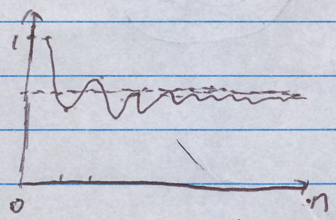
①
Long run Frequency/
Limiting Frequency Def

First define $\mathbb{1}\ w \in A = \begin{cases} 1 & \text{if } w \in A \\ 0 & \text{if } w \notin A \end{cases}$, $P(A) \doteq \lim_{n \to \infty} \frac{\sum_{i=1}^{n} \mathbb{1}\ w_i \in A}{n} = \frac{\#\{w \in A\}}{n}$



"As n goes larger, P(A) become more "stable". But this is an estimate probability. Because ① Require infinite experience. ② Not general.

lecture 7

U238

② Objects have inherent disposition toward outcome "propensity" -- Karl Popper, idea from Radioactive

Experiment we perform on object, the object allready had the probability and experiment perform, result will show. But ① For most random experiment, we don't know how to calculate the propensity of w's. ② Not general.

③ Subjective Def: Everyone use their own evidence, to come up with their own estimate. But there are more than one answer due to not objective.

Conclusion: No accepted definition of probability

What is random? In past, people think random is just illusion due to ignore, so there is nothing random, ~~as long~~ just we don't have enough information about the system. But the double slip light experiment in 1920 tell us there is something in the universe that is random.

Mathmatic Def of Probability

Assume $\exists \Omega \neq \phi$. P is a set function satisfy the 3 conditions

① $P(\Omega) = 1$ ② $\forall A \subseteq \Omega \quad P(A) \geq 0$

③ If $A_1, A_2, \ldots$ are disjoint, then $P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i)$

Thm 1 $P(A) = 1 - P(A^c)$

$\Omega = A \cup A^c$

$P(\Omega) = P(A \cup A^c)$

$P(\Omega) = P(A) + P(A^c)$ by ③

$1 = P(A) + P(A^c)$ by ①

$P(A) = 1 - P(A^c)$

Thm 2 $P(\phi) = 0$

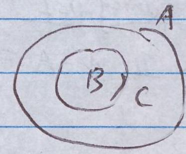
$P(\phi) = 1 - P(\phi^c)$ by Thm 1

$= 1 - P(\Omega)$ Set Theory

$= 1 - 1$ by ①

$= 0$

Thm 3 $A \subseteq B \Rightarrow P(A) \le P(B)$



$P(B) = P(A \cup C) = P(A) + P(C)$

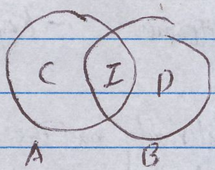
$P(B) - P(A) = P(C) \ge 0$ by ②.

$P(B) - P(A) \ge 0$

$P(B) \ge P(A)$

Thm 4

Law of inclusion-exclusion $P(A \cup B) = P(A + P(B) - (A \cap B)$



$P(A \cup B) = P(C) + P(I) + P(D)$ by ③

$= P(A - I) + P(I) + P(B - I)$

$= P(A) + P(B) - P(I)$

$= P(A) + P(B) - P(A \cap B)$

Thm 5 $|\Omega| < \infty$ if $P(w_i) = \frac{1}{|\Omega|}\ \forall w_i \Rightarrow P(A) = \frac{|A|}{|\Omega|}$

Let $n = |A| < \infty$

since $A \subseteq \Omega \Rightarrow |A| \le |\Omega|$

$A = \{w_1, w_2 \dots w_n\}$

$A = \{w_1\} \cup \{w_2\} \cup \dots \cup \{w_n\}$

$P(A) = P\left(\bigcup_{i=1}^{n} \{w_i\}\right) = \sum_{i=1}^{n} P(w_i)$ by ③

$= \sum_{i=1}^{n} \frac{1}{|\Omega|} = \frac{n}{|\Omega|} = \frac{|A|}{|\Omega|}$

Condition Probability

$\boxed{\text{Ex}}$ $n = 1000$ people

200 smokers A

60 cancer (lung) B

36 smoker and cancer $A \cap B$

Assume for illustration purpose

$P(A) = 0.2$

$P(B) = 0.06$

$P(A \cap B) = 0.036$

What is the probability of lung cancer among smokers?

$\frac{36}{200} \Rightarrow$