conference Interval

$$CI_{p,1-\alpha} :=$$

unknown

100yr of debate

$$\left[\hat{p} \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{p(1-p)}{n}}\right] \approx \left[\hat{p} \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right]$$

When repeated ... gives you $1-\alpha$ "coverage" of p

2-sited 1-proportion confidence interval

$\boxed{12/4}$

Statistical Inference

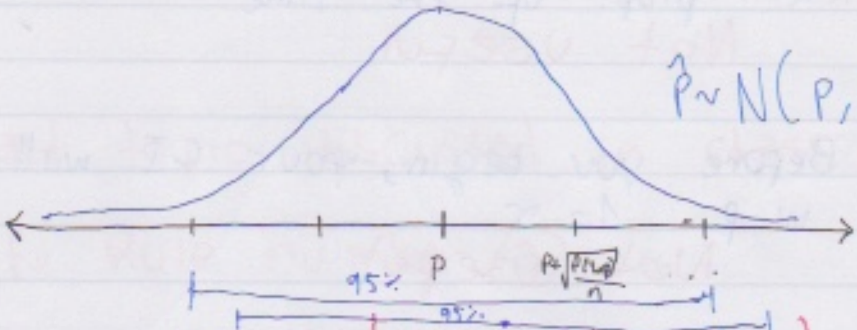
→ We don't ~~know~~ parameter so we want to...

1. Estimate its best guess
2. Provide range of possible (likely) values
3. Test theories ~~about~~ the parameter

$X_1, \dots, X_n \overset{iid}{\sim} \text{Bern}(p)$

$\hat{p} = \bar{x} = \dfrac{X_1 + \cdots + X_n}{n} = \dfrac{\#1\text{'s}}{n} \approx p$

$\hat{p} \sim N\left(p, \sqrt{\dfrac{p(1-p)}{n}}\right)$



look at last lecture for full info

$$P\left(p \in \left[\hat{p} \pm z_{\frac{\alpha}{2}} \sqrt{\frac{p(1-p)}{n}}\right]\right) = 1-\alpha$$

$$P\left(p \in \left[\hat{p} \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right]\right) \approx 1-\alpha$$

$$CI_{p,\,1-\alpha} := \left[\hat{p} \pm \overbrace{z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}}^{\text{margin of error}}\right]$$

Confidence interval for parameter p with coverage $1-\alpha$

$CI_{p,95\%} = [0.47, 0.57]$
$= [0.52 \pm 0.05]$

Interpretations of the CI

(1) If you sample many times and compute a CI for each, the $p \in CI$ $1-\alpha$ prop. of the time

Not useful

(2) Before you begin, your CI will contain p w.p. $1-\alpha$

Not useful

(3) $P(p \in CI_{p,1-\alpha})$
$= P(p \in [0.47, 0.57]) \in \{0,1\}$

Not useful ↳ Deg(0) or Deg(1)

Everyone wants this:

(4) $P(p \in CI_{p,1-\alpha}) = 1-\alpha$

only ~~the~~ available if you are subjectivist with the right prior information

Not a representative sample

Do you like mushrooms?

$n = 20$
sample size

Best guess of p: $\hat{p} = \frac{11}{20} = .55$

$\alpha = 5\%$ (95% coverage) $\Rightarrow z_{2.5\%} \approx 2$

$$CI_{p.95\%} = \left[.55 \pm 2\underbrace{\underbrace{\sqrt{\frac{.55 \cdot .45}{20}}}_{.11}}_{.22}\right] \approx [.33, .77]$$

does not give inference for the population of all humans.

Last topic discussed in class:

The Rule number 3

3 Test theories about the parameter

Human sex ratio / proportion

Do you think?

$P(\text{new human baby being male}) \neq 50\%$?

No. I think it's even.

$H_0 : P = 50\%$ → p is the probability of male being born

null hypothesis $= P_0$

We need "sufficient" evidence to reject the null hypothesis.

Occam's Razor:

simplest model is true.

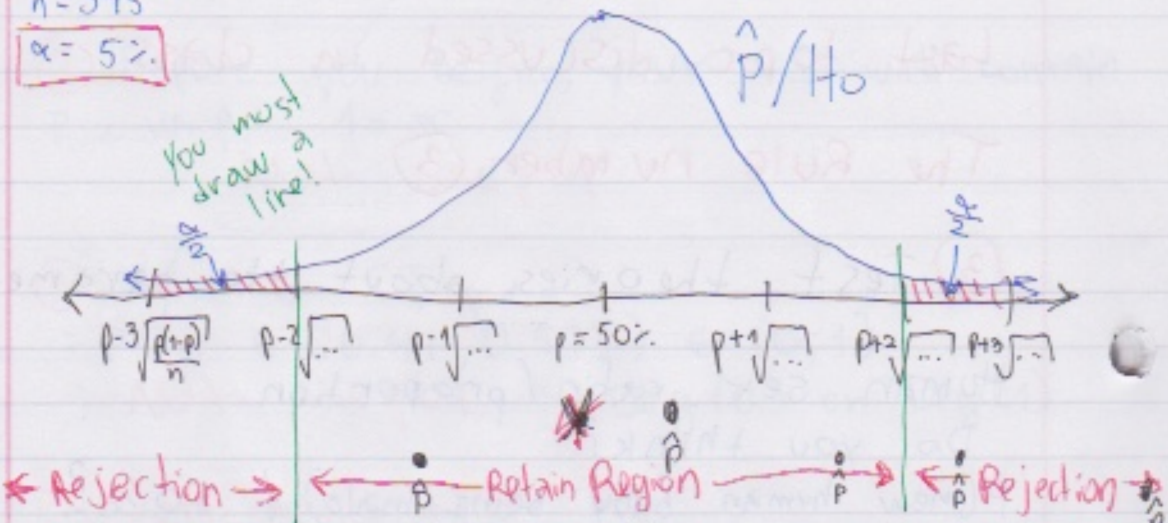
$H_0 : p = 50\%$, $H_a : p \neq 50\%$

hull hypothesis = P_0

Alternative Hypothesis

$n = 345$

$\boxed{\alpha = 5\%}$



We take a sample of size n.
(n is determined beforehand)
compare $\hat{p}$

Let Type (I) error rate

$\alpha := p(\text{Reject } H_0 \mid H_0 \text{ is true})$

$\alpha = 5\%$

Retainment Region

$$\left[p_0 \pm \frac{z_\alpha}{2} \sqrt{\frac{p_0(1-p_0)}{n}}\right]$$

to run the test, compute $\hat{p}$

Rejection Region is the complement of the retainment region

If $\hat{p} \in$ Retainment region $\Rightarrow$ Retain H_0
$\hat{p} \notin$ Retainment region $\Rightarrow$ Reject H_0

$$\left[0.5 \pm 2\sqrt{\frac{0.5(1-0.5)}{345}}\right.$$

$$= [.446, .554]$$

... Compute $\hat{p} = 169/345 = .48 \in$ Retainment Region
$\Rightarrow$ Retain H_0

Why do we need this?

Let's say you're testing if a coin is fair
$H_0 : p = 0.5 \rightarrow$ prop of heads

Situation ①: $n = 100$, # head = 51. Fair? Yes
Situation ②: $n = 100$, # head = 98. Fair? NO
Situation ③: $n = 100$, # head = 61. Fair?
At $\alpha = 5\%$ ...

The smaller the α, the harder it is to reject

15
5 blue 1 yellow
5 green 1 red
3 orange

Situation 3

Retainment Region:

$\left[p_0 \pm z_{\frac{\alpha}{2}} \sqrt{\frac{p_0(1-p_0)}{n}}\right]$

$=\left[0.5 \pm z \sqrt{\frac{0.5(1-0.5)}{100}}\right]$

$=[0.40, 0.60]$

$.61 \notin$ Retainment $\Rightarrow$ Reject H_0 (The coin is not fair)

M&M factory says 20% are blue.

Let's test this. $\alpha = 5\%$.

$H_0 : p_0 = 0.2$ $\quad n = 271$

$\hat{p} = \frac{58 \text{ blue}}{271} = .214$

$H_a : p_0 \neq 0.2$

2 or 2? I don't know

Retainment Region $= \left[p_0 \pm 2 \sqrt{\frac{p_0(1-p_0)}{n}}\right]$

$\hat{p} \in$ Retention Region $\Rightarrow$ Retain H_0

$= \left[0.2 \pm 2 \underbrace{\underbrace{\sqrt{\frac{0.2 \cdot 0.8}{271}}}_{.025}}_{.050}\right]$

$=[0.15, 0.25]$

Decision

TRUTH		Retain H_0	Reject H_0
	H_0 True	✓	Type I error
	H_0 False	Type II error	✓

decision theory

$1 - P(\text{Type II error})$
$= P(\text{Rejecting } H_0 \mid H_0 \text{ false})$
$=$ Power ← advanced class

$p(\text{Type I error})$
$= P(\text{Rejecting } H_0 \mid H_0 \text{ true})$

$= \alpha$

$P(\text{Type II error}) = P(\text{Retain } H_0 \mid H_0 \text{ false}) = \ldots$ advanced class