LRF def. coin flips

1m

II) Propensity theory

Objects have inherent disposition toward one or another. This "propensity" induces the l.r.f. (Karl Popper, 1957)

Inside this coin is an intelligence which forces it towards H 50% of the time and T 50% of the time. It's like a micromachine!

Popper was trying to assign prob's to events that are not repeatable

Where did he get this idea?

Consider U238 $\frac{1}{2}$ life of 4.5 Byr.

P(U238 atom exploding before 4.5 Byr) $= \frac{1}{2}$

Why? Due to hard-wired quantum mechanics.

Popper views all random events like this.

Problems

1. Cannot be calculated (unless you have the physical formulas).
2. Same as above

I, II are considered "objective" theories of prob. That is, they are independent of our beliefs. The tree falls in a forest $\Rightarrow$ yes it makes a noise!

(III) Subjective theory - people use evidence to come up with their own estimate of certainty. Ramsey, 1926

de Finetti, 1928

P_{Me}(OJ Simpson guilty) = 99%

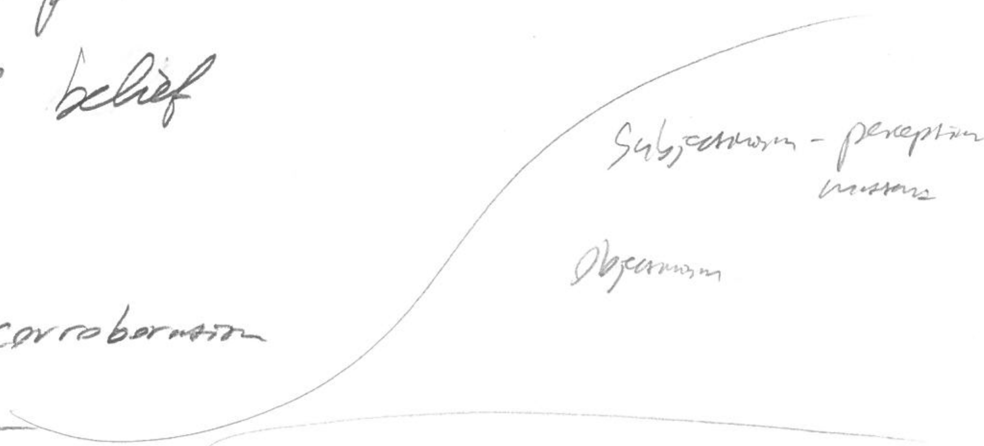
P_{Me}(Irma hits Miami) = 90%

most simulation models predict this

prob: degree of belief

P_{Me}(F = ma is true)

prob: degree of corroboration



⇒ No def. of prob. that is universally accepted

⇒ Def of phenomenon of prob. is an open problem

Another question: what is "randomness"? Is it real?

Flip a coin: what if you knew air pressure, etc. everything!

would it be $P(H) = \frac{1}{2}$? No ... you would know with certainty!

(Laplace's Demon) ⇒ No free will

Accepted... until..

1920's

Probabilistic!!

Not deterministic!

Einstein "dice with the universe"

Further, why was prob "invented" only in 1600's?

Pythagoras $\approx$ 500BC was studying geometry! Astrology...

"astragali"

In particular, we find the usual admiration for Newtonian mechanics, and the consequent belief in *universal determinism*. Indeed, Laplace's *Philosophical Essay on Probabilities* of 1814 gives one of the most famous formulations of the thesis of universal determinism. This is the formulation involving what is known as *Laplace's demon*. I will expound it in the next section.

Universal determinism and Laplace's demon

Laplace writes:

> We ought then to regard the present state of the universe as the effect of its anterior state and as the cause of the one which is to follow. Given for one instant an intelligence which could comprehend all the forces by which nature is animated and the respective situation of the beings who compose it – an intelligence sufficiently vast to submit these data to analysis – it would embrace in the same formula the movements of the greatest bodies of the universe and those of the lightest atom; for it, nothing would be uncertain and the future, as the past would be present to its eyes.
>
> (1814: 4)

The vast intelligence here described has come to be known as Laplace's demon. The idea is obviously founded on that of a human scientist (perhaps Laplace himself) using Newtonian mechanics to calculate the future paths of planets and comets. Extrapolating from this success, it was natural to suppose that a sufficiently vast intelligence could calculate the entire future course of the universe. Laplace himself relates his vast intelligence to human successes in astronomy. As he says:

> The human mind offers, in the perfection which it has been able to give to astronomy, a feeble idea of this intelligence. Its discoveries in mechanics and geometry, added to that of universal gravity, have enabled it to comprehend in the same analytical expressions the past and future states of the system of the world.
>
> (Laplace 1814: 4)

Assume $\exists \Omega \neq \emptyset$. "$P$" is a ~~set~~ function s.t.

(a) $P(\Omega) = 1$

(b) $P(A) \geq 0 \quad \forall A \subseteq \Omega$

(c) If $A_1, A_2, \dots$ disjoint $\Rightarrow P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i)$

you can add disjoint probs of

Thm I $\quad P(A) = 1 - P(A^c)$

$\Omega = A \cup A^c$ set theory

$P(\Omega) = P(A \cup A^c)$ by def of function

$P(\Omega) = P(A) + P(A^c)$ via (c)

$1 = P(A) + P(A^c)$ via (a)

$\Rightarrow P(A) = 1 - P(A^c)$ (algebra)

Thm 2 $\quad P(\emptyset) = 0$

$P(\Omega) = 1 - P(\Omega^c) = 1 - P(\emptyset) \Rightarrow 1 = 1 - P(\emptyset) \Rightarrow P(\emptyset) = 0$

Thm 3

$A \subseteq B \Rightarrow P(A) \leq P(B)$

$\Rightarrow C := B \setminus A$

$\Rightarrow A \cup C = B$ & $A \cap C = \emptyset$

$P(A \cup C) = P(B)$

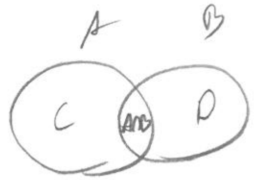
$P(A) + P(C) = P(B)$ (c)

$P(C) = P(B) - P(A) \geq 0$ (b)

$P(B) \geq P(A)$ ✓

Thm 5

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$ Inclusion-exclusion



$C = A \setminus B$

$D = B \setminus A$

$I = A \cap B$

$P(A) = P(C) + P(I) \Rightarrow P(C) = P(A) - P(I)$

$P(B) = P(D) + P(I) \Rightarrow P(D) = P(B) - P(I)$

$P(A \cup B) = P(C) + P(D) + P(I)$

$= (P(A) - P(I)) + (P(B) - \cancel{P(I)}) + \cancel{P(I)}$

$= P(A) + P(B) - P(A \cap B)$

Generally... $\cancel{P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i) - \sum_{i \neq j} P(A_i \cap}$

HW: Booles inequality

Thm 6

$|\Omega| < \infty$, if $P(\{\omega_i\}) = \frac{1}{|\Omega|}\ \forall \omega \Rightarrow P(A) = \frac{|A|}{|\Omega|}$

let $n = |A| < \infty$ since $|\Omega| < \infty$ $A \subseteq \Omega \Rightarrow |A| < \infty$

$A = \{\omega_1, \omega_2, \dots, \omega_n\}$

$A = \{\omega_1\} \cup \{\omega_2\} \cup \dots \cup \{\omega_n\}$

$\Rightarrow A = \bigcup_{i=1}^{n} \{\omega_i\}$

(c)

$P(A) = P(\,) = \sum_{i=1}^{n} P(\{\omega_i\}) = \sum_{i=1}^{n} \frac{1}{|\Omega|} = \frac{n}{|\Omega|} = \frac{|A|}{|\Omega|}$

Imagine $n=1000$ people (Ω)

200 smokers (A)

60 lung cancer (B)

36 S & l.c (AB)

$A \cap B$, A,B or $A \& B$

Via LRF Def...

$P(A) = \frac{200}{1000} = 0.2$

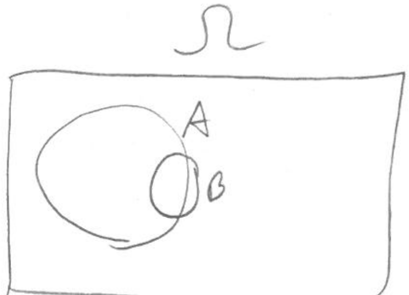
$P(B) = \frac{60}{1000} = 0.06$

$P(AB) = \frac{36}{1000} = 0.036$

What if I want to know the "probability of l.c. only among smokers"

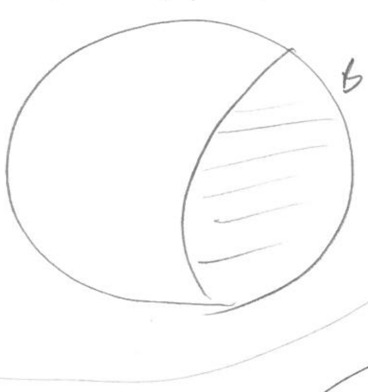
given smoking, what is $P(\text{l.c.})$? Denoted...

$P(B|A)$



Then you just need to look at A and ignore the rest of the Ω.

$\Omega' := A \subseteq \Omega$



$P(B|A) = \frac{36}{200} = 0.18$

What if we only had prob's?

Same shape but zoomed in...

Note