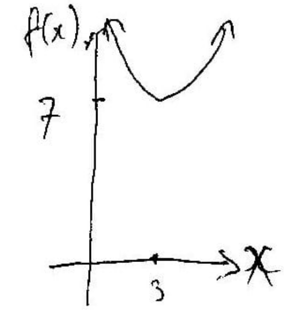
$X_1, X_2, \dots, X_n \overset{iid}{\sim} \text{Bern}(p)$

$T_n = X_1 + \dots + X_n \sim \text{Bin}(n,p)$.

$f(x) = 7 + (x-3)^2$.



$\min\{f(x)\} = 7$

$\max\{f(x)\} = \text{D.N.E}$

$\arg\min\{f(x)\} = 3$ s.t x where $f(x) = \min\{f(x)\}$

$\arg\max\{f(x)\} = \text{D.N.E}$.

$T := \min\{t : X_t = 1\}$ $\quad t_1, t_2, t_3, t_4$ $\quad 0, 0, 0, 1$

"Stopping time" "Discrete time"

$P(T=1) = p \quad \Rightarrow (1-p)^0 p^1$

$P(T=2) = (1-p)p$

$P(T=3) = (1-p)^2 p$

$\vdots$

$P(T=x) = (1-p)^{(x-1)} p$

$X \sim \text{Geometric}(p) := p(x) = (1-p)^{x-p} p$

$\text{Supp}[X] = \mathbb{N}$ (1 to ∞)

$p \in (0,1)$.

$\sum_{x \in \text{Supp}[X]} p(x) = 1$.

Want to show $\sum_{x=1}^{\infty} (1-p)^{x-1} p = 1$

$\Rightarrow \sum_{x=1}^{\infty} (1-p)^{x-1} = \frac{1}{p}$ $\Bigg\}$ to prove.

Proof: let $i = x - 1$

$\therefore x = i + 1$,

$$\sum_{i=0}^{\infty} (1-p)^{i} = \frac{1}{p}.$$

let $q = 1 - p$

$p = 1 - q$

$$\sum_{i=0}^{\infty} q^{i} = \frac{1}{p}.$$

$$S = \sum_{i=0}^{\infty} q^{i} \qquad q \in (0,1) \qquad \Leftarrow \text{Geometric Series.}$$

$$= 1 + q + q^2 + q^3 + \cdots$$

$$= 1 + q\left(1 + q + q^2 + \cdots\right)$$

$$\therefore S = 1 + qS.$$

$$S(1-q) = 1.$$

$$S = \frac{1}{1-q} = \frac{1}{p} \ \#$$

CDF

$$F(x) = P(X \leq x) = \sum_{i=1}^{x} (1-p)^{i-1} p.$$

$$= p(1) + p(2) + p(3) + \cdots + p(x).$$

$$= 1 - P(X > x)$$

$$= 1 - P(X_1 = 0, \ldots, X_x = 0)$$

fail. success.

$\frac{0}{0}$. $\frac{0}{1}$ $\frac{0}{2}$ $\frac{0}{x}$ $x+1$ $x+2$ $x+3$.

$F(x) = 1-(1-p)^x$

Proof: $p(1) = p$

$p(2) = (1-p)p$.

$F(1) = P(X \leq 1) = P(1) = p$.

$= 1-(1-p) = 1-1+p = p$.

$F(2) = P(X \leq 2) = P(1) + P(2)$

$= p + (1-p)p$

$= p(1+1-p)$

$= p(2-p)$.

$= 1-(1-p)^2$.

$= 1-(1-2p+p^2) = 1-1+2p-p^2$

$= p(2-p)$.

P(Dealth a royal flush) = .00000153

① What is the prob: first get dealth a Royal Flush on a million hand?

Step 1 ⇒ $X \sim \text{Geometric}(0.0000153)$.

Step 2 ⇒ $P(X = 1{,}000{,}000)$

$= .0000153 \cdot (1-0.0000153)^{999{,}999}$

② What is the prob: I get a royal flush before the 1,000,000?

$P(X < 1000001) = P(X \leq 1{,}000{,}000) = F(1{,}000{,}000)$.

$= 1-(1-0.00000153)^{1{,}000{,}000}$

$\approx 77.7\%$.

$X \sim Bern\left(\frac{1}{2}\right)$

$Supp[X] = \{0, 1\}$.

$p(x) = P(X = x)$

$x \in Supp[X]$.

X , x

process model. (abstract)

$\# \in Supp[X]$

concrete.

data: ~~...~~ realization of R.V's.

iid data: .... iid R.V's.

$X \sim Hyper(\overset{n}{4}, \overset{K}{3}, \overset{N}{8})$

$Supp[X] = \{0, 1, 2, 3\}$.

$X_1, \dots, X_6 \overset{iid}{\sim} Hyper(4, 3, 8)$.

$x_1 = 0$
$x_2 = 1$
$x_3 = 2$
$x_4 = 2$
$x_5 = 1$
$x_6 = 3$.

Data.

average $= \bar{x} = 1.5$.

$T_n = X_1 + \dots + X_n = \sum_{i=1}^{n} X_i$

$\bar{X}_n = \frac{X_1 + \dots + X_n}{n} = \frac{T_n}{n} = \frac{1}{n}\sum_{i=1}^{n} X_i$

average R.V.

Check How many 'Heads' $p(0) = \left(\frac{1}{2}\right)^8$.

$X \sim Bin\left(8, \frac{1}{2}\right)$

$Supp[X] = \{0, \dots, 8\}$.

2
4
5
5
2
3

Data.

$X_1, \dots, X_6 \overset{iid}{\sim} Bin\left(8, \frac{1}{2}\right)$

$\bar{x} = \frac{21}{6} = 3.5$

pick 1 if it is special.

$X \sim Geometric\left(\frac{3}{8}\right)$

1
6
2
8
1
2

$X_1, \dots, X_6 \sim Geometric\left(\frac{3}{8}\right)$

$\bar{x} = \frac{19}{6} = 3.167$