

Bishop 6.2

$$w = \sum_{n=1}^N \alpha_n t_n \phi(x_n) \quad \alpha_n \equiv \text{frequency of } n \text{ updating } w$$

$$\begin{aligned} y(x) &= \text{sign}(w^T \phi(x)) \\ &= \text{sign}\left(\sum_{n=1}^N \alpha_n t_n \phi(x_n)^T \phi(x)\right) \\ &= \text{sign}\left(\sum_{n=1}^N \alpha_n t_n h(x_n, x)\right) \end{aligned}$$

$$\therefore \alpha_n \rightarrow \alpha_n + 1$$

$$t_n (w^T \phi(x_n)) \geq 0$$

$$t_n \left( \sum_{n=1}^N h(x_n, x_n) \right) \geq 0$$

Learning algorithm depends on Gram Matrix

### Bishop 7.3

$$w^T x_1 + b = 1$$

$$w^T x_2 + b = -1$$

$$\arg\min_{w, b} \left\{ \frac{1}{2} \|w\|^2 + \underbrace{\lambda (w^T x_1 + b - 1)}_{\text{Lagrange multiplier}} + \underbrace{\eta (w^T x_2 + b + 1)}_{\text{Lagrange multiplier}} \right\}$$

Taking the derivative:

$$w + \lambda x_1 + \eta x_2 = 0$$

$$\lambda + \eta = 0$$

$$w = \lambda (x_1 - x_2)$$

$$2b = -w^T (x_1 + x_2)$$

$$b = -\frac{\lambda}{2} (x_1 - x_2)^T (x_1 + x_2)$$

$$= -\frac{\lambda}{2} (x_1^T x_1 - x_2^T x_2)$$

### Bishop 7.4

$$p = \frac{1}{\|w\|} \rightarrow \frac{1}{p^2} = \|w\|^2$$

$$L(w, b, \gamma) = \frac{\|w\|^2}{2} = \sum_n \alpha_n - \frac{\|w\|^2}{2}$$