Bishop 3.3

$$E_D(w) = \frac{(t - \phi w)^T}{2} R (t - \phi w)$$

$$w^* = (\phi^T R \phi)^{-1} \phi^T R t$$

Bishop 3.11

$$\sigma^2_{N+1}(x) = 1/\beta + \phi(x)^T S_{N+1} \phi(x)$$

$$S_{N+1} = (S_N^{-1} + \beta \phi_{N+1} \phi_{N+1}^T)^{-1}$$

$$= S_N - \frac{(S_N \phi_{N+1} \beta^{1/2})(\beta^{1/2} \phi_{N+1}^T S_N)}{1 + \beta \phi_{N+1}^T S_N \phi_{N+1}}$$

$$= S_N - \frac{\beta S_N \phi_{N+1} \phi_{N+1}^T S_N}{1 + \beta \phi_{N+1}^T S_N \phi_{N+1}}$$

$$\sigma^2_{N+1}(x) = 1/\beta + \phi(x)^T \left(S_N - \frac{\beta S_N \phi_{N+1} \phi_{N+1}^T S_N}{1 + \beta \phi_{N+1}^T S_N \phi_{N+1}} \right) \phi(x)$$

$$= \sigma^2_N - \frac{\beta \phi(x)^T S_N \phi_{N+1} \phi_{N+1}^T S_N \phi(x)}{1 + \beta \phi_{N+1}^T S_N \phi_{N+1}}$$

Bishop 3.14

When $\alpha = 0$:

$$S_N = (\beta \Phi^T \Phi)^{-1}$$

$$\psi(x) = \underbrace{V}\phi(x)$$

$V \rightarrow$ square matrix

$$\phi(x) = V^{-1}\psi(x)$$

$$\Psi = \Phi V^T \Leftrightarrow \Phi = V^{-T}\Psi$$

$$\begin{aligned} S_N &= \beta^{-1}(\Phi^T \Phi)^{-1} \\ &= \beta^{-1}(V^{-T}\Psi^T \Psi V^{-1})^{-1} \\ &= \beta^{-1} V^T V \end{aligned}$$

$$\begin{aligned} k(x, x') &= \beta \phi(x)^T S_N \phi(x') \\ &= \phi(x)^T V^T V \phi(x') \\ &= \psi(x)^T \psi(x') \end{aligned}$$

When $j = 1$:

$$\begin{aligned} \sum_{n=1}^{N} k(x, x_n) &= \sum_{n=1}^{N} \psi(x)^T \psi(x_n) \\ &= \sum_{n=1}^{N} \sum_{i=1}^{m} \psi_i(x)\psi_i(x_n) \\ &= \sum_{i=1}^{m} \psi_i(x)\delta_{i,1} \\ &= \psi_1(x) \\ &= 1 \end{aligned}$$

$\blacksquare$

Bishop 3.21

$$A u_i = \lambda_i u_i$$

$$\ln|A| = \ln\left(\prod_{i=1}^{m} \lambda_i\right)$$

$$= \sum_{i=1}^{m} \ln(\lambda_i)$$

$$\frac{d}{d\alpha} \ln|A| = \sum_{i=1}^{m} \frac{1}{\lambda_i} \frac{d}{d\alpha} \lambda_i$$

$$A = \sum_{i=1}^{m} \lambda_i u_i u_i^T \rightarrow A^{-1} = \sum_{i=1}^{m} \frac{1}{\lambda_i} u_i u_i^T$$

$$Tr\left(A^{-1} \frac{d}{d\alpha} A\right) = Tr\left(\sum_{i=1}^{m} \frac{i}{\lambda_i} u_i u_i^T \frac{d}{d\alpha} \sum_{j=1}^{m} \lambda_j u_j u_j^T\right)$$

$$= Tr\left(\sum_{i=1}^{m} \frac{1}{\lambda_i} u_i u_i^T \left\{\sum_{j=1}^{m} \frac{d\lambda_j}{d\alpha} u_j u_j^T + \lambda_j (b_j u_j^T + u_j b_j^T)\right\}\right)$$

$$= Tr\left(\sum_{i=1}^{m} \frac{1}{\lambda_i} u_i u_i^T \sum_{j=1}^{m} \frac{d\lambda_j}{d\alpha} u_j u_j^T\right) + Tr\left(\sum_{i=1}^{m} \frac{1}{\lambda_i} u_i u_i^T \sum_{j=1}^{m} \lambda_j (b_j u_j^T + u_j b_j^T)\right)$$

$$= \ldots$$

$$= Tr\left(\frac{d}{d\alpha} \sum_{i}^{m} u_i u_i^T\right)$$

But $\sum_{i}^{m} u_i u_i^T = I$

PTO →

$$\operatorname{Tr}\left(A^{-1}\frac{d}{dx}A\right)=\sum_{i=1}^{m}\frac{1}{n_i}\frac{dn_i}{dx}$$

Incomplete - ran out of time.