Stochastic Simulation

Bo Friis Nielsen

Institute of Mathematical Modelling

Technical University of Denmark

2800 Kgs. Lyngby – Denmark

Email: bfni@dtu.dk

The Bootstrap method

- A technique for estimating the variance (etc) of an estimator.
- Based on sampling from the empirical distribution.
- Non-parametric technique

Recall the simple situation

We have n observations x_i, $i = 1, \ldots, n$.

If we want to estimate the mean value of the underlying distribution, we (typically) just use the estimator $\bar{x} = \sum x_i / n$.

This estimator has the variance $\frac{1}{n}\mathbb{V}(X)$. To estimate this, we (typically) just use the sample variance.

A not-so-simple-situation

Assume we want to estimate the median, rather than the mean.

(This makes much sense w.r.t. robustness)

The natural estimator for the median is the sample median.

But what is the variance of the estimator?

The variance of the sample median

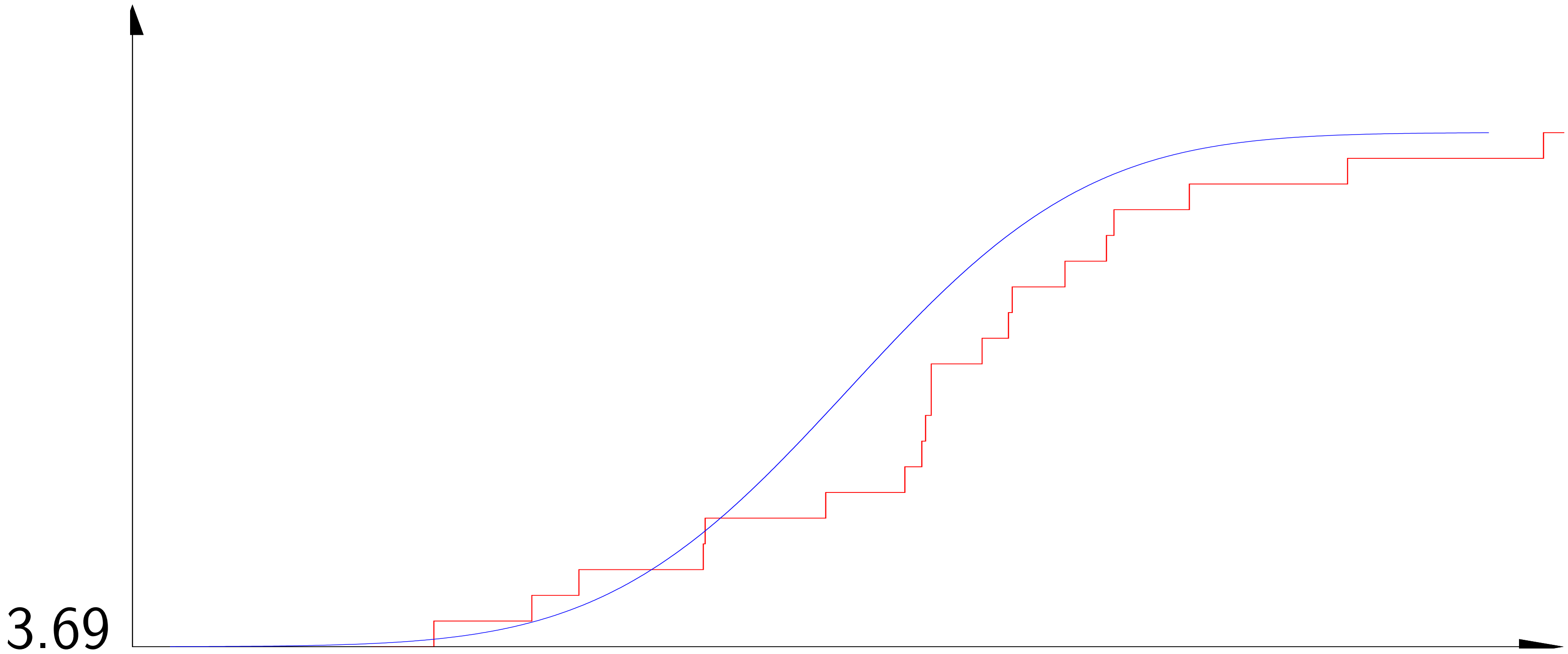
If we had access to the “true” underlying distribution, we could

1. Simulate a number of data sets like the one we had.
2. For each simulated data set, compute the median.
3. Finally report the variance among these medians.

We don’t have the true distribution. But we have the **empirical** distribution!

Empirical distribution

20 $N(0,1)$ variates (sorted): -2.20, -1.68, -1.43, -0.77, -0.76, -0.12, 0.30, 0.39, 0.41, 0.44, 0.44, 0.71, 0.85, 0.87, 1.15, 1.37, 1.41, 1.81, 2.65,



The Bootstrap Algorithm for the variance of a parameter estimator

Given a data set with N observations.

Simulate r (e.g., $r = 100$) data sets, each with N "observations" sampled form the empirical distribution F_e.

(To simulate such one data set, simple take N samples from the true data set *with* replacement)

For each simulated data set, estimate the parameter of interest (e.g., the median). This is a *bootstrap replicate* of the estimate.

Finally report the variance among the bootstrap replicates.

Advantages of the Bootstrap method

Does not require the distribution in parametric form.

Easily implemented.

Applies also to estimators which cannot easily be analysed.

Generalizes e.g. to confidence intervals.

Exercise 8

First do exercise 13 in Chapter 7 of Ross.

Write a subroutine that takes as input a "data" vector of observed values, and which outputs the median as well as the bootstrap estimate of the variance of the median, based on $r = 100$ bootstrap replicates.

Test the method: Simulate $N = 200$ Pareto distributed random variates with $\beta = 1$ and $k = 1.05$. Compute the mean, the median, and the bootstrap estimate of the variance of the sample median.

Compare the precision of the estimated median with the precision of the estimated mean.

Deliverable 1: Lab report on exercises

- Short documentation for exercises
- The reports are made by the groups, i.e. Campusnet group hand-in
  - BUT: Everybody, has to participate seriously in all parts
  - Annotated programs with output is sufficient
  - Comments to the results and considerations are important
  - The message is: You should document, that you have done the work and understood the important points, but you should not spend too much time on editing, as long as it is readable and understandable
- Deadline Monday June 12th (if this is a problem for some reason, negotiate with the TA's)