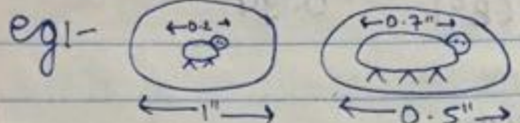
## Lecture 7:

Contd Lets take a look at zoom:

eg:-




$\therefore$ zoom $= \dfrac{\text{Prior scope size}}{\text{new scope size}}$

$= \dfrac{1}{0.5} = 2$

axiom $P(\Omega) = 1$

$\therefore P(B|A) = \dfrac{P(\Omega)}{P(A)} \cdot P(AB)$

$\boxed{P(B|A) = \dfrac{P(AB)}{P(B)}}$ $\Rightarrow$ def^n of conditional probability

$\therefore P(B) = P(AB) \cdot P(A|B) \rightarrow$ corollary

$\therefore P(B|A) = \dfrac{P(A|B) \cdot P(B)}{P(A)}$ } "Bayes Rule" (1763)

$\therefore P(B|A) = \dfrac{P(AB)}{P(A)} = \dfrac{0.036}{0.2} = 0.18 \approx 20\%$

$\therefore P(A|B) = \dfrac{P(AB)}{P(B)} = \dfrac{0.036}{0.06} = 0.6$ (good chance he is smoker)

(In english, what is the prob. that you are ~~at~~ a smoker and have a lung cancer?)

$\therefore$ P(lung cancer among non-smoker)

$\Rightarrow P(B|A^c) = \dfrac{P(BA^c)}{P(A^c)} = \dfrac{0.024}{1-0.2} = \dfrac{0.024}{0.8} = 0.03$

$(1 - P(A))$

$\therefore P(BA^c) = P(B) - P(AB)$

$= 0.06 - 0.036$

$= 0.024$

**Risk ratio**

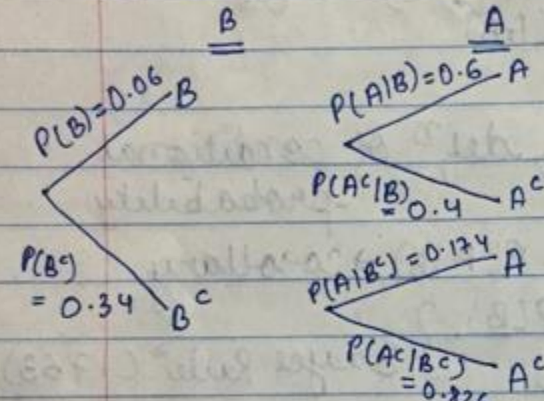
$\dfrac{P(B|A)}{P(B|A^c)} = \dfrac{0.18}{0.03} = 6$

Hence, the chance of lung cancer is increased by "6" times by changing from non-smoker to smoker

$$\therefore P(AB^c) = P(A) - P(AB)$$
$$= 0.02 - 0.036$$
$$= 0.164$$

$$P(A \mid B^c) = \frac{P(AB^c)}{P(B^c)} = \frac{0.164}{0.94} = 0.174$$

## Tree illustration




Joint outcomes

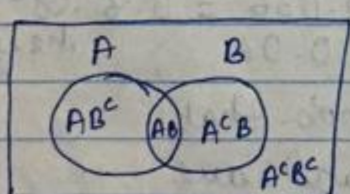
$P(AB) = P(A) \cdot P(A \mid B) = 0.036$

$P(A^cB) = 0.024$

$P(AB^c) = 0.164$

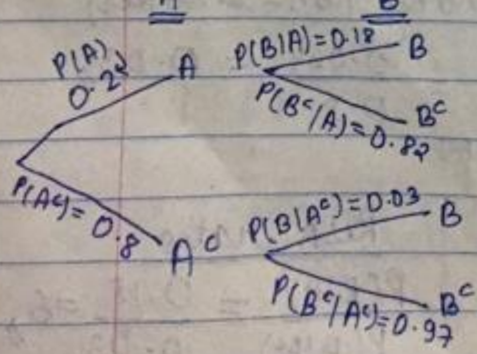
$P(A^cB^c) = 0.776$

Here, the branches refers to "conditional prob." and if we add them all we get 1.




$\therefore P(AB^c) + P(AB) + P(A^cB) + P(A^cB^c) = 1$

## Inversion illusion




Joint outcomes

$P(AB) = 0.036$

$P(AB^c) = 0.164$

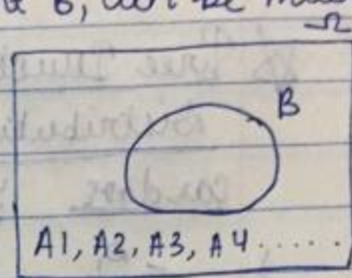
$P(A^cB) = 0.024$

$P(A^cB^c) = 0.776$

$\overline{\phantom{0.776}}$

1

Lets consider $A_1, A_2, \ldots$ mutually exclusive & collectively exhaustive & event B, can be made up of all the intersections.
Thus, this is "Law of total probability".




* $P(B) = P(B \cap \Omega)$

Since, $A_1, A_2, A_3 \cdots$ are collectively exhaustive it can be written as:

$= P(B \cap (A_1 \cup A_2 \cup A_3 \cdots))$

$= P((B \cap A_1) \cup (B \cap A_2) \cup \cdots))$

$= P(B \cap A_i) + P(B \cap A_J)$

$\therefore P(B) = \sum_{i=1}^{\infty} P(BA_i)$

Q) Is $B \cap A_i$, $B \cap A_J$ mutually exclusive?

$(B \cap A_i) \cap (B \cap A_J) \overset{?}{=} \phi$

$= (B \cap B \cap A_i \cap A_J) = B \cap \phi = \phi$

Q) Lets assume that birth of a girl and a boy are equally likely. Then

① P(if you had 2 kids and 1 of them is a girl, the other is also a girl)

Ω

| | |
|---|---|
| GG | GB |
| BG | BB |

$= \dfrac{P(\{GG\} \mid \{GG, GB, BG\})}{P(\{GG, GB, BG\})}$

$= \dfrac{1/4}{3/4} = \dfrac{1}{3}$

$\therefore \Omega = \{BB, BG, GB, GG\}$

## Tree Illustration of uniform discreet prob. Distribution