09/14/2017

(I) Long Run Frequency / Limiting Frequency Def

$$P(A) = \lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} \mathbb{1}_{\omega_i \in A}$$

(II) Karl Popper, 1957

objects have $\underbrace{\text{inherent dispositions}}_{\text{"propensity"}}$ towards outcomes

"propensity" → micromachine in coin

propencity induces the long run frequency

Radioactive U238

$P(\underbrace{\text{U 238 atom explodes in} < 4.5 \text{ Billion years}}_{\text{can be calculated explicitly if you understand quantum mechanics}}) = \frac{1}{2}$

can be calculated explicitly
if you understand quantum mechanics

Problems

1. For most random experiments we dont know how to calculate the propensites of ω's
2. Not general, can't do P(OJ Simpson guilty)

I, II are objectivist theories

Ramsey (1926) de Finetti (1928)

III Subjectivist Definition · everyone uses their own evidence, biases, intuition to come up w/ their own estimate of uncertainty

$P_{Adam}(H) = 0.5$, P(Newton's F=ma is true)

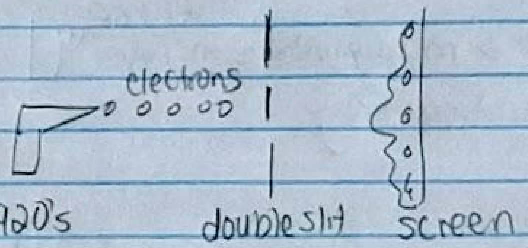
Problems

1. No one answer

Conclusion:
No accepted definition of probability

What is randomness?

choose $\omega \in \Omega$



Quantum mechanics
$\Rightarrow$ universe is random

Kolmogor 1930s
Mathematical Def. of Prob.

Assume $\exists\ \Omega \neq \phi$ P is a set function satisfying these 3 condition

Axioms
(a) $P(\Omega) = 1$
(b) $\forall A \subseteq \Omega,\ P(A) \geq 0$
(c) If $A_1, A_2 \ldots$ are disjoint $\Rightarrow P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i)$

Theorem 1
$P(A) = 1 - P(A^c)$
$\Omega = A \cup A^c$ and $[A, A^c]$ are disjoint
$P(\Omega) = P(A \cup A^c)$
$P(\Omega) = P(A) + P(A^c)$ by (c)
$1 = P(A) + P(A^c)$ by (A)
$P(A) = 1 - P(A^c)$ ✓

theorem II

$P(\emptyset) = 0$

$P(\emptyset) = 1 - P(\emptyset^c)$ by theorem I

$= 1 - P(\Omega)$ set theory

$= 1 - 1$ by Ⓐ

$= 0$ ✓

Theorem 3

$C = B \setminus A$

$C \cap A = \emptyset$

$A \subseteq B \Rightarrow P(A) \leq P(B)$

$P(B) = P(A \cup C) = P(A) + P(C)$

$P(B) - P(A) = P(C) \geq 0$ by ⓑ from set theory

$P(B) - P(A) \geq 0$

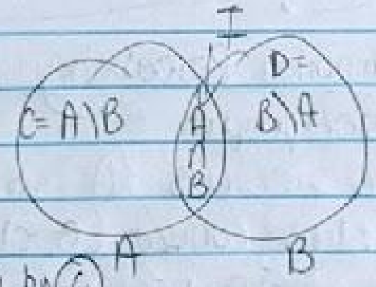
$P(B) \geq P(A)$

$P(A) \leq P(B)$ ✓

theorem 4

Law of inclusion - Exclusion

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$



$P(A \cup B) = P(C \cup I \cup D) = P(C) + P(I) + P(D)$ by ⓒ

C D I => disjoint

$P(C \cup I) = P(C) + P(I) = P(A)$ by ⓒ

$P(D \cup I) = P(D) + P(I) = P(B)$

$= P(A) - P(I) + \cancel{P(I)} + P(B) - \cancel{P(I)}$

$= P(A) + P(B) - P(I)$

$= P(A) + P(B) - P(A \cap B)$

Theorem 5

$|\Omega| < \infty$ if $P(\omega_i) = \frac{1}{|\Omega|} \; \forall \omega_i \Rightarrow P(A) = \frac{|A|}{|\Omega|}$

let $n = |A| < \infty$

since $|A \subseteq \Omega| \Rightarrow |A| \leq |\Omega|$

$A = \{\omega_1, \omega_2 \ldots, \omega_n\}$

$A = \{\omega_1\} \cup \{\omega_2\} \cup \ldots \cup \{\omega_n\}$

$P(A) = P\left(\bigcup_{i=1}^{n} \{\omega_i\}\right) =$

$\sum_{i=1}^{n} P(\{\omega_i\})$ by (c)

$= \sum_{i=1}^{n} \frac{1}{|\Omega|} = \frac{n}{|\Omega|} = \frac{A}{|\Omega|}$

Conditional Probability

n=1000 people

200 smokers (A = smoking)

60 lung cancer (B = lung cancer)

36 smoke + get lung cance (A ∩ B)

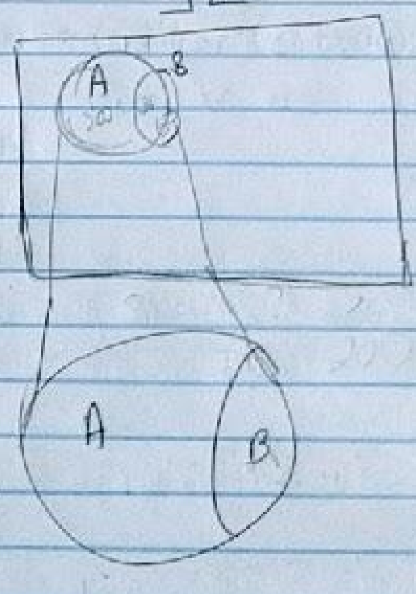
Assume for illustrative purposes

$P(A) = 0.2$

$P(B) = 0.06$

$P(A \cap B) = 0.036$

Ω



$\Omega' =$

What is the probability of lung cancer among smokers?

$\frac{36}{200}$

new universe

conditional probability $P(B|A)$

pipe denote "conditioning" on event A

"B given / conditional A"