CEE6400 Solution to Homework 3

1. Dingman 4.6

4-6. **The table below gives annual precipitation measured over a 17-yr period at five gages in a region. Gage C was moved at the end of 1974. Carry out a double-mass-curve analysis to check for consistency in that gage's record, and make appropriate adjustments to correct for any inconsistencies discovered. The data are in text-disk file "2MASCURV.XLS".**

Annual Precipitation (mm) at StationYear	**A**	**B**	**C**	**D**	**E**
1970	1010	1161	780	949	1135
1971	1005	978	1041	784	970
1972	1067	1226	1027	1067	1158
1973	1051	880	825	1014	1022
1974	801	1146	933	923	821
1975	1411	1353	1584	930	1483
1976	1222	1018	1215	981	1174
1977	1012	751	832	683	771
1978	1153	1059	918	824	1188
1979	1140	1223	781	1056	967
1980	829	1003	782	796	1088
1981	1165	1120	865	1121	963
1982	1170	989	956	1286	1287
1983	1264	1056	1102	1044	1190
1984	1200	1261	1058	991	1283
1985	942	811	710	875	873
1986	1166	969	1158	1202	1209

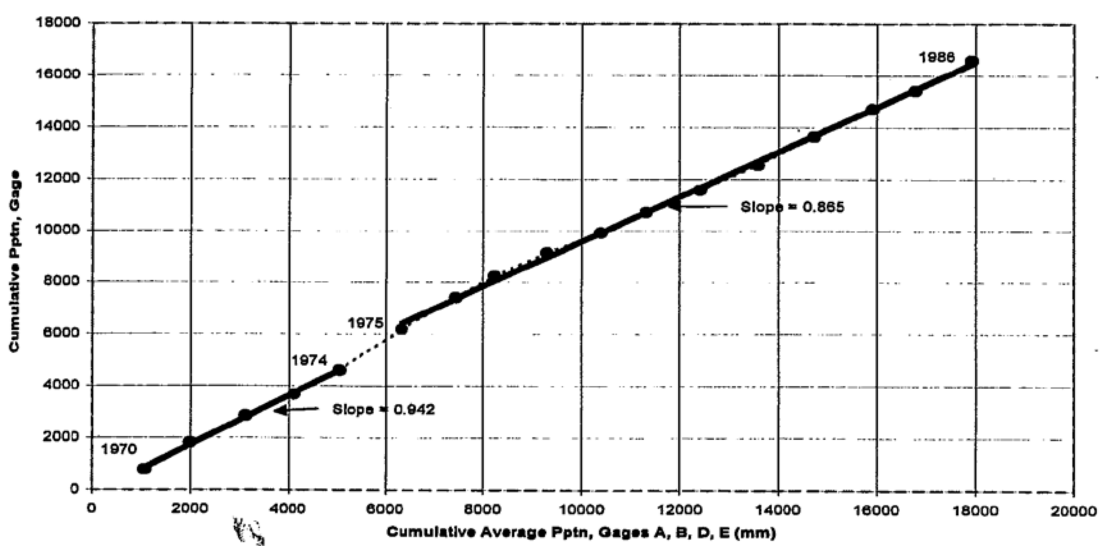
The following table gives the values needed for the double-mass curve plot.

Year	Sum A,B,D,E (mm)	Average A,B,D,E (mm)	Cumulative average A,B,D,E (mm)	Cumulative C (mm)
1970	4255	1064	1064	780
1971	3737	934	1998	1821
1972	4518	1130	3128	2848
1973	3967	992	4119	3673
1974	3691	923	5042	4606
1975	5177	1294	6336	6190
1976	4395	1099	7435	7405
1977	3217	804	8239	8237
1978	4224	1056	9295	9155
1979	4386	1097	10392	9936
1980	3716	929	11321	10718
1981	4369	1092	12413	11583
1982	4732	1183	13596	12539
1983	4554	1139	14735	13641
1984	4735	1184	15918	14699
1985	3501	875	16794	15409
1986	4546	1137	17930	16567

Using the regression routine of EXCEL, we find the following slopes:

Years	Slope
1970-1974	0.942
1975-1986	0.865

EXERCISE 4-6



Thus from Equation (4-9) we calculate

$$K = \frac{0.865}{0.942} = 0.918.$$

Multiplying K times the measured annual precipitation at C for 1970-1974 gives the following adjusted annual total precipitation at Gage C:

Year	Adjusted Precipitation (mm)
1970	716
1971	956
1972	943
1973	758
1974	857

2 a) Dingman 4.7

4-7. Consider a drainage basin of area = 1611 km^2 in which three precipitation gages have been operating for 20 yr. The average annual precipitation, $\hat{P}$, at the three gages is 1198 mm yr^{-1} and the overall variance of annual precipitation, $\hat{S}^2(p)$, is 47,698 mm^2 yr^{-2}. An analysis of the spatial correlation structure of annual precipitation in the region gives a value of c = 0.0386 km^{-1} [Equation (4B1-5c)]. Compute the variance reduction achieved by this measurement program and compare the variance reduction that would have been achieved by having four, rather than three, gages in the region during the 20-yr period.

As in Example 4-4, assume no autocorrelation in annual precipitation values [$r_1(p) = 0$]. Then from Figure 4-35 we find $F_1(20) = 0.05$.

To find $F_2(3)$ and $F_2(4)$ from Figure 4-36, we first compute $A \cdot c^2$:

$$A \cdot c^2 = 1611 \text{ km}^2 \times (0.0386 \text{ km}^{-1})^2 = 2.40.$$

Then from Figure 4-36 we find $F_2(3)$ = 0.72 and use Equation (4-26) to compute the variance and standard deviation achieved by 3 gages:

$$\hat{S}^2(\hat{P}) = 0.05 \times 0.72 \times 47{,}698 \text{ mm}^2 \text{ yr}^{-2} = 1717.128 \text{ mm}^2 \text{ yr}^{-2};$$

$$\hat{S}(\hat{P}) = 41.4 \text{ mm yr}^{-1}.$$

As in Example 4-4, we can then estimate the 95% confidence intervals for the regional mean annual precipitation, P, as

$$(1198 - 1.96 \times 41.4) \text{ mm yr}^{-1} < P < (1198 + 1.96 \times 41.4) \text{ mm yr}^{-1};$$

$$1117 \text{ mm yr}^{-1} < P < 1279 \text{ mm yr}^{-1}.$$

We now compare these values with what would have been achieved with $N = 4$ gages. In Figure 4-36 there is no curve for $N = 4$, but we find $F_2(5) = 0.65$. Linearly interpolating (see solution to Exercise 2-2), we find

$$F_2(4) = 0.72 + [(0.65 - 0.72)/(5 - 3)]\cdot(4 - 3) = 0.685.$$

The Equation (4-26) yields

$$\hat{S}(\hat{P}) = 0.05 \times 0.685 \times 47{,}698 \text{ mm}^2 \text{ yr}^{-2} = 1633.7 \text{ mm}^2 \text{ yr}^{-2}.$$

$$\hat{S}(\hat{P}) = 40.4 \text{ mm yr}^{-1}.$$

We can see immediately that little additional precision is obtained by operating a fourth gage. To complete the comparison, we estimate the 95% confidence intervals for the regional mean annual precipitation with 4 gages as

$$(1198 - 1.96 \times 40.4) \text{ mm yr}^{-1} < P < (1198 + 1.96 \times 40.4) \text{ mm yr}^{-1};$$

$$1119 \text{ mm yr}^{-1} < P < 1277 \text{ mm yr}^{-1}.$$

b) **The Compute, for the information given in Dingman 4.7 the uncertainty, quantified by standard deviation, associated with the estimate of area average precipitation in any one specific year (assuming that gages are randomly positioned). Compare your result to the uncertainty that would have resulted had there only been 1 rain gage.**

One year is treated as one statistical event for the purposes of using equation 4.34. in Bras, 1990.

MSE=F(N, A c^2)σ^2 with F(N, A c^2) from Figure 4.35. A c^2 = 2.40

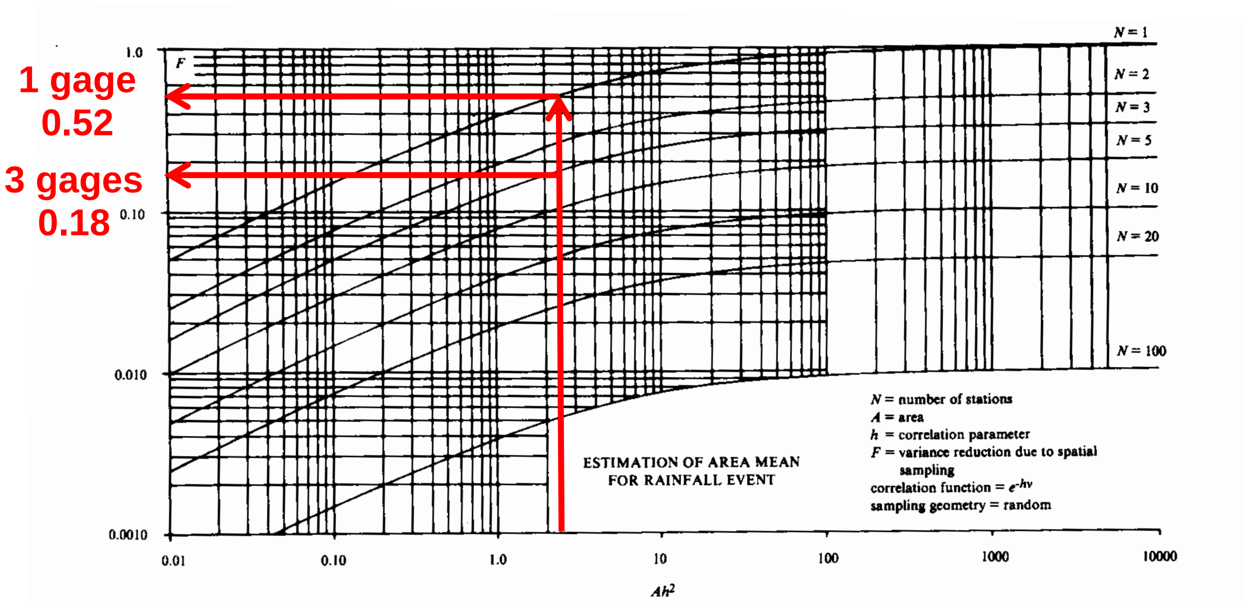


FIGURE 4.35 Variance reduction factor due to spatial sampling with random design used in the estimation of areal mean of rainfall event with $r(v) = e^{-hv}$. Source: I. Rodriguez-Iturbe and J. M. Mejia, "The Design of Rainfall Networks in Time and Space," *Water Resources Res.*, 10(4):725, 1974. Copyright by the American Geophysical Union.

Therefore, with 3 gauges

MSE = 0.18 x 47968 = 8634 mm^2

The uncertainty quantified in terms of standard deviation, is the square root of this

$\sqrt{\text{MSE}} = 93 \quad \text{mm}$

Had there been only one rain gage, we would have had

MSE = 0.52 x 47968 = 24943 mm^2

The uncertainty quantified in terms of standard deviation, is the square root of this

$\sqrt{\text{MSE}} = 158 \quad \text{mm}$

c) Explain physically what it means for the correlation structure of annual precipitation in the region to have a value of c=0.0386 km^{-1}, and comment on how physically realistic such a value is.

Physically, with c=0.0386 km^{-1} and with the exponential correlation model $\rho = e^{-cx}$ the correlation is 1/e=0.37 for separation distance x=1/0.0386 = 26 km and 1/e^2=0.13 for separation distance x=52 km and so on. This means that the rainfall field becomes essentially uncorrelated once separation distances get greater than 50 km. This seems rather short for annual rainfall, where wet years are likely to have persistence over spatial areas greater than 50 km. A smaller c would probably be better for this annual scale analysis. If the "event" was a storm, then c=0.0386 km^{-1} seems appropriate or even on the large side as storm cells have scales of 5 to 20 km.

3. Dingman 4.9

4-9. Re-do the depth-duration-frequency analysis of Example 4-6 as an intensity-duration-frequency problem. The data are in text-disk file "CHIANMAX.XLS".

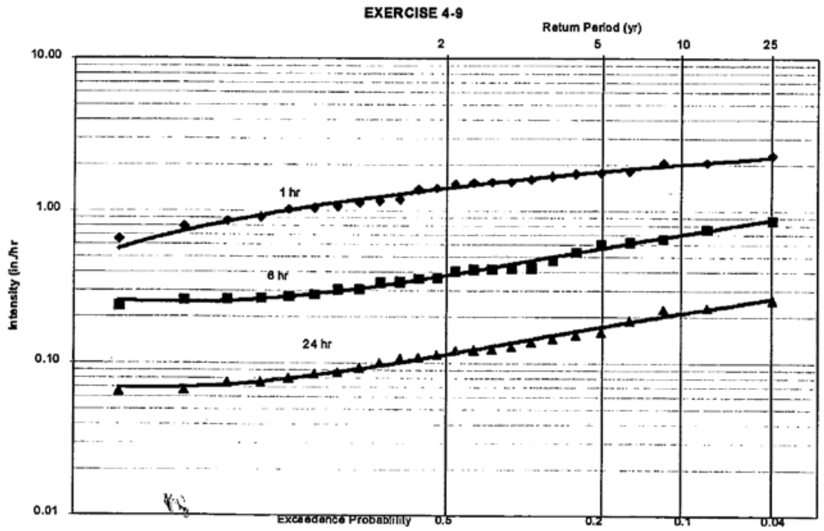
The intensity data in the "ChiAnnMax.xls" file are ranked from highest to lowest and the exceedence probabilities estimated via the Weibull plotting-position formula (see Box C-1):

$$EP(I) = i(I)/(N+1),$$

where $EP(I)$ is the estimated exceedence probability for rainfall intensity I, $i(I)$ is the rank of intensity I, and N is the number of years of record. (Note that for estimating exceedence probabilities, the computations are more conveniently done by ranking from highest to lowest, rather than lowest to highest as described in Box C-1).

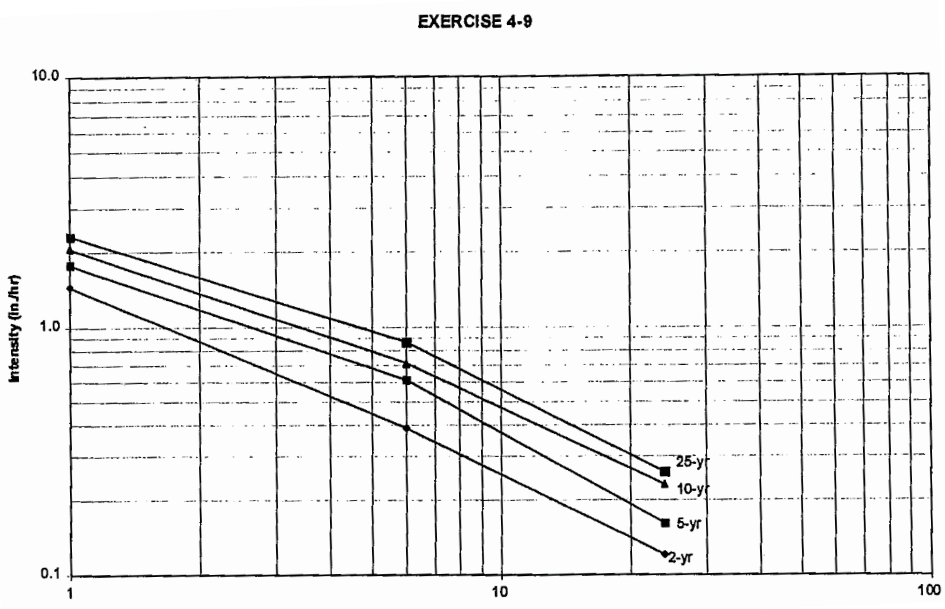
		Intensity (in./hr)		
Rank, $i(I)$	$EP(I)$	1-hr	6-hr	24-hr
1	0.040	2.32	0.87	0.26
2	0.080	2.08	0.76	0.23
3	0.120	2.06	0.66	0.22
4	0.160	1.82	0.62	0.19
5	0.200	1.78	0.61	0.16
6	0.240	1.75	0.54	0.15
7	0.280	1.69	0.47	0.15
8	0.320	1.61	0.42	0.14
9	0.360	1.55	0.42	0.13
10	0.400	1.55	0.42	0.12
11	0.440	1.53	0.41	0.12
12	0.480	1.50	0.40	0.12
13	0.520	1.41	0.37	0.11
14	0.560	1.38	0.36	0.11
15	0.600	1.19	0.34	0.11
16	0.640	1.16	0.34	0.10
17	0.680	1.13	0.31	0.09
18	0.720	1.07	0.30	0.09
19	0.760	1.04	0.28	0.09
20	0.800	1.02	0.27	0.08
21	0.840	0.91	0.27	0.08
22	0.880	0.86	0.26	0.08
23	0.920	0.79	0.26	0.07
24	0.960	0.65	0.24	0.07

As in Figure 4-49, the intensity values are plotted vs. exceedence probability and return period on probability paper:



Values of the 2-, 5-, 10-, and 25-yr rainfall intensities for the 3 durations are read from this graph, tabulated, and plotted as in Figure 4-50:

Return Period (yr)	1-hr	6-hr	24-hr
2	1.45	0.39	0.12
5	1.78	0.61	0.16
10	2.07	0.71	0.23
25	2.32	0.87	0.26



Estimation of area average precipitation from point measurements as an Introduction to ArcGIS

1.

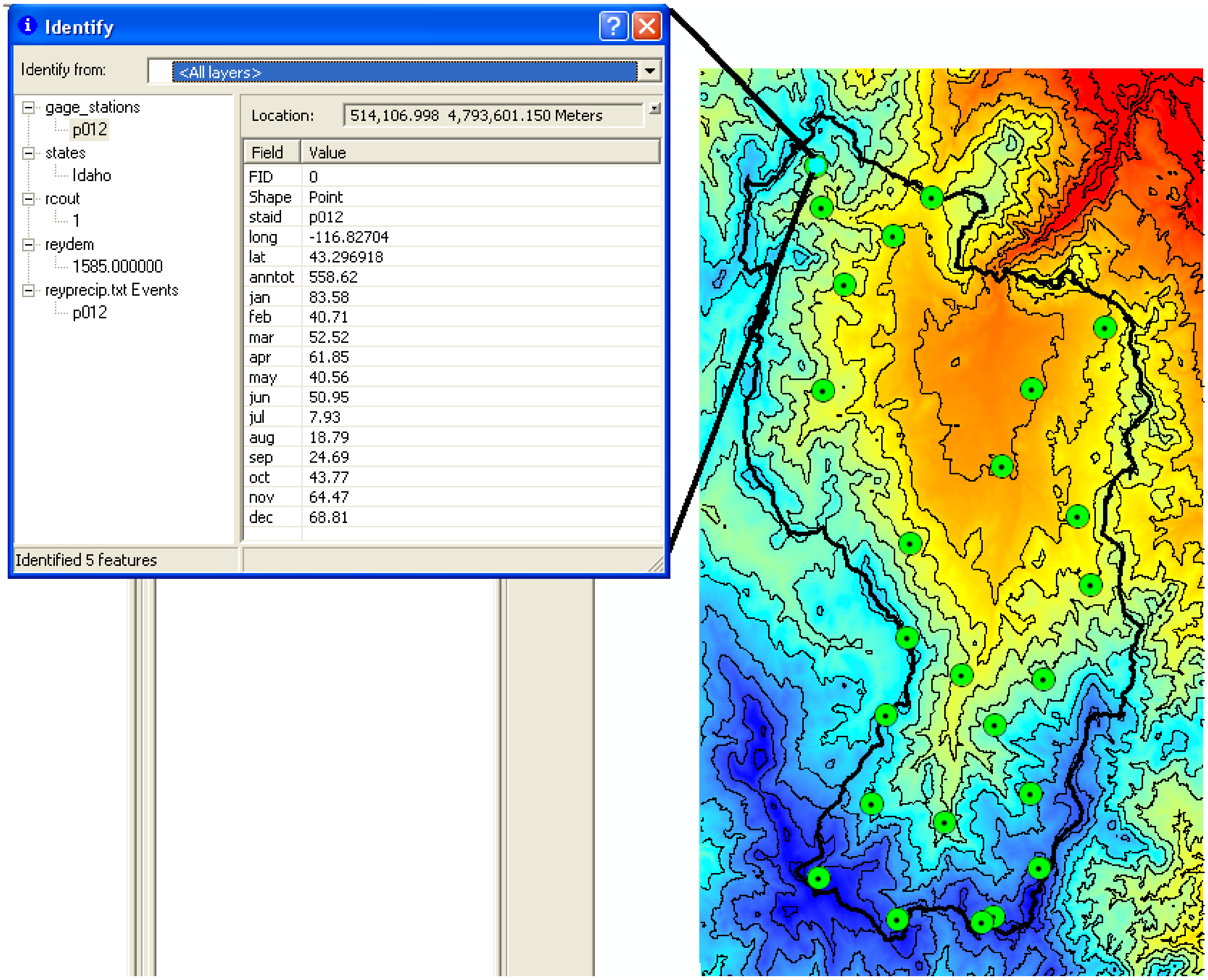
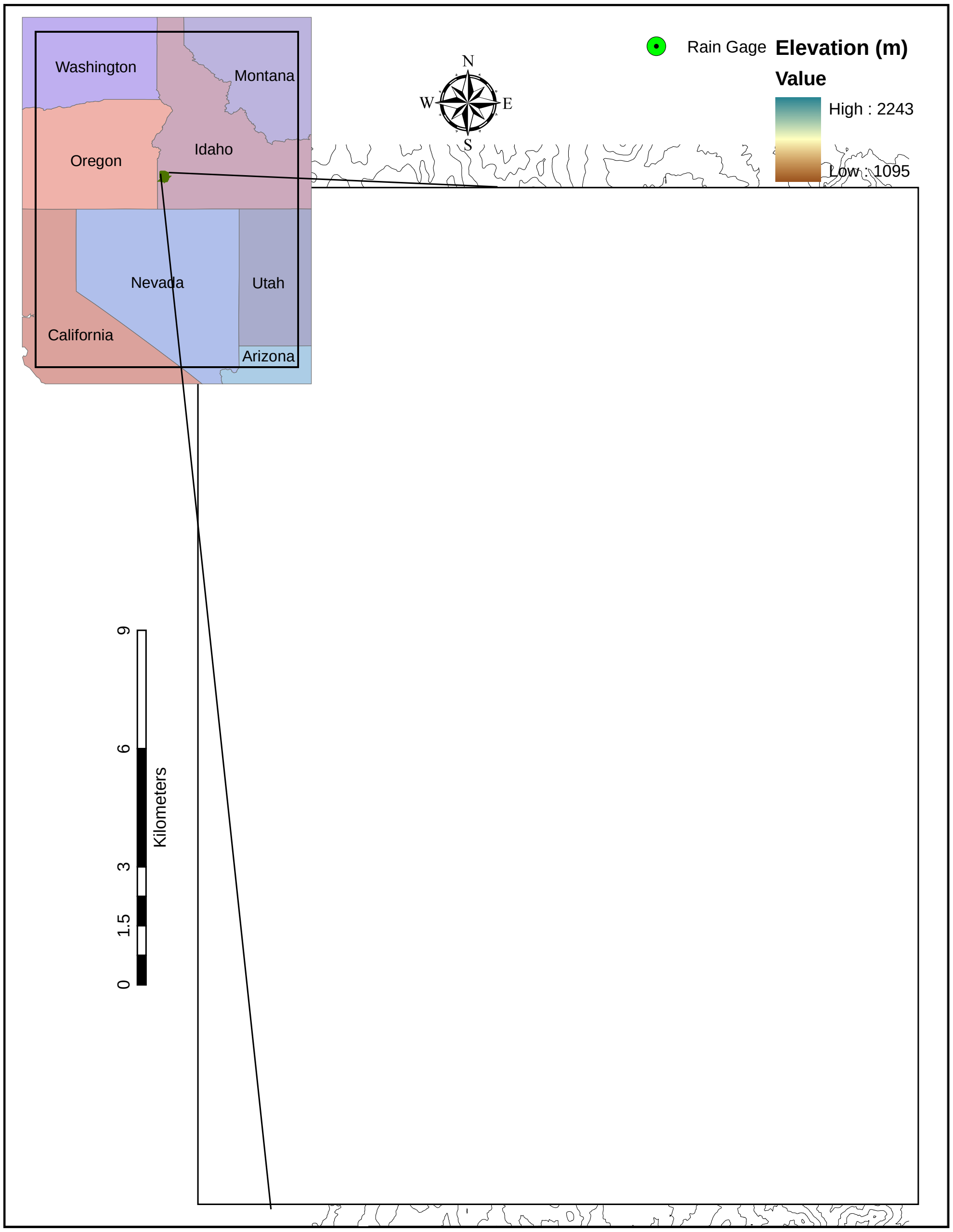


Table gives the station identifier (staid), elevation in meters and the mean annual precipitation for the southernmost and westernmost Reynolds Creek rain gages.

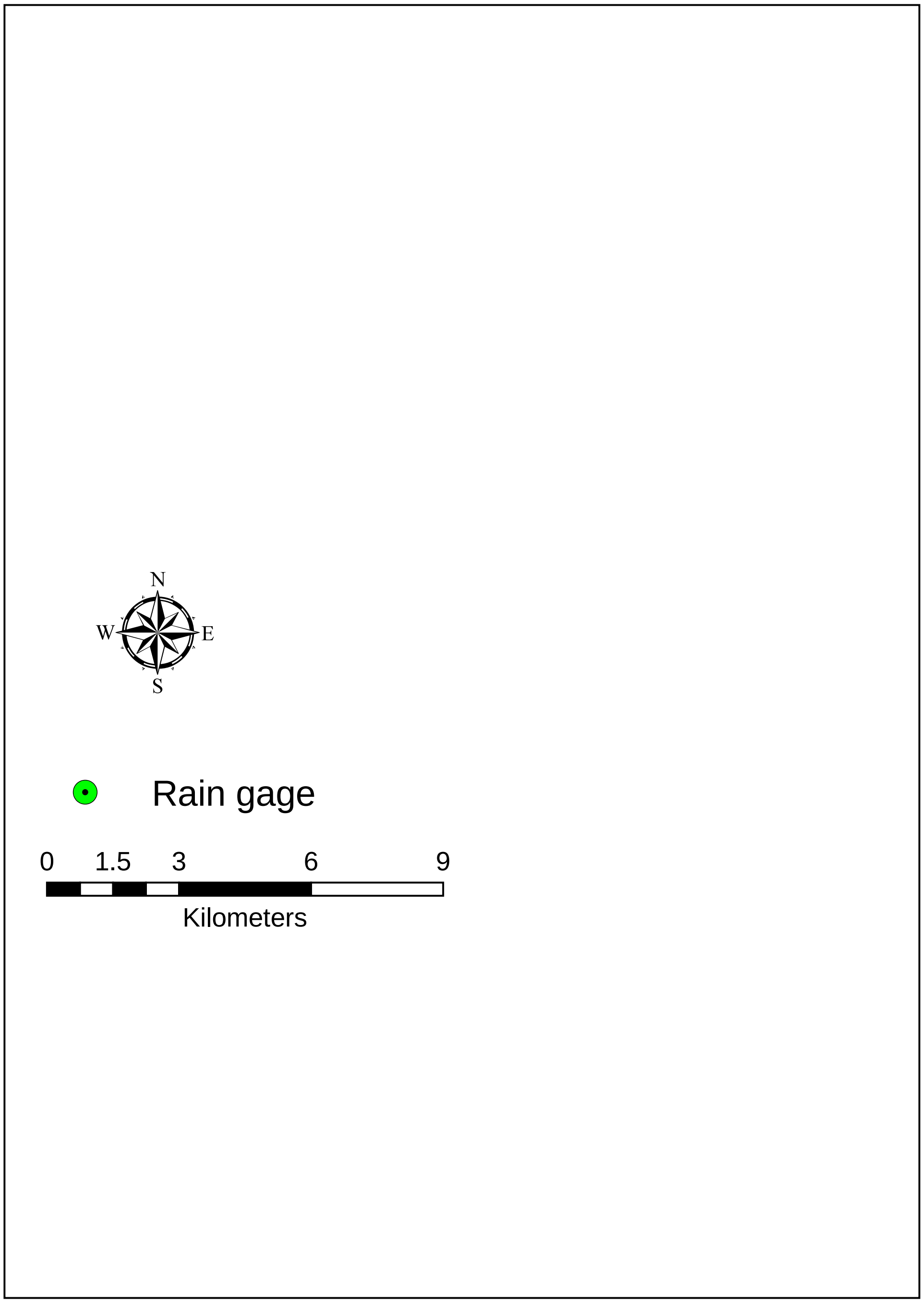
Station Identifier (staid)	Elevation (m)	Mean Annual Precipitation (mm)
P17614 (southernmost)	2095	763.62
P012 (northernmost)	1585	558.62

2.

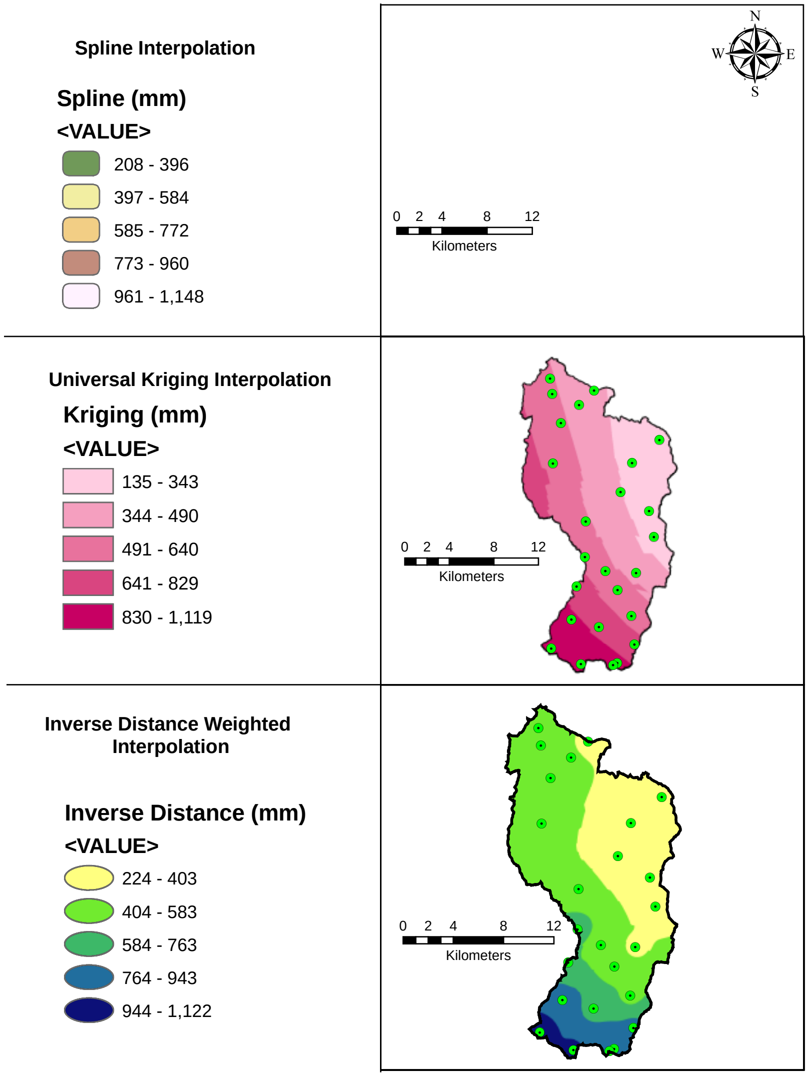


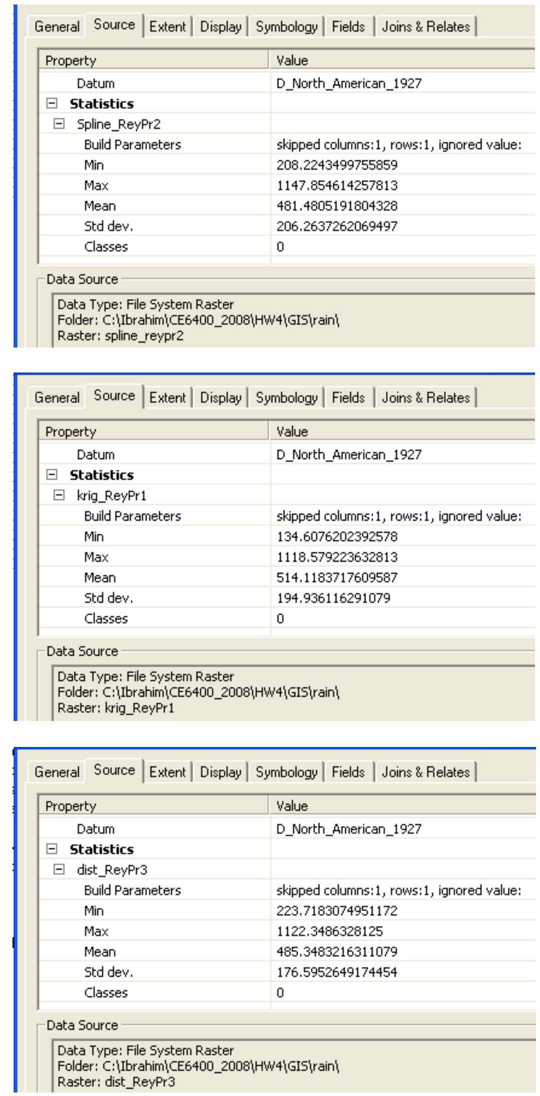
Reynolds Creek layout and its location in the Western U.S. The watershed elevation ranges between 1095 m and 2243 m. The contour lines interval is 100 m.

3.



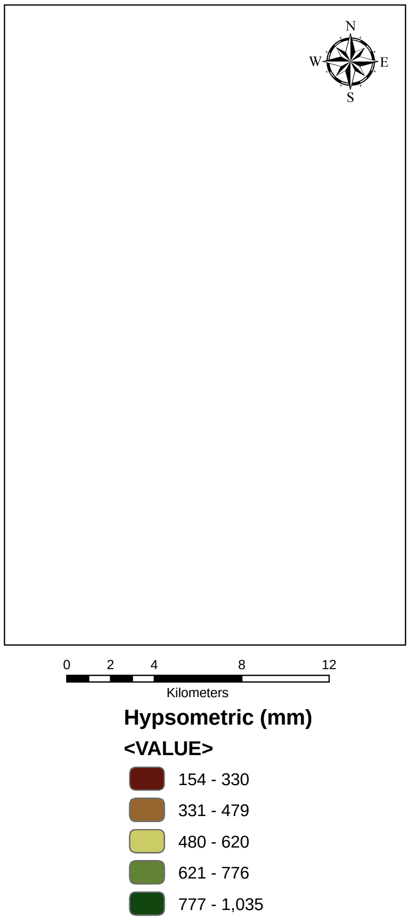
A layout showing the Thiessen polygons in Reynolds Creek watershed, ID. The precipitation station associated with the largest thiessen polygon is **P053**. The thiessen polygon basin average mean annual precipitation is **479 mm** (see spreadsheet for calculation – tab ThiessenCalcs).

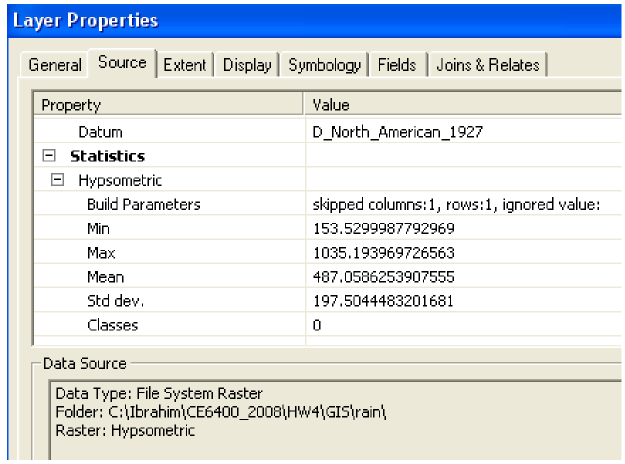


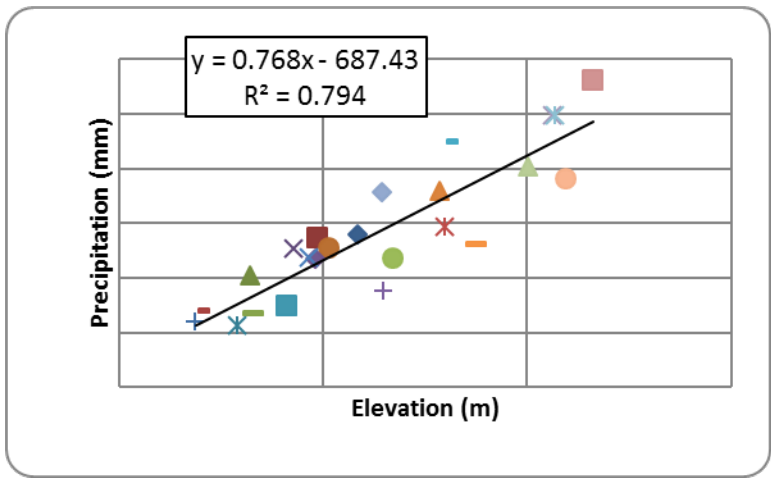


4) A layout showing the interpolated mean annual precipitation surface over Reynolds Creek using three methods (Spline, Kriging, and Inverse Distance method). The statistics for each method is shown on the right panel.

5)







A layout showing the mean annual precipitation surface over Reynolds Creek estimated from elevation. The mean basin average precipitation is **487 mm.** The statistics for the minimum, maximum and standard deviation is shown from the layer properties. The precipitation versus elevation relationship used is P = 0.768 E - 687 where, precipitation (P) is in **mm** and elevation (E) is in **m.** This relationship was obtained using the rain elev tab in the accompanying spreadsheet.