... Continuing <u>Set Theory</u> with Power Sets and Cardinality ...

$$2^A = \{B : B \subseteq A\} = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{1,2,3\}\}$$

$A = \{1,2,3\}$ $\quad |A| = 3$ $\quad |2^A| = 8 = 2^3$

<u>Special Set</u> denotes Ω

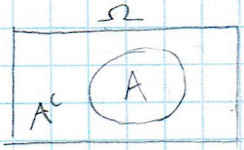
This is called the "Universe", "Sample Space", and/or "Scope"

$\Omega = F \cup M$ $\quad F \subseteq \Omega$ $\quad A \cap \Omega = A$ $\quad \emptyset \cup \Omega = \Omega$ $\quad A \setminus \Omega = \emptyset$

"defining my universe" $\quad F \subset \Omega$ $\quad A \cup \Omega = \Omega$ $\quad \emptyset \cap \Omega = \emptyset$

$A^c := \Omega \setminus A$

"A-complement"
or "everything that is not A"



$A \cup A^c = \Omega$
$A \cap A^c = \emptyset$

$\{A_1, A_2, A_3, \dots\}$ are "collectively exhaustive" if

$$A_1 \cup A_2 \cup \dots = \bigcup_{i=1}^{\infty} A_i = \Omega$$

$\{A_1, A_2, A_3, \dots\}$ are "mutually exclusive" if

$$A_i \cap A_j = \emptyset \;\forall i \neq j$$

<u>Ordered Pairs</u>

$\langle a,b \rangle := \{\{a\}, \{a,b\}\}$ $\quad \neq \quad$ $\langle b,a \rangle := \{\{b\}, \{a,b\}\}$

Side-Note: $\langle a,a \rangle := \{\{a\}, \underbrace{\{a,a\}}_{}\}$ ($\{a,a\} = \{a\}$) $= \{a\}$

<u>Set / Cartesian Product</u>

$A \times B := \{\langle a,b \rangle : a \in A, b \in B\}$

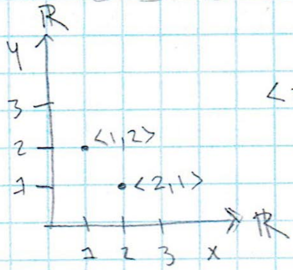
e.g. $A = \{1,2\}$, $B = \{3,4\}$ $\quad A \times B = \{\langle 1,3 \rangle, \langle 1,4 \rangle, \langle 2,3 \rangle, \langle 2,4 \rangle\}$

$|A| = 2$ $\quad |B| = 2$ $\quad |A \times B| = 4$ $\quad |A \times B| = |A||B|$ $\quad$ In general $|A_1 \times A_2 \times \dots \times A_n| = \prod_{i=1}^{n} |A_i|$

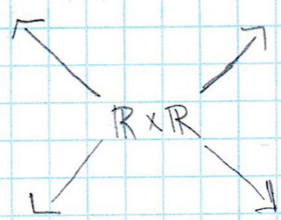
$A^2 := A \times A$

$|A^2| = |A|^2$

$|A^n| = |A|^n$



$\langle 3.7, 3.914 \rangle$



SET THEORY COMPLETE!

PROBABILITY

Ω is now called the "experimental space" or "outcome space" and its elements are called "outcomes" and denoted ω (little omega) ($\omega \in \Omega$). When an experiment is performed, are outcome is its result. For example, the coin toss experiment.

$$\Omega = \{\underset{\omega_1}{H}, \underset{\omega_2}{T}\}$$

"p" is the set function called "probability of"

$$P(\{H\}) = \frac{|\{H\}|}{|\Omega|} = \frac{1}{2} \qquad P(\{H,T\}) = \frac{|\{H,T\}|}{|\Omega|} = \frac{2}{2} = 1$$

Is this a good definition?

$P: \Omega \to [0,1]$

No! Why? This is not a good definition because it only describes elements of a set, therefore it can only solve the probability of 1 element at a time.

This is a good definition

$P: 2^{\Omega} \to [0,1]$

all subsets of Ω

This definition has all possible subsets of Ω, therefore can calculate all probability questions of Ω.

For Example

Die Roll Experiment

$$\Omega = \{1,2,3,4,5,6\}$$

$$P(\text{even \#}) = \frac{|\{2,4,6\}|}{|\Omega|} = \frac{3}{6} = \frac{1}{2}$$

Working Definition: The probability of "event" A is $P(A) := \frac{|A|}{|\Omega|}$

2^{Ω} is called "event space." A set $A \subseteq \Omega$ is called an event.