Average r.v. & Sum r.v.

Let $T_n = X_1 + X_2 + \dots + X_n$ — not necessarily iid

total r.v. aka **sum r.v.**

upper case X

$\bar{X}_n = \frac{T_n}{n} = \frac{X_1 + \dots + X_n}{n} = \frac{1}{n}\sum_{i=0}^{n} X_i$

average r.v.

n – sample size as in total number of conducted trials

Flip an unfair coin 3 times. Head=Success with prob. = 0.1. How many Heads?

e.g. $X_1, X_2, X_3 \overset{iid}{\sim} \text{Bernoulli}(0.1) = T_3 \sim \text{Binomial}(3, 0.1)$

A cup with 3 (unfair) coins. Head=Success with prob. = 0.1. Shake then capsize the cup. How many Heads?

$$T_3 \sim \begin{cases} 0 & \text{w.p. } \binom{3}{0}(.1)^0(1-p)^3 = 0.729 \\ 1 & \text{w.p. } \binom{3}{1}(.1)^1(1-p)^{3-1} = 0.243 \\ 2 & \text{w.p. } \binom{3}{2}(.1)^2(1-p)^{3-2} = 0.027 \\ 3 & \text{w.p. } \binom{3}{3}(.1)^3(1-p)^{3-3} = 0.001 \end{cases} \quad \Sigma=1$$

e.g. HTH 2 Heads

zero 1s, one 1s, two 1s, three 1s

$\text{Supp}[T_3] = \{0,1,2,3\}$

$\overset{X}{H} \to 1$

note: the **distribution** of an average r.v. $\bar{X}$ is different than that of a total r.v. T_n

$$\bar{X}_3 \sim \begin{cases} 0 & \text{w.p. } \binom{3}{0}(.1)^0(1-p)^3 = 0.729 \\ 1/3 & \text{w.p. } \binom{3}{1}(.1)^1(1-p)^{3-1} = 0.243 \\ 2/3 & \text{w.p. } \binom{3}{2}(.1)^2(1-p)^{3-2} = 0.027 \\ 1 & \text{w.p. } \binom{3}{3}(.1)^3(1-p)^{3-3} = 0.001 \end{cases} \quad \Sigma=1$$

e.g. HTH 2 out of 3 $\frac{1+0+1}{3}$

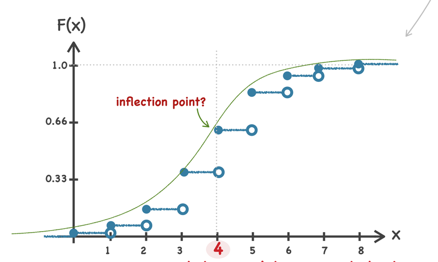
zero 1s, one 1s, two 1s, three 1s

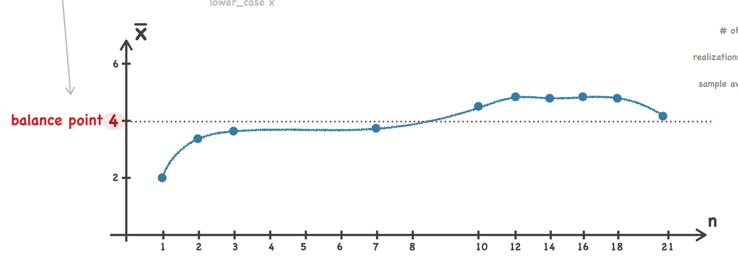
$\text{Supp}[\bar{X}_3] = \{\frac{0}{3}, \frac{1}{3}, \frac{2}{3}, \frac{3}{3}\}$

Def. lower case x, upper case X

sample average $\bar{x}$ is a realization of $\bar{X}$

$\bar{x} := \frac{1}{n}\sum_{i=0}^{n} x_i$ — lower case x





realized r.v. x, unrealized r.v. X, $p(x) := P(X=x)$

x , X , probability statement , PMF

$P(X=5'11'')$

prob. that you make a realization of this X and get x=5'11''

$P(X=5'11'') = ?$

r.v. for human adult height

$p(5'11'') = P(X=5'11'') = \text{PMF}$

We have to specify a model for X.
If no model, this is meaningless.

you need a function that links an element of support to the probabilities that come out of it, without such function you can not answer this question

function p(x): 1 w.p. 1/2; 0 w.p. 1/2 — support, probabilities

$X \sim \text{Binomial}(n,p) := \binom{n}{x} p^x (1-p)^{n-x}$

sample size; prob. of success; 1-p = prob of failure; n-x = # of failures; x = # of successes; p = prob. of success

1 cup: 8 fair coins , n=sample_size=8
shake then capsize the cup, how many HEADS ?

$X \sim \text{Binomial}(8, 1/2) := \binom{8}{x}(1/2)^x(1-1/2)^{8-x} = \binom{8}{x}(1/2)^8 = \frac{\binom{8}{x}}{2^8} = \frac{8!}{2^8}\frac{1}{x!(8-x)!}$

$\binom{8}{x} = \frac{8!}{x!(8-x)!}$

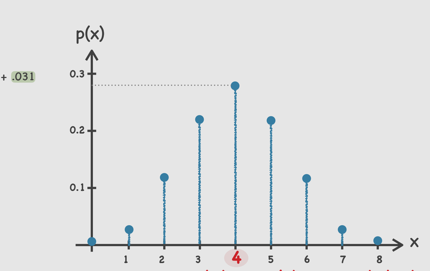
$(1/2)^8 \leftarrow x^n x^m = x^{n+m}$

$\binom{8}{x}(1/2)^8$

x	p(x) PMF P(X=x)	F(x) CDF P(X≤x)
0	.004	.004
1	.031	.035 = .004 + .031
2	.109	.144
3	.219	.363
4	.273	.636
5	.219	.855
6	.109	.964
7	.031	.995
8	.004	1
	Σ=1	

why equal ? 56 = 56 $\binom{8}{3} = \binom{8}{5}$

the sum over the support of the PMF is 1, otherwise it ain't no PMF :-)



1 cup: 8 coins , n=sample_size=8
shake then capsize the cup, how many HEADS ? – repeat to ∞

$X_1, X_2 \dots \sim \text{Binomial}(8, 1/2)$

realization build an empirical PMF and an empirical CDF. Show that they are about the same.

# of trials n :	1	2	3	4	5	6	7	8	9	10	11	12	13	14
realizations of X x :	2	5	4	4	3	5	4	5	6	5	7	4	5	3
sample average $\bar{x}$:	2	3.5	3.67				3.86			4.3		4.5		4.429

$\frac{x_1+x_2+x_3}{n} = \frac{2+5+4}{3}$

n :	15	16	17	18	19	20	21	…
x :	5	5	1	4	3	3	4	Supp[X]={0,…,8} (no Hs, all Hs)
$\bar{x}$:		4.5		4.3			4.14	≈ 4 balance point

after infinitely many trials the average number of Heads for all of the trials is 4, which makes sense since there are 8 coins in the cup

$\bar{X}$ sample average lower_case x

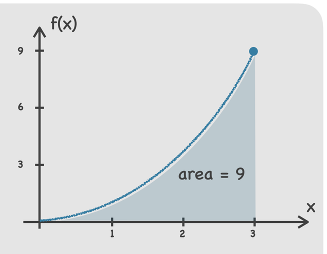
Mean

calculus

$f(x) = x^2 \quad x \in [0,3]$

function that takes a function $G[f] = \int_0^3 x^2\,dx = \frac{x^3}{3}\Big]_0^3 \; 9 - 0 = 9$

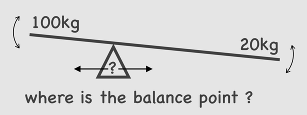
an integral is a function of a function



G: function → $\mathcal{R}$ (a single # e.g. 9)

it appears $\bar{x}$ (sample average) → **balance point or pivot point**

physics w = weight, d = distance



where is the balance point ?

$\sum_i w_i(d_i - d^*) = 0$

$\sum_i w_i d_i - \sum_i w_i d^* = 0$

$\sum_i w_i d_i = d^* \sum_i w_i$

$\frac{\sum_i w_i d_i}{\sum_i w_i} = d^*$ — analog — weights = probabilities PMF, distance = x — $\frac{\sum_i p_i x_i}{\sum_i p_i} = x^*$ (p_i = p(x), $\sum_i p_i = 1$)

$\frac{0\cdot100 + 1\cdot20}{100+20} = 0.17$

remember? summing all of the probabilities of the Support of a PMF = 1

$X \sim \begin{cases} 1 & \text{w.p. } 0.6 \\ 0 & \text{w.p. } 0.3 \\ 2 & \text{w.p. } 0.1 \end{cases}$ Σ=1

E[X] is another notation for μ – function of a function

interchangeable

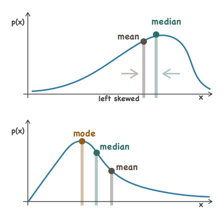
$E[X] := \mu := \sum_{x\in \text{Supp}[X]} x\,p(x)$ — E for Expectation, mu

the Sum of ALL members in the support of X multiplied by their probabilities

$E: X \to \mathcal{R}$ — space of r.v.'s, a single #

mean , expected value , expectancy , first moment

Law of Large #'s $\bar{X} \to \mu$ — you take the average of a lot of things and you get the expected value



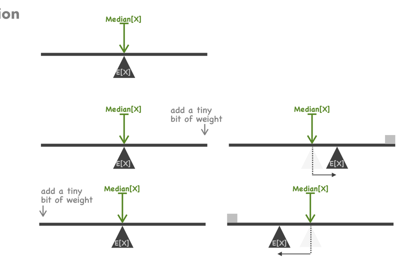
Warning!
Median and the Mean are NOT the same.
Median[X] ≠ E[X]

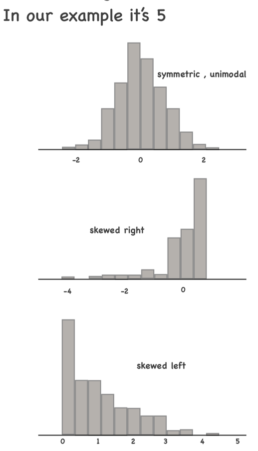
Median and the Mean are both measures of 'central tendency'

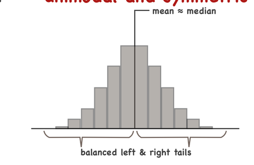
Median – says that there is 50% of the support below 4 and 50% above 4

Mean – says that if I balance a thing, where is the 'balancing point'. In our example it's 5

condition	types of distribution
Mean = Median E[X] = Median[X]	'symmetric'
Mean > Median E[X] > Median[X]	'skew right'
Mean < Median E[X] > Median[X]	'skew left'
| Mode[X] | = 1 one mode	'unimodal'
Mode[X] = Mean = Median Mode[X] = E[X] = Median[X]	'unimodal and symmetric'



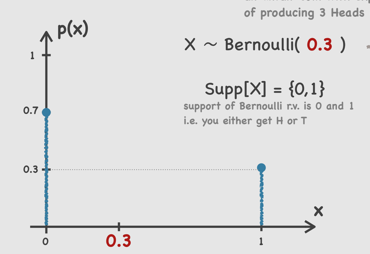




Mean – Bernoulli

an unfair coin with expectancy of producing 3 Heads in 10 flips

$X \sim \text{Bernoulli}(0.3)$



$\text{Supp}[X] = \{0,1\}$ — support of Bernoulli r.v. is 0 and 1 i.e. you either get H or T

$0.3 \in \text{Supp}[X]$? – NO, 0.3 is neither 0 nor 1

in general **$\mu \notin \text{Supp}[X]$**

X (r.v. function) taken through E (mean function) returns p (prob.)

$E[X] = 0\cdot p(0) + 1\cdot p(1) = p(1) = 0.3$ → or **p** for general Bernoulli

in the case of a single Bernoulli experiment E[X] aka μ = p
X (r.v. function) taken through E (mean function) returned p (prob.)

$$E[X \sim \text{Bernoulli}(p) := p^x(1-p)^{1-x}] = \mu = p$$

how about a 'mean' of a multiple iid Bernoulli experiments ?

$X_1, X_2, \dots, X_8 \overset{iid}{\sim} \text{Bernoulli}(0.3) = X \sim \text{Binomial}(8, 0.3)$

E[X] ≈ 2.4 — $8 \cdot 0.3 = 2.4$

$E[X] = \sum_{x=0}^{8} x\,p(x) = 0\cdot p(0) + 1\cdot p(1) + 2\cdot p(2) + 3\cdot p(3) + 4\cdot p(4) + 5\cdot p(5) + 6\cdot p(6) + 7\cdot p(7) + 8\cdot p(8)$

$x \in \text{Supp}[X] = \{0,\dots,8\}$

$\binom{8}{5}(0.3)^5(0.7)^3$ = 56 · .00243 · .343; $\binom{8}{8}(0.3)^8(0.7)^0$ = 1 · 0.000066 · 1

= 0·0 + 1·.198 + 2·.2965 + 3·.2541 + 4·.1361 + 5·.04668 + 6·.01 + 7·.0012 + 8·.000066
= 0 + .198 + .593 + .7624 + .5445 + .2333 + .06 + .00857 + .0005
≈ 2.4

How do we get the 'balance point'? We sum the products of all the values in the support of X and their corresponding probabilities. The above example had only 9 values in the support of X. However, the volume of calculations was quite large. Imagine Binomial(1000,0.3). Do we really need to perform 1000 product summations ?
Or is there a general formula which will allow us to skip this laborious process ?

Mean – Binomial

based on the above revelation:

if $E[X\sim\text{Bernoulli}(p)] = p$, $E[X\sim\text{Binomial}(n,p)] = np$????

Lets put the Binomial through the Mean function and see what comes out.

$E[X] = \sum_{x\in\text{Supp}[X]} x\,p(x) = \mu = \sum_{x=0}^{n} x \binom{n}{x} p^x(1-p)^{n-x}$ | expand the choose notation

$= \sum_{x=0}^{n} x \frac{n!}{x!(n-x)!} p^x(1-p)^{n-x}$ | $x! = x\cdot(x-1)!$; $\frac{n!}{(x-1)!(n-x)!}$ — $x\in\text{Supp}[0,\dots,n]$

$= \sum_{x=1}^{n} \frac{n!}{(x-1)!(n-x)!} p^x(1-p)^{n-x}$ | $(n-x)! = [(n-1)-(x-1)]! = (n-1-x+1)!$ — $0 \cdot \text{smthg} = 0$

$= n\sum_{x=1}^{n} \frac{(n-1)!}{(x-1)![(n-1)-(x-1)]!} p^x(1-p)^{((n-1)-(x-1))}$ | express in choose notation — $6! = 6\cdot5!$

n = n-1, k = x-1: $\frac{n!}{k!(n-k)!} = {}_nC_k = \binom{n}{k}$

$= n\sum_{x=1}^{n} \binom{n-1}{x-1} p^x(1-p)^{((n-1)-(x-1))}$ | create homogeneous terms

$= n\,p\sum_{x=1}^{n} \binom{n-1}{x-1} p^{x-1}(1-p)^{((n-1)-(x-1))}$ — $p^3 = p^2\cdot p = p^{3-1}\cdot p$

let: m = n-1, y = x-1; x = 1…n, y = 0…n-1 = 0…m

$= n\,p\sum_{y=0}^{m}\binom{m}{y} p^y(1-p)^{m-y}$ = 1 (as previously proved)

= **n p**

$$E[X \sim \text{Binomial}(p) := \binom{n}{x}p^x(1-p)^{n-x}] = \mu = n\,p$$

$X_1, X_2, \dots, X_8 \overset{iid}{\sim} \text{Bernoulli}(1/2) = X \sim \text{Binomial}(8, 1/2)$

E[X] ≈ 4 — $8 \cdot 0.5 = 4$

$E[X] = \sum_{x=0}^{8} x\,p(x) = 0\cdot p(0) + 1\cdot p(1) + 2\cdot p(2) + 3\cdot p(3) + 4\cdot p(4) + 5\cdot p(5) + 6\cdot p(6) + 7\cdot p(7) + 8\cdot p(8)$

$\binom{8}{5}(0.5)^8$ = 56 · .0039; $\binom{8}{8}(0.5)^8$ = 1 · 0.0039

= 0·0.004 + 1·.031 + 2·.109 + 3·.219 + 4·.273 + 5·.219 + 6·.109 + 7·.031 + 8·.004
= 0 + .031 + .218 + .657 + 1.092 + 1.095 + .654 + .217 + .032
≈ 4

balance point aka **mean** aka **expectation E[X]** aka **μ**

n · p = 8 · 0.5 = 4

self-note:
Conducting a single Bernoulli experiment (e.g. flip of a coin) means converting a theoretical r.v. X~Bernoulli into its practical 'realization' x equal to either 0 or 1. Shaking a cup of 8 coins is analogous to performing 8 simultaneous iid Bernoulli experiments which can also be expressed by X~Binomial. Every time we shake this cup of 8 coins, we can expect the number of Heads to be somewhere between 0 and 8. Since we know that there is 50% chance of getting a Head, it is reasonable to expect about 4 Heads per trial. This 'expectation' of 4 Heads is known under multiple names: 'mean' , 'expectation', 'first moment', and is denoted by the lower case greek letter μ or as E[X]. Performing the above 8 iid Bernoulli's can be mathematically expressed as the product of 8-flips and prob. of getting a Head. 8·0.5=4

end of class experiments…

1 cup: 7-coins , 4-marked , n=sample_size=3
shake the cup, quarry 3 coins, how many Marked?

$n\frac{K}{N} = 3\,\frac{4}{7} = 1.714$

X ~ Hypergeometric(n,K,N) (3,4,7) — P ⟶ n p = Binomial

n :	1	2	3	4	5	6	7	8	9	10	11	12	13	14
x :	1	2	1	2	2	2	2	2	3	2	1	2	1	2
$\bar{x}$:	1	1.5	1.33				1.71							1.786

n :	15	16	17	18	19	20	21	…
x :	2	3	1	2	2	1	0	Supp[X]={0,…,3}
$\bar{x}$:							1.714	≈ 1.714 balance point

1 cup: 6-coins , 2-marked , n=sample_size=1
shake the cup, quarry 1 coin, is it Marked?

X ~ Bernoulli(2/6) – balance point 2/6

1 cup: 6-coins , 4-marked , n=sample_size=1
shake the cup, quarry 1 coin, is it Marked?

X ~ Bernoulli(4/6) – balance point 4/6

∞ cups – in each cup: 6-coins , 2-marked , n=sample_size=1
shake the cups, stop on the 1st Marked coin:
quarry 1 coin, from the 1st cup , is it Marked? – NO
quarry 1 coin, from the 2nd cup , is it Marked? – NO
quarry 1 coin, from the 3rd cup , is it Marked? – Yes , STOP
— realization x=3

X ~ Geometric(2/6)

n :	1	2	3	4	5	6	7	8	9	10	11	12	13	14
x :	3	2	1	3	3	2	5	8	3	2	1	9	1	5
$\bar{x}$:		2.5	2				2.71							3.43

n :	15	16	17	18	19	20	21	…
x :	2	3	1	2	2	1	5	Supp[X]={1,…}
$\bar{x}$:							3.05	≈ 3 balance point

the alternative way to conduct this experiment:
1 cup with 6 coins in it. 2 of the coins are marked.
– shake the cup then quarry 1 coin, is it Marked ? – NO
– shake the cup then quarry 1 coin, is it Marked ? – NO
– shake the cup then quarry 1 coin, is it Marked ? – NO
– shake the cup then quarry 1 coin, is it Marked ? – Yes → STOP
record the number of shakes, repeat the entire process again

the alternative way of looking at this experiment:
The cup can be viewed as a single entity capable to deliver a marked coin with the probability of 1/3. It is as if we had a single 'unfair coin' with probability of Heads = 1/3. We keep flipping this 'unfair coin' until we get Heads. We record the number of flips. We repeat the experiment again.

$X\sim\text{Bernoulli}(1/3) := \begin{cases} 1 & \text{wp } 1/3 \quad (\text{Heads = marked coin}) \\ 0 & \text{wp } 2/3 \end{cases}$

Thus a single trial means that we keep conducting iid Bernoulli experiments over and over until we get the first success which is the 'stopping time' for each trial.

Mean – Geometric

$X \sim \text{Geometric}(0.2) := (1-0.2)^{x-1}\,0.2$

Min[X] = min{ Supp[X] } = 1 — the smallest value in the support

Warning! Mode and Min are NOT the same

look at all the probabilities find the largest one and tell me who is responsible — **Mode**[X] = $\arg\max_{x\in\text{Supp}[X]}\{p(x)\}$ = 1 — it's the most likely value you will get

arg_min() returns the smallest value in the support of X that is above the given probability

Quantile[X,p] := arg_min{ F(x)≥p } (CDF) ⟶ **Median**[X,0.5] = 4 (special case of Quantile)

aka

Percentile if p is a percent — e.g. 0.2·100 = 20%

70th Percentile = 6

90th Percentile = 11

what is a 95th percentile ?
Lets say 95% on the exam was 17. It means that at the most 5% students scored higher than 17, and at the minimum 95% of the students scored below 17.

Tertiles	Quartiles	Quintiles	Deciles
Quantile[X,0.33]	Quantile[X,0.25]	Quantile[X,0.20]	Quantile[X,0.10]
Quantile[X,0.66]	Quantile[X,0.50]	Quantile[X,0.40]	Quantile[X,0.20]
	Quantile[X,0.75]	Quantile[X,0.60]	…
		Quantile[X,0.80]	Quantile[X,0.90]

75th percentile – 25th percentile
IQR = Q[X,0.75] – Q[X,0.25] = 7 – 2 = 5
represents middle 50% of data

Max[X] = max{ Supp[X] } = d.n.e. — the largest value in the support

Range[X] = Max[X] – Min[X] = d.n.e.

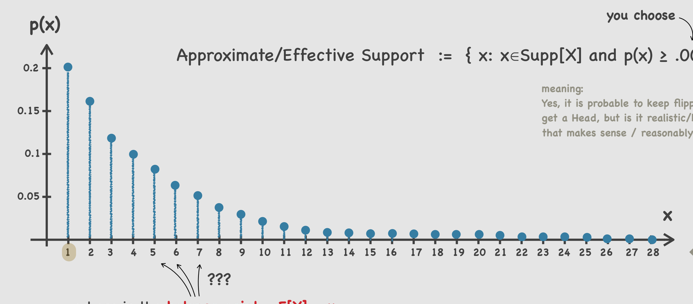
$0.8^{x-1}\,0.2$

x	p(x)	F(x)
1	.200	.200
2	.160	.360 = .200 + .160
3	.120	.480
4	.102	.582
5	.082	.664
6	.066	.730
7	.052	.782
8	.042	.824
9	.037	.861
10	.027	.888
11	.021	.909
12	.017	.926
13	.014	.940
14	.011	.951
15	.009	.960
16	.007	.967
17	.006	.973
18	.005	.978
19	.004	.982
20	.003	.985
21	.002	.987
22	.001	.988
23	.001	.989
24	.001	.990
25	.001	.991
26	.001	.992
27	.000	.992
28	.000	.993
…	Σ≈1	

Supp[X] = {1,…} → Why ≠ 1 ? In theory we could keep trying to ∞ and never get our success which is that necessary stopping factor.

Approximate/Effective Support := { x: x∈Supp[X] and p(x) ≥ .001 } — you choose

meaning: Yes, it is probable to keep flipping that proverbial coin and never get a Head, but is it realistic/believable ? Choose a value for p(x) that makes sense / reasonably realistic.



← values in the support of X

where is the **balance point – E[X] – μ**

$\mu = \sum_{x\in\text{Supp}[X]} x\,p(x) \to \sum_{x=1}^{\infty} x\,(1-p)^{x-1}p$ | pull the const p — Geometric PMF – p(x); x∈Supp = [1,…]

$= p\sum_{x=1}^{\infty} x\,(1-p)^{x-1}$ — reindexing trick: let y=x-1 → x=y+1; x=1…∞ → y=0…∞

$= p\sum_{y=0}^{\infty}(y+1)(1-p)^y$ | split Σ

$= p\left[\sum_{y=0}^{\infty} y(1-p)^y + \sum_{y=0}^{\infty} 1\,(1-p)^y\right]$ — geometric series $\sum_{i=0}^{\infty} a^i = \frac{1}{1-a}$

$= p\left[\sum_{y=0}^{\infty} y(1-p)^y + \sum_{y=0}^{\infty} 1\,\frac{1}{1-(1-p)}\right]$

$= p\sum_{y=0}^{\infty} y(1-p)^y + \cancel{p}\sum_{y=0}^{\infty}\frac{1}{\cancel{p}}$ | ps cancel

$= \sum_{y=0}^{\infty} y\,(1-p)^y p + 1$ | $x^3 = x^2\cdot x^1 = x^{3-1}\cdot x^1$

$= \sum_{y=0}^{\infty} y\,(1-p)^{y-1}(1-p)\,p + 1$ | pull the constant (1-p)

$= (1-p)\sum_{y=0}^{\infty} y\,(1-p)^{y-1}p + 1$

$= (1-p)\,\mu + 1$

$\cancel{\mu} = \cancel{\mu} - p\mu + 1$

$= -p\mu + 1$

$p\mu = 1$

thus our **balance point** is = 5 — $\frac{1}{0.2} = 5$

$$E[X \sim \text{Geometric}(p) := (1-p)^{x-1}p] = \mu = \frac{1}{p}$$

Custom r.v.s

Ever wondered how to build custom r.v.s?

Bet on Black. Payout is 1:1 (bet $1 win $1) — Roulette in U.S. total of 38 pockets: 18 black, 18 red, 2 green 0 , 00

Rademacher $X \sim \begin{cases} \$1 & \text{w.p. } 18/38 \\ -\$1 & \text{w.p. } 20/38 \end{cases}$ r.v. model of the payout of this bet

what is the expectation ?

$E[X] = \sum_{x\in\text{Supp}[X]} x\,p(x) = \$1\cdot p(\$1) + (-\$1)\cdot p(-\$1)$
$= \$1\cdot 18/38 + (-\$1)\cdot 20/38$
$= -\$0.053$

meaning if you play many times on average you are loosing ≈5.3 cents (per play)

the more you play the more you loose in a long run you'll loose everything – you can't win

$X_1, X_2, \dots, X_n \overset{iid}{\sim} \begin{cases} \$1 & \text{w.p. } 18/38 \\ -\$1 & \text{w.p. } 20/38 \end{cases}$

$\lim_{n\to\infty} T_n = -\infty$

Expectation is a 'long term' property and it means very little when you have only few random variables.

Bet on 'lucky' # 7. Payout is 35:1 (bet $1 win $35) — Roulette in U.S. total of 38 pockets: 18 black, 18 red, 2 green 0 , 00

$X \sim \begin{cases} \$35 & \text{w.p. } 1/38 \\ -\$1 & \text{w.p. } 37/38 \end{cases}$ r.v. model of the payout of this bet

what is the expectation ?

$E[X] = \sum_{x\in\text{Supp}[X]} x\,p(x) = \$35\cdot p(\$35) + (-\$1)\cdot p(-\$1)$
$= \$35\cdot 1/38 + (-\$1)\cdot 37/38$
$= -\$0.053$

hint: – play video draw poker duces wild
– black jack you loose 0.5% on average

Bet on 'dozen' # 1…12 Payout is 2:1 (bet $1 win $2) — Roulette in U.S. total of 38 pockets: 18 black, 18 red, 2 green 0 , 00

$X \sim \begin{cases} \$2 & \text{w.p. } 12/38 \\ -\$1 & \text{w.p. } 26/38 \end{cases}$ r.v. model of the payout of this bet

what is the expectation ?

$E[X] = \sum_{x\in\text{Supp}[X]} x\,p(x) = \$2\cdot p(\$2) + (-\$1)\cdot p(-\$1)$
$= \$2\cdot 12/38 + (-\$1)\cdot 26/38$
$= -\$0.053$

Bet on 'dozen' # 1…12 Payout is 2:1 (bet $1 win $2) — Roulette in Europe total of 37 pockets: 18 black, 18 red, 1 green 0

$X \sim \begin{cases} \$2 & \text{w.p. } 12/37 \\ -\$1 & \text{w.p. } 25/37 \end{cases}$ r.v. model of the payout of this bet

what is the expectation ?

$E[X] = \sum_{x\in\text{Supp}[X]} x\,p(x) = \$2\cdot p(\$2) + (-\$1)\cdot p(-\$1)$
$= \$2\cdot 12/37 + (-\$1)\cdot 25/37$
$= -\$0.027$

in a long run you will loose half the money playing the European Roulette over the US version.

if X models a payout of a game, "Fair Game" is if E[X] = 0

Basic r.v. transportation – Uber example

P(traffic on route A) = 0.3

If there is traffic on route A, take route B.
– route A is 7min
– route B is 12min

$W \sim \begin{cases} 7\text{min} & \text{w.p. } 0.7 \\ 12\text{min} & \text{w.p. } 0.3 \end{cases}$ not a Bernoulli b/c Supp[X] ≠ {0,1}

$E[W] = \sum_{x\in\text{Supp}[X]} x\,p(x) = 7\text{min}\cdot p(7\text{min}) + 12\text{min}\cdot p(12\text{min})$
$= 7\text{min}\cdot 0.7 + 12\text{min}\cdot 0.3$
$= 8.5$ min

– on average in all my trips the time spend in the taxi is ≈8.5 min
– this exact value pertains to an infinite amount of trips