Assignment 2: Plant Tissue Structure

Integrated Workshop

Due Friday, Oct. 20, 2023 at 11:59 PM EST

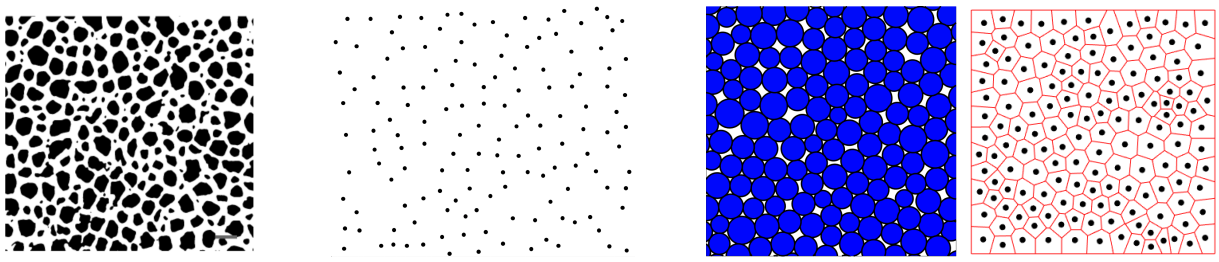

Fig. 1: (Left & second from left) MicroCT image of spongy mesophyll tissue and stomata positions taken from living leaf tissue, courtesy of Aleca Borsuk. In the image second from the left, we show a collection of $N_{\text{stomata}} = 143$ points that represent the positions of stomata on the underside of a leaf. The stomata are used for controlling leaf CO_2 intake, and are correlated with void space in the leaf interior (black space in the left image). (second from right) A disordered, jammed packing of $N_{\text{disk}} = 134$ disk-shaped particles. (right) A set of 150 points generated via Lloyd's algorithm with their corresponding Voronoi tesselations. The coordinates for the stomata, disk centers, and Lloyd's algoritm are stored in `stomata.dat`, `disk.dat`, and `lloyds.dat` in the Assignments folder on Canvas.

In the lectures from this module, we learned about the various structures in leaf and flower tissue. Now, we will use quantitative analyses of experimental and computational data to understand these structures. Using data from Aleca Borsuk, we will analyze the arrangement of stomata (leaf pores, see Fig. 1) using Voronoi tessellations and compare them to Voronoi tessellations from three other point patterns: random points, points taken from the centers of particles in a jammed, two-dimensional disk packing, and points generated via Lloyd's algorithm (second from right and right images in Fig. 1). We would like to know how correlated the positions in the stomata are to one another; if the stomatal locations are completely random, then the Voronoi tessellation will resemble that for random points. If the system resembles the disk packing, we will know that the stomata locations are more correlated with one another. If the system most closely resembles those generated from Lloyd's algorithm, the locations of the cells are determined by the constraint of uniform cell size and shape. We also know that stomata locations are correlated with void space found in the leaf mesophyll tissue (e.g. the sample in Fig. 1). Understanding where the stomata, and thus the void space, are placed will allow us to better model the leaf development process.

Voronoi code

For this problem, you will calculate several structural quantities related to the Voronoi tessellation of a few different point patterns in two spatial dimensions with fixed boundaries. To calculate the Voronoi tessellation, use the function `getVoronoiDiagram` as defined in the `getVoronoiDiagram.m` file provided on Canvas. The function call is defined as

```
[V,C,xbox,ybox] = getVoronoiDiagram(x,y,plotIt);
```

where `x` and `y` are $N \times 1$ column vectors of the x and y positions of the points that will define your Voronoi tessellation, respectively. The outputs of the function, `V` and `C`, contain information about the network of Voronoi edges and vertices. `V` is an $N_v \times 2$ matrix of Voronoi vertex positions, where N_v is the total number of vertices in the Voronoi diagram. `C` is a cell matrix, where the ith entry in the cell is itself a vector that contains the vertices associated with Voronoi polygon i. For example, suppose entry `2` in the cell `C` was the vector `[1; 3; 5; 7; 8; 9]`; this would mean that the polygon centered at point `2` has Voronoi vertices $1, 3, 5, 7, 8$ and 9 as polygonal vertices.

Some things to note about `getVoronoiDiagram`: the function as written will neglect Voronoi polygons that touch the boundary, so `C` will only list vertices that are not associated with a polygon that touches the boundary, and `xbox` and `ybox` are the defining point coordinates of polygons that do not touch the boundary. If the third input `plotIt` is 1, the code will plot the Voronoi diagram. If not, it will skip the plot.

Also provided is the code `triangularArea`, which takes in three points `p1`, `p2` and `p3`, and calculates the area of a triangle defined by the three points. In part (c) of Problem 1, you will need to calculate the area of the Voronoi polygons; since the polygons can be thought of the sum of triangles that have a vertex at the center, you can use this function to compute the total area of any polygon as long as you know the locations of the vertices.

Problems

1. For this problem, we will analyze the Voronoi tessellation obtained from the positions of stomata, taken from experiments on plant leaves, and compare the results to the Voronoi tessellations generated by synthetic data (random points, points generated through Lloyd's algorithm, and the locations of jammed disk packings). Write the code for this problem in a script titled `LASTNAME-FIRSTNAME-problem1.m`.

   (a) First compute the Voronoi diagram for $N = 150$ points sampled randomly from a uniform distribution in a box that is 1 x 1 (using the `rand()` function in MATLAB) and plot it. Save the plot as `LASTNAME-FIRSTNAME-random_points_voro.png`. You will need this Voronoi diagram for part (c), so save the vertices and connectivity information (i.e. `V` and `C`) as separate variables.

   (b) In the `Assignments` folder on Canvas, you will find four files that contain the x and y coordinates of three point patterns; points taken from the positions of stomata on the underside of a plant leaf (`stomata.dat`, provided by Aleca Borsuk), points taken from disordered, jammed disk packings (`disks.dat`), and points taken from

Lloyd's algorithm (`lloyds.dat`). Read in the $x-$ and $y-$coordinates from these files, and plot the Voronoi diagrams. Save the plots as `LASTNAME-FIRSTNAME-stomata_voro.png`, `LASTNAME-FIRSTNAME-disk_voro.png`, and `LASTNAME-FIRSTNAME-lloyds_voro.png`. **Note:** the coordinates as given have been normalized by the sizes of their constraining boxes, so all of the coordinates range from 0 to 1.

(c) In 1923, Lewis [1] noted that, in many different biological systems, the area of Voronoi cells with n neighbors, $A(n)$, obeys

$$\frac{A(n)}{A(\overline{n})} = \frac{n - n_0}{\overline{n} - n_0} \tag{1}$$

where $A(\overline{n})$ is an average area of cells with the average number of neighbors, $\overline{n}$, and n_0 is a fitting constant. Lewis found that $n_0 = 2$ for different biological systems. Make a plot of $A(n)/A(\overline{n})$ vs. $(n - n_0)/(\overline{n} - n_0)$ for all four data sets (the random points, the stomata and the locations of particle centers from a disordered packing of disks, and points generated from Lloyd's algorithm). Both $\overline{n}$ and $A(\overline{n})$ are constants for each data set. Plot the Lewis prediction for $n_0 = 2$ on the plot. Add a legend for the points from the four data sets and the prediction line. Save your plot as `LASTNAME-FIRSTNAME-lewis_law.png`.

(d) Plot the distributions $P(n)$ of the number of neighbors n for the four data sets; normalize your distributions as probability distribution functions (i.e. pdfs, where the area under the curves is normalized to be 1), give your plot a legend, and save the plot as `LASTNAME-FIRSTNAME-neighbor_distribution.png`.

(e) For the four data sets, plot the distributions $P(\mathcal{A})$ of the shape parameter $\mathcal{A} = \frac{p^2}{4\pi A}$, where A and p are the area and perimeter of the Voronoi cell. Normalize your distributions as probability distribution functions, give your plot a legend, and save the plot as `LASTNAME-FIRSTNAME-shape_distribution.png`

(f) Discuss your plots in parts (c), (d), and (e). Is Lewis's law satisfied? Given what you see from your comparisons of the four data sets what conclusions can you draw about the arrangement of stomata? Mention how you see this in your analysis of the data. Name the document `LASTNAME-FIRSTNAME-writeup.x`, where the file can be a .pdf, .doc or .txt file. **Include plot images in the write up document**.

Submission

Upload your MATLAB script, along with your 5 plots and write-up file to the Assignments section of Canvas by the due date.

References

[1] F. T. Lewis. The typical shape of polyhedral cells in vegetable parenchyma and the restoration of that shape following cell division. 58(15):537–554, 1923.