#22.

① Best guess $p \Rightarrow \hat{p} = \bar{x} = \frac{x_1 + \ldots + x_n}{n} = \frac{\#\,1's}{n}$.

② Provide window/range of p.

③ Test theorem of p.

$$P\left(p \in \left[\hat{p} \pm \sqrt{\frac{p(1-p)}{n}}\, z_{\frac{\alpha}{2}}\right]\right) = 1-\alpha$$

$$P\left(p \in \left[\hat{p} \pm \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\, z_{\frac{\alpha}{2}}\right]\right) \approx 1-\alpha$$

$$CI_{p,\,1-\alpha} := \left[\hat{p} \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right]$$

ppl who love mushroom. 5 ppl out of 16.

$$CI_{p,\,95\%} = \left[\frac{5}{16} \pm 2\sqrt{\frac{\frac{5}{16}\left(\frac{11}{16}\right)}{16}}\right] = [0.313 \pm 0.232] = [0.081,\ 0.545]$$

① If I take many sample and compare a $\hat{p}$ for each, $1-\alpha$ propability of the time they will cover (i.e cover p)
NOT USEFUL.

② Before obtaining the sample.
$P(p \in CI_{1-\alpha}) = 1-\alpha$ (Doesn't tell you anything about your CI)

③ $P\left(p \in \left[\hat{p} \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right]\right)$ = either 0 or 1 (not useful)

④ What everyone wants to say only true if you are subjectically and have specific prior info:
$P\left(p \in \left[\hat{p} \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right]\right) = 1-\alpha$.

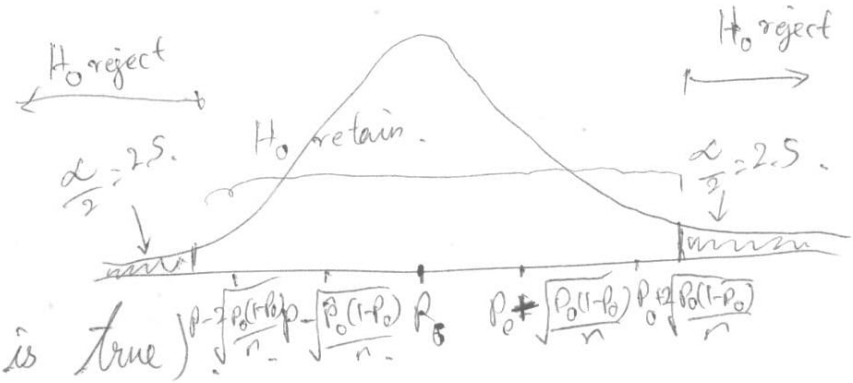
Do you think the proportion of human babies born male ≠ 50%

Yes: Simple model called the "null Hypothesis" (H_0)

$H_0: p = p_0 = 50\%$.

$H_a: \hat{p} \neq p_0 = 50\%$.

alternative hypothesis.



$\hat{p} \mid H_0$ (assuming H_0 is true)

Retainment region $:= \left[p_0 \pm \frac{z\alpha}{2} \sqrt{\frac{p_0(1-p_0)}{n}} \right]$

Rejection Region is the complement.

To test, check.

$\hat{p} \in$ Retainment Region $\Rightarrow$ Retain H_0.

$\hat{p} \in$ Retainment Region $\Rightarrow$ Reject H_0.

2 sided 1 proportion Hypothesis test.

$n = 345$, # of male $= 169$, $p = 50\% = .05$

$\hat{p} = \frac{169}{345} = 0.48$. Retain Region, $\alpha = 5\%$.

$\therefore \left[0.5 \pm 2\sqrt{\frac{0.5(0.5)}{345}} \right] = [.446, .554]$

Toss coin 100 times: Is this fair? $\alpha = 5\%$.

$H_0: p = 0.5$

$H_a: p \neq 0.5$ $\quad \hat{p} = \frac{61}{100} = 0.61$.

Ret Region $= \left[0.5 \pm 2\sqrt{\frac{0.5(0.5)}{100}} \right]$

$= [0.4, 0.6]$

$\hat{p} \notin$ Ret Region $\Rightarrow$ Reject H_0

Mars Inc said the proportion of Blue m&m's is 20%.
Let's test they are lying?

$H_0 : p = 0.2$, $H_a : p \neq 0.2$.

$\hat{p} = \frac{33}{206} = 0.16$. $n = 206$.

Retain Region $= 0.2 \pm 2\sqrt{\frac{0.2(0.8)}{206}} \approx$ ~~0.2557, 0.1443~~

$= (0.14426, 0.2557)$.

$\hat{p} \in$ Retain Region $\Rightarrow$ Return $H_0 \Rightarrow$ No reason to think they are lying.