CALCULUS I, Review Problems for test 3

1. The management of a department store has decided to enclose an 800 sq. ft. area outside the building for displaying potted plants and flowers. One side will be formed by the exterior wall of the store, two sides will be constructed of pine boards, and the fourth side that is opposite to the wall will be made of galvanized steel fencing. If the pine board fencing costs \$6/ft. and the steel fencing costs \$3/ft, determine the dimensions of the enclosure that can be erected at minimum cost? [**Ans:** $10\sqrt{2}$ by $40\sqrt{2}$]

2. Suppose we use Newton's Method to approximate the solution of the equation $x^3 = x + 1$.

(a) Find the iterative formula for x_{n+1}. [**Ans:** $x_{n+1} = x_n - \dfrac{x_n^3 - x_n - 1}{3x_n^2 - 1}$]

(b) Calculate three iterations of Newton's Method. Use $x_0 = 1$ as the initial guess. [**Ans:** $x_1 = 1.5$, $x_2 = 1.3478$, $x_3 = 1.3252$]

3. Evaluate each of the following integrals.

$$a)\ \int (3x^2 - 2x + 5)\, dx \quad b)\ \int \frac{3}{t^2}\, dt \quad c)\ \int \frac{y^3 + 2y + 3}{y}\, dy \quad d)\ \int \frac{3 + 4x^{3/2}}{\sqrt{x}}\, dx$$

[**Ans:** a) $x^3 - x^2 + 5x + C$ b) $\dfrac{-3}{t} + C$ c) $\dfrac{y^3}{3} + 2y + 3\ln|y| + C$ d) $6\sqrt{x} + 2x^2 + C$]

4. Evaluate each of the following integrals.

$$\text{a) } \int x(x^2-1)^4\, dx \qquad \text{b) } \int \frac{1}{\sqrt{2y+1}}\, dy \qquad \text{c) } \int \cos(3\theta)\, d\theta$$

$$\text{d) } \int \sin^3 x \cos x\, dx \qquad \text{e) } \int 8t\, e^{t^2}\, dt \qquad \text{f) } \int y^2 \sin(y^3)\, dy$$

[**Answer:** a) $\dfrac{1}{10}(x^2-1)^5 + C$] b) $\sqrt{2y+1} + C$] c) $\dfrac{1}{3}\sin(3\theta) + C$ d) $\dfrac{\sin^4 x}{4} + C$ e) $4e^{t^2} + C$ f) $\dfrac{-1}{3}\cos(y^3) + C$]

5. Find the particular solution of the equation $f'(x) = 2x^{-1/2}$ that satisfies the condition $f(1) = 6$. [**Ans:** $f(x) = 4\sqrt{x} + 2$]

6. A particle moves along the x-axis with acceleration function $a(t) = 5+4t-2t^2$. Its initial velocity is $v(0) = 3$ m /s. [4 pts]

(a) Find its velocity, $v(t)$, after t seconds. [**Answer:** $v(t) = 5t + 2t^2 - \frac{2}{3}t^3 + 3$]

(b) Assuming that the particle is located at the origin at $t = 1$, find the particle's position $s(t)$. [**Answer:** $s(t) = \frac{5}{2}t^2 + \frac{2}{3}t^3 - \frac{1}{6}t^4 + 3t - 6$]

7. Find the area of the region under the curve $y = \dfrac{1}{(3x+1)^2}$ over the interval [0,1]. [**Answer:** 1/4]

8. If $\int_0^5 f(x)\,dx = 7$ and $\int_2^5 f(x)\,dx = -1$, find $\int_0^2 f(x)\,dx$. [**Ans:** 8]

9. Approximate $\displaystyle\int_0^2 \frac{10}{x^2+1}\,dx$ by dividing the interval [0, 2] into N=4 equal subintervals and using

(a) the left endpoint approximation. [Ans: 13.038]

(b) the right endpoint approximation. [Ans: 9.038]

(c) the midpoint approximation. [Ans: 11.0878]

10. Evaluate each of the following integrals.

$$a)\ \int_1^4 \sqrt{x}\,dx \quad b)\ \int_{-1}^1 (y^2-2)\,dy \quad c)\ \int_{-1}^0 (2x - e^x)\,dx \quad d)\ \int_1^8 (1 - \frac{1}{t^{2/3}})\,dt$$

[**Ans:** a) $\dfrac{14}{3}$ $b)\ -10/3$ $c)\ \dfrac{1}{e} - 2$ $d)\ 4$]

11. (a) Find $\dfrac{d}{dx}\displaystyle\int_3^x (2t^2+5)^2\,dt$. [**Ans:** $(2x^2+5)^2$]

(b) Find $\dfrac{d}{dx}\displaystyle\int_{x^3}^4 \sqrt{y^3+1}\,dy$. [**Ans:** $-3x^2\sqrt{x^9+1}$]

12. Find $\int_0^{10} (8 - x)\, dx$ by using (a) geometry and (b) The Fundamental Theorem of Calculus.

[**Ans:** 30]

13. Find $\int_0^4 |x - 3|\, dx$ [**Ans:** 5]

14. Evaluate each of the following definite integrals.

a) $\displaystyle\int_0^1 (2x+1)^4\, dx$ b) $\displaystyle\int_{-2}^{-1} \frac{y}{(y^2+2)^3}\, dy$

c) $\displaystyle\int_0^1 x\sqrt{1-x^2}\, dx$ d) $\displaystyle\int_0^1 \frac{2}{5t+3}\, dt$

[**Ans:** a) 121/5 b) -1/48 c) 1/3 d) $\frac{2}{5}$ ln(8/3)]

15. Sketch the region and find the area of:

a) the region bounded by $y = x^2 - 6x$, and the x-axis. [**Answer:** 36]

b) the region bounded by $y = x + 6$, $y = x^2$, $x = 0$ and $x = 2$. [**Answer:** 34/3]

c) the region bounded by $y = x + 6$ and $y = x^2$. [**Answer:** 125/6]