10/18/17

$$\frac{n!}{x!(n-x)!}\lim_{N\to\infty}\frac{pN!\,((1-p)N)!\,(\cancel{N-n})!}{(pN-x)!\;\;((1-p)N-(n-x))!\;\cancel{N!}}$$

$$=\binom{n}{x}\lim_{N\to\infty}\frac{pN!}{(pN-x)!}\frac{((1-p)N)!}{((1-p)N-(n-x))!}$$

n-x terms

n terms $\to$ $(pN)(pN-1)(pN-2)\dots(pN-x+1)$; $((1-p)N)((1-p)N-1)\dots((1-p)N-(n-x)+1)$

$$=\binom{n}{x}\lim_{N\to\infty}\underbrace{\frac{\cancel{pN}}{N}}_{p}\lim_{N\to\infty}\underbrace{\frac{\cancel{pN-1}}{N-1}}_{p}\lim_{N\to\infty}\underbrace{\frac{\cancel{pN-2}}{N-2}}_{p}\cdots\lim_{N\to\infty}\underbrace{\frac{\cancel{pN-x+1}}{N-x+1}}_{p}.$$

$$\lim_{N\to\infty}\underbrace{\frac{\cancel{(1-p)N}}{N-x}}_{1-p}\cdots\lim_{N\to\infty}\underbrace{\frac{\cancel{(1-p)N-(n-x)+1}}{N-n+1}}_{1-p}$$

$$=\binom{n}{x}p^{x}(1-p)^{n-x}.$$

$$X\sim \text{Binomial}(n,p):=\binom{n}{x}p^{x}(1-p)^{n-x}.$$

$\text{Supp}[X]=\{0,1,2,\dots,n\}$.

Parameter Space $n\in\mathbb{N}$

$p\in(0,1)$

$\sum_{x\in \text{Supp}[X]} p(x)=1$. To prove $p(x)$ is valid.

$$\sum_{x=0}^{n}\binom{n}{x}p^{x}(1-p)^{n-x}=\binom{n}{n}((p)+(1-p))^{n}$$

$$=1^{n}=1.$$

Binomial Theorem $(a+b)^n = \sum_{i=0}^{n} \binom{n}{i} a^i b^{n-i}$

$X_1 \sim \text{Bern}(\frac{1}{3})$

$X_2 \sim \text{Bern}(\frac{1}{3})$

$X_1 \overset{d}{=} X_2$.

Generally, X_1 & X_2 are "indepedent" if $(X_1, X_2$ indepe$)$

(a) $P(X_2 = x_2 \mid X_1 = x_1) = P(X_2 = x_2)$

$\forall x_1 \in \text{Supp}[X_1]$ and $\forall x_2 \in \text{Supp}[X_2]$

(b) $P(X_1 = x_1 \mid X_2 = x_2) = P(X_1 = x_1)$

(c) $P(X_1 = x_1, X_2 = x_2) = P(X_1 = x_1)\, P(X_2 = x_2)$.

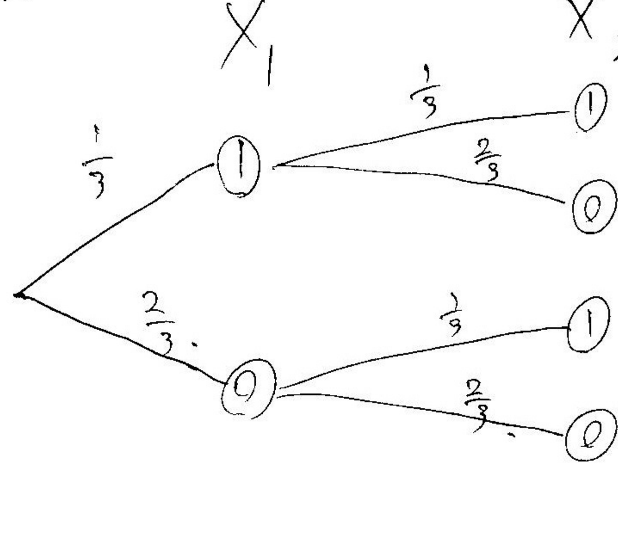
$X_1 X_2 \overset{iid}{\sim}$ means $X_1 X_2 \overset{ind}{\sim}$ & $X_1 \overset{d}{=} X_2$.

$X_1 X_2 \overset{iid}{\sim}$ ~~Bi~~ $\text{Bern}(\frac{1}{3})$

$T_2 = X_1 + X_2$.

$\text{Supp}[X] = \{\underbrace{0}_{\text{both } 0}, \underbrace{1}_{\text{either } 0}, \underbrace{2}_{\text{both } 1}\}$.

$$T_2 \sim \begin{cases} 0 & \text{imp } \frac{2}{3} \cdot \frac{2}{3} = \frac{4}{9}. \\ 1 & \text{imp } \frac{2}{3} \cdot \frac{1}{3} + \frac{1}{3} \frac{2}{3} = \frac{4}{9}. \\ 2 & \text{imp. } \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9}. \end{cases}$$

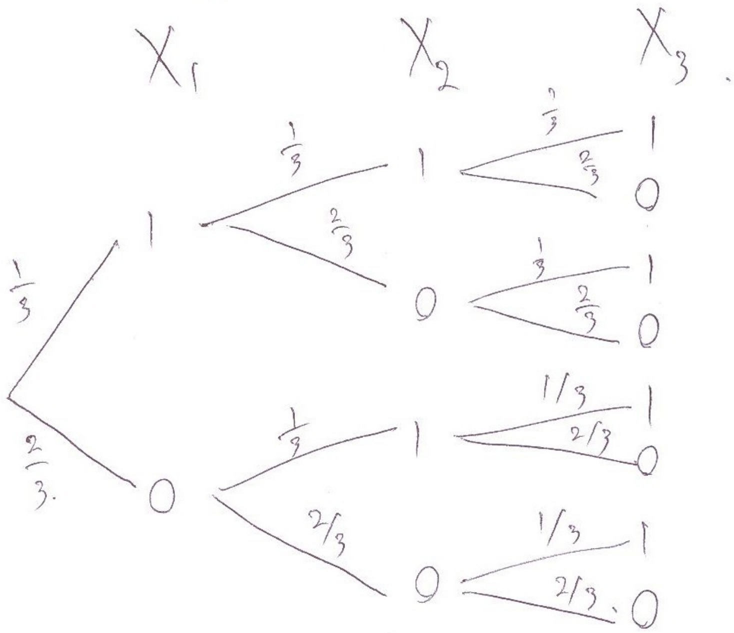


		T_2.
$P(X_1 = 1, X_2 = 1)$	$= \frac{1}{9}$	2.
$P(X_1 = 1, X_2 = 0)$	$= \frac{2}{9}$	1
$P(X_1 = 0, X_2 = 1)$	$= \frac{2}{9}$	1
$P(X_1 = 0, X_2 = 0)$	$= \frac{4}{9}$.	0

$X_1, X_2, X_3 \overset{iid}{\sim} Ber(\frac{1}{3})$.

$T_3 = X_1 + X_2 + X_3$.

$Supp[X] = \{0, 1, 2, 3\}$.



	T_3
$(\frac{1}{3})^3(\frac{2}{3})^0 = \frac{1}{27}$	3
$(\frac{1}{3})^2(\frac{2}{3})^1 = \frac{2}{27}$	2
$(\frac{1}{3})^2(\frac{2}{3})^1 = \frac{2}{27}$	2
$(\frac{1}{3})^1(\frac{2}{3})^2 = \frac{4}{27}$	1
$(\frac{1}{3})^2(\frac{2}{3})^1 = \frac{2}{27}$	2
$(\frac{1}{3})^1(\frac{2}{3})^2 = \frac{4}{27}$	1
$(\frac{1}{3})^1(\frac{2}{3})^2 = \frac{4}{27}$	1
$(\frac{1}{3})^0(\frac{2}{3})^3 = \frac{8}{27}$	0

$$p(T_3 = 1) = \binom{3}{1}\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^2 = \frac{12}{27}.$$

$$p(T_3 = 2) = \binom{3}{2}\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^1 = \frac{8}{27}.$$

$$p(T_3 = 3) = \binom{3}{3}\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^0 = \frac{1}{27}.$$

$$p(T_3 = 0) = \binom{3}{0}\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^3 = \frac{8}{27}.$$

$X_1, \dots, X_n \overset{iid}{\sim} Ben\left(\frac{1}{3}\right) \leftarrow p.$

$$T_n = \sum_{i=1}^{n} X_i$$

$$Supp[T_n] = \{0, 1, 2, \dots, n\}.$$

$$T_n \sim \begin{cases} 0 & wp \binom{n}{0}\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^n \\ 1 & wp \binom{n}{1}\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^{n-1} \\ 2 & wp \binom{n}{2}\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^{n-2} \\ \vdots & \\ n-1 & wp \binom{n}{n-1}\left(\frac{1}{3}\right)^{n-1}\left(\frac{2}{3}\right)^1 \\ n & wp \binom{n}{n}\left(\frac{1}{3}\right)^n\left(\frac{2}{3}\right)^0. \end{cases}$$

$$\binom{n}{x} p^x (1-p)^{n-x}$$

$$= Bin(n, p).$$

Binomial can be ~~comp~~ conceptualized as.

① . $T = \lim_{N \to \infty} Hgper(n, p, N)$

② . $T = \sum_{i=1}^{n} X_i$ s.t $X_1, \dots, X_n \overset{iid}{\sim} Ben(p)$.

$$p(x) = p(X = x)$$

$$F(x) = P(X \leq x).$$

CDF of binomial.

$$F(x) = \sum_{i=0}^{n} \binom{n}{i} p^i (1-p)^{n-i}$$

no closed form.