We demonstrate that despite significant uncertainty in the time evolution of the shock, we can still break the degeneracy of the outgoing and incoming merger scenarios by comparing the observed and the simulated position of the radio relic. The key to the success of this method lies our ability in estimating the upper and lower bounds on the possible position of the relic. The uncertainty of the time evolution of the velocity of the shock stem from how the velocity depends on a number of physical quantities, including the local gravitational potential, matter density, temperature, pressure among others (Ensslin et al. 1998, Shu .F. , more citations?). The exact time evolution of the shock velocity requires detailed numerical simulation similar to Springel & Farrar (2007), Vazza et al. (2012), Kang et al. (2007), etc. We draw physical insight from the simulation of the merger shock of the Bullet cluster from Springel & Farrar (2007), which shows that, right after the collision of the subclusters, the shock speed is comparable to the merger speed of the two subclusters in the center of mass frame. The shock speed mildly drops by only $\sim 14\%$ in the 300 Myr period after the formation of the shock versus a $\sim 65\%$ drop for the main subcluster, which was slowed down by gravity.

We approximated the upper and lower bounds of the shock speed with the simulated speeds of the subclusters. We simplified the calculation by working in the center of mass frame where the shock speed drops only slightly with time. We estimated the upper and lower bounds of the time-averaged velocity ($\langle v_{relic} \rangle$) of the shock between the collision of the subclusters and the observed time as:

$$\frac{(v_{3D}(t_{obs}) + v_{3D}(t_{col}))}{2} < \langle v_{relic} \rangle \lesssim v_{3D}(t_{col}) \tag{1}$$

The lower bound is set to be the average speed of the subcluster since the relic is observed to have traveled further away than the NW subcluster from the center of mass. The upper bound can be approximated as the relative collision speed of the subclusters due to how the shock is powered by the collision. This is consistent with the fact that shock is a pressure wave and does not experience gravitational deceleration while some dissipative processes may have slowed down the shock wave. It is unlikely that there would be significant energy injected into the shock to speed up the shock .

$$s_{proj} = \langle v_{relic} \rangle (t_{obs} - t_{col}) \cos(\hat{\alpha}) \tag{2}$$

where s_{proj} is the bound projected separation. We compare the bounds with the observed position of the NW relic from Lindner et al. (2013). We plot the most extreme values from the realizations for the two possible scenario in Figure (X), while we leave detailed analysis in Appendix X.

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