VIETNAM NATIONAL UNIVERSITY, HO CHI MINH CITY
UNIVERSITY OF TECHNOLOGY
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# MATHEMATICAL MODELING (CO2011)

**Assignment (Semester: 231, Duration: 06 weeks)**

# "Stochastic Programming and Applications"

**(Version 0.1, in Preparation)**

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# 1 Abstract

When you have a problem that requires you to find the optimal solution to a goal, while taking into account the limitations of your resources and the trade-offs of your choices, you may have a **linear programming problem**. This type of problem can be expressed using linear functions of some variables for both the goal and the limitations. Linear programming problems are very useful for modeling many practical situations in different fields, such as:

- A farmer who wants to maximize the profit from planting crops, while considering the available land, water, seeds, and fertilizer.
- A manufacturer who wants to minimize the cost of producing goods, while meeting the demand and quality standards of the customers.
- A transportation company who wants to optimize the routes and schedules of its vehicles, while reducing the fuel consumption and travel time.

A **stochastic programming problem** is a linear programming problem in which some of the parameters or variables are **uncertain**. The uncertainty can be expressed using probability distributions. The goal of a stochastic programming problem is to find the optimal solution that maximizes the expected value of the goal function, while satisfying the constraints with a certain probability.

In this report, we will introduce the basic concepts of ...

# 2 Introduction to Stochastic Programming and Optimization

## 2.1 What is Stochastic Programming?

An optimization problem is said to be a **stochastic program** if it satisfies the following properties:

1. There is a unique objective function.
2. Whenever a decision variable appears in either the objective function or one of the constraint functions, it must appear only as a power term with an exponent of **1**, possibly multiplied by a constant.
3. No term in the objective function or in any of the constraints can contain products of the decision variables.
4. The coefficients of the decision variables in the objective function and each constraint are ***probabilistic*** in nature.
5. The decision variables are permitted to assume fractional as well as integer values.

These properties ensure, among other things, that the effect of any decision variable is proportional to its value.

## 2.2 Approach to solve a Stochastic Programming problem

As mentioned above, stochastic problems involve some parameter or variables which are uncertain and described by probability distributions. To yield the optimal result of a stochastic problem, it requires a combination of mathematical modeling, statiscal analysis and optimization techniques to effectively handle the uncertainty that lie within the problem.

There are different methods to solve this kind of problem that depend on the complexity, the common methods are **Stochastic Linear Programming (SLP)**, **Stochastic Integer Programming (SIP)**, **Stochastic Dynamic Programming (SDP)**. This report will discuss specifically about **SLP** problems.

# 3 Background - Basic concepts

## 3.1 The Simplex Method

The **Simplex Method**, developed by George Dantzig, incorporates both *optimality* and *feasibility* tests to find the optimal solution(s) to a linear program (if one exists).
Geometrically, the Simplex Method proceeds from an initial extreme point to an adjacent extreme point until no adjacent extreme point is more optimal.

To implement the Simplex Method we first separate the *decision* and *slack* variables into two non-overlapping sets that we call the **independent** and **dependent** sets. For the particular linear programs we consider, the original independent set will consist of the decision variables, and the slack variables will belong to the dependent set.

**The Algorithm:**

1. Tableau Format: Place the linear program in Tableau Format, as explained later.
2. Initial Extreme Point: The Simplex Method begins with a known extreme point, usually the origin (0, 0).
3. Optimality Test: Determine whether an adjacent intersection point improves the value of the objective function. If not, the current extreme point is optimal. If an improvement is possible, the optimality test determines which variable currently in the independent set (having value zero) should *enter* the dependent set and become nonzero.
4. Feasibility Test: To find a new intersection point, one of the variables in the dependent set must *exit* to allow the entering variable from Step 3 to become dependent. The feasibility test determines which current dependent variable to choose for exiting, ensuring feasibility.
5. Pivot: Form a new, equivalent system of equations by eliminating the new dependent variable from the equations do not contain the variable that exited in Step 4. Then set the new independent variables to zero in the new system to find the values of the new dependent variables, thereby determining an intersection point.
6. Repeat Steps 3 - 5 until the extreme point is optimal.

## 3.2 Place holder

# 4 Solution to PROBLEM 1

## 4.1 Summary of the Problem

In this problem, the objective is to minimize the production of an industrial firm F. To be specific, the firm needs to produce $n$ products. Each product requires different number of parts which have to be ordered from $m$ 3rd-party suppliers and each part has a cost of $b$. Before the demand for the products are known, the number of parts to be ordered has to be decided. After that, the production stage begins and the demand is revealed. With each product, there is an associating selling price $q$. If the parts are overbought (exceed the necessary number for production), they have to be reselled with a price of $s$ to minimize the cost. Moreover, when a demand of a product is not fulfilled, it costs additionally $l$ per unit to satisfy that demand.

This is a 2-stage stochastic linear programming (2-SLP) problem where the demand of the products acts as the uncertainty. The goal of the two stages is to minimize the cost of preordering parts and the cost of production, respectively. The detailed formulation process and solution will be provided below.

## 4.2 Model formulation

### 4.2.1 Notations

for ease of reading, the tables below summarize the notation of parameters and decision variables used in the formulation of this report.

| Symbol | Definition |
|---|---|
| $n$ | Number of products |
| $m$ | Number of parts |
| $i, j$ | Index of products and parts, respectively |
| $b$ | The set of preorder cost of parts |
| $q$ | Selling price per unit of product $i$ |
| $l$ | The set of additional cost to satisfy a demand per unit |
| $A$ | The matrix representing parts needed for each product |
| $a_{ij}$ | Number of part $j$ needed for product $i$ |
| $s$ | The set of salvage part selling price |
| $D$ | The set of the demand of the product |
| $S$ | Number of scenarios |
| $p_s$ | Probability of scenario $s$ |

| Symbol | Definition |
|---|---|
| $x$ | The set of parts $j$ to be ordered |
| $z$ | The set of manufactured product $i$ |
| $y$ | The set of salvage part $j$ |

### 4.2.2 First stage

As mentioned above, the first stage is about making sure the number of parts preordered will be enough to meet the unknown demand in the future and not overbought or underbought as that will damage the profit of the firm. The decision variable in this stage is $x$ and also a *here-and-now* decision. It can be formulated as:

$$\min\ g(x,y,z) = b^T x + Q(x)$$
$$\text{s.t. } x, y, z \geq 0$$

where $b^T x$ is the cost of purchasing vital parts for manufacturing and $Q(x)$ is the expected value of the optimal production cost (which is the solution to the second stage problem which will be presented right below).

### 4.2.3 Second stage

In this stage, the demand $d = (d_1, d_2, ..., d_n)$ for the products are realized, thus, the best production plan can be achieved by solving the stochastic linear program with decision variable $x$ and $y$:

$$min_{x,y} Z = \sum_{i=1}^{n} (l_i - q_i) z_i - \sum_{j=1}^{m} s_j y_j$$
$$\text{s.t. } y_j = x_j - \sum_{i=1}^{n} a_{ij} z_i, \quad j = 1, ..., m$$
$$0 \leq z_i \leq d_i, \quad i = 1, ..., n$$
$$y_j \geq 0, \quad j = 1, ..., m$$

### 4.2.4 Final model

### 4.2.5 Solution

In the description of the assignment, specific value of the parameters are given as follows:

- n = 8
- m = 5
- S = 2
- $p_s$ = 1/2
- Values of b, l, q, s and A are randomized
- Random demand vector $D$ and its density $p_i$ follows the binomial distribution **Bin(10,1/2)**

# 5 Solutions

## 5.1 Problem I. [Industry - Manufacturing]

Consider an industrial firm **F** where a manufacturer produces n products. There are different parts (sub-assemblies) to be ordered from in total $m$ 3rd-party suppliers (the sites). This picture shows a transportation plan of the industrial firm **F** with $m = 3$ from suppliers and $n = 4$ production locations (products, or warehouses).

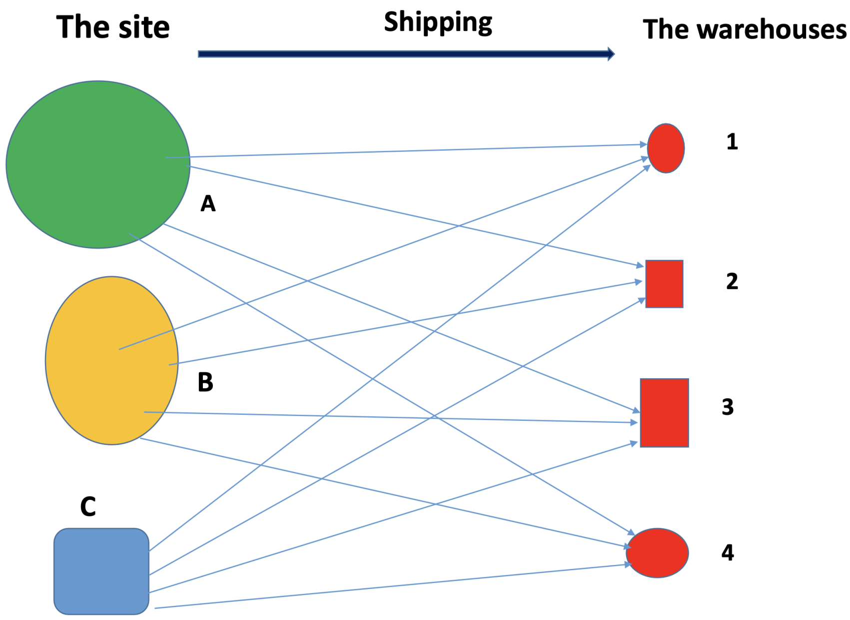


Figure 1: Transportation plan of the industrial firm **F**

## 5.2 Problem II

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# 6 Member list & Workload

| No. | Fullname | Student ID | Contribution | Percentage of work |
|---|---|---|---|---|
| 1 | Trần Đình Đăng Khoa | 2211649 | - Report, problem 1 | 25% |
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| 3 | Nguyễn Hồ Phi Ưng | 2252897 | - Problem 1 | 20% |
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| 5 | Vũ Minh Quân | 2212828 | - Problem 2 | 15% |

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