Problem Set 3

Bishop 5.3

$$p(T|X,W,\Sigma) = \prod_{n=1}^{N} \mathcal{N}(t_n | y(x_n,w),\Sigma)$$

$$\ln p(T|X,W,\Sigma) = -N/2(\ln|\Sigma| + K\ln(2\pi)) - 1/2\sum_{n=1}^{N}(t_n-y_n)^T\Sigma^{-1}(t_n-y_n)$$

$$E(w) = 1/2\sum_{n=1}^{N}(t_n-y_n)^T\Sigma^{-1}(t_n-y_n)$$ ← Error Function / ↑ Likelihood Function

Maximizing likelihood function w.r.t Σ:

$$-N/2\ln|\Sigma| - 1/2\sum_{n=1}^{N}(t_n-y_n)^T\Sigma^{-1}(t_n-y_n)$$

$$= -N/2\ln|\Sigma| - 1/2\,\mathrm{Tr}\left[\Sigma^{-1}\sum_{n=1}^{N}(t_n-y_n)(t_n-y_n)^T\right]$$

$$\therefore \Sigma = 1/N\sum_{n=1}^{N}(t_n-y_n)(t_n-y_n)^T$$

Bishop 5.4

$$p(t=1|x) = \sum_{k=0}^{1} p(t=1|k)\,p(k|x)$$

$$= (1-\epsilon)y(x,w) + \epsilon(1-y(x,w))$$

$$p(t|x) = p(t=1|x)^t(1-p(t=1|x))^{1-t}$$

$$\therefore E(w) = -\sum_{n=1}^{N}(t_n\ln((1-\epsilon)y(x_n,w)+\epsilon(1-y(x_n,w))) + (1-t_n)\ln(1-(1-\epsilon)y(x_n,w) - \epsilon(1-y(x_n,w))))$$

Bishop 5.26

$$\Omega_n = \tfrac{1}{2} \sum_k \left(\sum_i \tau_{ni} \frac{\partial y_{nk}}{\partial x_{ni}} \right)^2$$

$$= \tfrac{1}{2} \sum_k \left(\sum_i \tau_{ni} J_{nki} \right)^2 \quad \leftarrow \text{Jacobian } J \text{ from textbook}$$

$$\beta_{nj} = \sum_i w_{ji} \alpha_{ni}$$

$$= \sum_i w_{ji} \mathcal{G} x_{ni}$$

$$= \sum_i w_{ji} \sum_{i'} \tau_{ni'} \frac{\partial x_{ni}}{\partial x_{ni'}}$$

$$= \sum_i w_{ji} \tau_{ni}$$

$$\Omega_n = \tfrac{1}{2} \sum_k (\mathcal{G} y_{nk})^2$$

$$= \tfrac{1}{2} \sum_k \alpha^2_{nk}$$

$$\phi_{nkr} = \mathcal{G} \delta_{nkr}$$

$$= \mathcal{G} \left(h'(a_{nr}) \sum_l w_{lr} \delta_{nkl} \right)$$

$$= h''(a_{nr}) \beta_{nr} \sum_l w_{lr} \delta_{nkl}$$

$$+ h'(a_{nr}) \sum_l w_{lr} \phi_{nkl}$$

$$\partial \Omega_n / \partial w_{rs} = \sum_k (\mathcal{G} y_{nk}) \mathcal{G} (\delta_{nkr} z_{ns})$$

$$= \sum_k \alpha_{nk} (\phi_{nkr} z_{ns} + \delta_{nkr} \alpha_{ns})$$

$$\delta_{nkr} = h'(a_{nr}) \sum_l w_{lr} \delta_{nkl}$$