9/18

I Long run Frequency Definition of Probability

$$P(A) := \lim_{n \to \infty} \frac{1}{4} \sum_{i=1}^{n} \mathbb{1}_{a_i \in A}$$

II Propensity Theory of Probability

Karl Popper, 1957

Objects have an inherited disposition towards one ω or the other

→ Consider Uranium 238. P(U238 atom exploding ≤ 4.5 Billion years) = $\frac{1}{2}$

Problems

1. Difficult or impossible to compute propensity

I, II are called "objective" or Function of physical reality

III Subjective Theory of Probabily

→ People use their own evidence and intuition to come up with their estimate of uncertainty.

Problems

→ 1 Everyone has a different probability
- Ramsey, 1926, de Finetti, 1920

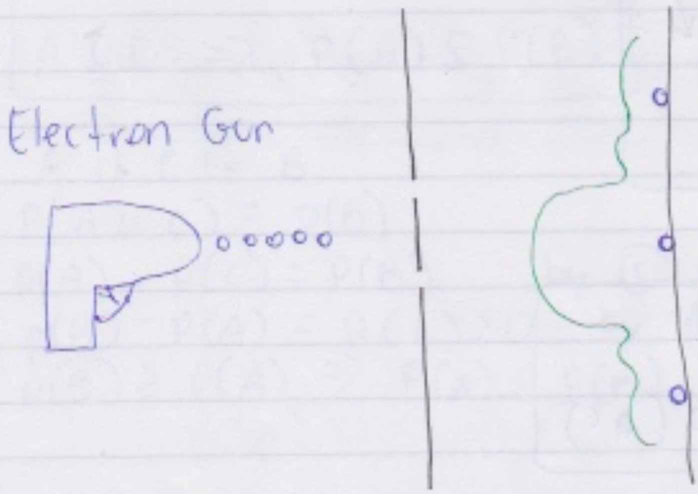
Question?

P(F = MR is true) = degree of "corroboration"

⇒ There is no universally accepted / unproblematic definition of probability

Laplace says randomness is an illusion. It is only due to your ignorance and your inability to do the necessary computation.

Double slit Experiment



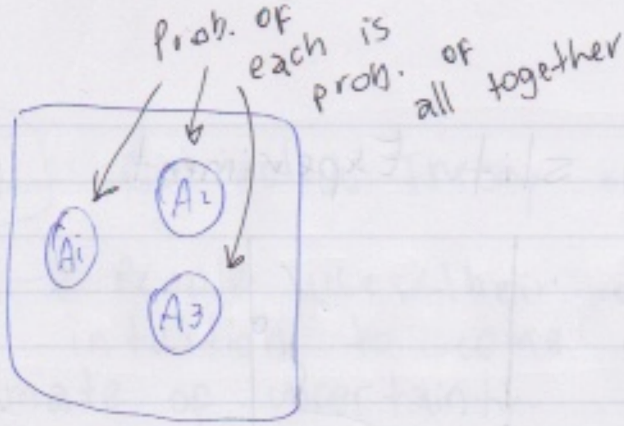
Probability (as a mathematical theory) seems to be invented in the 1600's

Mathematical Theory of Probability

Kolmogorer, 1930's

Assume $\exists\ \Omega = \emptyset$. P is a set function so that

a) $P(\Omega) = 1$
b) $\forall A \quad P(A) \geq 0$
c) If $A_1, A_2, A_3, \ldots$ disjoint $\Rightarrow P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i)$



$$\boxed{\text{Theory I} \quad P(A) = 1 - P(A^c)}$$

$\Omega = A \cup A^c \Rightarrow A, A^c$ are disjoint

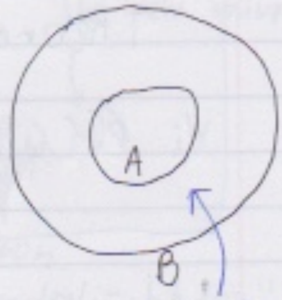
$$P(\Omega) = P(A \cup A^c)$$
$$P(\Omega) = P(A) + P(A^c) \quad \text{by } c$$
$$1 = P(A) + P(A^c) \quad \text{by } a$$
$$\Rightarrow P(A) = 1 - P(A^c)$$

$$\boxed{\text{Theory II} \quad P(\emptyset) = 0}$$

$$P(\emptyset) = 1 - P(\emptyset^c)$$
$$P(\emptyset) = 1 - P(\Omega)$$
$$P(\emptyset) = 1 - 1 \quad \text{by } a$$
$$= 0$$

Theorem III

$$\boxed{A \subseteq B \Rightarrow P(A) \leq P(B)}$$



$C = B \setminus A$

A, C disjoint

$A \cup C = B$

$P(A \cup C) = P(B)$

$P(A) + P(C) = P(B)$ by (c)

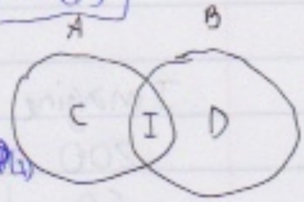
$P(B) - P(A) = P(C) \geq 0$ by (b)

$P(B) \geq P(A) \Rightarrow P(A) \leq P(B)$

Law of "inclusion-exclusion"

Theorem IV

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$



$A \cup B = C \cup D \cup I$

$P(A \cup B) = P(C \cup D \cup I) = P(C) + P(D) + P(I)$

$= (P(C) + P(I)) + (P(D) + P(I)) - P(I)$

$= P(C \cup I) + P(D \cup I) - P(I)$

$= P(A) + P(B) - P(A \cap B)$

$C = A \setminus B$

$D = B \setminus A$

$I = A \cap B$

$$\boxed{\text{Theorem V} \quad |\Omega| < \infty \quad \text{If} \quad \forall i \;\; P(\{\omega_i\}) = \frac{1}{|\Omega|} \Rightarrow \forall A \;\; P(A) = \frac{|A|}{|\Omega|}}$$

$|A| = n$

$\Rightarrow A = \{\omega_1, \omega_2, \dots, \omega_n\}$

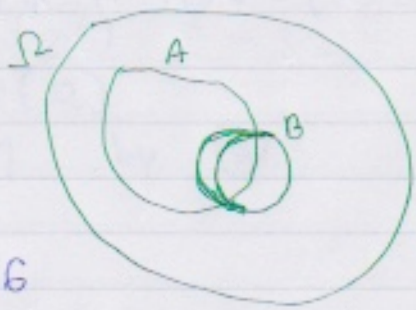
$A = \{\omega_1\} \cup \{\omega_2\} \cup \dots \cup \{\omega_n\}$

$P(A) = P(\{\omega_1\}) \cup P(\{\omega_2\}) \cup \dots \cup (\{\omega_n\})$ by ⓒ

$= \frac{1}{|\Omega|} + \frac{1}{|\Omega|} + \dots + \frac{1}{|\Omega|} = \frac{n}{|\Omega|} = \frac{|A|}{|\Omega|}$

Imagine $n = 1000$ people
200 smokers (A: smoking)
60 lung cancer (B: lung cancer)
36 smokers & lung cancer ($A \cap B$)

$\boxed{\text{Assume}}$



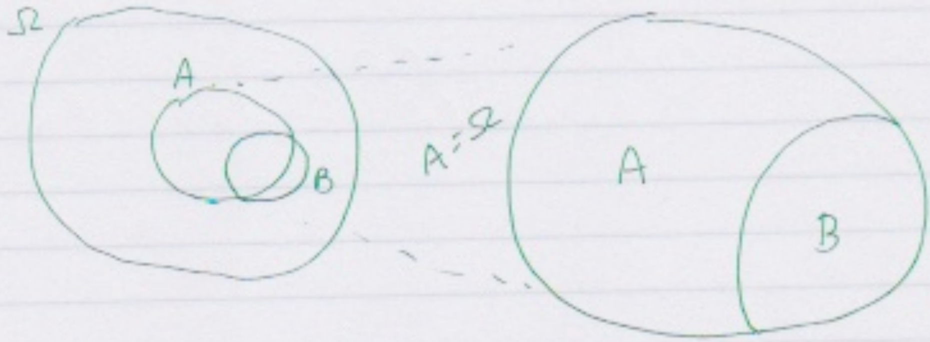
$P(A) = 0.2$
$P(B) = 0.06$
$P(A \cap B) = 0.036$

$P(\text{lung cancer}) = P(B) = 0.06$

the "new universe"

$P(\text{lung cancer among smokers}) = P(B \backslash A)$

"given"
"conditional on"



$P(B \backslash A) = \frac{36}{200} = 0.18$