STAEDTLER® No. 937 811E Engineer's Computation Pad

CEE 6400 FINAL SOLUTION

1. a) Sufficient info for A. Priestley Taylor and C. combination. B and D require surface temperature

b) $E = \alpha \dfrac{\Delta}{\Delta + \gamma} E_r$ — PRIESTLEY TAYLOR

$\alpha = 1.26$

$e_s(T) = 0.611 \exp\left(\dfrac{17.3\, T}{T + 237.3}\right)$ 7-4

with $T = 22^\circ C$

$e_s = 2.6515 \text{ kPa}$

$\Delta = \dfrac{4098\, e_s}{(237.3 + T)^2} = 0.1616 \text{ kPa } {}^\circ C^{-1}$

$\gamma = \dfrac{C_a\, P}{0.622\, \lambda_v}$

$\lambda_v = 2.501 \times 10^6 - 2370\, T$

$= 2.4489 \times 10^6 \text{ J kg}^{-1}$

$C_a = 1005 \text{ J kg}^{-1}\, {}^\circ C^{-1}$

$\therefore \gamma = \dfrac{1005 \times 85}{0.622 \times 2.45 \times 10^6}$ $\dfrac{\text{J kg}^{-1}\, {}^\circ C^{-1} \text{ kPa}}{\text{J kg}^{-1}}$

$= 0.05608 \text{ kPa } {}^\circ C^{-1}$

$E_r = \dfrac{R_n}{\rho_w \lambda_v} = \dfrac{90}{1000 \times 2.45 \times 10^6}$ $\dfrac{\text{J s}^{-1} \text{ m}^{-2}}{\text{kg m}^{-3} \text{ J kg}^{-1}}$

$= 3.675 \times 10^{-8} \text{ m s}^{-1}$

$\times 3600 \times 24 \times 1000 = 3.18 \text{ mm/day}$

$$\therefore \; E_{\text{Priestly Taylor}} = 3.18 \times 1.26 \times \underbrace{\frac{0.1616}{0.1616 + 0.05608}}_{0.7424}$$

$$= \underline{2.97 \text{ mm/day}}$$

COMBINATION / PENMAN

$$E = \frac{\Delta}{\Delta + \gamma} E_r + \frac{\gamma}{\Delta + \gamma} E_a$$

$$E_a = K_E \, u \left(e_s(T_a) - e_a \right)$$

$$K_E = \frac{0.622 \, k^2 \rho_a}{p \, \ln(z/z_0)^2 \rho_w}$$

$$\rho_a = \frac{p}{R_a T} = \frac{85 \times 10^3}{287.04 \times (273 + 22)} \qquad \frac{N\,m^{-2}}{J\,kg^{-1}\,K^{-1}\,K} \quad Nm$$

$$= 1.003 \text{ kg m}^{-3}$$

$$\therefore \; K_E = \frac{0.622 \times 0.4^2 \times 1.003}{85 \; \ln\left(\frac{2}{0.0003}\right)^2 \cdot 1000} \qquad \frac{\cancel{kg\,m^{-3}}}{kPa \; \cancel{kg\,m^{-3}}}$$

$$= 1.516 \times 10^{-8} \text{ kPa}^{-1}$$

$$q = \frac{0.622\, e}{p}$$

$$\therefore \; e = q\, p / 0.622 = 0.009 \times \frac{85}{0.622}$$

$$= 1.23 \text{ kPa}$$

$$\therefore \; E_a = \underset{kPa^{-1}}{1.51 \times 10^{-8}} \times \underset{m\,s^{-1}}{2.5} \times \underset{kPa}{(2.65 - 1.23)}$$

$$= 5.39 \times 10^{-8} \text{ m s}^{-1}$$

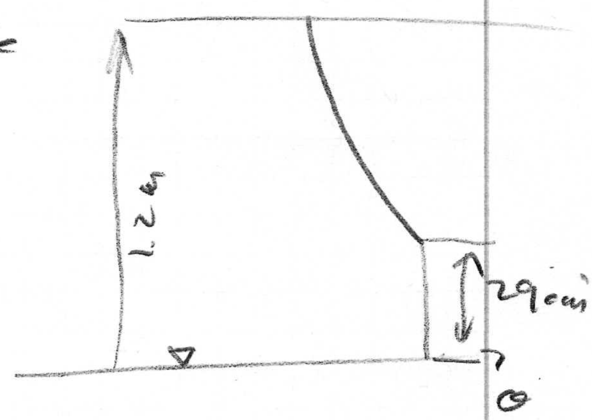
$$\therefore \quad E_a = 5.39 \times 10^{-8} \times 3600 \times 24 \times 1000$$

$$= 4.66 \quad mm/day$$

$$\therefore \quad E_{penman} = 0.7424 \times 3.18 + (1 - 0.7424) \times 4.66$$

$$= 3.56 \quad mm/day$$

2. a) 29 cm above water table



b) $\psi = \psi_a \left(\frac{\theta}{n}\right)^{-b}$

$\therefore \frac{\theta}{n} = \left(\frac{\psi}{\psi_a}\right)^{-1/b}$

$\therefore \theta = n \left(\frac{\psi}{\psi_a}\right)^{-1/b}$ $\quad n = 0.4$

$b = 7$

$\psi = -z$ where z = ht above W.T.

$\psi_a = -29$

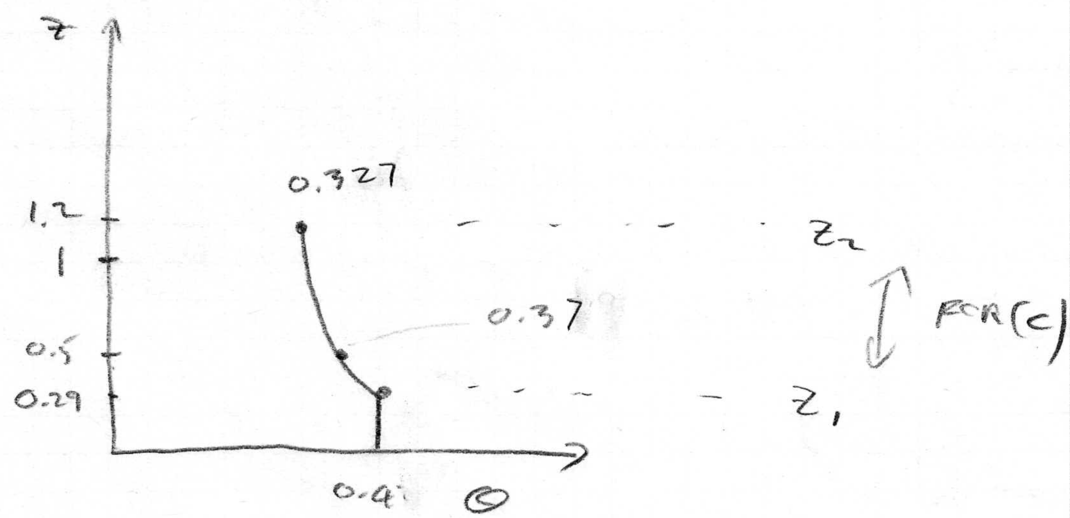
$\therefore \theta(29) = n = 0.4$

$\theta(z=50) = 0.4 \times \left(\frac{50}{29}\right)^{-1/7}$

$= 0.37$

$\theta(z=120) = 0.4 \times \left(\frac{120}{29}\right)^{-1/7}$

$= 0.327$ at surface



c) $D = \int_{z_1}^{z_2} (n - \theta)\, dz$

$$= \int_{z_1}^{z_2} \left(n - n\left(\frac{z}{|\psi_a|}\right)^{-1/b}\right) dz$$

$$= n\left(z_2 - z_1 - \left(\frac{1}{|\psi_a|}\right)^{-1/b} \left[\frac{z^{1-1/b}}{1-1/b}\right]_{z_1}^{z_2}\right)$$

$$= n\left(z_2 - z_1 - \left(\frac{1}{|\psi_a|}\right)^{-1/b} \left[\frac{z_2^{1-1/b} - z_1^{1-1/b}}{1-1/b}\right]\right)$$

$n = 0.4$

$z_1 = 29$ cm

$z_2 = 120$ cm

$|\psi_a| = 29$ cm

$b = 7$ $\quad 1 - \frac{1}{b} = 0.857$

$\therefore D = 4.22$ cm

d) Infiltration excess runoff occurs when the infiltration rate is limiting. Mathematically $w > f_c$ so runoff $r = w - f_c$. Saturation excess runoff occurs when the soil profile saturates. This would occur here if more than 4.22 cm of water infiltrated. Upon saturation all rainfall becomes runoff

e) 0.25 m is lower than the top of the capillary fringe of 0.29 m or 29 cm.

So $D = 0$.

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3 a) $\bar{\lambda}$ = average $\ln \frac{a}{S}$ = $\frac{1}{A}\int_A \ln \frac{a}{S}\, dA$

From the average of the figure

$\bar{\lambda} = 8$

b) $K = K_0 e^{-fz}$

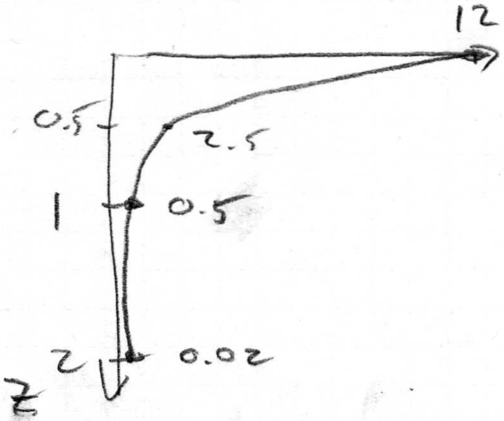
$f = 3.125$

So at $z = 0.5$

$K = 12\, e^{-3.125 \times 0.5} = 2.5$ m/hr

at $z = 1$ $K = 0.527$ m/hr

at $z = 2$ $K = 0.0237$ m/hr



c) average soil moisture deficit

$\bar{D} = z_w\, \theta_e = 1.3 \times 0.25$

$= 0.325$ m

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d) Eqn 88

$$r = T_0 \, e^{-\bar{D}/m} \, e^{-\bar{\lambda}}$$

$$= 3.84 \, e^{-0.325/0.08} \, e^{-8}$$

$$= 2.216 \times 10^{-5} \text{ m/hr}$$

$$Q_b = r A \times \frac{1}{3600 \text{ s/hr}}$$

$$= 2.216 \times 10^{-5} \times \frac{400 \times 10^{6}}{3600}$$

$$= \underline{2.46 \text{ m}^3/\text{s}}$$

e) Eqn 87

$$D = \bar{D} - m\left(\ln \frac{a}{s} - \bar{\lambda}\right)$$

This implies saturation when $D = 0$

or $\ln \frac{a}{s} - \bar{\lambda} > \bar{D}/m$

or $\ln \frac{a}{s} > \bar{\lambda} + \bar{D}/m$

Saturation threshold $= 8 + \dfrac{0.325}{0.08}$

$$= \underline{12.06}$$

f) From the histogram

area fraction $= \dfrac{13 - 12.06}{2} \times 0.1$

$$= \underline{0.0469}$$

g) At end of storm 5 cm has infiltrated in non saturated areas so

$\bar{D} = 0.325 - 0.05 = 0.275$ m

with this $\bar{D}$ saturation threshold is

$\ln\frac{a}{S} = 8 + \frac{0.275}{0.08} = 11.44$

Saturated area fraction $\frac{13 - 11.44}{0.2} \times 0.1 = 0.078$

$= 7.8\%$

h) Runoff from saturated area

$= 400 \times 10^6 \ m^2 \times 0.0469 \times 0.05$

$= 938000 \ m^3$

Runoff from area that saturates, using half of 0.05 m runoff due to linear decline

$= 400 \times 10^6 \times (0.078 - 0.0469) \times 0.025$

$= 312000 \ m^3$

Total Runoff $= 1250 \times 10^3 \ m^3$