Math 241 Lecture 21

How to use CLT if $X_1, \dots X_n \overset{iid}{\sim}$ and n large

2 $\bar{X} \overset{d}{\approx} N(\mu, (\frac{\sigma}{\sqrt{n}})^2)$

3 $T \overset{d}{\approx} N(n\mu, (\sigma\sqrt{n})^2)$

$X_1, \dots, X_{100} \overset{iid}{\sim} \begin{cases} 1 & wp\ \frac{1}{2} \\ -1 & wp\ \frac{1}{2} \end{cases} \Rightarrow \mu = 0,\ \sigma^2 = 1 \Rightarrow \sigma = 1$

what is prob of being more than 10 steps away from the origin after 100 steps?

$T = X_1 + X_2 + \dots + X_{100} \approx N(n\mu, (\sqrt{n}\,\sigma)^2) = N(0, 10^2)$

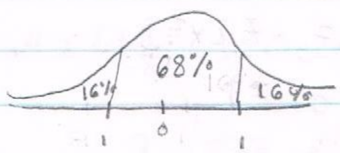
$P(T > 10 \text{ or } T \leq 10) = P(T \geq 10) + P(T \leq 10) = P\left(\frac{T-0}{10}\right.$

$= P(|T| \geq 10)$

$= P\left(\frac{T-0}{10} \geq \frac{10-0}{10}\right) + P\left(\frac{T-0}{10} \leq \frac{-10-0}{10}\right)$

$\begin{array}{r} 100 \\ -68 \\ \hline 32 \end{array}$

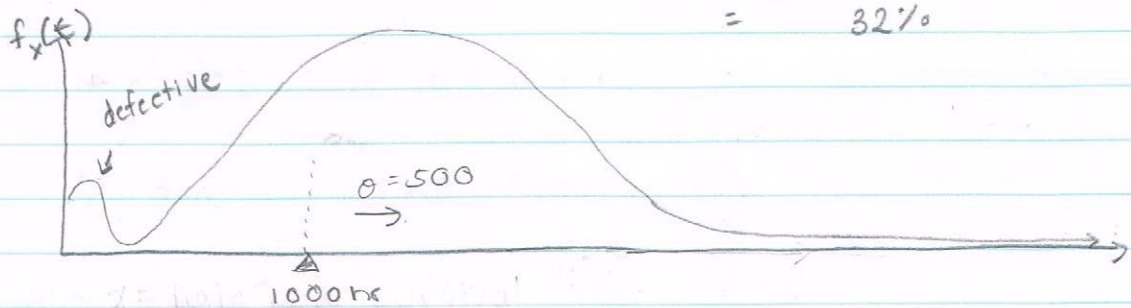
$\frac{32}{2} = 16$



$= P(Z \geq 1) + P(Z \leq 1)$

$= 16\% + 16\%$

$= 32\%$



X = light bulb survival

If you get 50 bulbs. what is the prob the avg lifetime is more than 1300hr?

$P(\bar{X} > 1300)$

$\bar{X} \overset{d}{\approx} N(\mu, (\frac{\sigma}{\sqrt{n}})^2) = N(1000, (\frac{500}{\sqrt{50}})^2 = N(1000, 7672)$

by CLT

$P(\bar{X} > 1300) \approx P\left(\frac{\bar{X} - 1000}{70.7} > \frac{1300-1000}{70.7}\right) = P(Z > 4.24) \approx 0$

$X_1, \dots, X_{10000} \overset{iid}{\sim} \text{Bern}(0.02) \Rightarrow \mu = 0.02, \quad \sigma = \sqrt{0.02(1-0.02)} = 0.14$

Shipments are late 2% of the time. In 10,000 orders, what is the prob. more than 3% are late?

$P(\bar{x} > 0.03) \approx P\left(\frac{\bar{x} - 0.02}{0.0014} > \frac{0.03 - 0.02}{0.0014}\right) = P(z > 7.14) \approx 0$

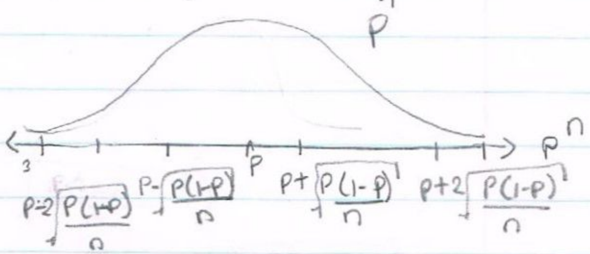
$\bar{x} \overset{d}{\approx} N\left(\mu, \left(\frac{\sigma}{\sqrt{n}}\right)^2\right) = N\left(0.02, \left(\frac{0.14}{\sqrt{10000}}\right)^2\right) = N(0.02, 0.0014^2)$

by CLT

$\bar{x} \overset{d}{\approx} N\left(\mu, \left(\frac{\sigma}{\sqrt{n}}\right)^2\right) \quad \hat{p} \overset{d}{\approx} N\left(p, \left(\frac{p(1-p)}{n}\right)^2\right)$ *

estimate $\hat{p} = \bar{x} = \frac{1+1+0+0+0}{5} = 0.4 \approx p$

sample proportion



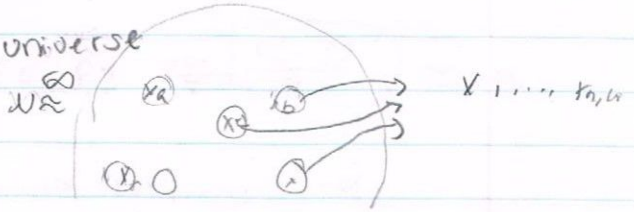
PROB ↑

STAT ↓

just Bern

Statistical Inference

3 goals:



Need $x_1, \dots, x_n \overset{iid}{\sim} x$ Bern(p)

confidence

hypothesis

1. Estimate bern goes opp of.
11. Estimate range/rand of P ($\hat{p}$)
111. test theories about P

sample n large cc & near

"Representation sampe"

"simple random sample"

Uniform discrete prob $\frac{1}{n}$

$P\left(p \in \hat{p} \pm \frac{p(1-p)}{n}\right)$

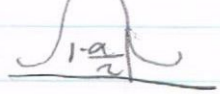
$= P\left(\hat{p} - \sqrt{\frac{p(1-p)}{n}} \le p_n \le \hat{p} + \sqrt{\frac{p(1-p)}{n}}\right)$

$= P - \left(\frac{1-1-p}{2}\right) \le p - \hat{p} = \sqrt{\frac{p(1-p)}{n}}$

$= P\left(-1 \le \frac{\hat{p} - p}{p(1-p)} \le 1\right) = P(-1 \le -z \le 1) = P(1 \ge z \ge -1)$

$= P(z \in [-1, 1]) = .68$

$z_{\frac{\alpha}{2}} := F_z^{-1}\left(1 - \frac{\alpha}{2}\right) \Rightarrow 1 - \frac{\alpha}{2} = \int_{-\infty}^{z_\alpha} f_z(z)\,dz$



$\alpha = 5\% \Rightarrow \frac{\alpha}{2} \Rightarrow 2.5\% \quad F_{1-\frac{\alpha}{2}} = 97.5\% \Rightarrow 94$

$97.5\% = \int f_z(z)\,dz$

$I \; p, 1-\alpha$

$$\left(p^{\wedge} \pm \frac{z\alpha}{2} \sqrt{\frac{p(1-p)}{n}}\right)$$ when repeated it gives you $1-\alpha$ "(overase p)"

$$\left[x \hat{p} + \frac{z\alpha}{2} \frac{\hat{p}(1-\hat{p})}{n}\right]$$ 2 sided
1 propotition
confidence interval