Tree Cutting problem

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1 Problem Formalisation

1.1 Parameters definition

We consider a tree cutting problem. The tree starts at height 1. If we decide not to cut it, it can grow randomly according to the weather or it also has a probability to be sick and die. We chose the following parameters for the model :

State space : height varies from 1 to 8

Initial state : the height of the tree is set to one.

Actions : "cut" or "no-cut".

Dynamics :

– height max = 8

- The tree may get a disease. At each state, it has a probability to get sick which is different for each state (here we modeled this probability by a $Uniform[0, 0.3]$ Random Variable).

- Otherwise, the probability to grow from a set k to $k + d$ is set to be the same for all the states between $k + 1$ to $k + d$.

- The sum of those defined probabilities should be equal to 1 and we verify it in the our script.

– If cut : a new tree is planted with an initial height of one unit.

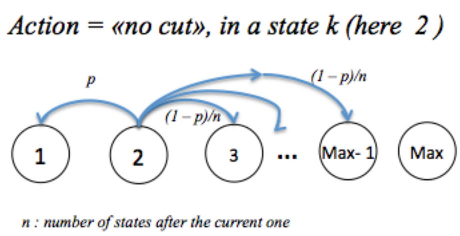
Reward :

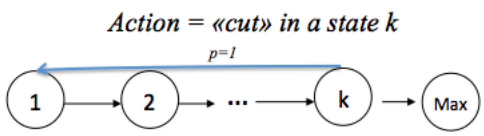
– If no cut : a fixed amount of maintenance cost, equal to 1 in our case

– If cut : the value of each unit of wood is equal to 3. The cost of planting a new one is equal to 3.

Discount factor : $r = 0,05$, and $\gamma = \frac{1}{1+r}$

- You can find below a graph showing the logic of the tree :





1.2 Code organization

- We model the tree by a Class called *Tree*.

- We model the simulation by another Class called *treeMDP*. This Class gives us the ability to get the transition probabilities according to a policy and thus some actions. Also, we have an example of a naive policy where we cut only when the $state = 0 \ mod(4)$.

- Finally, I also computed the function *tree sim* inside this class to get the simulation.

2 Policy Evaluation

Given the naive policy we defined, We compute the Monte Carlo Method to evaluate the Value function for state = 5, with 2000 iterations :

$$V_n(x_0) = \frac{1}{n} \sum_{k=1}^{n} \left[\sum_{t=1}^{T_{\max}} \gamma^{t-1} R_t^{(k)}, \right]$$

— x_0 : initial state (here 5)
— $R_t^{(k)}$: Reward at each moment for each trajectory

After that, we compute the Value vector using the following formula :
$V^\pi = (I - \gamma P^\pi)^{-1} R^\pi$
— π : naive policy
— P : Probability Matrix given a policy π
— R : Reward Vector given a policy π
— V : Value Vector given a policy π

Below we plot the graph of the error between the Monte Carlo approximation and the DP value according to the number of iteration :

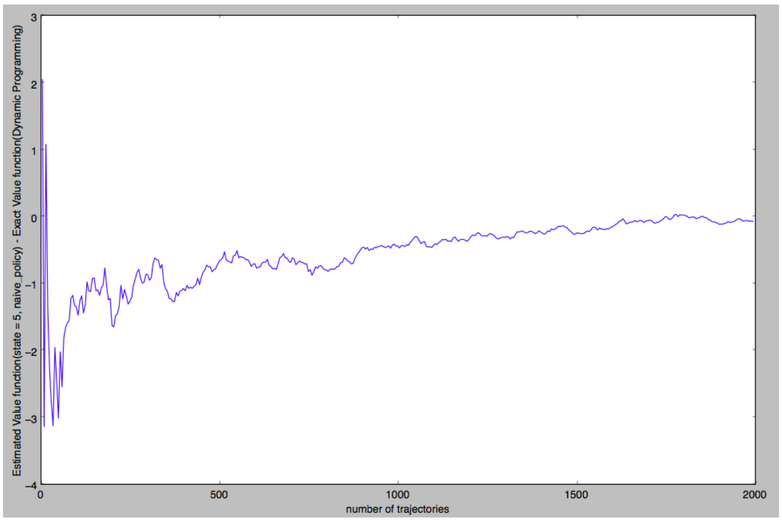


Figure 1 Error between Simulation Monte Carlo Value and DP Value

Result : We can see that the MC method converges to the exact value. The speed of convergence is in $\frac{1}{\sqrt{n}}$, which is true with the Central Limit Theorem.

3 Optimal Policy

We compute the optimal policy using Value iteration and policy iteration methods (see code).

Given the results, we can see that the optimal policy given our $treeMDP$ (transition probabilities, rewards, state of actions, states) is the following :

$['no - cut', 'no - cut', 'no - cut', 'no - cut', 'cut', 'cut', 'cut', 'cut']$

We Compare the speed of convergence between the two methods :

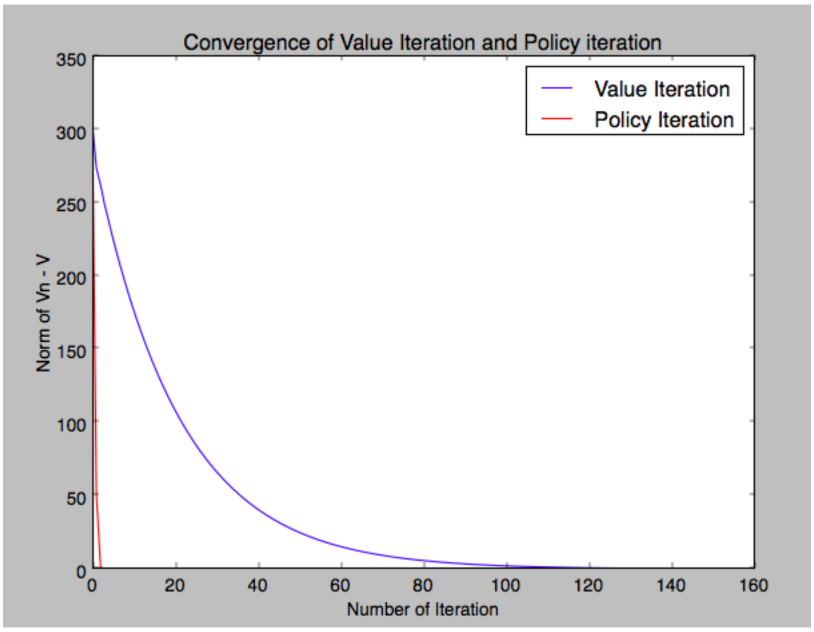


Figure 2 Convergence of Value and Policy Iteration

There is a trade off to do between the Policy and Value iteration methods We can see that the Convergence of the Value Iteration is slower in terms of number of iterations. The Policy Iteration converges with only 4 iterations. The Value Iteration converges only asymptotically. However an iteration with Value iteration takes less time : it is less expansive computationally. In our case, the policy iteration is not very consuming and converges directly, so it is the best way to do it.

4 Going further

The model is based on different parameters(max height, value of the tree, cost of planting, cost of maintenance) :

max height : when we increase the max height we observe an increase of the Value function. Also, in the optimal policy, we should start cutting later in the growth.

value a tree : The lowest the price of the wood is, the more we have to wait before cutting the tree also.

Cost of planting : The more this cost is high, the more we also have to wait, even if it means to never cut the tree because it would be very expansive to plant a new one.

Cost of maintenance : When this cost increases, the optimal policy wants us to cut more often the tree.