CEE6400 Physical Hydrology, Homework 2. Climate System and Global Hydrology

Solution

<u>Problem 1: Dingman 3.1</u>

The sensitivity to planetary temperature was evaluated using R

The following lines define the variables used

```
Sref=1.74*10^17 # Reference Solar Flux in W
apref=0.3  # Reference albedo
A=5.14*10^14 # Area of the system in m2
SBconst=5.78*10^-8 # Stefan-Boltzmann constant W/m2/k4
Tp=253.6 # Reference temperature in kelvin
```

Define sequences of planetary albedo ap and solar flux S

```
ap=seq(from = 0.2, to = 0.4, by = 0.02) # different ap values
S=seq(from = 1.24*10^17, to = 2.24*10^17, by = 1*10^16) # different S values
```

Evaluate planetary temperature using these sequences

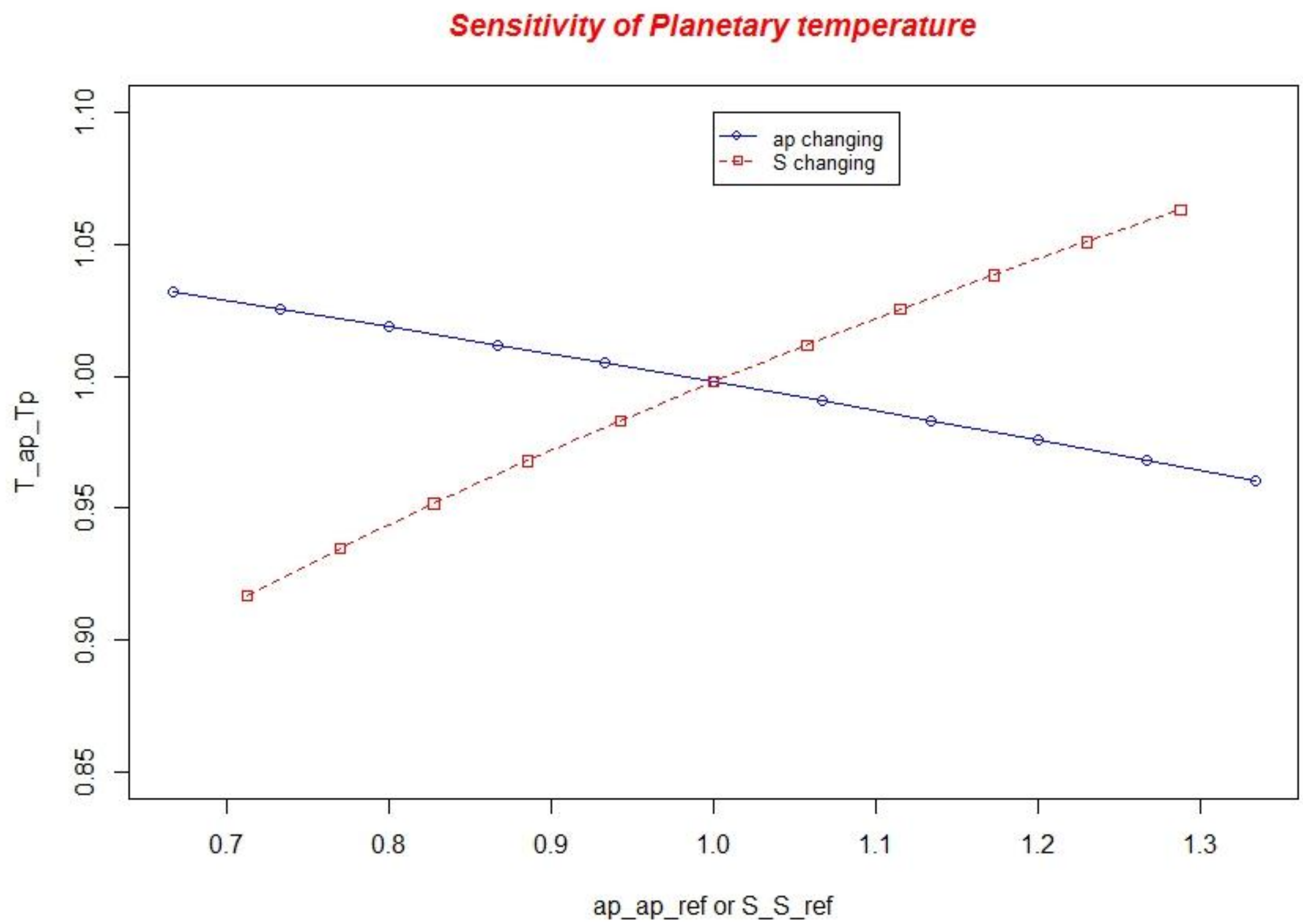
```
T_ap=(Sref*(1-ap)/(SBconst*A))^(1/4) #estimating T value for different ap values
T_S=(S*(1-apref)/(SBconst*A))^(1/4) #estimating T value for different S value
```

Express sensitivities in relative terms

```
ap_ap_ref=ap/apref  # relative change of ap with respect to  apref
T_ap_Tp=T_ap/Tp     #relative change of T_ap with respect to Tp

S_S_ref=S/Sref    # relative change of S with respect of Spref
T_S_Tp=T_S/Tp     #relative change of T_S with respect of Tp
```

Plot the results

```
plot(ap_ap_ref, T_ap_Tp, type="o", col="blue", ylim=c(0.85,1.1),xlab="ap_ap_ref or S_S_ref")
lines(S_S_ref, T_S_Tp, type="o", pch=22, lty=2, col="red")
legend(1, 1.1, c("ap changing ","S changing"), cex=0.8,
   col=c("blue","red"), pch=21:22, lty=1:2);
title(main="Sensitivity of Planetary temperature", col.main="red", font.main=4)
```



Comment:

The sensitivity if planetary temperature T_p to changes in Albedo is seen to be less (line less steep) than changes in T_p due to changes is S for the same percentage change in each.

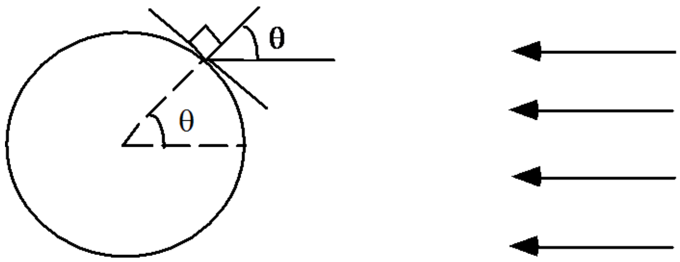
Problem2:

$$\frac{S}{A}(1-a_p) = \sigma T_P^4$$

$$\frac{S}{A} = \frac{I_{sc}\cos\theta}{\pi}$$

$$\frac{I_{sc}\cos\theta}{\pi}(1-a_p) = \sigma T_P^4$$

$$T_P = \left[\frac{I_{sc}\cos\theta\,(1-a_p)}{\pi\,\sigma}\right]^{\frac{1}{4}}$$



Given:

σ = 5.78x10^{-8} W m^{-2}K^{-4}

I_{sc} = 1367 W/m^2

a_p= 0.3

The radiational temperature at 20° N

θ = 20°

Evaluating

```
SBconst=5.78*10^-8 # Stefan-Boltzmann constant W/m²/k⁴
Isc=1367 # Solar flux W/m²
ap=0.3   # albedo
ang1=20*(pi/180) # angle converted into radian
Temp1=(Isc*cos(ang1)*(1-ap)/(pi*SBconst))^(1/4)-273  # temperature in degree Celsius
Temp1: -7.72
```

T_P = -7.7° C

The radiational temperature at 40° N

θ = 45°

```
ang2=40*(pi/180)
Temp2=(Isc*cos(ang2)*(1-ap)/(pi*SBconst))^(1/4)-273
Temp2: -20.93
```

T_P = -20.9° C

Problem3: Dingman 3.2

Derive Equation 3B2–4 for T_l and T_u in terms of T_s and parameters:

Step 1: $S + W = a_p S + \sigma {T_u}^4 A + (1 - f)\, \sigma {T_s}^4 A$ (3B2–1)

$$\Rightarrow {T_u}^4 = \frac{S+W-a_p S-(1-f)\,\sigma A {T_s}^4}{\sigma A}$$

Step 2: $k_u S + \sigma {T_l}^4 A + 0.5 Q_e = 2\sigma {T_u}^4 A$ (3B2–2)

$$\Rightarrow {T_l}^4 = \frac{2\sigma {T_u}^4 A - k_u S - 0.5 Q_e}{\sigma A}$$

Step 3: $${T_l}^4 = \frac{2\sigma A\left(\frac{S+W-a_p S-(1-f)\,\sigma A {T_s}^4}{\sigma A}\right) - k_u S - 0.5 Q_e}{\sigma A}$$

$$\Rightarrow {T_l}^4 = \frac{2S+2W-2a_p S-(1-f)2\sigma {T_s}^4 A - k_u S - 0.5 Q_e}{\sigma A}$$

Step 4: $K_l S + \sigma {T_u}^4 A + f\sigma {T_s}^4 A + 0.5 Q_e + Q_h + w = 2\sigma {T_l}^4 A$ (3B2–3)

$$\Rightarrow {T_s}^4 = \frac{2\sigma {T_l}^4 A - K_l S - \sigma {T_u}^4 A - 0.5 Q_e - Q_h - W}{f\sigma A}$$

Step 5: $${T_s}^4 = \frac{2\sigma A\left(\frac{2S+2W-2a_p S-(1-f)\,2\sigma A {T_s}^4 - k_u S - 0.5 Q_e}{\sigma A}\right) - K_l S - \sigma A\left(\frac{S+W-a_p S-(1-f)\,\sigma A {T_s}^4}{\sigma A}\right) - 0.5 Q_e - Q_h - W}{f\sigma A}$$

$$\Rightarrow T_s = \left[\frac{(3-3a_p-2k_u-K_l)S-1.5Q_e-Q_h+2W}{(3-2f)\sigma A}\right]^{\frac{1}{4}}$$

From Step 1:

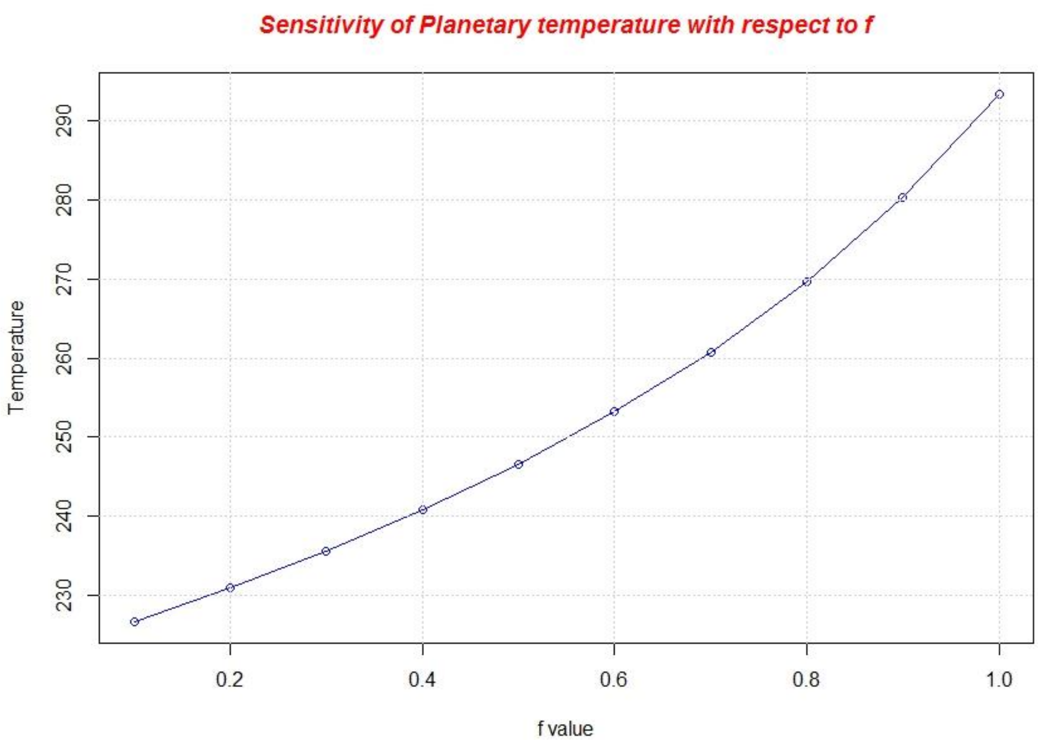
$$\Rightarrow T_u = \left[\frac{(1-a_p)S+W-(1-f)\sigma {T_s}^4 A}{\sigma A}\right]^{\frac{1}{4}}$$

From Step 3:

$$\Rightarrow T_l = \left[\frac{(2-2a_p-k_u)S+2W-0.5Q_e-2(1-f)\sigma {T_s}^4 A}{\sigma A}\right]^{\frac{1}{4}}$$

Problem 4: Dingman 3.3

```
A=5.14*10^14 # Area of the system in m2
SBconst=5.78*10^-8 # Stefan-Boltzmann constant W/m2/k4
S=1.74*10^17 # in W
Qe=4.08*10^16 # in w
Qh=8.67*10^15 # in w
W=1.07*10^13 # in w

ap=0.3
ku=0.18
kj=0.075
kl=0.075

f1=seq(from = 0.1, to = 1, by = 0.1) # different values of f
T=(((3-3*ap-2*ku-kl)*S -1.5*Qe-Qh+2*W)/((3-2*f1)*SBconst*A))^(1/4) # calculating T value

##plotting
plot(f1, T, type="o", col="blue", xlab="f value",ylab="Temperature")
grid()
title(main="Sensitivity of Planetary temperature with respect to f", col.main="red", font.main=4)
```



Comment:

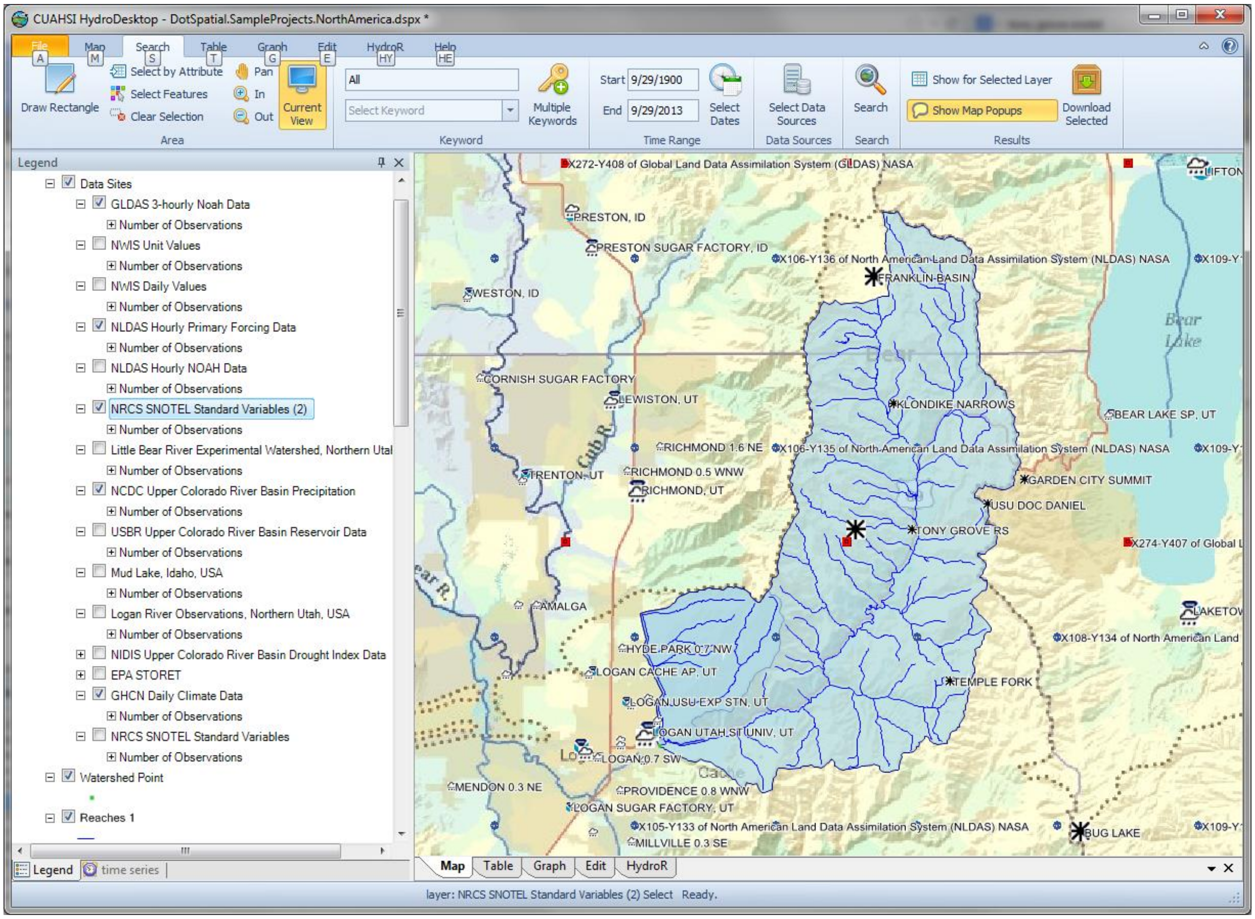
Box 3-2 gives $f = 0.95$ as a representative value. This gives $T_s = 288.3\ K = 15.14$ °C which is close to what is observed. f is bounded by one which places a bound on the additional greenhouse warming possible in this model. If f were to increase to 1 (i.e., no surface radiation losses directly to space) T_s would increase to $295.2\ K = 22.04$ °C , a 7 °C warming.

Problem 5: Dingman 3.4

a) Determine the long term average precipitation, runoff and evaporation for a location of interest.

This problem was done for the Logan River. Precipitation data is the hardest to find.

The CUAHSI data discovery tool, HydroDesktop - see http://hydrodesktop.codeplex.com finds the following data in and around the Logan River Watershed. There are online tutorials on using this tool



Precipitation sources with these are

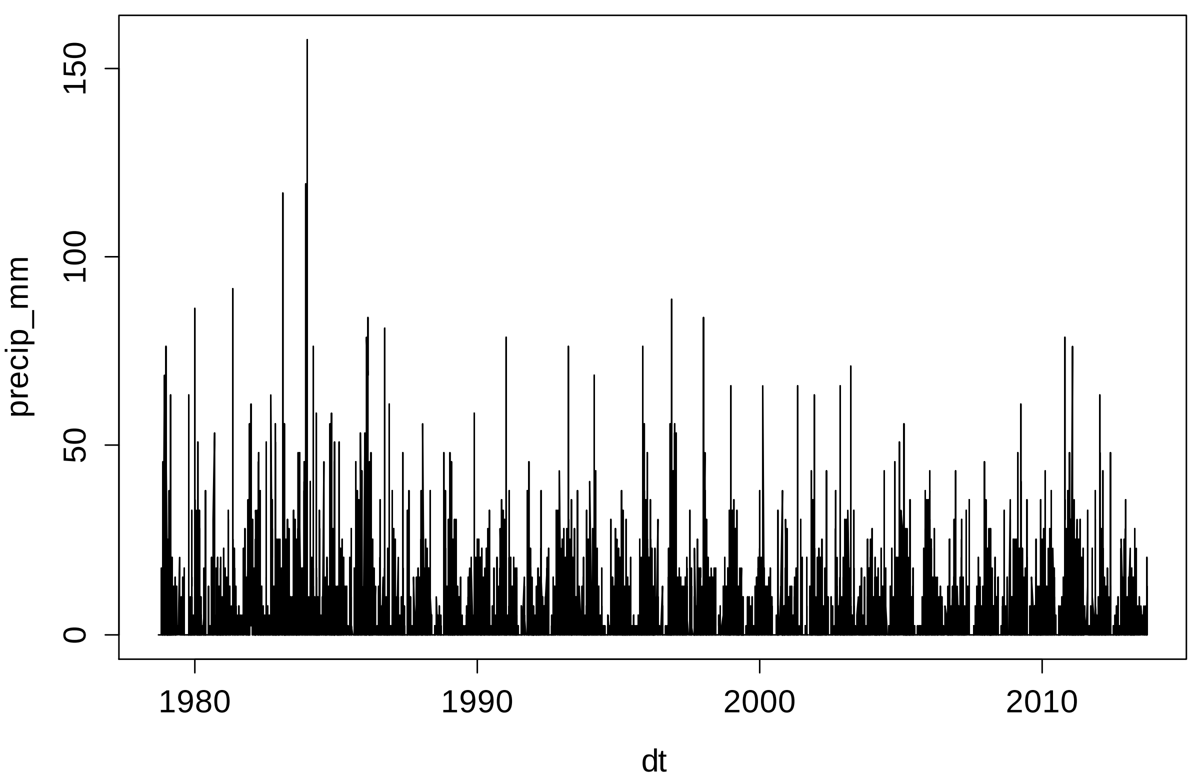
- SNOTEL - Tony Grove, Franklin Basin and Bug Lake have long term records
- NLDAS and GLDAS gridded data. This is data created by a reanalysis model, not real measurements.
- GHCN. These are weather stations mostly in the valleys. Logan Utah State University, Logan Radio KVNU, etc.

I used Tony Grove SNOTEL data, because the GHCN data suffers from a lot of missing values and NLDAS data is not direct observations.

I exported the file SNOTELTonyGrove.csv from HydroDesktop then used the following R code to determine mean annual precipitation

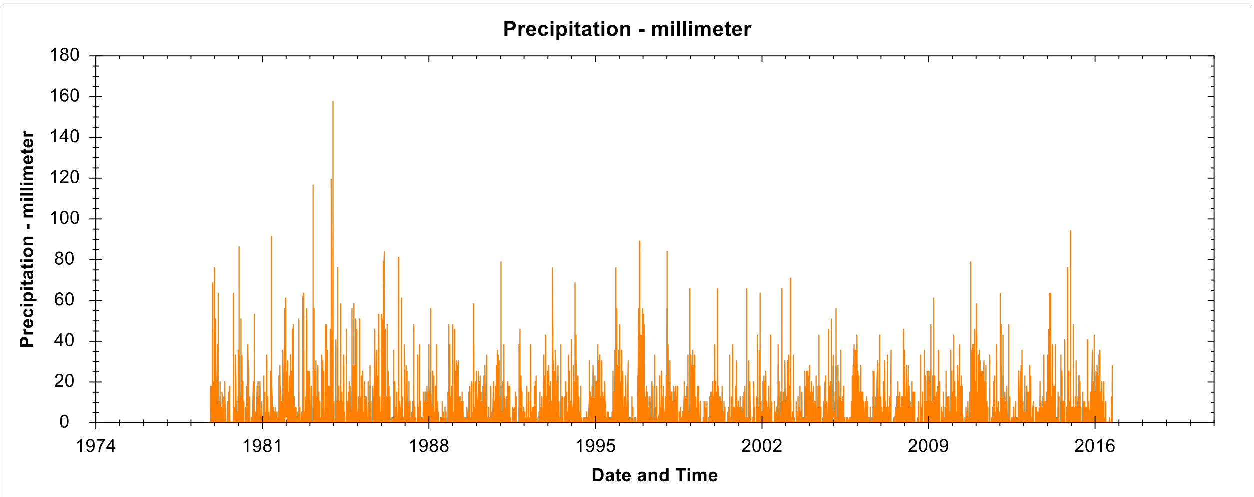
```
TonySnow =
read.csv("SNOTELTonyGrove.csv",colClasses=c("numeric","character","character","characte
r","numeric","character","character"))
dt=as.Date(TonySnow$LocalDateTime,format="%m/%d/%Y")
TonySnow$DataValue=ifelse(TonySnow$DataValue<0,NA,TonySnow$DataValue)  # This
replaces negatives (often -9999 indicating missing) by NA for no data
sum(is.na(precip_mm))  # This counts the no data values
# 34 missing days in 30 years is not bad, so I am ignoring these

precip_mm=TonySnow$DataValue*25.4
plot(dt,precip_mm,type="l")
annprecip=mean(precip_mm,na.rm=T)*365.25  # This is an average of daily values so must
be multiplied by 365.25 to get annual average.
```

1237 mm

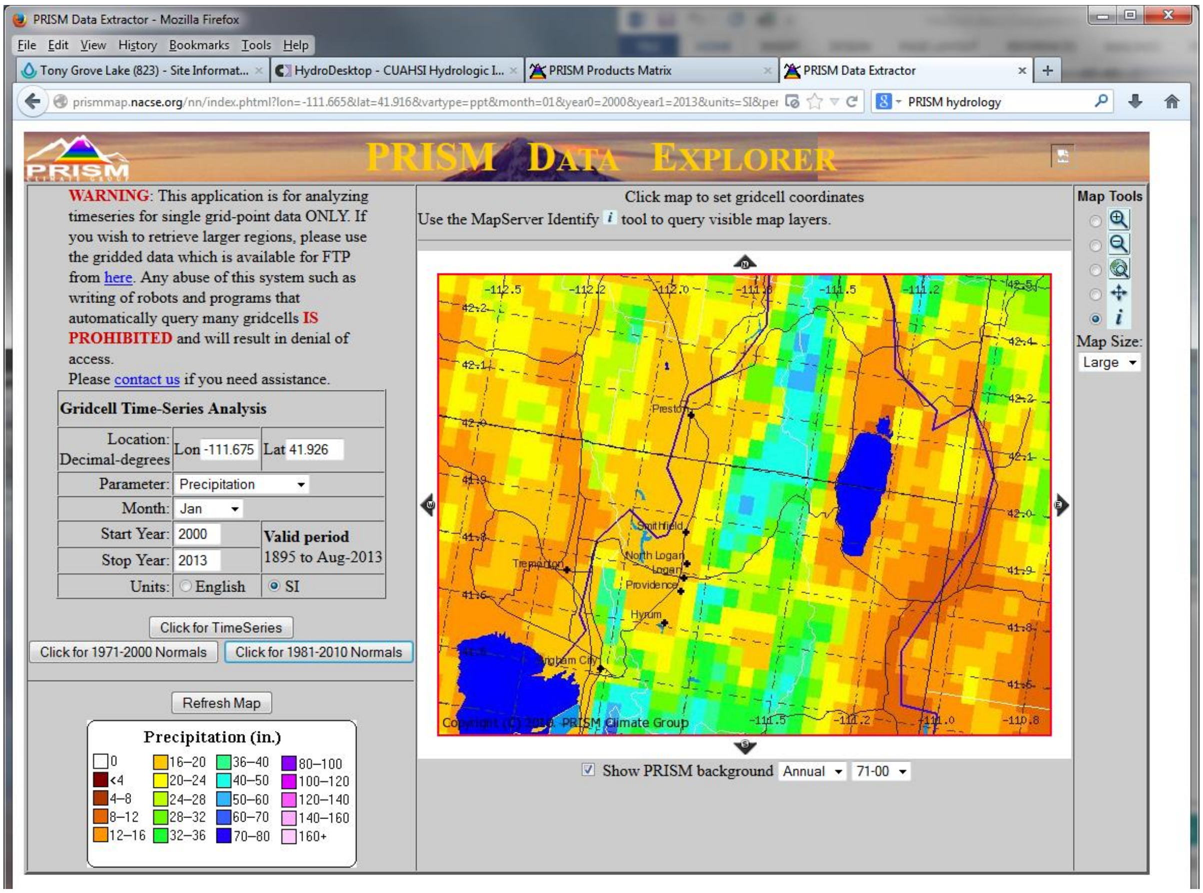


Tony Grove Daily Precipitation

Alternatively, you can use HydroDesktop Graph tools and use HydroR to do the R computations



There is also PRISM data available from http://www.prism.oregonstate.edu/. I was able to use the Explorer on this web page to drill down into the area around the Logan River Watershed



PRISM reports the following 1981-2010 Climate Normals, clicking on high and low grid cells in this area.

Cell Query Results - Mozilla Firefox

prismmap.nacse.org/nn/query_normals.phtml?lon=-111.675&lat=41.926&vartype=ppt&month=01&year0=2000&year1=2013&units=SI&

Climate Normals

POR:	1981-2010
Grid Resolution:	30-arcsec (~800m)
Units:	SI (degrees C / mm)
Selected gridcell	
Longitude:	-111.675
Latitude:	41.926

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Ann
tmax	-2.32	-1.24	2.33	6.24	10.74	15.52	21.01	20.40	15.01	8.22	1.02	-2.89	7.84
tmin	-10.96	-10.86	-8.38	-5.59	-0.91	3.69	8.65	7.33	3.42	-1.65	-7.67	-10.96	-2.82
ppt	187.63	147.08	154.79	118.30	108.18	52.12	29.18	31.79	57.21	94.18	162.28	175.02	1317.76

Cell Query Results - Mozilla Firefox

prismmap.nacse.org/nn/query_normals.phtml?lon=-111.602&lat=41.796&vartype=ppt&month=01&year0=2000&year1=2013&units=SI&

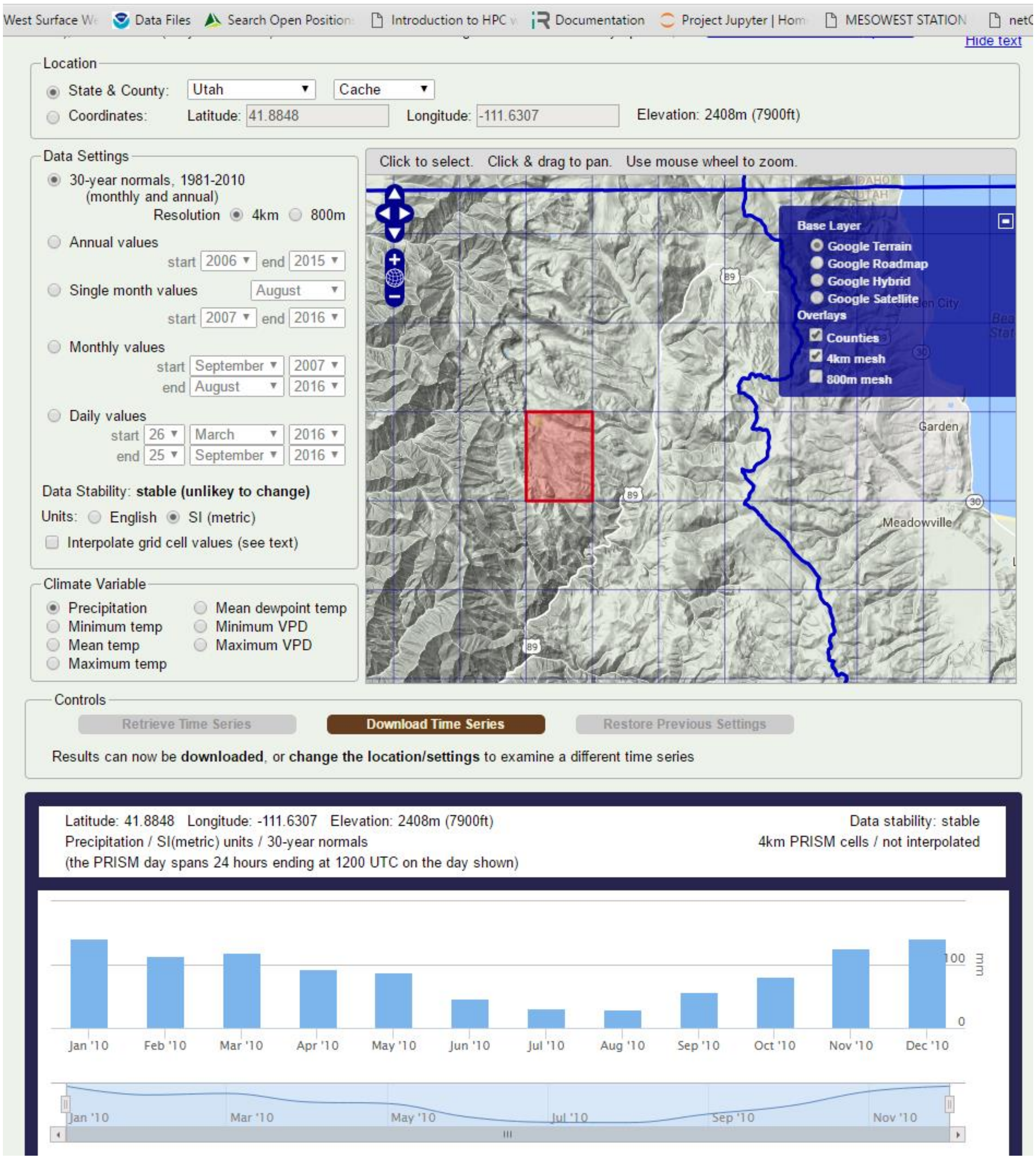
Climate Normals

POR:	1981-2010
Grid Resolution:	30-arcsec (~800m)
Units:	SI (degrees C / mm)
Selected gridcell	
Longitude:	-111.602
Latitude:	41.796

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Ann
tmax	-0.01	2.25	7.28	11.61	16.74	22.00	27.53	26.87	21.07	13.70	5.04	-0.09	12.83
tmin	-9.79	-8.91	-5.16	-1.70	2.42	6.21	11.24	10.26	5.48	0.58	-5.08	-9.03	-0.29
ppt	71.93	64.87	66.97	65.66	62.18	40.14	27.87	26.46	47.37	63.70	69.50	75.50	682.15

The high of **1317 mm** and low of **682 mm** represent mountain and valley values. The high of 1317 mm is comparable to Tony Grove SNOTEL.

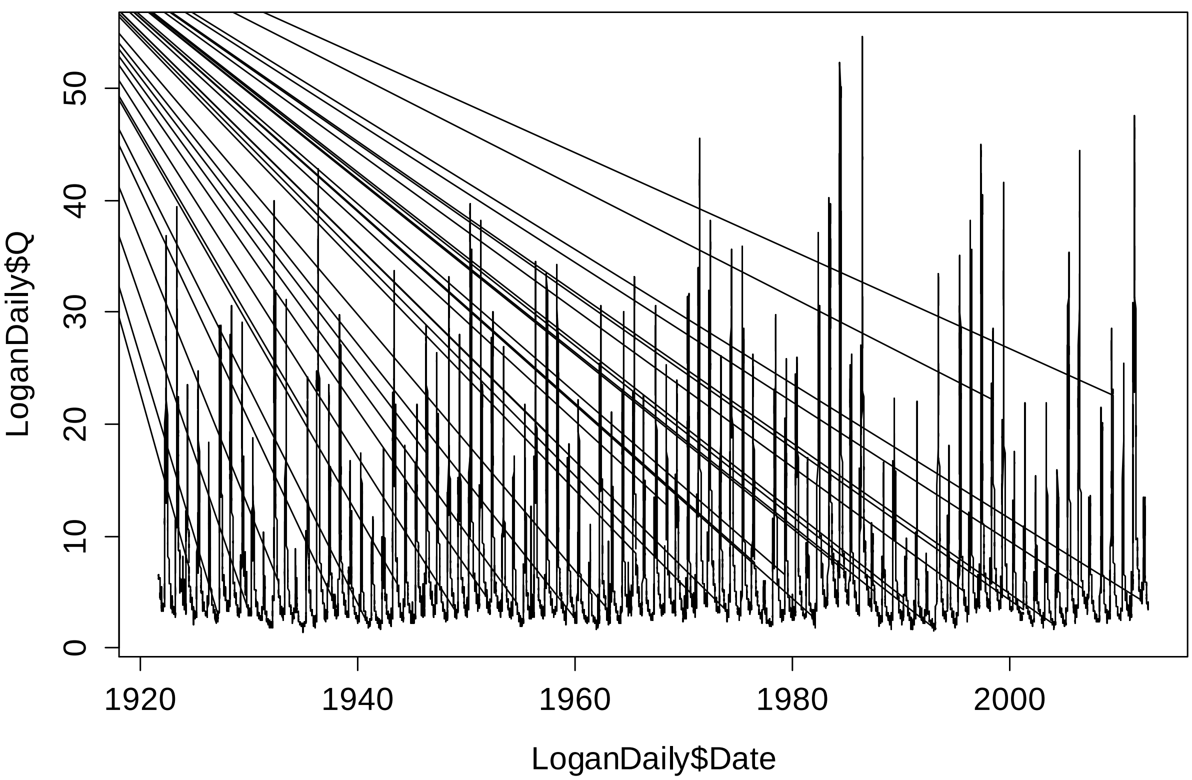
Select the grid cell containing the Tony Grove Lake SNOTEL station and plot the 30 year normals



The following R code determined the long term runoff

```
library(dataRetrieval)
siteNumber="10109001"  # Logan River
LoganDaily = getDVData(siteNumber,"00060","1900-01-01","2013-09-30") # Retrieve data for Logan river
plot(LoganDaily$Date,LoganDaily$Q,type="l")
# Note the above retrieved data from1921-10-01 to 2012-09-30, the last day data is currently available
# values are daily in m^3/s
A=214*1609.34*1609.34 # Area in square meters
streamflow=(LoganDaily$Q)*3600*24*1000/A # stream flow in mm/day
mean(streamflow)*365.25
```

397 mm



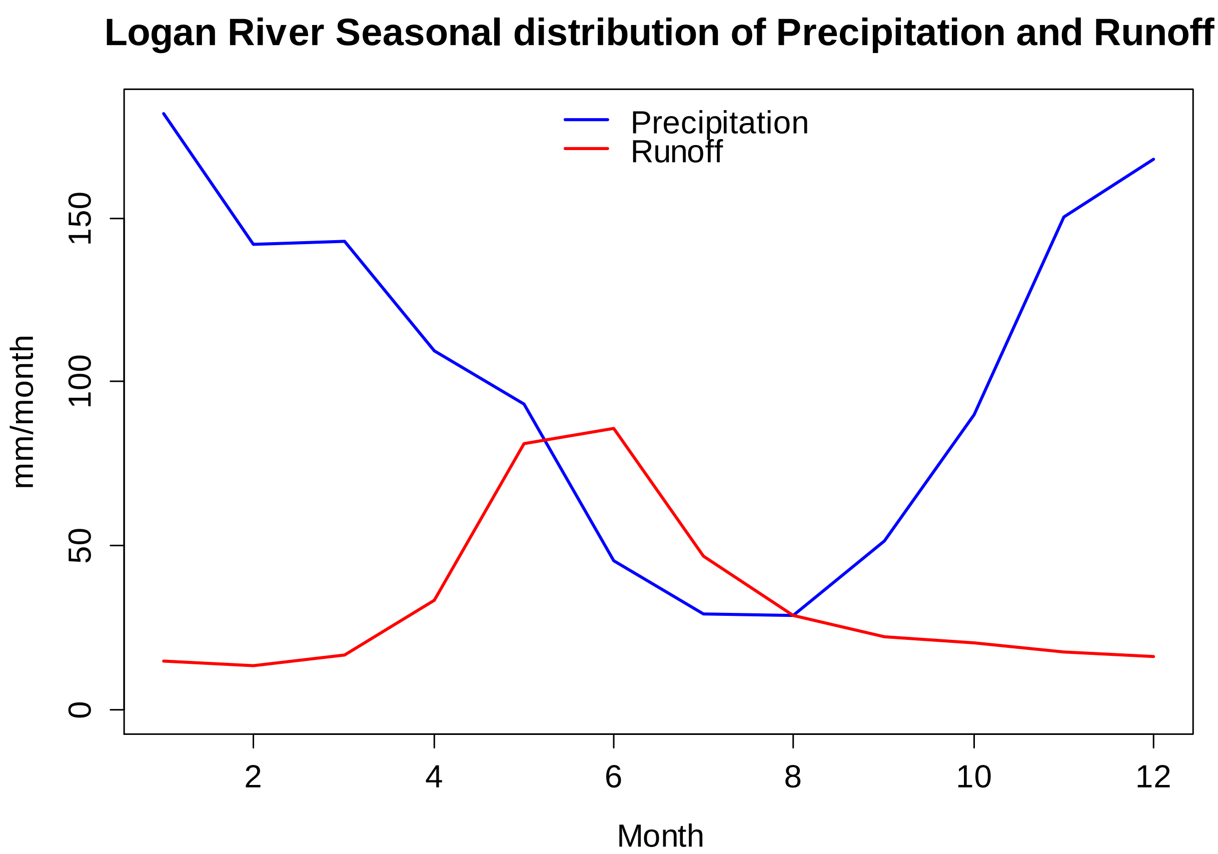
Long term annual rainfall 682 - 1317 mm. Lets take this as **1000 mm**
Long term runoff **397 mm**
Long term evaporation E=P-Q = 1000-397 = **603 mm**

b) Determine the seasonal distribution of precipitation and runoff

Here R was used on the SNOTEL precipitation and Logan streamflow

```
# For streamflow
yy=as.numeric(format.Date(LoganDaily$Date,"%Y"))
yrmo=yy*100+LoganDaily$Month
yrmoseq=unique(yrmo)
Qmonth=rep(NA,length(yrmoseq))
for(i in 1:length(yrmoseq)){
  ii=yrmoseq[i]
  Qmonth[i]=sum(streamflow[yrmo==ii])  # This sums the daily streamflows for a month
}

# Now average the months for all years
year=trunc(yrmoseq/100)
month=yrmoseq-year*100
Qmm=rep(NA,12)
for(mm in 1:12){
  Qmm[mm]=mean(Qmonth[month==mm])
}

# For Precipitation
yy=as.numeric(format.Date(dt,"%Y"))
yrmo=yy*100+as.numeric(format.Date(dt,"%m"))
yrmoseq=unique(yrmo)
Pmonth=rep(NA,length(yrmoseq))
for(i in 1:length(yrmoseq)){
  ii=yrmoseq[i]
  Pmonth[i]=sum(precip_mm[yrmo==ii],na.rm=T)  # This sums the daily precipitation for a 
month
}

# Now average the months for all years
year=trunc(yrmoseq/100)
month=yrmoseq-year*100
Pmm=rep(NA,12)
for(mm in 1:12){
  Pmm[mm]=mean(Pmonth[month==mm])
}

plot(1:12,Pmm,type="l",col="blue",xlab="Month",ylab="mm/month",ylim=c(0,max(Pmm)),l
wd=2)
lines(1:12,Qmm,type="l",col="red",lwd=2)
```

```
legend("top", c("Precipitation","Runoff"), col=c("blue","red"),bty="n",lwd=2,lty=1)
title(main="Logan River Seasonal distribution of Precipitation and Runoff")
```



This pattern is typical of winter snowfall and spring runoff. High precipitation values in the winter are stored and appear as runoff later in the spring.

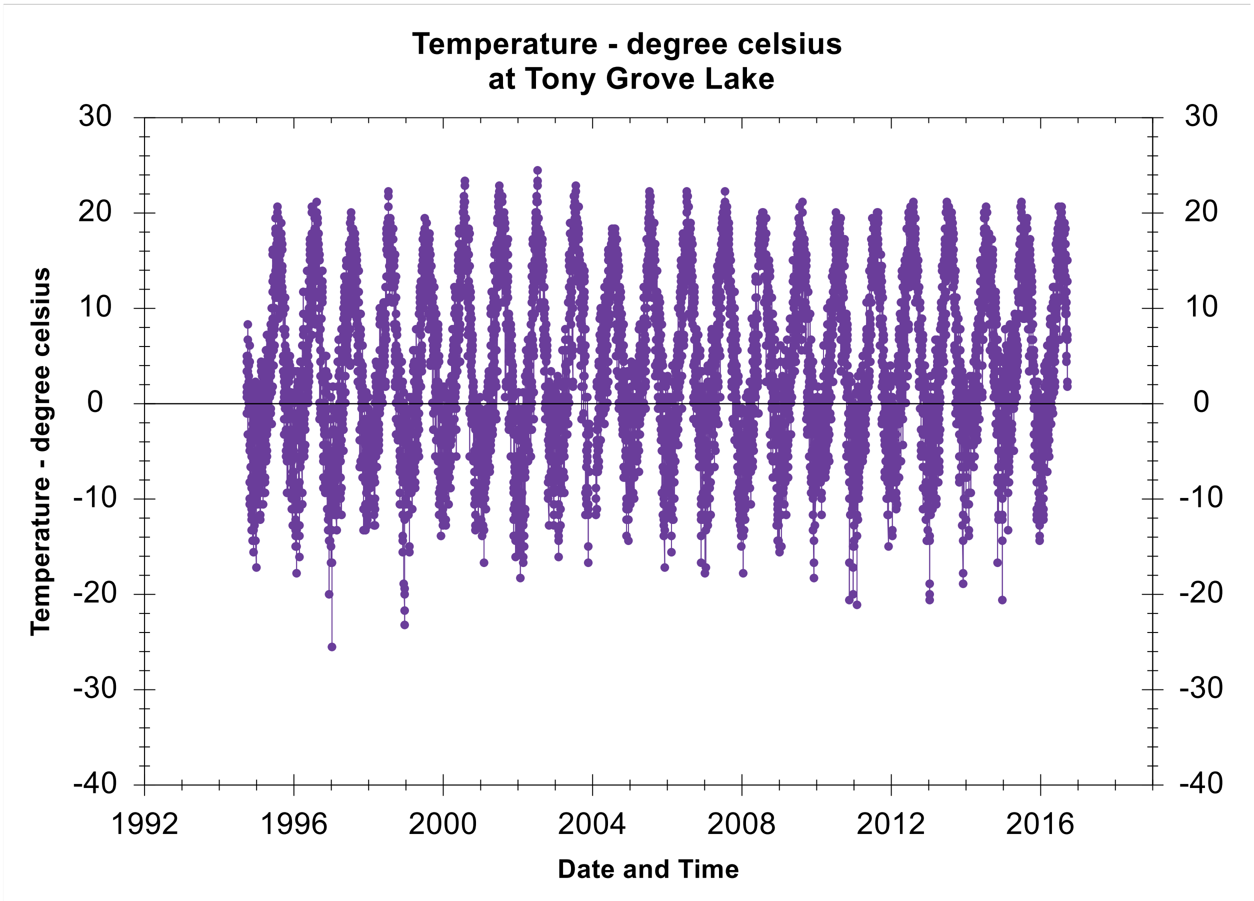
c) Runoff ratio

w=Runoff_avg/Precip_avg =397/100 = **0.397**

It is hard to see exactly where Logan is in the Global Figures in Dingman, but these values are somewhat close to those figures.

Problem 6:

a) For a watershed of interest estimate the mean annual air temperature and regional PET using equation 2B4.2.



Long term mean = 3.9 oC

PET = 1.2 * 10^10 * exp(-4620/T) , T = 3.9 + 273 = 276.9

PET = 680 mm/y

b) Then determine a best-fit value of the storage parameter w in the Budyko equation [equation 2.12] by programming the equation in Excel or R, and adjusting the value of w until the calculated value of RO most closely approximates the value for your watershed.

Using:

P = 1000 mm/ y

PET = 680 mm /y

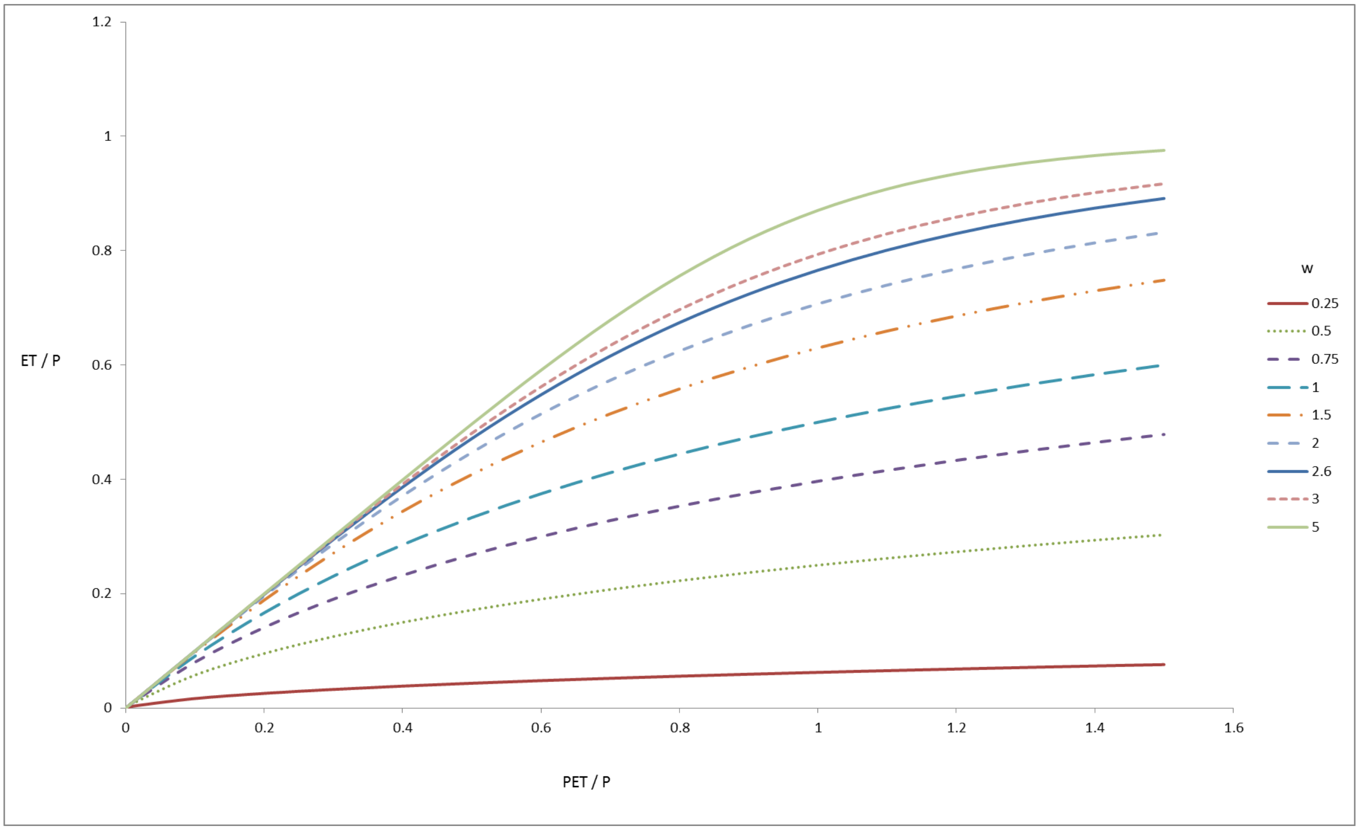
RO = 397 mm /y

$$RO = P * [1 - \frac{PET}{(P^w + PET^w)^{\frac{1}{w}}}]$$

Using Excel solver, w = 2.6

Re-writing eq. 2.10, $\frac{ET}{P} = \frac{PET/P}{(1+ (PET/P)^w)^{\frac{1}{w}}}$

Then plot the curve for different values of w



For Logan PET/P = 0.68,

From the graph ET/P (@PET/P = 0.68) = 0.6 ➔ ET = 0.6*P = 600 mm

c) Estimate the elasticity of runoff to precipitation via equation 2B4.1. In addition to giving the numeric value, write a few sentences that explain in layman's terms what this means.

From equation 2B4.1, for Logan Watershed,

e(RO, P) = 2.5

This means that for a 1% unit change in precipitation, the change in runoff is 2.5%, indicating an amplification in change as precipitation becomes runoff.

d) Estimate the relative change in runoff due to a temperature increase of 1 ^{o}C via equation 2B4.7

From equation 2B4.7

$dRO/RO = -0.0009\, dT = -0.001dT$

For $dT = 1\ ^{o}C$, Runoff change => -0.001

This means that for a 1 ^{o}C change in temperature, the change in runoff is0.1%. This is quite small. Interpreting (c) and (d) together, this analysis suggests that changes in streamflow are more driven by changes in precipitation than changes in temperature.