

Math 241 Lecture 07

Sept 25th

smokers
lung cancer

$$P(A) = 0.2$$

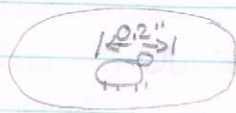
$$P(B) = 0.06$$

$$P(AB) = 0.036$$

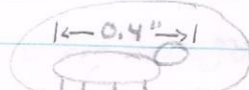
$$P(\text{lung cancer among smokers}) = P(B|A) \propto P(AB)$$

$$AB \propto$$

$$B|A$$



$$| \leftarrow 1'' \rightarrow |$$



$$| \leftarrow 0.5'' \rightarrow |$$

$$\text{Zoom factor} = \frac{\text{previous size}}{\text{new scope size}} = \frac{1''}{0.5''} = 2$$

$$= \frac{P(\Omega)}{P(A)} \quad \text{zoom}$$

$$P(AB) \Rightarrow P(B|A) = \frac{P(AB)}{P(A)}$$

Definition of Conditional probability

$$\Rightarrow P(AB) = P(A)P(B|A) \Rightarrow P(AB) = \frac{P(B|A)P(A)}{P(B)} \quad \text{"Bayes Rule" 1763}$$

$$= \frac{.036}{.2} = .18$$

$$P(\text{smoking among those who get lung cancer}) = P(A|B) = \frac{P(AB)}{P(B)} = \frac{0.036}{.06} = .6$$

$$P(B|A) = \frac{P(AB)}{P(A)} = \left(\frac{0.036}{0.2} \right) = 0.18$$

$$P(\text{lung cancer among non smokers}) = P(B|A^c) = \frac{P(BA^c)}{P(A^c)} = \frac{0.024}{0.8} = 0.3$$

$$P(BA^c) =$$

$$B = AB \cup A^c B$$

$$P(B) = P(AB) + P(A^c B)$$

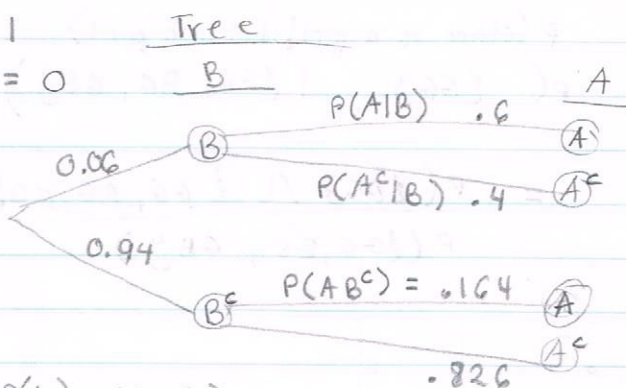
$$P(A^c B) = P(B) - P(AB) = .06 - .036 = 0.024$$

Risk
Ratio

$$\frac{P(B|A)}{P(B|A^c)} = \frac{.18}{.03} = 6 \text{ times more likely to get lung cancer if you smoke vs you don't, (risk ratio)}$$

$$P(A|A) = 1$$

$$P(A|A^c) = 0$$



Joint

$$AB = .036$$

$$A^cB = .024$$

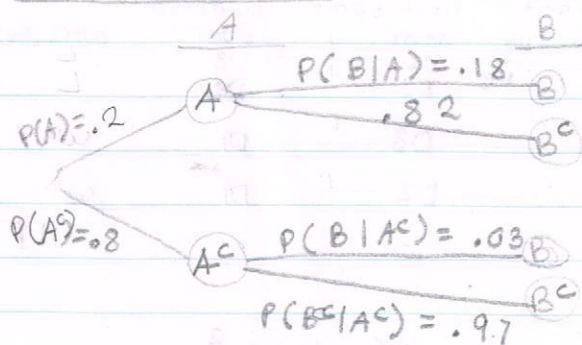
$$AB^c = .164$$

$$A^cB^c = .776$$

$$P(AB^c) = P(A) - P(AB)$$

$$= .2 - .036 = .164$$

Tree Inversion



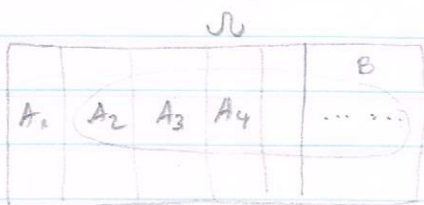
$$P(AB) = .036$$

$$P(AB^c) = .164$$

$$P(A^cB) = .024$$

$$P(A^cB^c) = .776$$

Consider A_1, A_2, \dots are mutually exclusive & collectively exhaustive and event B



$$P(B) = P(B \cup \dots)$$

$$= P(B \cap (A_1 \cup A_2 \cup \dots))$$

$$= P((B \cap A_1) \cup (B \cap A_2) \cup \dots)$$

$$= P(B \cap A_1) + P(B \cap A_2) + \dots$$

$$P(B) = \sum_{i=1}^{\infty} P(B|A_i)$$

Law of total probability

$$(B \cap A_i) \cap (B \cap A_j)$$

$$= B \cap B \cap A_i \cap A_j = \emptyset$$

2 kids, 1 is a girl. $P(\text{other is a girl})$

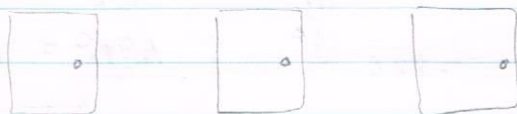
$$\Omega$$

GG	BG
GB	BB

$P(\text{other is a girl} | \text{if 1 is a girl})$

$$P(\{GG\} \cap \{GG, BG, GB\}) = \frac{1}{3}$$

$$\forall i \quad P(u_i) = \frac{1}{4} = \frac{P(\{GG\} \cap \{GG, BG, GB\})}{P(\{GG, BG, GB\})} = \frac{\frac{1}{4}}{\frac{3}{4}} = \boxed{\frac{1}{3}}$$



3 doors: 2 goats, 1 car

The probability of switching is $\frac{2}{3}$

