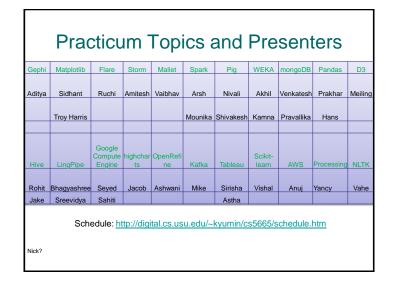
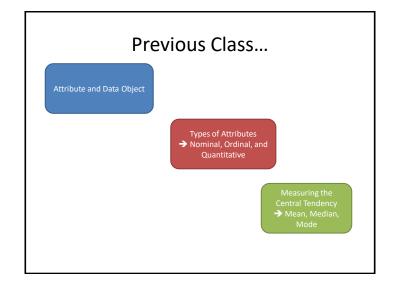
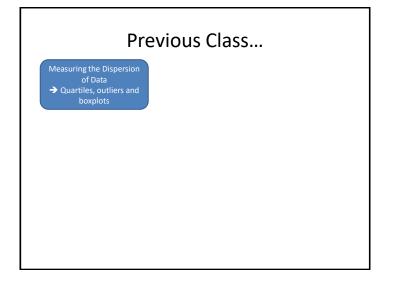
Introduction to Data Science

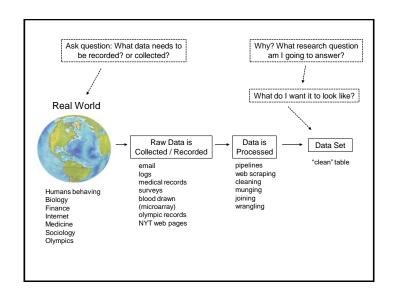
CS 5665
Utah State University
Department of Computer Science
Instructor: Prof. Kyumin Lee

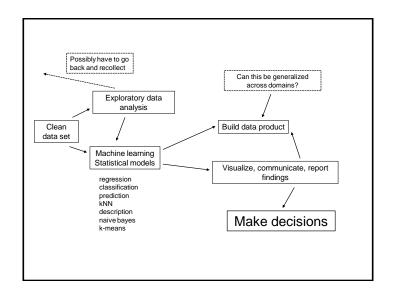


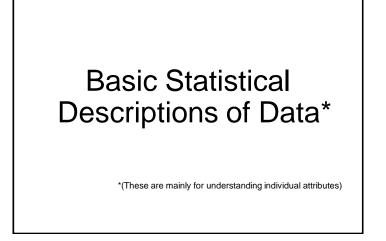




Data Science: The Context



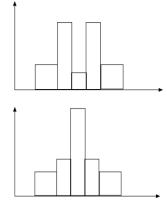




Histogram

- Histogram: Graph display of tabulated frequencies, shown as bars
- It shows what proportion of cases fall into each of several categories

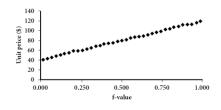
Histograms Often Tell More than Boxplots



- The two histograms shown in the left may have the same boxplot representation
 - The same values for: min, Q1, median, Q3, max
- But they have rather different data distributions

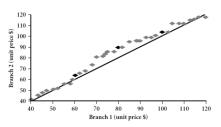
Quantile Plot

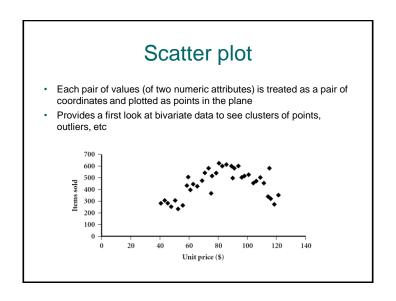
- Displays all of the data (allowing the user to assess both the overall behavior and unusual occurrences)
- Plots quantile information
 - For a data x_i, data sorted in increasing order, f_i indicates that approximately f_i*100% of the data are below the value x_i

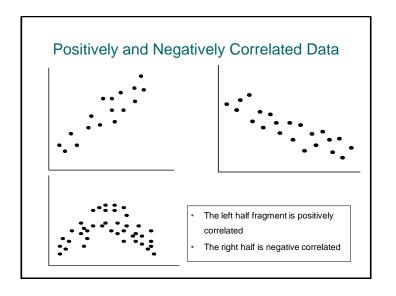


Quantile-Quantile (Q-Q) Plot

- Graphs the quantiles of one univariate distribution against the corresponding quantiles of another
- Allows the user to view whether there is a shift in going from one distribution to another
- Example shows unit price of items sold at Branch 1 vs. Branch 2 for each quantile. Unit prices of items sold at Branch 1 tend to be lower than those at Branch 2.









Measuring Data Similarity and Dissimilarity*

*(These are mainly for understanding the relationship between objects with multiple attributes)

Similarity and Dissimilarity

- Similarity
 - Numerical measure of how alike two data objects are
 - Value is higher when objects are more alike
 - Often falls in the range [0,1]
- Dissimilarity (e.g., distance)
 - Numerical measure of how different two data objects are
 - Lower when objects are more alike
 - Minimum dissimilarity is often 0
 - Upper limit varies
- · Proximity refers to a similarity or dissimilarity

Data Matrix and Dissimilarity Matrix

- Data matrix

- Dissimilarity matrix
 - n data points, but registers only the distance
 - A triangular matrix
 - Single mode (one entity distance)

$$\begin{bmatrix} 0 \\ d(2,1) & 0 \\ d(3,1) & d(3,2) & 0 \\ \vdots & \vdots & \vdots \\ d(n,1) & d(n,2) & \dots & \dots & 0 \end{bmatrix}$$

Proximity Measure for Nominal Attributes

- Can take 2 or more states, e.g., red, yellow, blue, green (generalization of a binary attribute)
- Method 1: Simple matching
 - m: # of matches, p: total # of attributes describing the objects, i and j: two objects

$$d(i,j) = \frac{p-m}{p}$$

- Method 2: Use a large number of binary attributes
 - creating a new binary attribute for each of the M nominal states

Proximity Measure for Binary Attributes

· A contingency table for binary data

Object
$$i$$
 0 sum $q + r$ $q + r$ $q + r$ sum $q + s$ $r + t$ p

- Distance measure for symmetric binary variables:
- $d(i,j) = \frac{r+s}{q+r+s+t}$ $d(i,j) = \frac{r+s}{q+r+s}$
- Distance measure for asymmetric binary variables:
- Jaccard coefficient (similarity measure for asymmetric binary variables):
- $sim_{Jaccard}(i, j) = \frac{q}{q + r + s}$

Dissimilarity between Binary Variables

Example

Name	Gender	Fever	Cough	Test-1	Test-2	Test-3	Test-4
Jack	M	Y	N	P	N	N	N
Mary	F	Y	N	P	N	P	N
Jim	M	Y	Y	N	N	N	N

- Gender is a symmetric attribute
- The remaining attributes are asymmetric binary
- Let the values Y and P be 1, and the value N be 0
- Let's measure for only asymmetric binary variables

$$d(jack,mary) = \frac{0+1}{2+0+1} = 0.33$$
$$d(jack,jim) = \frac{1+1}{1+1+1} = 0.67$$
$$d(jim,mary) = \frac{1+2}{1+1+2} = 0.75$$

Standardizing Numeric Data

- Z-score: $z = \frac{x \mu}{\sigma}$
 - X: raw score to be standardized, μ : mean of the population, σ : standard deviation
 - the distance between the raw score and the population mean in units of the standard deviation
 - negative when the raw score is below the mean, "+" when above

Standardizing Numeric Data

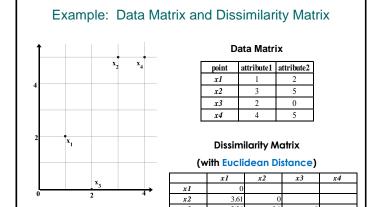
• An alternative way: Calculate the mean absolute deviation

where
$$s_f = \frac{1}{n}(|x_{1f} - m_f| + |x_{2f} - m_f| + ... + |x_{nf} - m_f|)$$

$$m_f = \frac{1}{n}(x_{1f} + x_{2f} + ... + x_{nf})$$

- standardized measure (z-score):

$$z_{if} = \frac{x_{if} - m_f}{s_f}$$



Distance on Numeric Data: Minkowski Distance

· Minkowski distance: A popular distance measure

$$d(i,j) = \sqrt[h]{|x_{i1} - x_{j1}|^h + |x_{i2} - x_{j2}|^h + \dots + |x_{ip} - x_{jp}|^h}$$

where $i = (x_{i1}, x_{i2}, ..., x_{ip})$ and $j = (x_{j1}, x_{j2}, ..., x_{jp})$ are two *p*-dimensional data objects, and *h* is the order (the distance so defined is also called L-*h* norm)

- Properties
 - d(i, j) > 0 if $i \neq j$, and d(i, i) = 0 (Positive definiteness)
 - d(i, j) = d(j, i) (Symmetry)
 - $d(i, j) \le d(i, k) + d(k, j)$ (Triangle Inequality)
- A distance that satisfies these properties is a metric

Special Cases of Minkowski Distance

- h = 1: Manhattan (city block, L₁ norm) distance
 - E.g., the Hamming distance: the number of bits that are different between two binary vectors

$$d(i,j) = |x_{i_1} - x_{j_1}| + |x_{i_2} - x_{j_2}| + ... + |x_{i_p} - x_{j_p}|$$

• h = 2: (L₂ norm) Euclidean distance

$$d(i,j) = \sqrt{(|x_{i1} - x_{j1}|^2 + |x_{i2} - x_{j2}|^2 + \dots + |x_{ip} - x_{jp}|^2)}$$

Example: Minkowski Distance

x2		3	5	
x3		2	0	
x4		4	5	
t				
4		x,	x ₄	
2	x ₁			
	1	x ₃		
0		2	4	

attribute 1 attribute 2

Dissimilarity Matrices Manhattan (L.)

	·· (-)			
L	x1	x2	x3	x4
x1	0			
x2	5	0		
x3	3	6	0	
x4	6	1	7	0

Euclidean (L_2)

4
x4
0

Ordinal Variables

- · An ordinal variable can be discrete or continuous
- Order is important, e.g., rank
- Can be treated like interval-scaled
 - replace x_{if} by their rank $r_{if} \in \{1,...,M_f\}$
 - map the range of each variable onto [0, 1] by replacing *i*-th object in the *f*-th variable by

$$z_{if} = \frac{r_{if} - 1}{M_f - 1}$$

 compute the dissimilarity using methods for intervalscaled variables

Attributes of Mixed Type

- A database/dataset may contain all attribute types
 - Nominal, symmetric binary, asymmetric binary, numeric, ordinal
- One may use a weighted formula to combine their effects

$$d(i,j) = \frac{\sum_{f=1}^{p} \delta_{ij}^{(f)} d_{ij}^{(f)}}{\sum_{f=1}^{p} \delta_{ij}^{(f)}}$$

- $-\delta_{ij}^{(f)}=0$ if x_{if} or x_{if} is missing, or $x_{if}=x_{jf}=0$ and attribute f is asymmetric binary; otherwise =1
- f is binary or nominal:

$$d_{ij}^{(f)} = 0$$
 if $x_{if} = x_{if}$, or $d_{ij}^{(f)} = 1$ otherwise

- -f is numeric: use the normalized distance
- f is ordinal
 - Compute ranks r_{if} and

$$Z_{if} = \frac{I_{if} - 1}{M}$$

• Treat z_{if} as interval-scaled