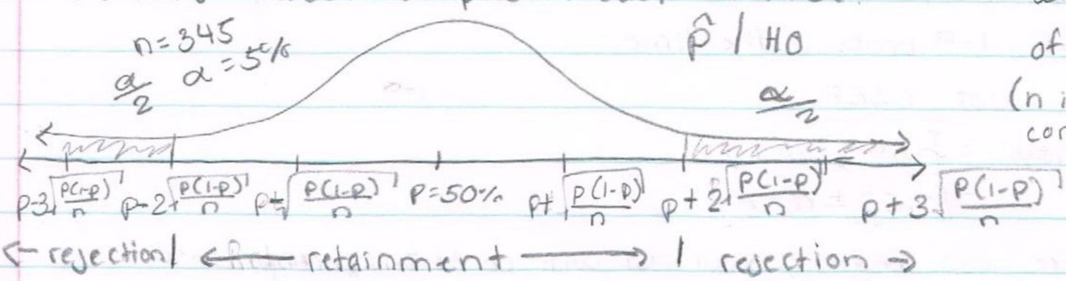
$H_0 : p = 50\%$, $H_a : p \neq 50\%$

null hypothesis, alternative hypothesis

We need "sufficient" evidence to reject the null hypothesis.

Occam's Razor: simplest model is true

we take a sample of size n. (n in decimal) compare $\hat{p}$



let $\alpha := P(\text{reject } H_0 \mid H_0 \text{ is false})$

$\alpha = 5\%$.

Retainment Region

$\left[p_0 \pm z_{\frac{\alpha}{2}}\sqrt{\frac{p_0(1-p_0)}{n}}\right]$ If $\hat{p} \in$ retainment region $\Rightarrow$ retain H_0

$\hat{p} \notin$ retainment region $\Rightarrow$ reject H_0

Rejection Region is the compliment of the retainment region.

$= \left[0.5 \pm 2\sqrt{\frac{0.5(1-0.5)}{345}}\right]$

169 babies male

$= [0.446, 0.554]$

To run the test compare $\hat{p} = 169/345 = 0.48 \in$ retainment region?

$\Rightarrow$ retain H_0

why do we need this?

Testing if coin is fair $H_0: p = 0.5$

situation 1 $n=100$, # heads $= .51$ fair? YES

situation 2 $n=100$, # heads $= .98$ fair? NO

Situation 3 $n=100$, # heads $= .61$ fair? ?

At $\alpha = 5\%$

$$= \left[0.5 \pm 2\sqrt{\frac{.5(1-.5)}{100}}\right]$$

$$= [0.40, 0.6]$$

$.61 \notin$ retainment region (the coin is less fair)

M&Ms factory says 20% are blue.

Lets test this $\alpha = 5\%$

$H_0: p_0 = 0.2$ $n = 271$

$H_0: p_0 \neq 0.2$ retainment region $= p_0 \pm 2\sqrt{\frac{p_0(1-p_0)}{n}}$

$$= 0.2 \pm 2\underbrace{\sqrt{\frac{(0.2)(0.8)}{271}}}_{0.025}$$

0.050

$$= [0.15, 0.25]$$

$$\hat{p} = \frac{50}{271} = 0.214$$

Decision

TRUTH	Retain H_0	Reject H_0
H_0 true	✓	type I error
H_0 false	type 2 error	✓

P(type I error)
= P(Rejecting H_0 | H_0 true)
= α

p(type II error) = p(retain H_0 | H_0 false) = ... advanced class

1 - P(type II error)
= P(rejecting H_0 | H_0 false)
= POWER = ... advanced class