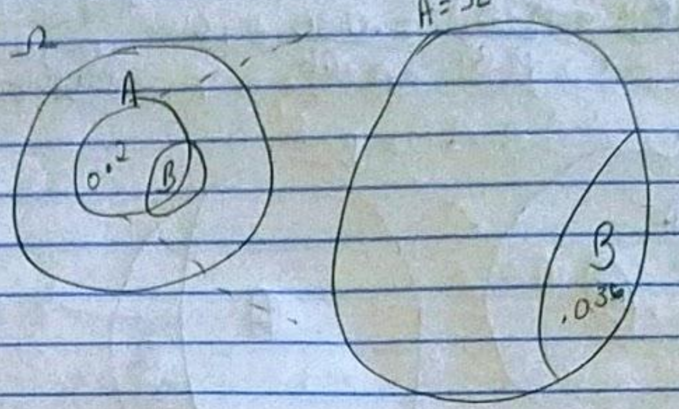
Lecture 7 09/19/2017

smoking

$P(A) = 0.2$

$P(B) = 0.06$

lung cancer



$P(AB) = 0.036$

$\downarrow$

$A \cap B$

P(lung cancer) among smokers

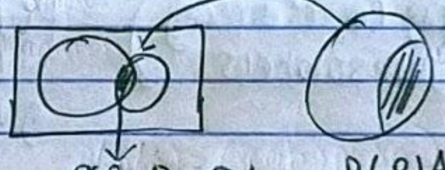
$P(B) = 0.06$

$P(B|A) = \frac{P(AB)}{P(A)} = \frac{.036}{.2} = .18$

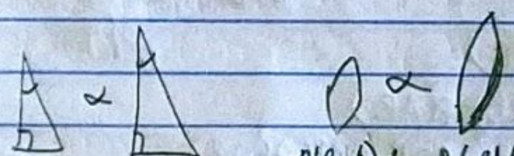
"given"

"conditionalor"

NOTE



$P(B,A) \propto P(B|A)$



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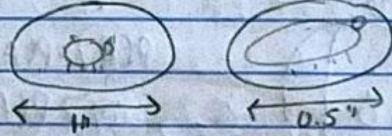


$\text{Zoom} = \frac{\text{prior scope size}}{\text{new scope size}} = \frac{1}{0.5} = 2$

$P(B|A) = (\text{scale})\, P(AB)$

$P(B|A) = \frac{P(\Omega)}{P(A)} P(AB)$

$\Rightarrow P(B|A) = \frac{1}{P(A)} P(AB)$

ZOOM

$\boxed{P(B|A) = \frac{P(AB)}{P(A)}}$

Def. of conditional Probability

only valid if $P(A) \neq 0$

$\boxed{\text{NOTE}}$: $P(BA) = P(B|A)P(A)$ corollary

IF $P(A|B) = \frac{P(AB)}{P(B)} = \frac{P(BA)}{P(B)}$

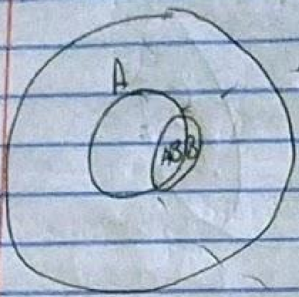
$P(A|B) = \frac{P(AB)}{P(B)}$

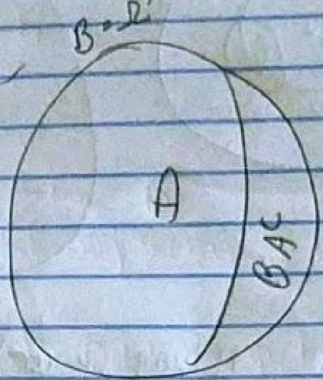
$\Rightarrow P(AB) = P(B)P(A|B) \rightarrow \div P(A)$

$\boxed{P(B|A) = \frac{P(A|B)\,P(B)}{P(A)}}$

Bayes Rule (1763)

$P(A|B) = \frac{P(AB)}{P(B)} = \frac{.036}{.06} = .6$ those who have lung cancer majority smoke

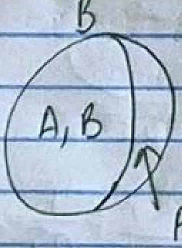




$P(\text{lung cancer among non-smokers}) = P(B|A^c) = \frac{P(B, A^c)}{P(A^c)} = 1 - P(A)$

given

$= \frac{.024}{0.8} = .03$



$\Rightarrow$ $P(B) = P(AB) + P(A^cB)$

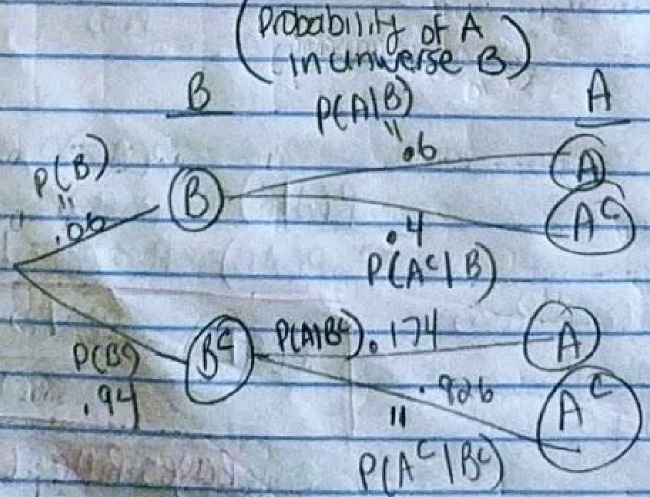
$\Rightarrow P(A^cB) = P(B) - P(AB)$

$= .06 - .036 = .024 = P(A^c \cap B)$

changing the universe

risk ratio $\left\{ \frac{P(B|A)}{P(B|A^c)} = \frac{.18}{.03} = 6 \right.$ $\frac{\text{Probability of lung cancer among smokers}}{\text{Probability of lung cancer among nonsmokers}}$

Tree (illustration)



$P(AB) = P(B)\,P(A|B) = .036$

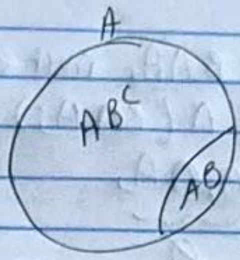
$P(A^cB) = P(B)\,P(A^c|B) = .024$

$P(AB^c) = P(B^c)(P(A|B^c)) = .164$

$P(A^cB^c) P(B^c) P(A^c|B^c) = .776$

$= 1$

$$P(A \mid B^c) = \frac{P(AB^c)}{P(B^c)} = \frac{.164}{.94} = .174$$

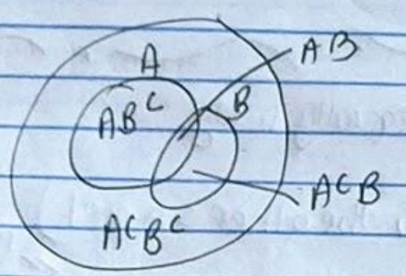


$P(A) = P(AB^c) + P(AB)$

$P(AB^c) = P(A) - P(AB)$

$= .2 = .036$

$= .164$



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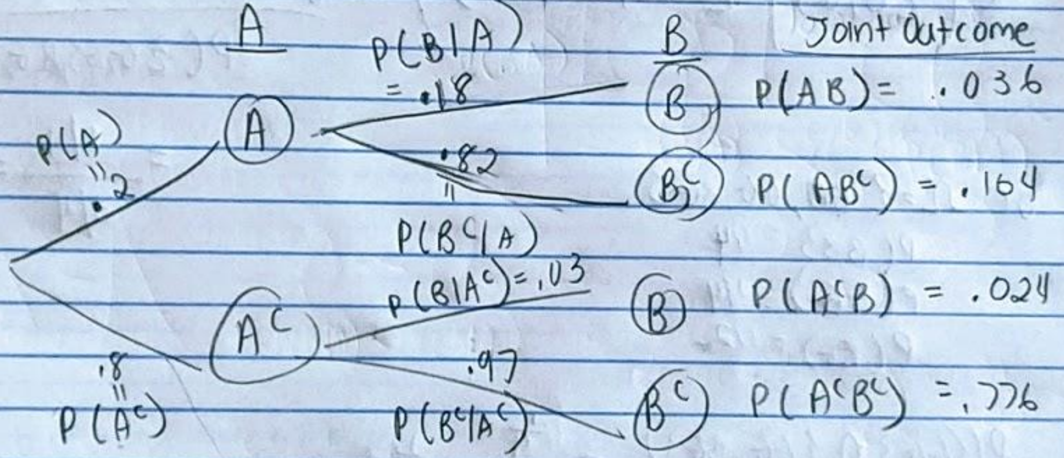

not have lung cancer when given nonsmokers

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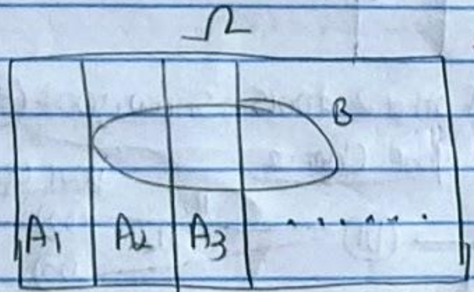
consider $A_1, A_2 \ldots$ mutually exclusive and coll exhaustive and event B



Law of Total Probability

$P(B) = P(B \cap \Omega)$

$= P(B \cap (A_1 \cup A_2 \cup \ldots))$

$= P((B \cap A_1) \cup (B \cap A_2) \cup \ldots)$

$= P(B \cap A_1) + P(B \cap A_2) +$

$$P(B) = \sum_{i=1}^{\infty} P(B, A_i)$$

Are $B \cap A_c$, $B \cap A_j$ mut exclusive?

$$(B \cap A_i) \cap (B \cap A_j) \stackrel{?}{=} \emptyset$$

$$= \underbrace{B \cap B}_{B} \cap \underbrace{A_i \cap A_j}_{\emptyset}$$

$$= B \cap \emptyset = \emptyset$$

Assume girl births and boy births are equally likely

P(If you have 2 kids and 1 is a girl, the other is a girl) $= P(\{G,G\})$

$\Omega = \{B,B$
B,G
G,B
$G,G\}$

$$P\left(\{G,G\} \mid \{GG, BG, GB\}\right) = \frac{P(\{GG\} \cap \{GG, GB, BG\})}{P(\{GG, BG, GB\})}$$

$$= \frac{1/4}{3/4} = \frac{1}{3}$$

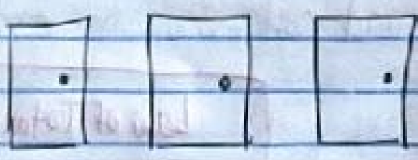
or $\Omega = \{BB, GG, BG\}$

$P(BB) = 1/4$

$P(GG) = 1/4$

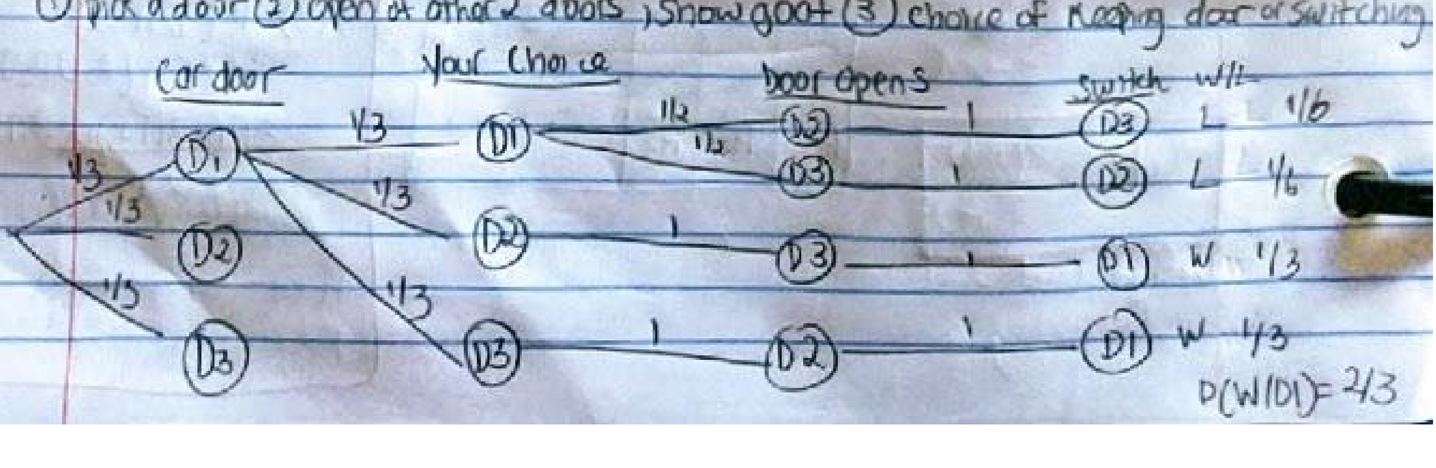
$P(BG) = 1/2$

$$\frac{P(\{GG\} \cap \{GG, BG\})}{P\{GG, BG\}} = \frac{\frac{1}{4}}{\frac{1}{4} + \frac{1}{2}} = \frac{1}{3}$$



two goats and a car

① pick a door ② open 1 of other 2 doors, show goat ③ choice of keeping door or switching



$P(W \mid D1) = 2/3$