Oct 30, 2017

Custom r.v's

Roulette in America

\$1 Bet on Black Payout is 1:1

$$X \sim \begin{cases} \$1 & wp \ 18/38 \\ -\$1 & wp \ 20/38 \end{cases}$$

$$E[X] = \sum_{x \in Supp[X]} x\,p(x)$$

$$= (\$1)\left(\frac{18}{38}\right) + (-\$1)\frac{20}{38} = \frac{1}{19} = -\$\,0.053$$

"the average will converge to -\$0.053"

$\overline{X} \to M$

Liu of large #'s

$M \in Supp[X]$
generally speaking.

$$\lim_{n\to\infty} T_n = \underbrace{X_1 + \dots + X_n}_{n(\bar{x})} = -\infty$$

$n \to \infty$, $\bar{x} \to -\$0.053$

Bet on "Lucky 7" Payout 35:1

$$X \sim \begin{cases} \$35 \ (win) & wp \ 1/38 \\ -\$1 & wp \ 37/38 \end{cases}$$

$$E[X] = \sum_{x \in Supp[X]} x\,p(x)$$

$$= 35 \cdot \frac{1}{38} + (-\$1)\frac{37}{38} = -\$0.053$$

Bet on 'first dozen' 1-12 payout 2:1

$$X \sim \begin{cases} \$2 & wp \ 12/38 \\ -\$1 & wp \ 26/38 \end{cases}$$

$$E[X] = 2 \cdot \frac{12}{38} + (-1)\frac{26}{38} = -\$0.053$$

Roulette in Europe

$1 Bet on Black Payout is 1:1

$$x \sim \begin{cases} \$1 & \text{wp } 18/37 \\ -\$1 & \text{wp } 19/37 \end{cases}$$

$$E[X] = \sum_{x \in supp[X]} x\,p(x) = (\$1)\frac{18}{37} + (-\$1)\frac{19}{37} = -\$0.027$$

Average

European roulette is "more ~~fa~~ fair" than American roulette.

"fair Game" $E[X]=0$.

$P(\text{traffic}) = 0.3$

if traffic, Uber takes 12 min

if no traffic, 7 min

Model the M car W.

$$W \sim \begin{cases} 12 \text{ min} & \text{wp } 0.3 \\ 7 \text{ min} & \text{wp } 0.7 \end{cases} \qquad E[W] = 12(0.3) + 7(0.7) = 8.5 \text{ min}$$

Uber charges \$.40/min Model B, the price paid for ~~th~~ time in taxi

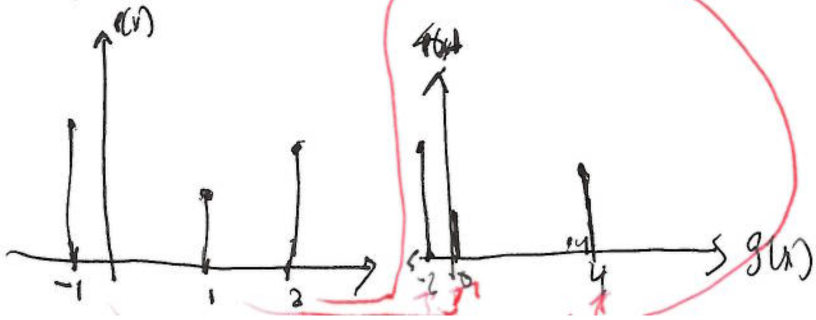
$$B \sim \begin{cases} \$0.40 * 12 = \$4.80 & \text{wp } 0.3 \\ \$0.40 * 7 = \$2.80 & \text{wp } 0.7 \end{cases}$$

$$E[B] = (4.80)0.3 + 2.80(.7) = \$3.12$$
$$= \$.40 \cdot 12 \cdot 0.3 + \$.40 \cdot 7 \cdot 0.7$$
$$= \$.40(12 \cdot 0.3 + 7 \cdot 0.7)$$
$$= \$.40\, E[W]$$

$$\boxed{Y = aX,\ a \in \mathbb{R} \quad E[Y] = E[aX] = E[g(x)]} = \sum_{x \in Supp[X]} ax\,p(x) = a \sum_{x \in Supp[X]} x\,p(x) = a \cdot E[X]$$

$E[g(x)]$

$$E[g(x)] = \sum_{x \in supp[X]} g(x)p(x)$$



Base Fare is \$3

Model T, the total price

$$T \sim \begin{cases} 3 + 4.80 = 7.80 & \text{wp } 0.3 \\ 3 + 2.80 = 5.80 & \text{wp } 0.7 \end{cases}$$

$$E[T] = 7.80(0.3) + 5.80(.7) = 6.12$$
$$= 3 + 4.80(0.3) + (3 + 2.80)\cdot 0.7$$
$$= \cancel{3.30}$$
$$= 3\cdot 0.3 + 4.80\cdot 0.3 + 3\cdot 0.7 + 2.80\cdot 0.7$$
$$= 3(0.3 + 0.7) + 4.80\cdot 0.3 + 2.80\cdot 0.7$$
$$= 3 + E[B]$$

$Y = X + c \quad c \in \mathbb{R}$

$$E[Y] = E[X + c] = \sum_{x \in Supp[X]} (x + c)p(x) = \underbrace{\sum x\, p(x)}_{E[X]} + \sum c\, p(x)$$
$$= E[X] + c\sum p(x)$$
$$= E[X] + c$$

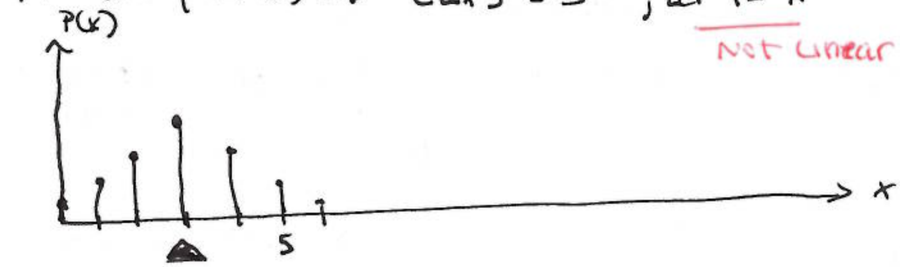
Linear transformations

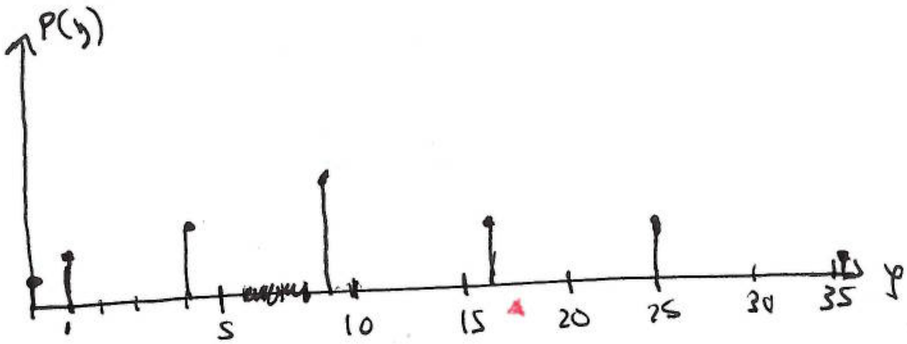
$Y = aX + c, \quad a, c \in \mathbb{R}$

$E[Y] = aE[X] + c$

$X \sim Bin(6, \frac{1}{2}) \Rightarrow E[X] = 3$, Let $Y = X^2$

Not Linear



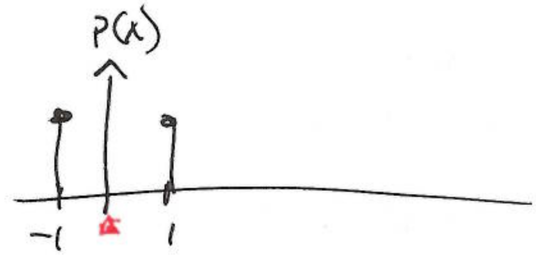


Does $E[X] \overset{?}{\neq} (E[X])^2$

$$E[Y] = \sum_{x=0}^{6} x^2 \binom{n}{x} p^x (1-p)^{n-x}$$
$$= 17.5$$

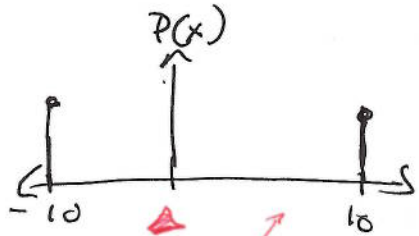
$\Rightarrow E[g(x)] \neq g(E[X])$ generally.

$x \sim \text{Ratenmacher} = \begin{cases} 1 & \text{wp } \frac{1}{2} \\ -1 & \text{wp } \frac{1}{2} \end{cases}$



~~Y = 10X~~

$y = 10x = \begin{cases} 10 & \text{wp } \frac{1}{2} \\ -10 & \text{wp } \frac{1}{2} \end{cases}$



100 square units away from pivot.

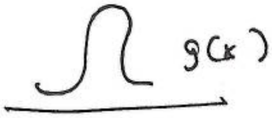
Y is more
a) spreadout
b) deviant
c) variable

$E[X] = 0$

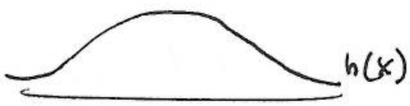
$E[Y] = 10\,E[X] = 0$

E[x] is a summary metric of r.v

$E[X] \Rightarrow E[Y] \not\Rightarrow X \overset{d}{=} Y$



$= \int_{\mathbb{R}} g(x)\,dx = 3$



$\int_{\mathbb{R}} h(x) = 3$

Theory of Error / Distance Functions

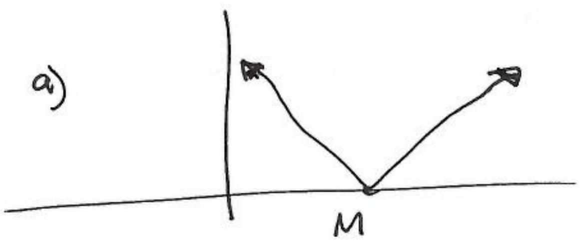
measure error from X to M

a) $e(X, M) = |X - M|$ L1 error metric

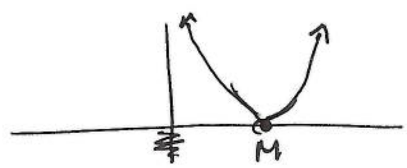
you can minimize

b) $e(X, M) = (X - M)^2$ L2 error metric

a)



b)



Let $L = (X - M)^2$

"squared error loss"

$Var[X] = E[L] = E[(X-M)^2] = E[(X - E[X])^2]$

$Var[X] = \sum_{x \in supp[X]} (X - M)^2 P(X)$

$= (1-0)^2 \frac{1}{2} + (-1-0)^2 \frac{1}{2}$

$= 1 \cdot \frac{1}{2} + 1 \cdot \frac{1}{2} = \boxed{1}$

more spreadout

$Var[Y] = \sum_{y \in supp[X]} (y - M_y)^2 P(y)$

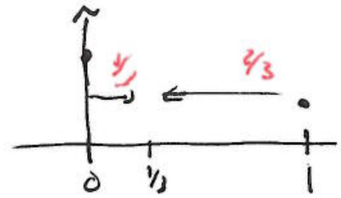
$= (10-0)^2 \frac{1}{2} + (-10-0)^2 \frac{1}{2}$

$= 100 \cdot \frac{1}{2} + 100 \cdot \frac{1}{2} = \boxed{100}$

November 14
TEST

November 13
HW 5 Due

$X \sim \text{Bern}\left(\frac{1}{3}\right)$



$\text{var}[x] = E[(x-\mu)^2]$

$= \sum_{x \in \text{supp}[x]} (x-\mu)^2 p(x) = \left(1-\frac{1}{3}\right)^2 \frac{1}{3} + \left(0-\frac{1}{3}\right)^2 \frac{2}{3}$

$= \left(\frac{2}{3}\right)^2 \frac{1}{3} + \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)$

$= \frac{4}{9} + \frac{3}{6} = \frac{2}{3}$

$x \sim \text{Bern}(p)$

Variance = "The average square distance from pivot"

$\sigma^2 = \text{Var}[x] = \sum (x-\mu)^2 p(x)$

$= (1-p)^2 p + (0-p)^2 (1-p)$

$= (1-2p+p^2)p + p^2(1-p)$

$= p - 2p^2 + \cancel{p^3} + p^2 - \cancel{p^3} = p - p^2 = \boxed{p(1-p)}$

μ, σ^2