Lecture 23 Math 241 Dec 6th

TRUTH	Decision: Retain H_0	Decision: Reject H_0
H_0 true	✓	type I error, prob $= \alpha$
H_0 false	type II error, prob $= \beta$	prob $1-\beta$ ✓

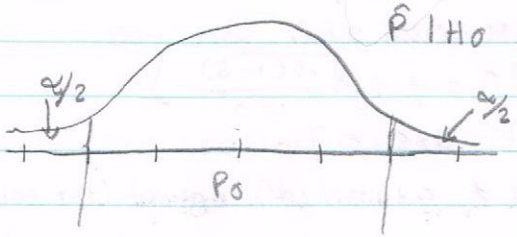
$\beta = P(\text{Retain } H_0 \mid H_0 \text{ false})$

$\alpha = P(\text{Reject } H_0 \mid H_0 \text{ true})$

$\text{POWER} = P(\text{Reject } H_0 \mid H_0 \text{ false})$

$\alpha \downarrow \Rightarrow \beta \uparrow$

$\alpha \uparrow \Rightarrow \beta \downarrow$



Clinical Trial

H_0: drug does not work

H_a: drug works

Type I error: release a drug that does not work / not effective

Type II error: do not release a drug that is efficacious

Cost of I: side effects

Cost of II:

Fire Alarm System

H_0: No fire

H_a: Fire

Type I error: false alarm , Cost: annoyance

Type II error: fire but alarm doesn't sound, cost: lives

American Justice System

H_0: Innocent

H_a: Guilty

Type I error: innocent person goes to jail, cost: live ruined

Type II error: guilty person goes free , cost: depends on crime

New Scientic Theory

H_0 : old theory $\quad \alpha = 1\%$ or 5%

H_a : new theory

Return to human sex ratio $\quad H_0: p = 0.5$ (male), $H_a: p \neq 0.5$

Previously $\hat{p} = 165/345 = .48$

Lets do the experiment again

In USA 2008 $n = 4{,}247{,}000$

babies born: 2,173,000 males

retainment region $= \left[0.5 \pm 2\sqrt{\frac{0.5(1-0.5)}{4{,}247000}}\right] = [0.495, 0.505]$

Ret Region $= \left[p_0 \pm \frac{2\alpha}{2}\sqrt{\frac{p_0(1-p_0)}{n}}\right]$

CI $p, 1-\alpha = \left[\hat{p} \pm \frac{2\alpha}{2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right]$ $\quad n\uparrow \Rightarrow B\downarrow \Rightarrow$ POWER $\uparrow$

$\hat{p} = \frac{2173000}{4247000} = .512 \notin$ retainment region $\Rightarrow$ reject $H_0 \Rightarrow$ gender ratio is not equal

H_0 : $p = 0.500001$ Retain H_0 OR Reject H_0

H_a : $p \neq 0.500001$

Alien Problem

H_0 : aliens don't exist $\quad$ H_0 : aliens do exist

H_a : aliens exist $\quad$ H_a : alien don't exist

α high $\quad$ α low

α high

* Course closed *