CSCI 3104 Summer 2017 - Assignment # 6

Due: July 14th 5:30pm

Be sure to justify all of your work.

1. Gru has decided to give you yet another task. He gives you a system of inequalities with variables $x_1, x_2, \ldots, x_n$, where each inequality is of the form $x_{i_1} < x_{i_2}$ for $i_1, i_2 \in \{1, \ldots, n\}$. He proceeds to give you an example of such a system of inequalities: $n = 5$ and $x_1 < x_3$, $x_1 < x_4$, $x_2 < x_5$, $x_5 < x_4$.
   Your task, he explains, is to write down an algorithm (high level description is sufficient) which does two things: (1) it should check whether a solution to a given system of inequalities exists, and (2) if a solution exists, it should find the solution. (**Gru's hint:** Convert the system of inequalities into a graph).

2. Gru argues that he could simplify our algorithm for finding SCCs by using the original graph G (instead of G^T) in the second call to `DFS`, and considering the vertices in *increasing* order of finish times. Will Gru's idea produce the correct result? If yes, provide justification of why it works; if not, provide a counterexample.

3. Gru gives you a graph G, a set of edge weights w, and an MST T of G. He would like to know if T is also an MST of G', where G' is formed by decreasing the weight of exactly one of the edges in T. In other words, denote the edge chosen to be $(x, y) \in T$, k be a positive number representing the decreased edge weight, and a weight function defined as

$$w'(u, v) = \begin{cases} w(u, v) & \text{if } (u, v) \neq (x, y) \\ w(x, y) - k & \text{if } (u, v) = (x, y) \end{cases}$$

   Prove that T is an MST for G', whose edge weights are given by w'.

4. Your final task this week (assigned, of course, by Gru himself) is to show that a graph G has a unique MST if, for every cut of G, there exists a unique light edge crossing the cut. In addition, Gru asks you to disprove the converse via a counterexample.