LECTURE 01 : 08 | 29 | 2017

* Set Theory (1840's) :-

Set are the fundamental unit of all of Mathematics.

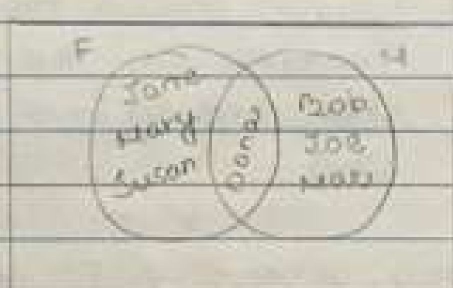
A set is a collection of elements which are unordered and unique.

assigned or defined as

Ex: $F := \{ \text{Jane}, \text{Mary}, \text{Susan}, \text{Dana} \}$

denotes the set on the left — begin — elements — end

$M := \{ \text{Bob}, \text{Joe}, \text{Max}, \text{Dana} \}$



** Sets can have infinite elements

Ex: $\mathbb{N} := \{ 1, 2, 3, \ldots \}$ continuation of the pattern

Natural numbers

$\mathbb{Z} := \{ \ldots, -2, -1, 0, 1, 2, \ldots \}$

* Operators on Sets :-

$\in$: Jane $\boxed{\in}$ F → "Jane is an element of the set F."

$\notin$: Bob $\boxed{\notin}$ F → "Bob is not an element of set F."

$\subseteq$: {Jane, Mary} $\boxed{\subseteq}$ F → "The set {Jane, Mary} is a subset of F"

Def: Subset means that all the elements in the set on the LHS are in the set on the RHS.

$\not\subseteq$: {Bob, Jane} $\not\subseteq$ F → "The set {Bob, Jane} is not a subset of F."

* F' := {Jane, Mary, Susan, Dana}

$\therefore$ $\boxed{F' = F}$? [$F \subseteq F'$ and $F' \subseteq F$]

if, $A = B \rightarrow A \subseteq B$ and $B \subseteq A$

* Proper subset :-

When A is a subset of B for example, then set A is not equal to B.

Ex: $\{Jane, Mary\} \neq F$

$\{Jane, Mary\} \subset F$

proper set

$\therefore A \subset B \rightarrow A \subseteq B$

but $A \neq B$

Ex: $\{Jane\} \subset F \longrightarrow$ TRUE

$\{Jane\} \in F \longrightarrow$ FALSE

$Jane \in F \longrightarrow$ TRUE

$Jance \notin F \longrightarrow$ TRUE

** $\in, \notin, =, \neq, \subset, \subseteq \rightarrow$ returns "TRUE"/"FALSE" results and are described as predicate functions.

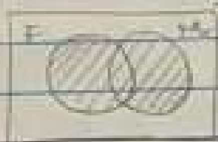
Ex: $\in (Jane, F) =$ TRUE

* Union :-

∴ $F \cup M$ = { Jane, Mary, Susan, Dana, Bob, Joe, Max }

union

* Union → combines all elements in both sets, but no repetations are done.



→ one sort of "ADDITION" but not "ADDITION".

** "AND" / "OR" → non-exclusive or

Ex: { Dana } ∪ { Dana } = { Dana }

Dana ∈ (M ∪ F) → TRUE

$\mathbb{N}_0 := \mathbb{N} \cup \{0\} = \{0, 1, 2, \ldots\}$

all natural number set

* Intersection :-

∴ $F \cap M$ = { Dana }

intersection

intersection set → "AND"

common part is taken.

Ex: $F \cap \{Bob, Joe\} = \{\ \}$

$\phi := \{\ \}$ ← empty/null set

If, A, B have infinite elements then can :

$\therefore A \cap B = \phi \rightarrow$ TRUE

$A = \{2, 4, 6, \ldots\}$

$B = \{1, 3, 5, \ldots\}$

As, they donot share anything in common, so, if

$A \cap B = \phi$

$\Rightarrow$ A, B are Mutually Exclusive or Disjoined.

Ex: $\phi \not\subset F \rightarrow$ TRUE ← vacuously true ← empty sets is a subset of every set.

$\phi \in F \rightarrow$ FALSE

$\phi \in F \rightarrow$ TRUE

* Set subtraction :-

$\therefore F \setminus M = \{Jane, Mary, Susan\}$

all elements in F except those elements of M

$M \setminus F = \{Bob, Joe, Max\}$

↳ not the same as $F \setminus M$

Ex: if,

$A \cap B = \phi$ then,

$A \setminus B = A$ and $B \setminus A = B$

↳ $A \subseteq B \rightarrow A \setminus B = \phi$

if,

$A \setminus B = \phi$ then $A \cap B = A$

Null Set : $\left\{ \begin{array}{l} \phi \setminus \phi = \phi \\ \phi \cap \phi = \phi \\ \phi \cup \phi = \phi \end{array} \right\}$

* Set Building Notation :-

$$E := \{2n \overset{\text{such that}}{:} n \in \mathbb{Z}\}$$ n is an integer

$$= \{\ldots, -6, -4, -2, 0, 2, 4, 6, \ldots\}$$

∴ "E is a new set with element 2n such that n is an integer"

* Power Set :-

The power set is the set of all possible subsets of the given sets.

Ex: $2^A := \{ B : B \subseteq A \}$

let, $A = \{1, 2, 3\}$ → 3 digit binary

then.

null set

$2^A = \{ \phi, \{1\}, \{2\}, \{3\}, \{1,2\},$
$\{2,3\}, \{1,3\}, A \}$

whole set A

∴ 2^A is defined to be the set B such that B is a subset of A.

* Size of a set (cardinality):-

$|A| = 3$

absolute value sign

[$|A|$ = number of element of set

Ex: $|2^A| = 8$

$|F \cup M| = |F| + |M|$
$= 7 \neq 4 + 4$

× ——— × ——— × ———