$\boxed{8/30}$

... continuation

Power Set & Review from 8/28

$2^A = \{B : B \subseteq A\}$

$A = \{1, 2, 3\}$

$2^A = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{1,3\}, A\}$

$|A| = 3$ (cardinality)

$|F \cup M| \overset{?}{=} |F| + |M| \quad 7 \neq 4+4$

$|F \setminus M| \overset{?}{=} |F| - |M| \quad 3 \neq 4-4$

$|F \cap M| \overset{?}{=} |F| - |M| \quad 1 \neq 4-4$

$|2^A| = 8 = 2^3 = 2^{|A|}$

let's say we have

$\{ \overbrace{\frac{F}{1}\ \frac{F}{2}\ \frac{F}{3}}^{\emptyset},\ \frac{T}{1}\ \frac{F}{2}\ \frac{F}{3}, \dots$ (follow logic)

All true/false permutations $\overbrace{\frac{F|T}{1}}^{2} \cdot \overbrace{\frac{F|T}{2}}^{2} \cdot \overbrace{\frac{F|T}{3}}^{2}$

$|2^A| = 2^{|A|}$

Special set Ω
called "universe",
"space of discourse",
"scope". What you are limited to.

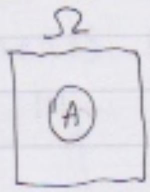
You define it

$\Omega := F \cup M$

Note:

$F \subseteq \Omega$
$M \subseteq \Omega$

All sets are subsets of Ω



$\forall A$

$\boxed{A \cap \Omega = A}$

$\boxed{A \cup \Omega = \Omega}$

$\boxed{\emptyset \cup \Omega = \Omega}$

$\boxed{\emptyset \cap \Omega = \emptyset}$

A^c : " A-complement"

everything that is not A

$\boxed{A^c := \Omega \setminus A}$

$(A^c)^c = A$

$A \cup A^c = \Omega$

$A \cap A^c = \emptyset$

A, A^c are $\boxed{\text{neautrally exclusive}}$, or $\boxed{\text{distinct}}$

$\boxed{A, A^c \text{ are collectevly exhaustive}}$

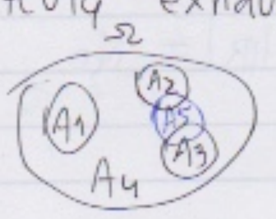
They have everything in the universe

$\{A_1, A_2, \dots\}$

are mutually exclusive if $A_i \cap A_j = \emptyset \quad \forall i \neq j$

$\{A_1, A_2, \dots\}$

are collectevly exhaustive if $A_1 \cup A_2 \cup \dots = \bigcup_{i=1}^{\infty} A_i = \Omega$



$|A| + |A^c| \stackrel{?}{=} |\Omega|$

Question:

$\mathbb{N} = \{1, 2, \ldots\}$ $\aleph_0$ = countable infinity

$|\mathbb{N}| = \aleph_0$

$\mathbb{Z} = \{\ldots, -1, 0, 1, \ldots\}$

$|\mathbb{Z}| = \aleph_0$

Definition $|A| = |B|$

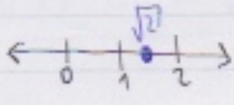
$\exists\, f : A \to B$ 1:1 onto $\therefore$

$\mathbb{Q} := \left\{ \frac{p}{q} : p \in \mathbb{Z},\ q \in \mathbb{N} \right\}$

$|\mathbb{Q}| = \aleph_0$

$\sqrt{2} \notin \mathbb{Q}$ $\mathbb{R} := \mathbb{Q} \cup \{\text{all holes}\}$

$\pi \notin \mathbb{Q}$ $|\mathbb{R}| \stackrel{?}{=} \aleph_0$



$[a, b] := \{x : x \geq a \ \& \ x \leq b\}$

$(a, b) := \{x : x > a \ \& \ x < b\}$

$A = (0, 1)$

$|A| = ?$

He expressed them as binary decimals

diagonal

$$\text{If } |(0,1)| = \aleph_0 \Rightarrow \begin{array}{l} 0.000\ldots \\ 0.010\ldots \\ 0.111\ldots \end{array}$$

x^* = diagonal flipped

$= 0.100\ldots$

$x^* \neq \{x_1, x_2, \ldots\}$

$\Rightarrow |\mathbb{R}| = c$ (uncountable)

Stuff for test

$\langle a, b \rangle := \{ \{a\}, \{a,b\} \}$

is called an ordered pair

$(a,b) = \{ \{b\}, \{a,b\} \} \neq$

$\underbrace{A \times B}_{\substack{\uparrow \\ \text{cartesian product}}} := \{ (a,b) : a \in A, b \in B \}$

$A = \{1, 2\}, \quad B = \{3, 4\}$

$A \times B = \{ (1,3), (1,4), (2,3), (2,4) \}$

$|A \times B| = 4 = |A|\,|B|$ ← cool Actually a rule

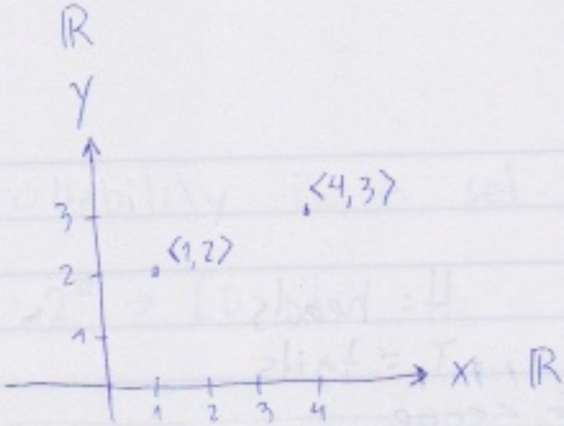
$|A_1 \times A_2 \times \ldots \times A_n| = \prod_{i=1}^{n} |A_i|$

$A^2 := A \times A$

$A^3 := A \times A \times A$

$|A^2| = |A \times A| = |A|\,|A| = |A|^2$

$|A^n| = |A|^n$



$\mathbb{R} \times \mathbb{R}$
cartesian plane

Actually
Now
we're getting
into probability

Probability

Ω is called "sample space" or "outcome space"
its elements are called "outcomes"
denoted the ω's (ω = lower case Ω)

We define an experiment is drawing an ω from Ω

experiment.

coin toss: H = heads
T = tails

$\Omega = \{H, T\}$ ← scope

$\omega_1 = H$
$\omega_2 = T$

$$\boxed{P(\{H\}) = \frac{|\{H\}|}{|\Omega|}}$$

there's a problem here

H is an element
Now a set

$P(H)$
$P: \Omega \to [0, 1]$

$$2^{\Omega} = \{\emptyset, \{H\}, \{T\}, \underbrace{\{H, T\}}_{= \Omega}\}$$

↑
set of all "elements"

An event $A \subseteq \Omega$

All possible events are $\in 2^{\Omega}$

Probability is a set function

$P: 2^{\Omega} \to [0,1]$

$\boxed{\text{Dice Roll experiment}}$

$\Omega = \{1, 2, 3, 4, 5, 6\}$

$P(\text{even } \#)$

$A = \text{even } \# = \{2, 4, 6\} \subseteq \Omega$, $\{2,4,6\} \in 2^{\Omega}$

$|2^{\Omega}| = 2^{|\Omega|} = 2^6 = 64$ $\boxed{\text{Probability questions}}$

Trivial questions:

$P(\emptyset) = \frac{|\emptyset|}{|\Omega|} = \frac{0}{6} = 0$

$P(\Omega) = \frac{|\Omega|}{|\Omega|} = \frac{6}{6} = 1$

Working definition of $P(A) := \frac{|A|}{|\Omega|}$

Let $A :=$ even #'s

$P(A) = \frac{|A|}{|\Omega|} = \frac{|\{2,4,6\}|}{|\Omega|} = \frac{3}{6} = \frac{1}{2}$

experiment

Flip two coins

$\Omega = \{ \langle H, H \rangle, \langle H, T \rangle, \langle T, H \rangle, \langle T, T \rangle \}$

$\Omega^1 = \Omega^2 = \{ \langle H, H \rangle \cdots$

$\Omega \times \Omega$))

$P(\text{Heads \& Heads}) = P(\{\langle H, H \rangle\}) = \frac{|\{\langle H, H \rangle\}|}{|\Omega|} = \frac{1}{4}$

$p(\text{one Heads \& one Tails}) = p(\{\langle H, T \rangle, \langle T, H \rangle\}) =$

$\frac{|\{\langle H, T \rangle, \langle T, H \rangle\}|}{|\Omega|} = \frac{2}{4} = \frac{1}{2}$