MASARYK UNIVERSITY
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Time Series Prediction Using Neural Networks

BACHELOR THESIS

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Declaration

Hereby I declare, that this paper is my original authorial work, which I have worked out by my own. All sources, references and literature used or excerpted during elaboration of this work are properly cited and listed in complete reference to the due source.

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Advisor: doc. RNDr. Tomáš Brázdil, Ph.D.

Acknowledgement

I would like to thank my advisor for the guidance he has provided.

Abstract

This thesis compares existing methods for predicting time series in real time using neural networks. Focus is put on recurrent neural networks (RNNs) and online learning algorithms, such as Real-Time Recurrent Learning and truncated Backpropagation Through Time. In addition to the standard Elman's RNN architecture, Clockwork-RNN is examined. Methods are compared in terms of prediction accuracy and computational time, which is critical in real-time applications. Part of the work is experimental implementation of the tested models and working applications in robotics and network traffic monitoring.

Keywords

time series, prediction, neural network, recurrent network, backpropagation, backpropagation through time, real-time recurrent learning, clockwork recurrent network

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1 Introduction

Each year more and more data is collected from high velocity data streams. Machine learning algorithms are used to analyse and create models of the data. Patterns found in the data structure are then exploited to make predictions about the future which can be used to guide decision making. In this thesis, I compare various artifical neural network (ANN) architectures and learning algorithms for online prediction of time series.

One of the most popular machine learning algorithms are ANNs which are capable of approximating unknown functions and thus are good candidates for use in prediction. Inspired by biological neural networks, ANNs consist of artificial neurons wired together to form a network.

Common architectures, such as feedforward networks (FFN) and simple recurrent networks (SRN) trained by backpropagation through time (BPTT) and Real-Time Recurrent Learning (RTRL), as well as latest Clockwork-RNN are evaluated in this thesis. Algorithms are compared in terms of prediction accuracy as well as computational time and the trade-off between them.

Time series are sequences of data points in time, usually created by measuring output of some process in discrete time intervals. The goal of prediction is to successfully estimate output of the process in next time step or several steps. It is assumed that the process is at least partially observable and to some extent, future values can be determined by observing past values. Prediction then reduces to problem of process approximation.

Since value of stored data decreases over time, effort is put into gaining insight from data as they come. Many applications don't require storing data at all, or storing it would be impractical, therefore it is advantageous to create models and update them with every data point. Continuous online learning is well suited for such tasks and can adapt to changing environments without human intervention.

Quality and speed of prediction is tested in two application scenarios. The first is robotic simulator implemented in 3D physics library BEPUPhysics. Manipulator with three actuated joints is created, the goal is to predict future position of robot's arm. The predic-

tion network should act as a virtual model of the body, which can be useful in control systems, e.g. internal model control.

The second scenario is monitoring utilisation of computer resources, namely processor, memory, disk, and network usage. Network should constantly predict future values of these variables and detect anomalies. Should an unexpected event occur, it needs to be logged for an administrator to examine.

2 Prediction

2.1 Time Series

Time series are sequences of data points measured over time. In this thesis, data points are real-valued vectors implemented as arrays of floating point numbers. Data sequence is created by measuring output of a process at discrete, regular time intervals. Process may also receive input in every time step, which affects its future behaviour, or it may be purely generative receiving no input at all. For generalisation's sake, let's assume every process receives input which is a real-valued vector just like output. Generative processes then simply receive input vector of zero length. In every time step, process

Figure 2.1: Schema of a process.

receives input, updates its internal state and produces output. Internal state of the process is usually hidden and can be observed only partially from output of the process.

2.2 Prediction

The goal of prediction is to approximate the process as closely as possible and hence minimise forecast error, i.e. difference between actual and forecasted value. Naive approaches include methods like averaging past data points, returning previous data point or linear extrapolation. While these methods may suffice for some very simple time series, more sophisticated methods are required to cope with real-world time series. Sophisticated methods use historical data to estimate future value by finding a model of the process that empirically fits past data.

The problem of prediction can be alternatively viewed as problem of function approximation.

$$(State_{t+1}, Output_{t+1}) = Process(State_t, Input_t)$$

Process is a function from current internal state and input to next internal state and output. Unfortunately, internal state is unknown, so next output can only be estimated from history of inputs and outputs. History can be defined as an ordered set of past inputs and outputs.

$$History_t = ((Input_t, Output_t), (Input_{t-1}, Output_{t-1}), \dots (Input_0, Output_0))$$

Prediction is then a function from current history to next output.

$$Output_{t+1} \approx Prediction(History_t)$$

2.3 Prediction Horizon

Number of steps the prediction is made into the future is called prediction horizon. Some applications require estimating output of the process more than one step into the future. This can be achieved by chaining one step predictions or by approximating function:

$$Output_{t+h} \approx Prediction(History_t, FutureInputs)$$

2.4 Prediction Chain

As an alternative for predicting multiple steps of the future, one step prediction can be applied recursively multiple times, i.e. predict next step, from the prediction predict next step etc. Intermediary results are then available for use. In case of processes that receive input, chaining predictions is suitable only if the future inputs are known up until prediction horizon.

$$Output_{t+h} \approx Prediction(Prediction(\dots Prediction(State_t, Input_t), Input_{t+1}) \dots Input_{t+h})$$

3 Artificial Neural Networks

Inspired by biological neural networks, ANNs are groups of elementary processing units called artificial neurons connected together to form a directed graph. Nodes of the graph represent biological neurons and connections between them represent synapses. Unlike in biological neural networks, connections between artificial neurons cannot be added or removed after the network was created. Instead, connections are weighted and the weights are adapted by learning algorithm.

Input signal propagates through the network in the direction of connections until it reaches output of the network. In supervised learning, learning algorithm adapts the weights in order to minimize the difference between output of the network and desired output provided by teacher.

3.1 Artificial Neuron

The complex behavior of biological neurons was simplified to create a mathematical model of artificial neurons, also called units. Each unit consists of a real valued activation representing some property of the unit. Unit receives activations of other units via input connections, computes its output activation and sends it to other units.

Connections between units are stored in a matrix w, where w_{ij} denotes weight of the connection from unit i to unit j. Every unit j has a potential p_j which is calculated as weighted sum of all of its N input units and bias.

$$p_j = \sum_{i=1}^{N+1} w_{ij} a_i$$

Bias term, also known as threshold unit, is usually represented as an extra input unit whose activation always equals one. Presence of bias term enables shifting the activation function along x-axis by changing the weight of connection from threshold unit.

Activation of the unit a_j is then computed from the potential p_j transformed by a non-linear activation function act.

$$a_j = act\left(p_j\right)$$

Commonly used non-linear activation function ranging from 0 to 1 is sigmoid function thanks to its easily computable derivative which is used by learning algorithms.

$$\sigma(x) = \frac{1}{1+e^{-x}}$$

$$\frac{d\sigma(x)}{dx} = \sigma(x)\left(1-\sigma(x)\right)$$

3.2 Feedforward Neural Networks

Feedforward neural network is an ANNs where information moves in one direction, from input to output, i.e. without any backward or recurrent connections. Multilayer perceptron (MLP) is a class of feedforward networks consisting of three or more layers of units. Layer is a group of units receiving connections from the same units. Units inside a layer are not connected to each other.

MLP consists of three types of layers: input layer, one or more hidden layers and output layer. Input layer is the first layer of network and it receives no connections from other units, but instead holds network's input vector as activation of its units. Input layer is fully connected to first hidden layer. Hidden layer i is then fully connected to hidden layer $i+1$. Last hidden layer is fully connected to output layer. Activation of output units is considered to be output of the network.

MLPs are often used to approximate unknown functions from their inputs to outputs. MLP's capability of approximating any continuous function with support in the unit hypercube with only single hidden layer and sigmoid activation function was first proved by George Cybenko [3].

3.2.1 Backpropagation

Backpropagation, or backward propagation of errors, is the most used supervised learning algorithm for adapting weights of feedforward

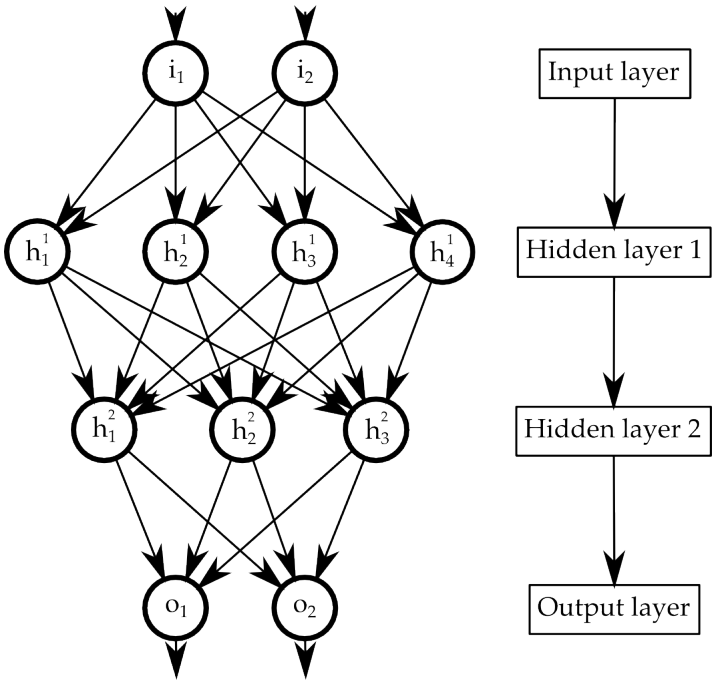


Figure 3.1: On the left, MLP consisting of input layer with two units, two hidden layers with four and three units respectively, and output layer with two untis. Schematic diagram of the MLP's layers on the right.

ANNs. Weights of the network are tuned so as to minimize summed squared error

$$E = \frac{1}{2}(t - o)^2$$

where t denotes target output provided by teacher and o is network's prediction of the output for the corresponding input.

Let's assume the error is a function of network's weights, then backpropagation can be seen as optimization problem and standard gradient descent method can be applied. Local minimum is approached by changing weights along the direction of negative error gradient

$$-\frac{\partial E}{\partial w}$$

proportionally to α, which is constant positive value called learning

rate.

$$new\ w_{ij} = w_{ij} - \alpha \frac{\partial E}{\partial w_{ij}}$$

The central part of the algorithm is finding the error gradient. Let's assume there is a MLP with L layers, first being input and last being output layer. Layer k has U_k units and holds a matrix of weights w_{ij}^k representing weights of connections from unit i in layer $k-1$ to unit j in layer k. The computation can be then divided into three steps:

1. Forward propagation. Input vector is copied to activation of input layer. Layer by layer, from first hidden to output layer, activation of units is calculated. Activation of the output layer a^L is considered output of the network.

2. Backward propagation. Compute error gradient Δ_i^L w.r.t. unit i for each output layer unit i as

$$\Delta_i^L = \left(target_i - a_i^L\right) \frac{\partial act\left(p_i^L\right)}{\partial p_i^L}$$

   For units i of hidden layers $h = L-1,\ L-2,\ \ldots 2$, error term is

$$\Delta_i^h = \sum_{j=1}^{U_{h+1}} \Delta_j^{h+1} w_{ji}^h \frac{\partial act\left(p_i^h\right)}{\partial p_i^h}$$

3. Weights update. Change weights in layer k according to

$$new\ w_{ij}^k = w_{ij}^k + \alpha \Delta_i^{k+1} a_j^k$$

3.2.2 Time-Delay Neural Networks

Time-Delay Neural Network (TDNN) is a modification of feedforward network designed to capture dynamics of modelled process [5]. As FFNs have no internal memory to store information about past, they are insufficient for processing temporal sequences. To overcome this, memory of past is introduced by means of extending network's input with sliding window of previous inputs, also known as tapped

delay line. The information about the past is thus stored in the network input itself. Because there are no modifications to the network, standard backpropagation algorithm can be used.

Size of the sliding window determines how many past inputs are stored and consequently how much past data can be correlated to future output. Too small window may not capture necessary dynamics of the system, because network is blind to anything that happened before, and thus create inferior results. Whereas too large window can drastically prolong learning time.

To generalise TDNN to processes that also receive input, prediction of future process output can be defined as approximation of function *Predict* with window size w

$$Output_{t+1} = Predict(Output_t, \ldots, Output_{t-w}, Input_t, \ldots Input_{t-w})$$

3.3 Recurrent Neural Networks

Recurrent network is a class of ANNs which allows units to form a directed graph with cycles. This allows the network to store an internal state and consequently process sequences of inputs and thus perform temporal tasks.

3.3.1 Elman's Simple Recurrent Network

One of the simplest and most popular RNN architectures is Elman's simple recurrent network (SRN). SRN resembles a three-layer feed-forward network due to its structure composed of input, hidden and output layer, with addition of a context layer. Input and context layer project to hidden layer, which projects to output layer. Context layer is a copy of hidden layer's activation in previous time step. Therefore, context layer acts as network's memory of previous activity.

Recurrent network's dynamics can be formulated by two equations:

$$a^{hid}(t) = act\left(W^{in}a^{in}(t) + W^{hid}a^{hid}(t-1)\right)$$

$$a^{out}(t) = act\left(W^{out}a^{hid}(t)\right)$$

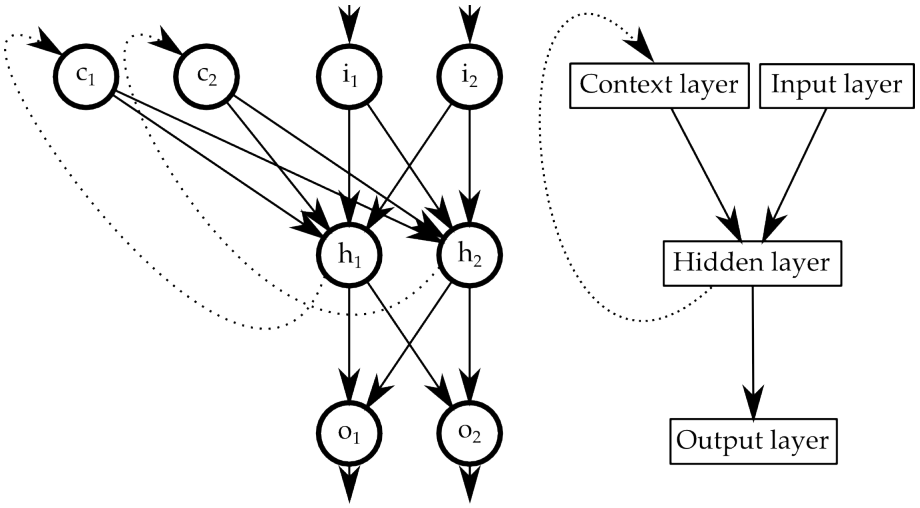


Figure 3.2: On the left, Elman's recurrent network with input, hidden, context, and output layer, each containing two units. On the right, schema of layers in Elman network. Dotted link signifies copying activity of source's units to target units.

where $a^{in}(t)$, $a^{hid}(t)$ and $a^{out}(t)$ are column vectors of activations of units in input, hidden and output layer respectively in time step t. W^{in}, W^{hid} and W^{out} are matrices [1] of weights of connections from input to hidden layer, hidden to hidden layer and hidden to output layer respectively. act is an element wise activation function.

3.3.2 Backpropagation Through Time

Standard backpropagation algorithm is not suited for networks with cycles in them. Fortunately, RNN can be modified to look like a feed-forward network by unfolding the network in time as shown in figure 3.3 and then trained with Backpropagation Through Time (BPTT) algorithm first laid out by Rumelhart, Hinton and Williams in 1986 [4].

The unfolding process begins with a SRN in current time step t, denoted as SRN_t. Since context layer of a SRN is just a copy of

1. Element at row i and column j of the matrix W holds weight of connection from unit j to unit i

hidden layer activation from previous step, cycles in the network can be avoided by replacing context layer with an identical copy of the SRN network from previous step, SRN_{t-1}. Hidden layer of SRN_{t-1} is then connected to hidden layer of SRN_t. This procedure is repeated until time step 0 is reached, in which case the context layer is not replaced, but rather stays set to its initial activity. The number of SRN copies represents depth of the unfolded network and each copy of the network uses exact same set of weights.

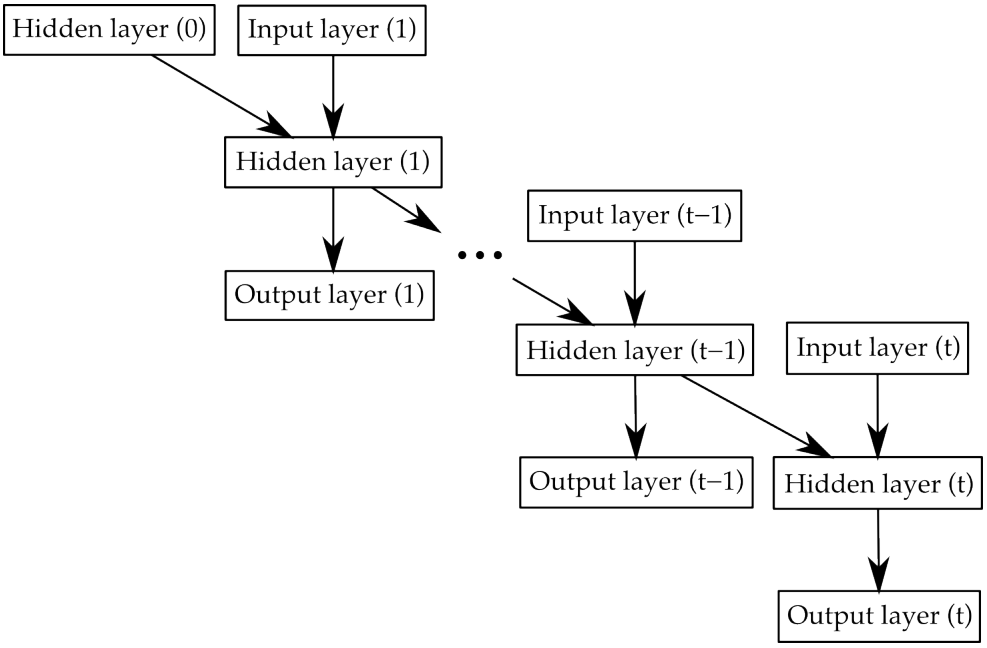


Figure 3.3: Elman's recurrent network unfolded in time.

Once the SRN has been unfolded into a feedforward network, backpropagation can be used. The algorithm again consists of 3 steps:

1. Forward propagation. Signal is propagated through the unfolded network in the standard fashion, from top to bottom. In this case, from the SRN copy furthest in the past to the most recent copy.

2. Backward propagation. Error gradient $\Delta_i^{out}(t)$ of every output layer of SRN copy in time t is computed w.r.t. output unit i as:

$$\Delta_i^{out}(t) = \left(target_i(t) - a_i^{out}(t)\right) \frac{\partial act\left(a_i^{out}(t)\right)}{\partial a_i^{out}(t)}$$

For every unit i in hidden layer of unfolded SRN in time step t, let $\Delta_1 \dots \Delta_N$ be error terms of units that receive connections from hidden layer in time t. Error term $\Delta_i^{hid}(t)$ w.r.t. hidden unit i is then

$$\Delta_i^{hid}(t) = \left(\sum_{j=1}^{N} \Delta_j w_{ij}^h\right) \frac{\partial act\left(a_i^{hid}(t)\right)}{\partial a_i^{hid}(t)}$$

3. Update weights in the original SRN. For every unit i of the original network k that is connected to units j in layer l, update the weight according to

$$new\ w_{ij}^l = w_{ij}^l + \alpha \sum_{m=1}^{t} \Delta_i^k(m)\, a_j^l(m)$$

3.3.3 Truncated Backpropagation Through Time

Number of SRN copies in the unfolded network is equal to current time step t. Should this algorithm be used in online manner, it would be impractical, since its memory footprint would grow linearly with time. To overcome this, online version of the BPTT algorithm called Truncated Backpropagation Through Time (TBTT) can be used. TBPTT works analogously to BPTT, except the maximum depth of the unfolded network is limited.

3.3.4 Real-Time Recurrent Learning

Real-Time Recurrent Learning (RTRL) algorithm is a gradient descent method suitable for online learning of recurrent networks.

Let's assume the recurrent network's total number of U units is divided into U_{in} input units, U_{hid} hidden units and U_{out} output units. For convenience, let's denote potential and activation of all units by p_i and a_i, where $i = 1 \dots U_{in}$ represents indices of input units, $i = U_{in} + 1 \dots U_{in} + U_{hid}$ represents indices of hidden units and $i = U_{in} +$

$U_{hid}+1 \ldots U_{in}+U_{hid}+U_{out}$ represents indices of output units. All weights of connections from unit i to unit j can be then denoted by w_{ij}.

We wish to minimise error E in time step t

$$E(t) = \frac{1}{2}\left(a_i(t) - target_i(t)\right)^2$$

where i enumerates indices of output units and *target* holds teacher given desired activations of output units. We do this by adjusting weights along the negative gradient of error

$$-\frac{\partial E(t)}{\partial w_{ij}} = \sum_{k=U_{in}+U_{hid}+1}^{U} \left(target_i(t) - a_i(t)\right) \frac{\partial a_i(t)}{\partial w_{ij}}$$

$\partial a_i(t)/\partial w_{ij}$ can be computed by differentiating the network dynamics equation, resulting in the derivative v_{ij}^k of hidden or output unit k w.r.t. weight w_{ij}

$$v_{ij}^k(t+1) = \frac{\partial a_k(t+1)}{\partial w_{ij}} = act'(p_k(t)) \left[\left(\sum_{j=U_{in+1}}^{U} w_{kj} \frac{\partial a_j(t)}{\partial w_{ij}} \right) + \delta_{ki} act_j(t) \right]$$

where δ_{ki} is Kronecker's delta

$$\delta_{ki} = \begin{cases} 1, & \text{if } k = i, \\ 0, & \text{if } k \neq i. \end{cases}$$

This creates dynamical system with variables v_{ij}^k for all hidden and output units [6]. Since the initial state of the network is independent from its weights, we can set $v_{ij}^k(0) = 0$. Network's weights are then updated according to negative gradient of the error

$$new\ w_{ij} = w_{ij} - \alpha \sum_{k=U_{in}+U_{hid}+1}^{U} \left[\left(a_k(t) - target_k(t)\right) v_{ij}^k \right]$$

3.4 Clockwork Recurrent Network

SRNs have trouble capturing capturing long-term dependencies in input sequences due to vanishing gradient [**?**]. Clockwork recurrent

neural network (CW-RNN) is a modification of Elman's SRN designed to solve this problem by having hidden layer split into M modules running at different clocks [**?**]. Each module i is assigned a clock rate T_i. In time step t only modules with period T_i that satisfies $(t \ mod \ T_i) = 0$ compute its activation, other modules retain their previous activation.

Like in SRN, input layer is connected to hidden layer, context layer stores activation of hidden layer from previous time step and hidden layer is connected to output layer. The difference is that module of hidden layer with clock rate T_i connects to module in context layer with period T_j only if $T_i <= T_j$ as shown in figure 3.4. This

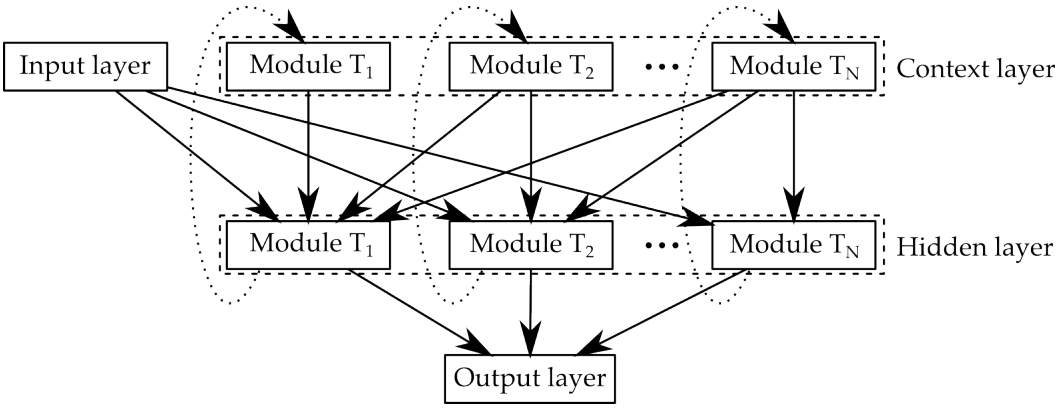


Figure 3.4: Clockwork-RNN.

allows slower modules to focus on long-term information in the input sequence, while faster modules focus on short-term information with context provided by slower modules.

To adapt network weights, BPTT learning algorithm can be used. The algorithm works similarly with the only difference compared to SRN being that error propagates only from active modules executed at time t. Error of inactive modules is retained from previous step.

4 Implementation

4.1 Neural Prediction Framework

Neural Prediction Framework (NPF) is a tool built for experimenting with various neural network models in online time series prediction tasks.

4.1.1 MemoryBlock

MemoryBlock is an object encapsulating an array of floating point numbers and providing convenience methods. MemoryBlock is used all around NPF to store data points, network weights or activations. MemoryBlockView inherits from MemoryBlock and provides access to its elements inside specified range.

4.1.2 NeuralLayer

NeuralLayer is an object storing activation of units in a single layer and weights of connections coming from units of other layers. NeuralLayer is also capable of propagating input signals forward and propagating errors backward using standard backpropagation algorithm.

4.1.3 NeuralNetwork

NeuralNetwork is an abstract class providing common interface shared between all derived networks.

4.1.4 FeedforwardNetwork

FeedForwardNetwork implements a MLP with variable number of hidden layers with variable number of units in them.

4.1.5 CWRecurrentNetwork

CWRecurrentNetwork implements a CW-RNN with variable number of hidden layer modules with specified clock rates.

4.1.6 SimpleRecurrentNetwork

SimpleRecurrentNetwork derives from CWRecurrentNetwork to implement a SRN, since Elman's SRN is just a special case of CW-RNN network with single hidden layer module with clock rate equal one.

4.1.7 LearningAlgorithm

LearningAlgorithm is an abstract class providing common interface for all learning algorithms.

4.1.8 Backpropagation, TBPTT, RTRL

Backpropagation, TBPTT, RTRL classes are descendants of LearningAlgorithm implementing respectively named algorithms. Backpropagation class is used for training a FeedforwardNetwork. RTRL is capable of training a SimpleRecurrentNetwork. TBPTT supports both SimpleRecurrentNetwork and CWRecurrentNetwork.

4.2 Network Monitor

Network monitor works with assistance of unix system resources monitoring tool Dstat [2]. Output of Dstat is redirected to a neural network of choice for prediction. As a proof of concept, the network is constantly training and predicting incoming and outgoing network traffic. Should the difference between prediction and actual value exceed predefined threshold, this incident is logged to a file as an anomaly.

4.3 Robotic Arm Simulator

Robotic arm simulator is implemented in BEPUPhysics 3D physics library [1]. Manipulator with three degrees of freedom from BEPU demo is constructed in 3D world. Three controlled joints are situated in base, "shoulder" and "elbow" of the arm. Simulator renders the scene, calculates physics and supplies information about current rotation of joints as well as position of claw to NPF, which provides

prediction of claw position in next time step. As a proof of concept, line showing direction from current position of claw to the predicted position is drawn. Further use could be found in control of the arm.

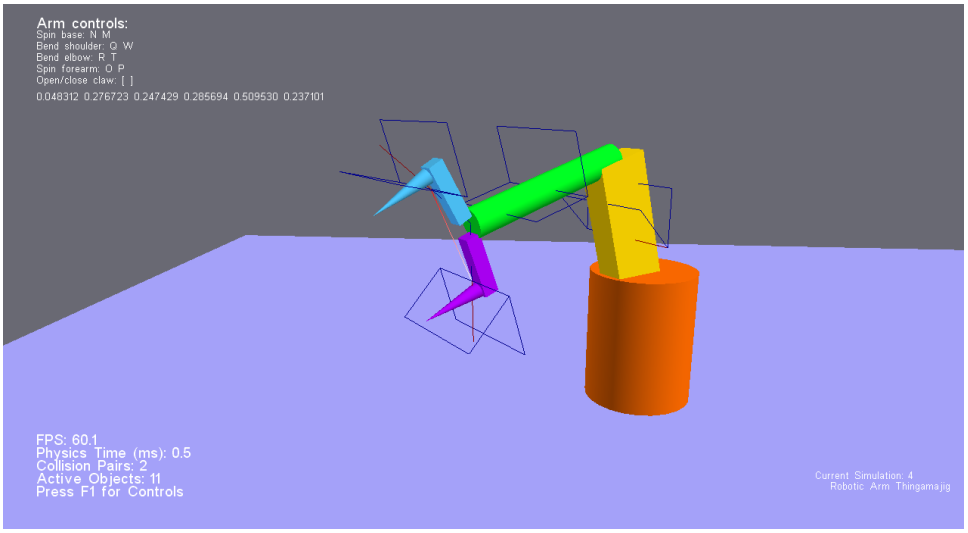


Figure 4.1: Robotic arm in BEPUPhysics 3D physics simulator.

5 Experiments

In the following experiments, I attempt to find the best performing network and its hyperparameters for prediction in three different scenarios with restricted computation time.

Networks are tested on the same prepared datasets to ensure equal testing conditions. Units of all networks use logistic activation function. Weights of connections in all networks are initialised randomly with uniform distribution on interval [-0.25, 0.25]. Since the initial setting of network influences its overall performance, all experiments described below are run ten times and only the average of them is presented as the result.

Experiments measuring computation time are run on 1.7GHz Intel Core i5 2557M. While the absolute computation times should vary depending on system configuration, I expect them to retain their ratio relative to each other.

As a measure of prediction accuracy, total error E_{total} collected over T time steps is used. Network that predicts entire sequence flawlessly would have zero total error.

$$E_{total} = \sum_{\tau=1}^{T} \frac{1}{2} \sum_{i=1}^{N} (Target_i - Output_i)^2$$

Networks are trained online in every time step. In order to capture both their accuracy and ability to adapt quickly, there is no traditional division into training and testing period. Instead, networks are evaluated online as they learn in all time steps.

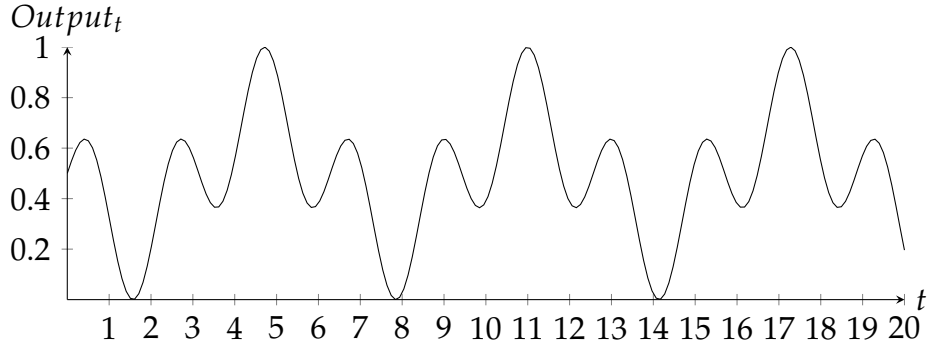
5.1 Testing Scenarios

Networks are tested on three different datasets with varying complexity collected from three scenarios. Each scenario is designed to test a specific aspect of prediction, such as modelling generative processes, noisy network traffic or substantially time constrained robotic manipulator.

5.1.1 Goniometric function

The first simple testing scenario is predicting next step of a generative process with no inputs whose output in time t is defined as

$$Output_t = \frac{1 + sin(t)cos(2t)}{2}$$



The goniometric function's range is [0,1] which matches logistic activation function's range [0,1], therefore no preprocessing is needed. This function is periodical with period of 2π.

5.1.2 Network Usage

Second testing scenario is predicting computer network usage measured by monitoring tool Dstat. Each second, Dstat measures usage of system resources such as CPU load, memory usage, disk throughput, number of open sockets as well as network incoming and outgoing traffic. Prediction has to be computed in less time than it takes Dstat to collect network usage, which poses a loose time constraint of computation time being less than one second. Dataset consists of 3,600 data points created by monitoring real application server for one hour.

Dstat measures 26 variables of the monitored system, two of which are incoming and outgoing network traffic. Values of these two variables in next time step shall be predicted. Since the values observed by Dstat have huge dynamic range, preprocessing is necessary. Values are linearly squashed into interval $[0, 1]$ by dividing the observed value by the expected maximum.

5.1.3 Robotic Arm

Third testing scenario is predicting position of robotic arm's claw in BEPUPhysics 3D physics library. Manipulator with three controlled joints is constructed. At each simulation step, network receives rotations of all joints and their respective control signals. The goal is to predict next position of the claw in 3D coordinate system.

This task is heavily time constrained. The 3D simulator is required to run at 60 frames per second for smooth control, which leaves only $\frac{1}{60}$ of a second for computation of the prediction. Dataset consisting of 18,000 data points is created by controlling the robot with random commands for a period of five minutes.

5.2 Tested Models

Learning and momentum rates are informally chosen to the best performing value for each dataset.

5.2.1 Time Delay Neural Network

The role of sliding window size is examined w.r.t. prediction accuracy and computation time. TDNNs with a single hidden layer consisting of 64, 128 and 256 units are examined. In goniometric function scenario, TDNN is tested with learning and momentum rate equal 0.05 and 0.9 respectively. In network traffic scenario, learning and momentum rates are 0.001 and 0.9. In manipulator scenario, learning and momentum rate equal 0.01 and 0.9.

5.2.2 SRN trained by TBPTT

SRNs with 64, 128 and 256 hidden units trained using TBPTT are tested with varying unfolding depth in order to find its impact on precision and computation time.

5.2.3 SRN trained by RTRL

Due to RTRL's severe computational complexity, SRNs with only 8 and 16 hidden units are tested for prediction accuracy and computa-

tion time. In all scenarios, learning rate is set to 0.1 and momentum to 0.9.

5.2.4 CW-RNN trained by TBPTT

Experiments examine role of unfolding depth in CW-RNN with 64, 128 and 256 hidden units on its precision and computation time. Hidden layer of the network is divided into four equally big modules with exponential clock rates: 1, 2, 4 and 8.

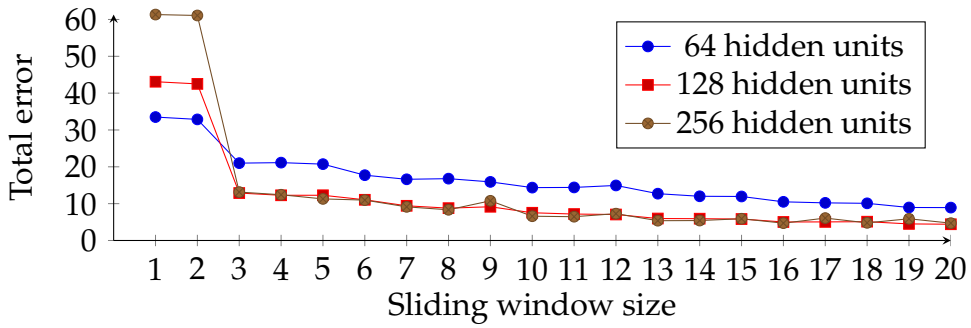
6 Results

6.1 Goniometric Function Results

6.1.1 Role of TDNN's Sliding Window Size

The results show steep decrease of error when increasing window size from two to three past time steps and mild decrease of error when extending window further. Presumably, three steps are threshold when information in the sliding window becomes sufficient to capture dynamics of the simple goniometric function. Network with only 64 hidden units performed notably worse than bigger networks. However, there is no significant gain in increasing network size beyond 128 hidden units.

Figure 6.1: Total error of predicting goniometric function with TDNN and various sliding window sizes.



6.1.2 Role of TBPTT Unfolding Depth

In both SRN and CW-RNN, increasing unfolding depth decreases total prediction error. Increasing hidden layer size helps both networks find better error minimum. In this scenario, SRN provides better results than CW-RNN with the same hidden layer size.

Figure 6.2: Total error of predicting goniometric function with SRN trained by TBPTT with various unfolding depth.

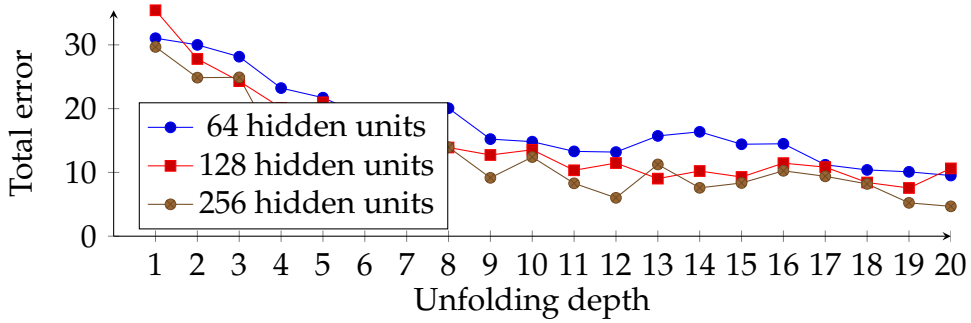
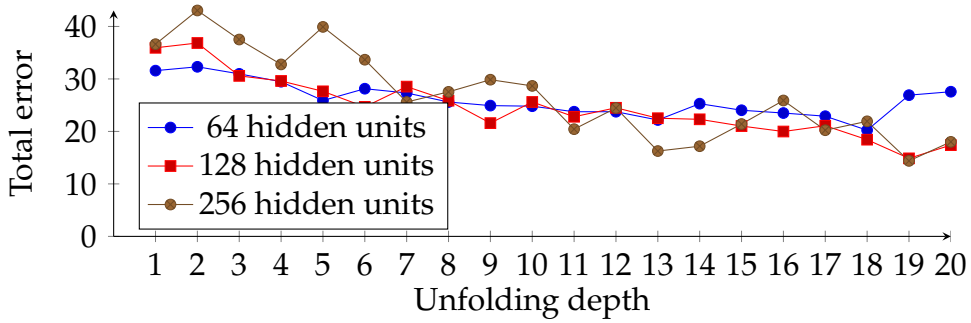


Figure 6.3: Total error of predicting goniometric function with CW-RNN trained by TBPTT with various unfolding depth.



6.1.3 Computation Time Tradeoff

The most accurate network in predicting goniometric function was TDNN with 128 hidden units and sliding window of 20 past steps. In this simple task, TDNN shows remarkable performance while also being the least computationally extensive.

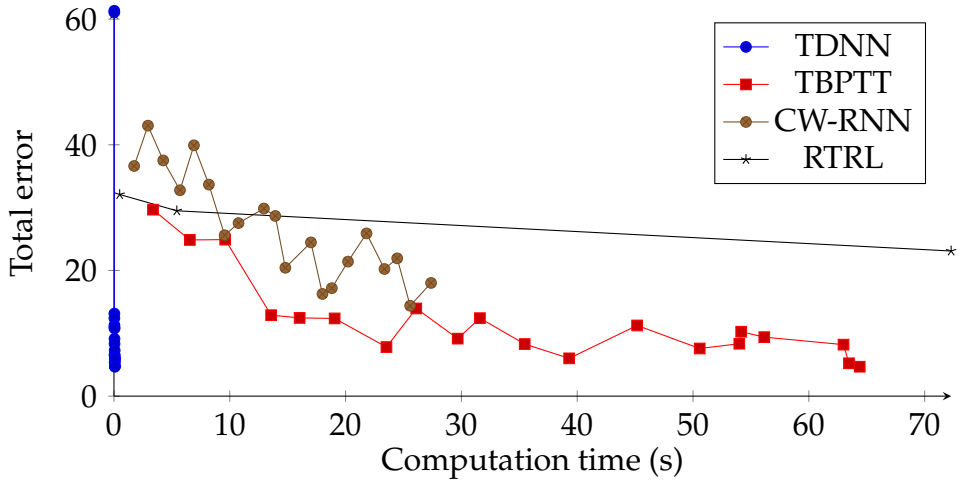
SRN trained by TBPTT achieves the second best result, though with much higher computation time. While CW-RNN requires approximately only a half of SRN's computation time with the same network size and unfolding depth, in this scenario SRN offers lower error per computation time.

SRN trained by RTRL didn't perform particularly well, primarily

because of its high computation complexity which limits the maximum practical network size to 32 units.

In the following figure, all four methods are compared at how decreasing total error affects computation time. Depicted below are TDNN with 128 hidden units and various sliding window sizes, SRN and CW-RNN with 256 hidden units trained by TBPTT with various unfolding depth, and SRN trained by RTRL with 8, 16 and 32 hidden units.

Figure 6.4: Tradeoff between time and error in goniometric function scenario.



6.2 Network Usage Results

Number of hidden units, sliding window size and unfolding depth show little or no effect on reducing total error. This behaviour could be caused by inherent unpredictability of the network traffic or inability of the methods to find the correct model.

The best performing method was TDNN with 256 hidden units and sliding window of 18 past steps, but not by any significant margin.

Figure 6.5: Total error of predicting network traffic with TDNN and various sliding window sizes.

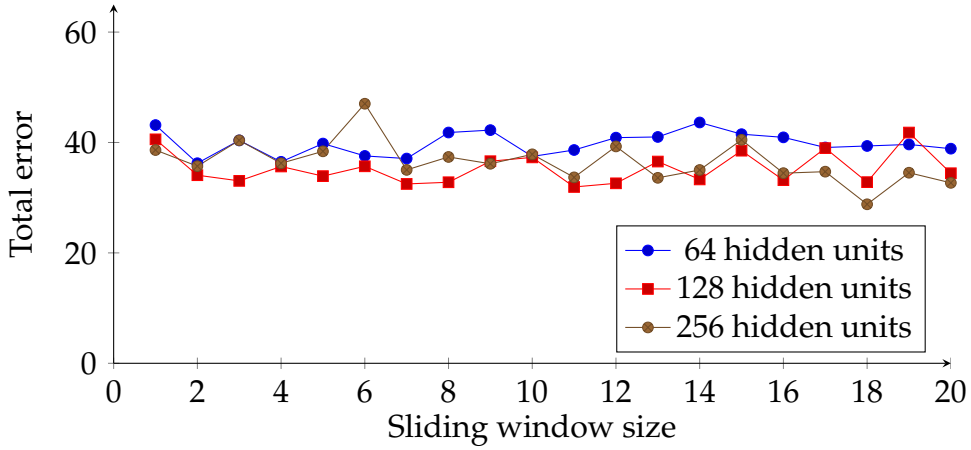


Figure 6.6: Total error of predicting network traffic with SRN trained by TBPTT with various unfolding depth.

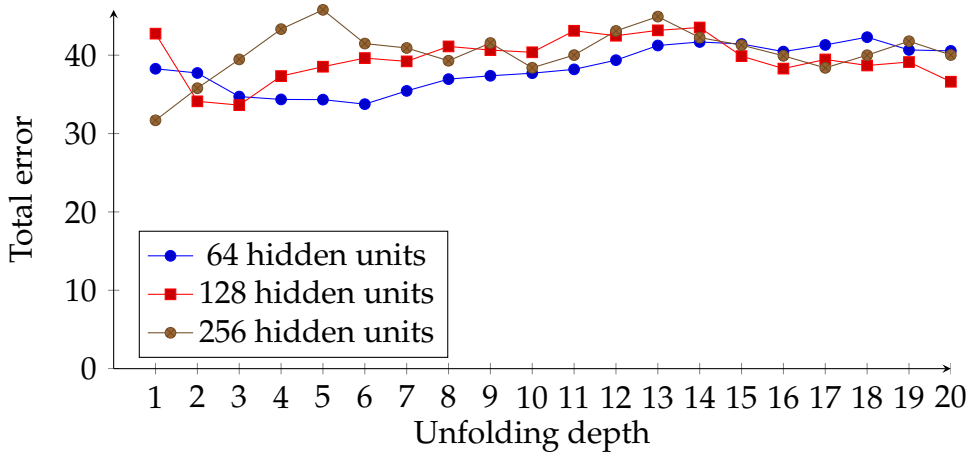
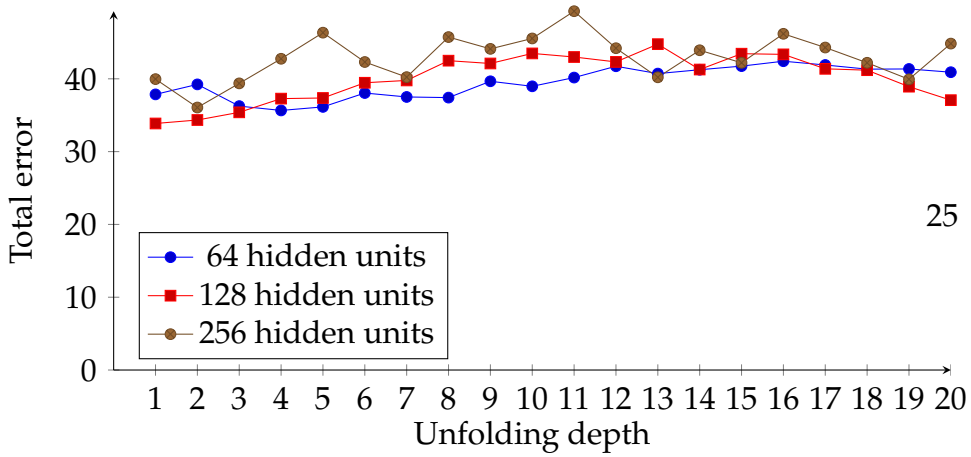


Figure 6.7: Total error of predicting network traffic with CW-RNN trained by TBPTT with various unfolding depth.

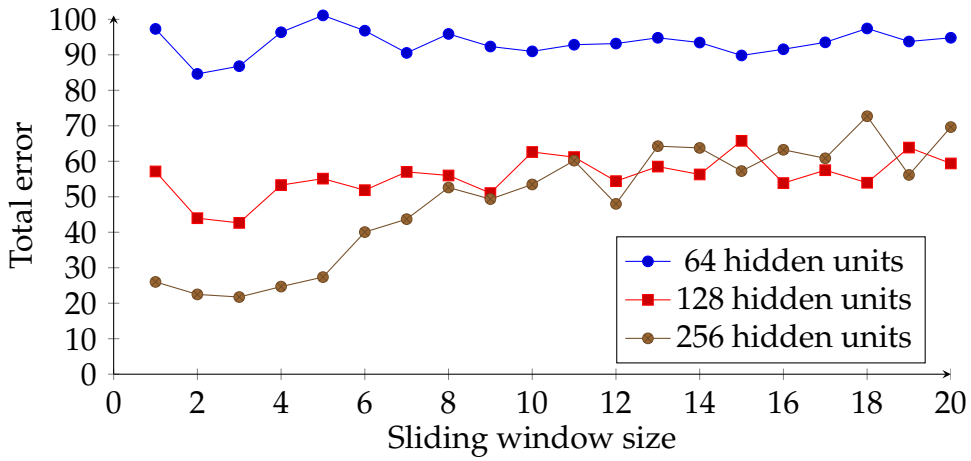


6.3 Robotic Arm Results

6.3.1 Role of TDNN's Sliding Window Size

Regardless of number of hidden units, TDNNs find their error minimum with window of size two or three. Enlarging the window further has negative impact and increases error. The larger the network, the more susceptible it is to this phenomenon. This may suggest that the state of manipulator is sufficiently captured in two or three consecutive steps. Increasing the number of hidden units is distinctly decreasing total error.

Figure 6.8: Total error of predicting manipulator's claw position with TDNN and various sliding window sizes.



6.3.2 Role of TBPTT Unfolding Depth

As unfolding depth increases, both SRN and CW-RNN show similar pattern of first sharply decreasing the total error, finding their minimum, and then gradually rising error. Networks with more hidden units perform better in this scenario.

By a small margin, the best performing method is SRN with 256 hidden units trained by TBPTT with unfolding length of 3 followed by CW-RNN with 256 hidden units and unfolding length of 5.

Figure 6.9: Total error of predicting manipulator's claw position with SRN trained by TBPTT with various unfolding depth.

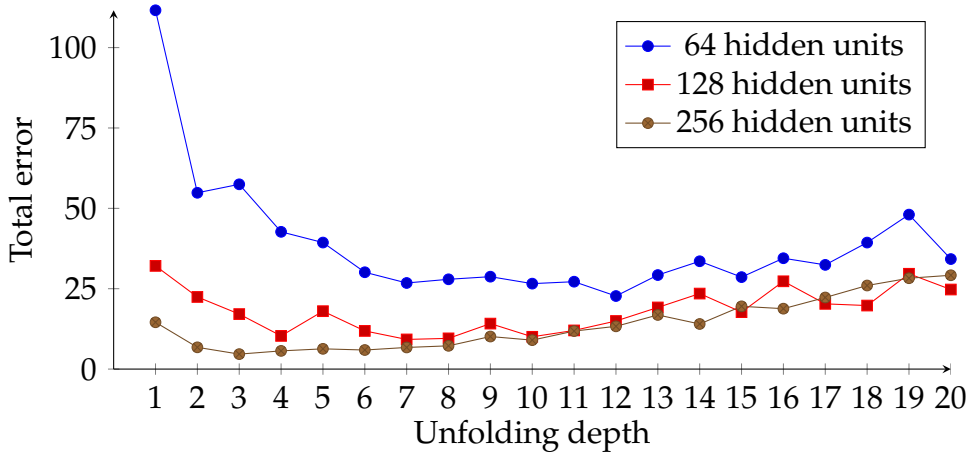
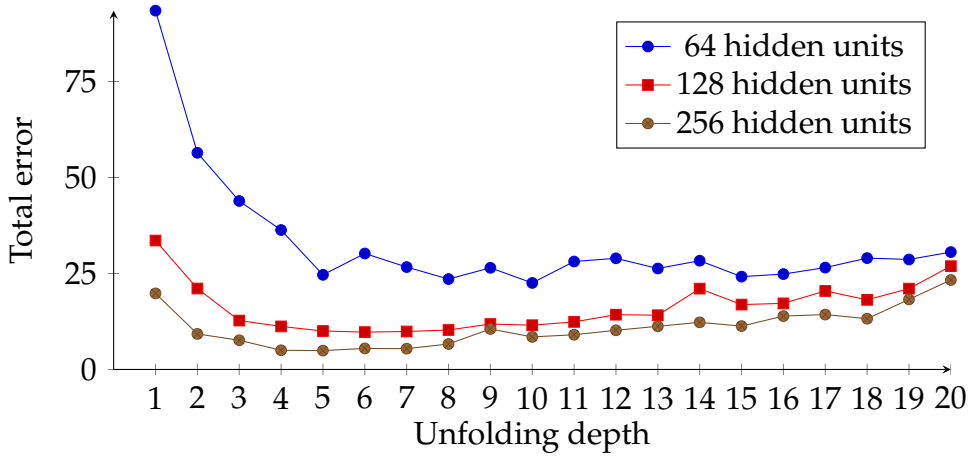


Figure 6.10: Total error of predicting manipulator's claw position with CW-RNN trained by TBPTT with various unfolding depth.



6.3.3 Computation Time Tradeoff

In this heavily time constrained scenario, many neural network models fail to meet the maximum computation time of 300 seconds. RTRL ends up particularly bad on this front, with its network consisting of

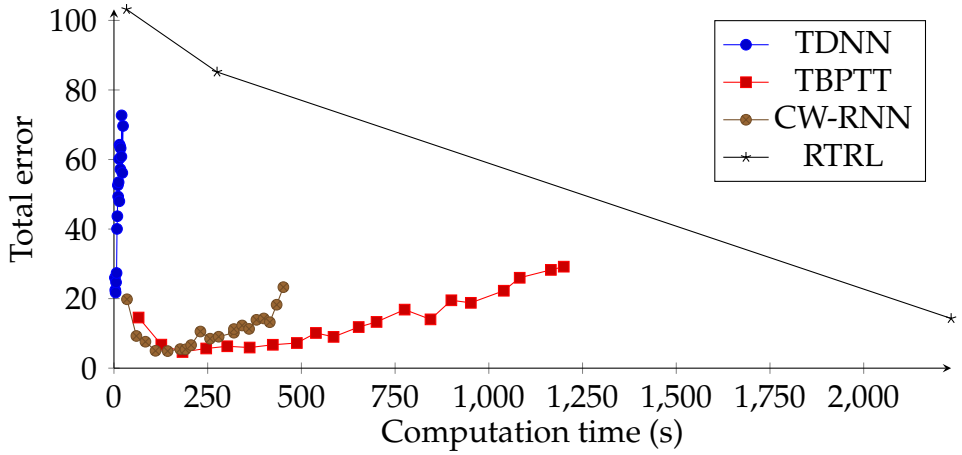
32 hidden units using up more than 7 times the maximum computation time. Smaller SRNs trained by RTRL that meet the requirement are not precise enough.

TDNN again shows its lightweight computation complexity and beats all networks in this regard, however it scores only third in precision.

SRN trained by TBPTT has a slight edge over CW-RNN in precision, but on the other hand, CW-RNN has a much lower computation complexity. Since both methods can fit into required maximum computation time, the final choice of the method should be based on whether one prefers increased precision or lower computation time.

In the following figure, TDNN with 256 hidden units and varying sliding window size, SRN and CW-RNN with 256 hidden units trained by TBPTT with varying unfolding depth, and SRN with 8, 16 and 32 hidden units trained by RTRL.

Figure 6.11: Tradeoff between time and error in robotic arm scenario.



7 Conclusion

TODO

A Appendix

Source code of all tested models and experiments can be found at https://github.com/karolkuna/Time-Series-Prediction-Using-Neural-Networks

B Computation Time

B.0.4 Computation Time

As expected, increasing window size is linearly increasing the computation time. Similarly, computation time is increased linearly by adding hidden units.

Figure B.1: Computation time of predicting goniometric function with TDNN and various sliding window sizes.

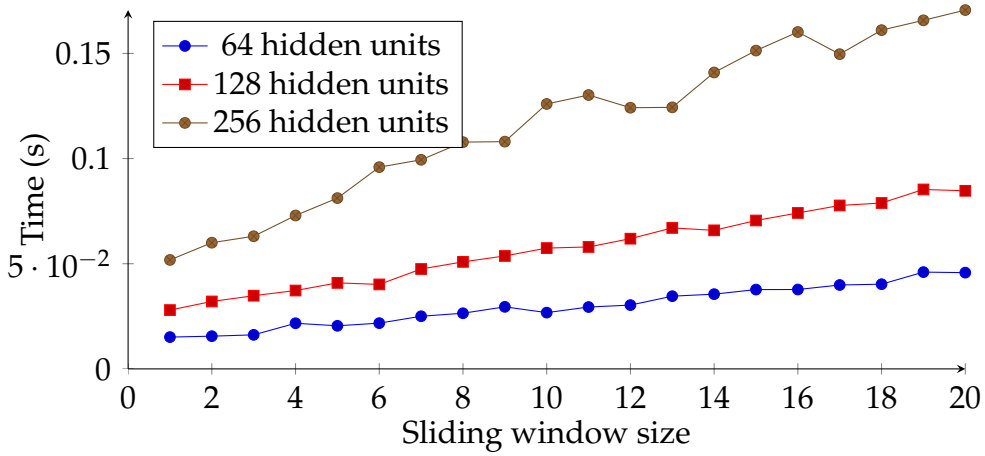


Figure B.2: Computation time of predicting network traffic with TDNN and various sliding window sizes.

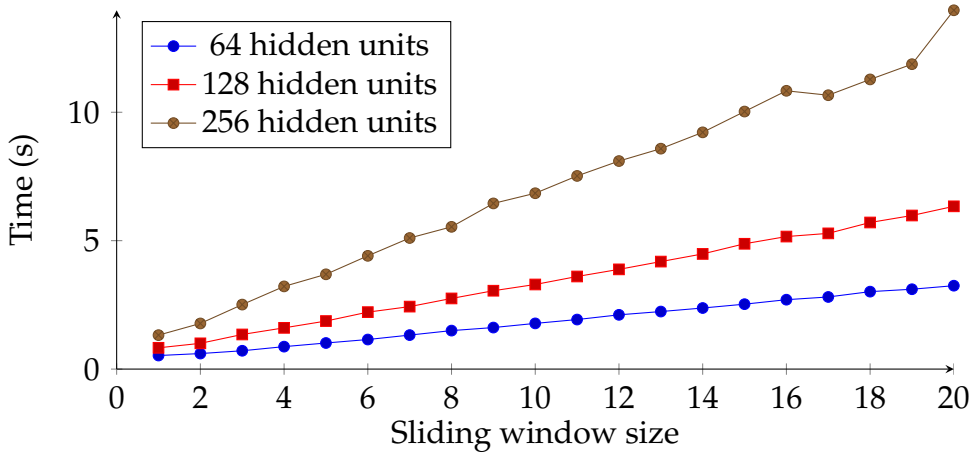
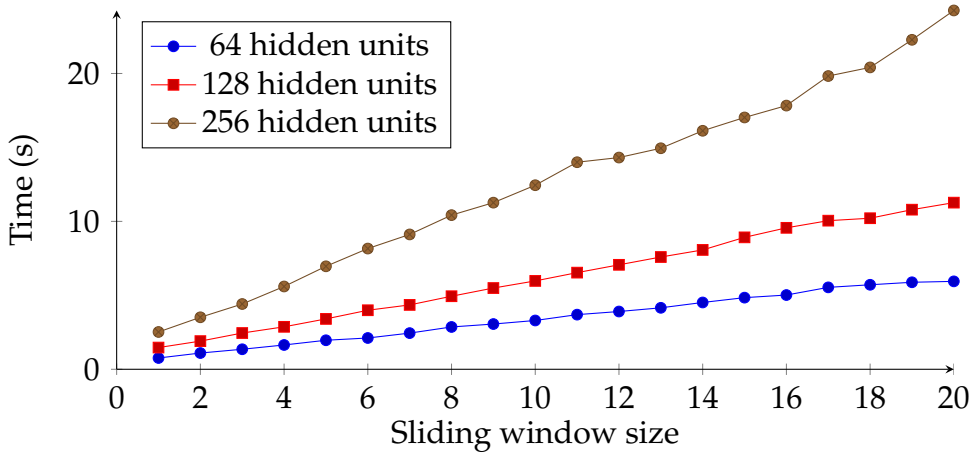


Figure B.3: Computation time of predicting manipulator's claw position with TDNN and various sliding window sizes.



B.0.5 Computation Time

Increasing unfolding linearly increases the computation time. On the other had, computation time is increased quadratically by adding

more hidden units.

Figure B.4: Computation time of predicting goniometric function with SRN trained by TBPTT with various unfolding depth.

Figure B.5: Computation time of predicting network traffic with SRN trained by TBPTT with various unfolding depth.

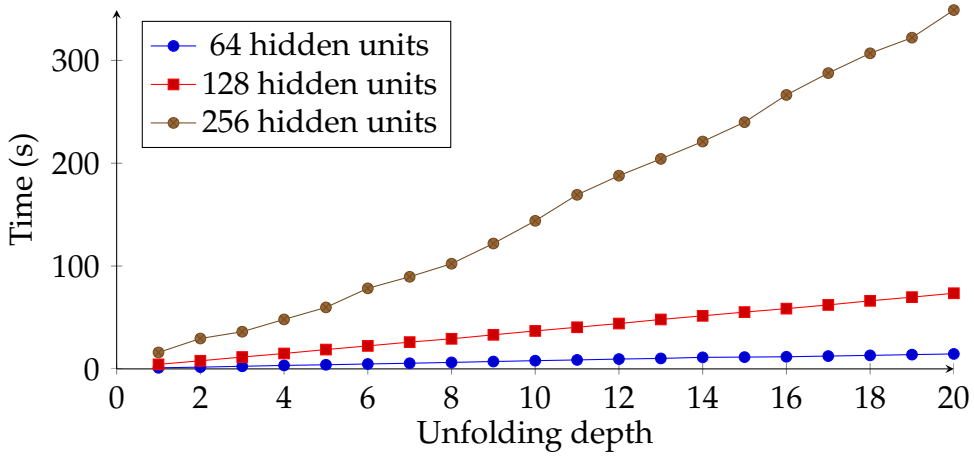
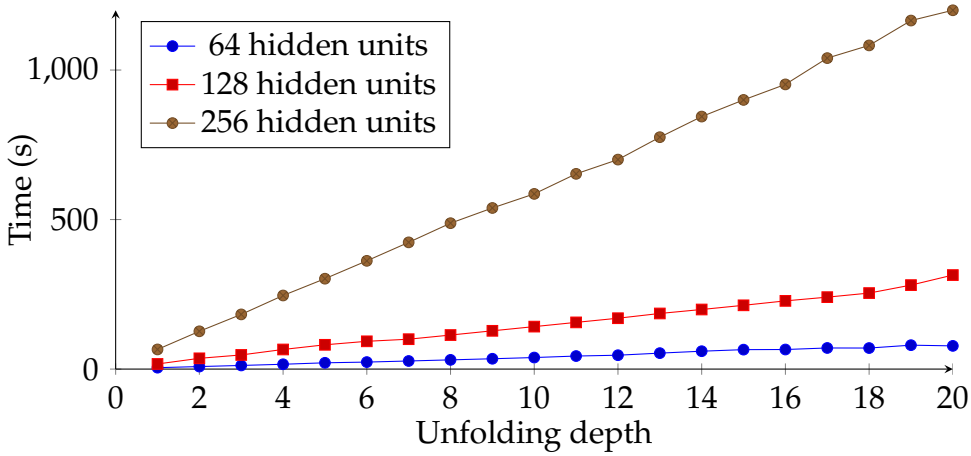


Figure B.6: Computation time of predicting manipulator's claw position with SRN trained by TBPTT with various unfolding depth.



B.0.6 Computation Time

RTRL's $O(n^4)$ time complexity appears to be the limiting factor in online use. Even with 16 hidden units, the network approaches maximum allowed execution time of 360 seconds in manipulator scenario.

Figure B.7: Computation time of prediction in all three scenarios with SRN trained by RTRL with various number of hidden units.

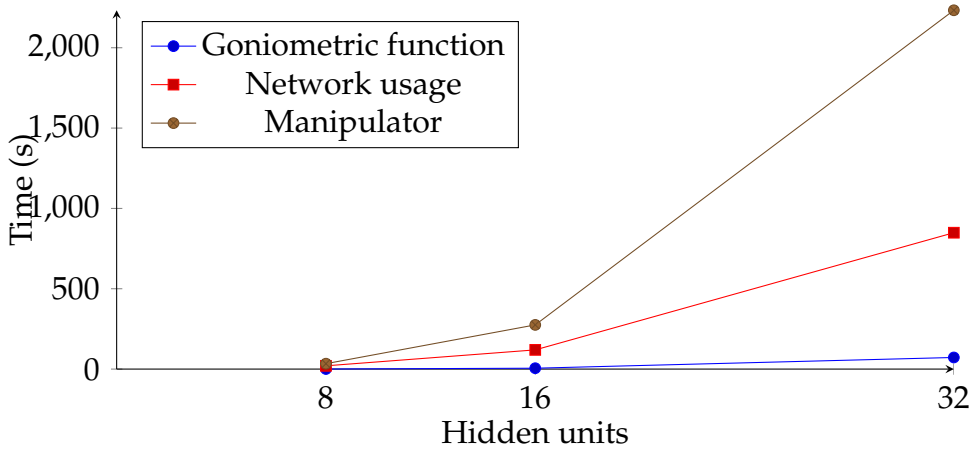


Figure B.8: Computation time of predicting goniometric function with CW-RNN trained by TBPTT with various unfolding depth.

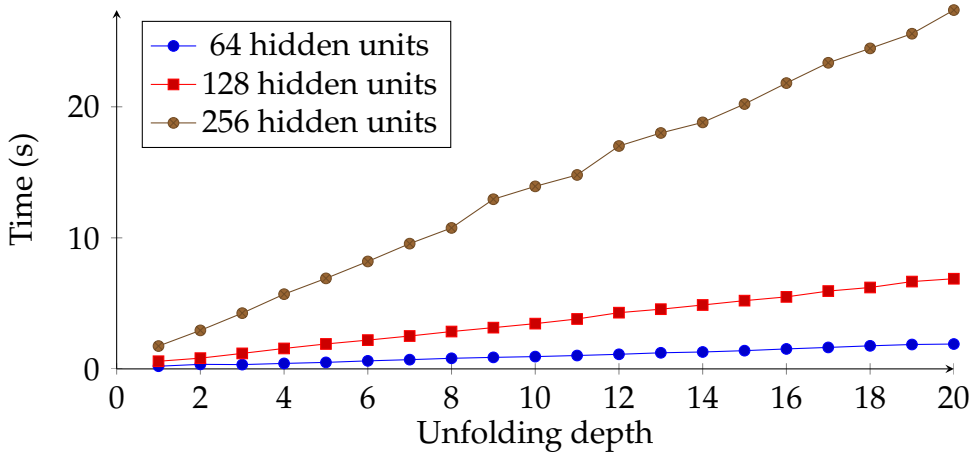


Figure B.9: Computation time of predicting network traffic with CW-RNN trained by TBPTT with various unfolding depth.

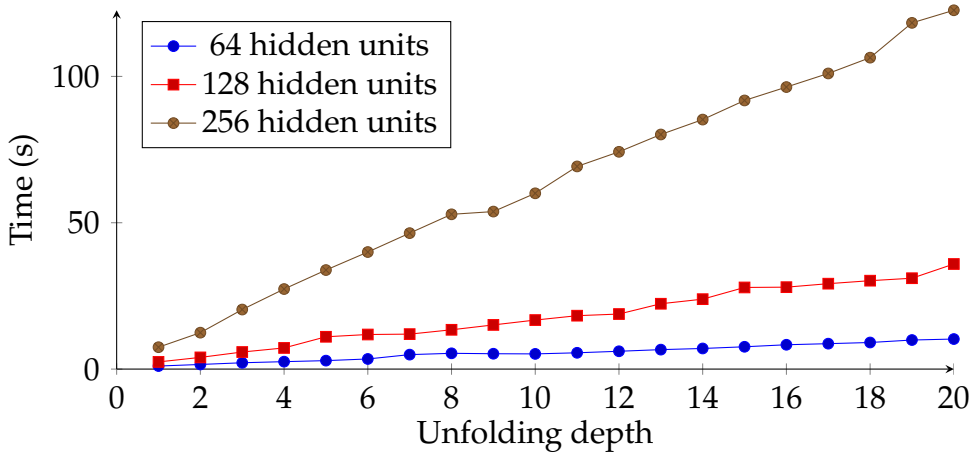
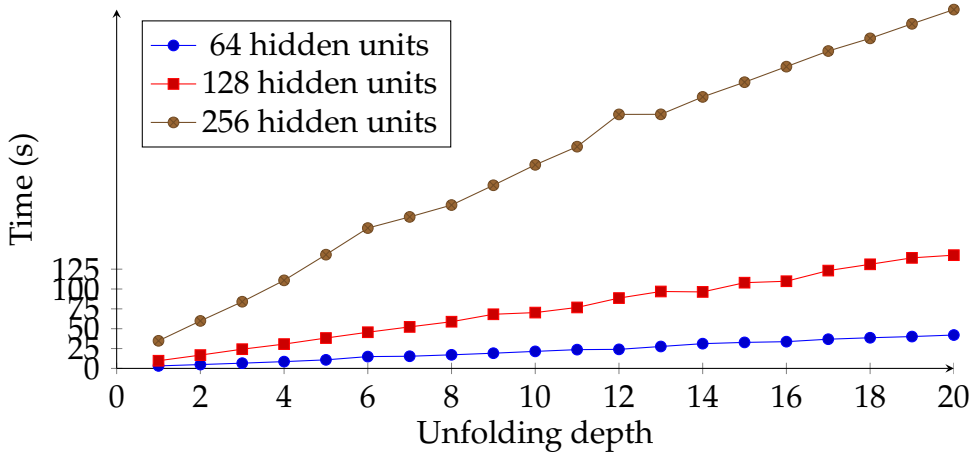


Figure B.10: Computation time of predicting manipulator's claw position with CW-RNN trained by TBPTT with various unfolding depth.



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